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Parameter Estimation of Weibull Distribution Using Constrained Search Space: An Application to Elevator Maintenance

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Abstract

The Weibull distribution is widely used in reliability estimation across industries, but accurately identifying its parameters remains a challenging task. This research proposes an efficient method for estimating Weibull distribution parameters by combining the maximum likelihood method with optimization theory. First, the parameter estimation problem is formulated as an optimization problem. A constrained search space partitioning framework is introduced, leveraging parameter-specific minimum and maximum bounds for the shape, location, and scale parameters. By dividing the search space into smaller subspaces for each parameter, the method constrains the search direction, significantly reducing estimation time. To address the local optima problem common in heuristic algorithms, a randomness operator is integrated into the optimization process. The proposed constrained search space partitioning framework is implemented using a conventional g-best version of the particle swarm optimization algorithm with historical fault data. Experimental results demonstrate that the proposed scheme outperforms state-of-the-art methods and conventional optimization-based approaches in terms of estimation accuracy and computational efficiency.

Keywords: preventive maintenance; parameter estimation; Weibull distribution; particle swarm optimization

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1. Introduction

An elevator is a sophisticated mechatronic system with various interconnected components [1]. Its safe and reliable functioning relies on the proper operation of all internal parts. However, over time, continuous use can lead to inevitable component failures. Even a minor malfunction can compromise the overall safety of the elevator system. Preventive

maintenance is crucial in mitigating the risk of failures by addressing potential issues of mechanical systems beforehand [2]. With effective reliability analysis, the frequency of breakdowns can be minimized, resource wastage reduced, and overall operational efficiency enhanced [3].

The Weibull distribution technique is a famous method for formulating the maintenance schedule of different mechanical equipment, including elevators [4,5]. The Weibull distribution is used to model failure behavior through its parameter thresholds to estimate failure risks, calculate confidence intervals, and optimize maintenance package scheduling for preventive maintenance [6]. Kovacs et al. incorporated a customized Weibull model into a digital support system to estimate and display the remaining service life and project the required number of spare parts [7]. In Ref. [8], Weibull distribution was used to analyze failure data from production systems, which enabled the prediction of likely failure intervals and informed the scheduling of preventive maintenance actions. In a further study, the Weibull distribution was used to model and analyze wind energy characteristics and wind speed distributions in suburban wind farms, and the parameters of the Weibull distribution were estimated using the least squares method [9].

Wu et al. [10] analyzed the failure rate distribution of circuit breaker components using the Weibull distribution and applied an average cost rate optimization model to determine the maintenance intervals for each component. Duan et al. [11] utilized estimated failure time distributions to develop a cost function, aiming to minimize the long-term expected cost rate, thereby identifying optimal preventive maintenance intervals. To model failures and repairs for preventive maintenance, Chien et al. [12] proposed a generalized Polya process method. A Bayesian auto-regression model and multivariate analysis were used to devise a probabilistic fault prediction method by Zhao et al. [13]. Probabilistic Boolean networks were used by Pedro et al. [14], to schedule the preventive maintenance of the ultrasound welding process. A condition-based maintenance scheme was proposed by Duan et al. [15] formulating a semi-Markov decision process-based optimization problem. Yang et al. [16] presented a two-phase preventive maintenance policy that optimized inspection, repair, and replacement intervals to maximize expected net revenue under performance-based contracting. To optimize the system performance and profitability, Ref. [17] proposed an imperfect preventive maintenance policy combined with minimal repair at failure.

Although the Weibull distribution technique successfully schedules preventive maintenance, estimating Weibull parameters is a complicated problem [18]. Many researchers have employed swarm optimization-based heuristic algorithms to estimate model parameters [19]. Alptekin et al. [20] presented a particle swarm optimization (PSO) method-based Weibull distribution parameter estimation using factorial design to optimize key PSO parameters for enhanced accuracy and efficiency. Particle swarm optimization was employed to estimate Weibull distribution parameters for wind energy forecasting, utilizing a novel square frequency error objective function to enhance parameter accuracy [21]. Lu et al. [22] presented the approximate parameter values and bounds for the Weibull distribution, which were determined using least squares estimation, providing optimal starting points for particle swarm optimization for parameter estimation. In a further study, Weibull distribution parameters were estimated by analyzing historical fault data from an elevator system to establish a mixed failure rate model, and the optimized maintenance period was determined using Monte Carlo simulation [23]. Acitas et al. [24] proposed maximum likelihood (ML) estimates of Weibull parameters, which were used to create a search space for the PSO method to deal with slow convergence problems. Swarm optimization-based methods can effectively identify the parameters using experimental data, but slow convergence and local optima problems reduce their efficiency.

This paper presents a heuristic algorithm-based approach for estimating Weibull distribution parameters by integrating a modified maximum likelihood method. A constrained search space partitioning (CSSP) framework enhanced with a randomness operator is proposed to optimize parameter estimation. First, the Weibull parameter identification problem is formulated as an optimization problem using maximum likelihood. Traditional heuristic algorithms like particle swarm optimization (PSO) often suffer from slow convergence and local optima traps, degrading their performance. To address these limitations, the CSSP method partitions the search space into smaller subspaces based on parameter-specific bounds. By deriving minimum and maximum values for each parameter (shape, location, scale), the method reduces the search space dimensionality, accelerating convergence and improving optimization accuracy. Additionally, a randomness operator is introduced to relocate swarm particles across the search space when stagnation occurs, effectively escaping local optima. While CSSP is compatible with any heuristic algorithm, this study employs PSO to demonstrate its efficacy. The combined framework enhances PSO's efficiency by concentrating the search on high-probability regions, simultaneously reducing convergence time and avoiding suboptimal solutions.

2. Motivation and Problem Formulation

The Weibull distribution technique is widely used for reliability analysis, including failure analysis, failure prediction, aging, and reliability assessment of mechanical systems such as elevators. Its highly adaptable distribution model allows for efficient characterization of failure distributions across all three phases of the bathtub curve. This study utilizes the Weibull distribution to characterize the failure patterns of elevator components. The cumulative distribution function, probability density function, and failure rate function of the Weibull distribution are as follows:

$$F(x;\mu,\eta,\beta) = 1 - \exp\left\{-\left(\frac{x-\mu}{\eta}\right)^{\beta}\right\}, \tag{1}$$

$$f(x;\mu,\eta,\beta) = \frac{\beta}{\eta^{\beta}}(x-\mu)^{\beta-1}\exp\left\{-\left(\frac{x-\mu}{\eta}\right)^{\beta}\right\}, \tag{2}$$

$$x > \mu, \eta > 0, \beta > 0$$

$$h(x;\mu,\eta,\beta) = \frac{\beta}{\eta}\left(\frac{x-\mu}{\eta}\right)^{\beta-1}, x > \mu. \tag{3}$$

Key symbols used in the Weibull distribution formulation are defined in Table 1. The point at which the log of the likelihood function, $\ln(L)$, attains its maximum value is the maximum likelihood estimate for the parameters μ, η, and β:

$$\ln(L) = n\ln\beta - n\ln\eta + (\beta-1)\sum_{i=1}^{n}\ln z_i - \sum_{i=1}^{n} z_i^{\beta} \tag{4}$$

where $x_i \in x_i \mid i \in 1,2,3,\ldots,n$ is the set of samples from the distribution $f(x;\mu,\eta,\beta)$, and

$$z_i = \frac{(x_i-\mu)}{\eta}. \tag{5}$$

Maximum likelihood estimates of the unknown parameters are found by taking the partial derivatives of the log-likelihood function with respect to the parameters and setting them to zero. However, these equations are nonlinear and involve complex terms such as powers and logarithms, making it impossible to solve them explicitly. Solving these nonlinear equations is essentially an optimization problem. This work focuses on improving the

widely adopted g_{best}-PSO for its prevalent use in reliability engineering and fair comparison with prior Weibull estimation methods [20,24]. Traditional methods, such as gradient-based approaches, struggle because the log-likelihood function can have multiple local maxima, making it difficult to find the global maximum. Additionally, these methods are sensitive to initial guesses and may fail to converge. Therefore, heuristic optimization techniques are more suitable for solving such problems because they do not rely on derivatives or gradients. Thus, this work focuses on enhancing the PSO-based optimization technique frequently used in reliability estimation by proposing a constrained search space based on historical fault data and likelihood estimation.

Although heuristic algorithms such as particle swarm optimization (PSO) have proven effective for parameter estimation, there is a tendency for swarm particles to become stuck in local optima, leading to inaccurate results and slow convergence.

Table 1. Nomenclature.

Symbol	Meaning	Symbol	Meaning
μ	Location parameter (minimum life/failure-free period)	η	Scale parameter (characteristic lifetime)
β	Shape parameter ($\beta < 1$: early failures, $\beta > 1$: wear-out)	x, t	Random variable (time-to-failure)
$F(x)$	Cumulative distribution function (CDF)	$f(x)$	Probability density function (PDF)
$h(x)$	Failure rate function (hazard rate)	L	Likelihood function
$\ln L$	Log-likelihood function	n	Sample size (failure observations)
z_i	Standardized variable: $(x_i - \mu)/\eta$	X_k^i	Position of particle i at iteration k
V_k^i	Velocity of particle i at iteration k	P_{best}^i	Personal best position of particle i
G_{best}	Global best swarm position	ω	Inertia weight (momentum control)
c_1, c_2	Acceleration coefficients	r_1, r_2	Random numbers $\in [0, 1]$
pn	Number of particles in swarm	nd	Segments per dimension in CSSP
k_r	Stagnation threshold (iterations)	ρ	Randomness operator
Q	Quadrant index matrix	S_s	Constrained search space range
α	Significance level	$q_{\alpha/2}$	Upper $\alpha/2$ normal quantile
MEAN	Mean absolute error	MSE	Mean squared error
DEF	Total deficiency (summed MSE)		

Proposed Solution

The estimation of the maximum likelihood parameters for reliability models such as the Weibull distribution is an optimization problem. By maximizing the log-likelihood function $\ln(L)$ (Equation (4)) with respect to the parameters β, μ, and η, the maximum likelihood estimates can be found. To solve this optimization problem, a heuristic algorithm such as particle swarm optimization can be used. Since the Weibull distribution is a multi-parameter identification problem and the occurrence of elevator failures is random, a complex search space is formed for the heuristic algorithm to identify the model parameters. This complex search space usually results in slow convergence and local optima problems in heuristic algorithms.

The initial conditions for the heuristic algorithm, such as PSO, directly affect its performance in terms of convergence time. Therefore, to improve slow convergence, the search space is partitioned into multiple subspaces, with each parameter having its own designated search domain. Each subspace is limited by the extrema values of the corresponding parameters, i.e., $\beta_{\min}$ and $\beta_{\max}$, $\mu_{\min}$ and $\mu_{\max}$, and $\eta_{\min}$ and $\eta_{\max}$ for the shape, location, and scale parameters, respectively, thus creating a constrained search space. A novel method is proposed to derive the extrema values for each subspace using maximum likelihood estimates. The fault data collected from the elevator parts over time is then used to solve the complex search space to find the optimal parameter values.

To address the local optima problem, the optimization process is introduced with a randomness operator to scatter swarm particles across subspaces. This method ensures more effective exploration of the search space and avoids becoming stuck in local optima. The formation of CSSP and parameter estimation process is presented in Figure 1. The following section will present the constrained search space partitioning approach with the randomness operator for particle swarm optimization.

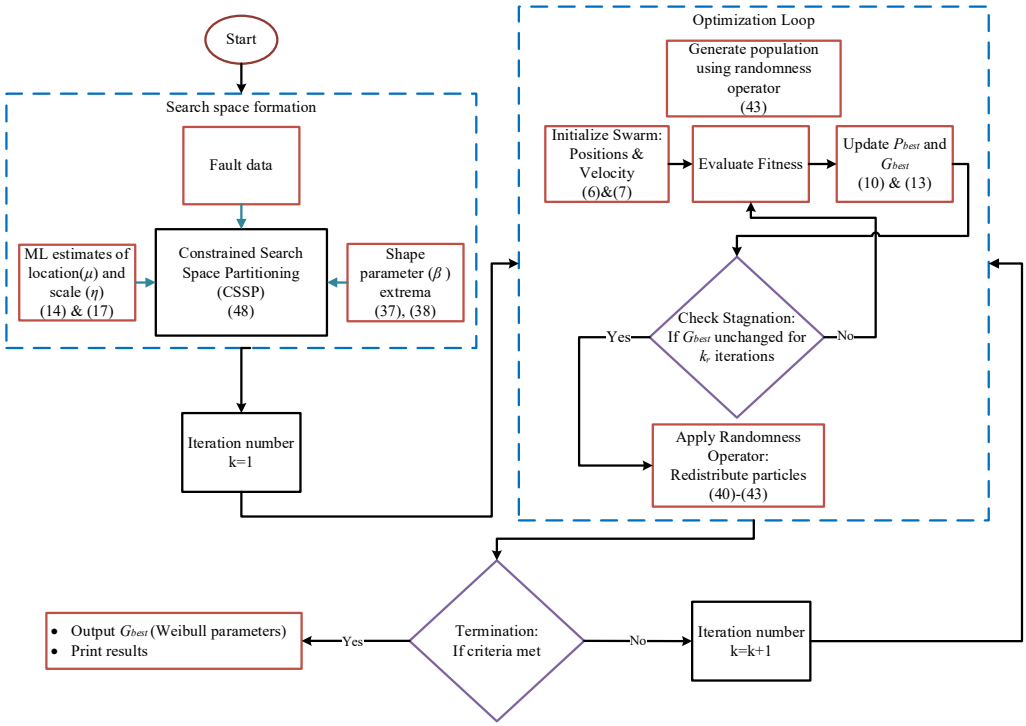


Figure 1. Constrained Search Space Partitioning (CSSP)-based parameter estimation.

3. Multidimensional Particle Swarm Optimization for 3-p Weibull Distribution

Particle swarm optimization (PSO) is an optimization technique rooted in swarm intelligence involving a collection of randomly positioned particles (potential solutions). These swarm particles search for new solutions within the search space by relocating based on their own past experiences and the collective experience of the entire swarm. While the global best-based PSO has known limitations [25], CSSP's partitioned search space inherently reduces its vulnerability to premature convergence. Let $X_k^i = \left(x_k^{(i,1)}, x_k^{(i,2)}, \ldots, x_k^{(i,n)}\right)$, and $V_k^i = \left(v_k^{(i,1)}, v_k^{(i,2)}, \ldots, v_k^{(i,n)}\right)$ represent the position and velocity of the i-th particle at k-th iteration operating in n dimensions, such that $X_k^i = \left\{x_k^i \mid k \in \mathbb{N}, 1 \leq k \leq pn\right\}$. For 3-p Weibull distribution position and velocity of particle is defined as:

$$x_k^i = \left(x_k^{(i,\beta)}, x_k^{(i,\mu)}, x_k^{(i,\eta)}\right), \tag{6}$$

$$v_k^i = \left(v_k^{(i,\beta)}, v_k^{(i,\mu)}, \ldots, v_k^{(i,\eta)}\right). \tag{7}$$

Based on the objective function, the personal best and global best, P_{best} and G_{best}, are defined as follows:

$$P_{\text{best}}^i = \left(p^{(i,\beta)}, p^{(i,\mu)}, p^{(i,\eta)}\right), \tag{8}$$

$$G_{\text{best}} = \left(g^{(i,\beta)}, g^{(i,\mu)}, g^{(i,\eta)}\right). \tag{9}$$

The vector P_{best} tracks the best positions each particle has achieved while G_{best} denotes the overall best solution found so far. The particle's new velocity and position at iteration t are updated using the following equations:

$$V_{k+1}^{i} = \omega V_{k}^{i} + cg_k + sc_k \tag{10}$$

where

$$\begin{aligned} cg_k =& c_1 r_1 \Big[\Big(p^{(i,\beta)} - x_k^{(i,\beta)} \Big), \Big(p^{(i,\mu)} - x_k^{(i,\mu)} \Big), \\ & \Big(p^{(i,\eta)} - x_k^{(i,\eta)} \Big) \Big] \end{aligned} \tag{11}$$

$$\begin{aligned} sc_k =& c_2 r_2 \Big[\Big(g^{(i,\beta)} - x_k^{(i,\beta)} \Big), \Big(g^{(i,\mu)} - x_k^{(i,\mu)} \Big), \\ & \Big(g^{(i,\eta)} - x_k^{(i,\eta)} \Big) \Big] \end{aligned} \tag{12}$$

and the positions of each particle are updated iteratively using the following:

$$X_{k+1}^{i} = X_{k}^{i} + V_{k+1}^{i} \tag{13}$$

where, ω is the inertia weight, c_1 and c_2 are acceleration coefficients and $r_1, r_2 \in [0, 1]$. The velocity and position are updated in each iteration until the global optimum is reached. The inertia weight ω propels the particle in its current direction; the term cg_k, known as the cognitive component, incorporates personal experience, and sc_k, the social component, reflects collective behavior of the multidimensional particle swarm optimization. The proposed multidimensional particle swarm optimization can act separately on each search dimension, this allows the construction of a squeezed search space for each dimension separately using the confidence intervals to solve the slow convergence of swarm optimization-based metaheuristic algorithms. The following presents the construction of the search space using likelihood estimates.

4. Formulation of Constrained Search Space Partitioning (CSSP)

In this research, we introduce a novel approach to particle swarm optimization (PSO) called constrained search space partitioning (CSSP). The method enhances the efficiency of the optimization process by dividing the search space into smaller subspaces for each parameter and introducing a randomness operator in the optimization process. Each parameter's subspace is separately dealt with, taking into account its minimum and maximum values. By analyzing these parameter-specific constraints, the search space can be "squeezed" or confined within the boundaries of each parameter's range. As a result, the overall search space becomes non-spherical and adapts to the characteristics of the individual parameters. The randomness operator is introduced whenever the swarm particles are stuck in the local optima, which relocates the swarm particles by distributing them on entire search space by dealing each subspace. This approach significantly reduces the optimization time by focusing the search in more relevant regions of the space, thus improving the convergence rate while the randomness operator solves the local optima problem, hence enhancing the overall performance of the PSO algorithm.

To construct the constrained search space, we need estimates of the upper and lower bounds of the location scale and the shape parameters of the Weibull distribution. For location and scale parameters, the upper and lower bounds can be found using Tikku's modified maximum likelihood confidence intervals. However, for a search space constrained in all subspaces, the extreme values of the shape parameter β must be known. This research proposes a novel method to find the estimates of the upper and lower bounds of

the shape parameter. For the location and scale parameters of the Weibull distribution, the asymptotic $100(1-\alpha)\%$ confidence interval is given as follows:

$$\hat{\mu}_{\min} < \mu < \hat{\mu}_{\max} \tag{14}$$

$$\mu_{\min} = \hat{\mu} - q_{(\alpha/2)}\,\mathrm{se}(\hat{\mu}) \tag{15}$$

$$\mu_{\max} = \hat{\mu} + q_{(\alpha/2)}\,\mathrm{se}(\hat{\mu}) \tag{16}$$

and

$$\begin{aligned} \hat{\eta} - q_{(\alpha/2)}\,\mathrm{se}(\hat{\eta}) < \eta \\ < \hat{\eta} + q_{(a/2)}\,\mathrm{se}(\hat{\eta}) \end{aligned} \tag{17}$$

For the standard normal distribution, $q_{a/2}$ is the upper $\alpha/2$ quantile. The maximum likelihood estimators $\hat{\mu}$ and $\hat{\eta}$ for location and scale, parameters are driven as:

$$\hat{\mu} = \hat{\mu}_n - (\Delta/m_l)\hat{\eta} \tag{18}$$

$$\hat{\eta} = \frac{-B + \sqrt{B^2 + 4nc}}{2\sqrt{n(n-1)}} \tag{19}$$

where

$$m_l = \sum_{l=1}^{n} \delta_l \tag{20}$$

$$\delta_l = (\beta - 1)s_{l0} + \beta s_l, \tag{21}$$

$$\Delta_l = (\beta - 1)\alpha_{l0} - \beta\alpha_l, \tag{22}$$

$$\Delta = \sum_{l=1}^{n} \Delta_l \tag{23}$$

$$B = \sum_{l=1}^{n} \Delta_i \left(y_{(i)} - \hat{\mu}_n \right), \tag{24}$$

$$c = \sum_{l=1}^{n} \delta_i \left(y_{(i)} - \hat{\mu}_n \right)^2, \tag{25}$$

$$s = t_{(l)}^{-2}, \alpha_{l0} = 2t_{(l)}^{-1}, s_l = (\beta - 1)t_{(l)}^{\beta-2} \tag{26}$$

$$\hat{\mu}_n = (1/m_l) \sum_{l=1}^{n} \delta_l y_{(i)} \tag{27}$$

$$\alpha_l = (2 - \beta)t_{(l)}^{\beta-1} \tag{28}$$

$$t_{(i)} = \left[-\log\left(1 - \frac{i}{n+1} \right) \right]^{1/p}, i = 1, 2, \ldots, n \tag{29}$$

It can also be demonstrated that the distribution of $\hat{\theta} = (\hat{\mu}, \hat{\eta})'$ is asymptotically normal, with a mean of $\theta = (\mu, \eta)'$ and a variance-covariance matrix of I^{-1}, where

$$I = \frac{np^2}{\sigma^2} \begin{bmatrix} \left(1 - \frac{1}{p}\right)^2 \Gamma\left(1 - \frac{2}{p}\right) & \Gamma\left(2 - \frac{1}{p}\right) \\ \Gamma\left(2 - \frac{1}{p}\right) & 1 \end{bmatrix}, p > 2 \tag{30}$$

for given p. Since the diagonal elements of the matrix I^{-1} correspond to the variance of $\hat{\mu}$ and $\hat{\sigma}$, we obtain:

$$\mathrm{se}(\hat{\mu}) = \sqrt{I_{11}^{-1}} \tag{31}$$

and

$$\text{se}\,(\hat{\eta}) = \sqrt{I_{22}^{-1}} \tag{32}$$

The variance–covariance matrix I^{-1} follows from the MLE's asymptotic normality under regularity conditions. For Weibull distribution, the matrix I is derived from second-order partial derivatives of the log-likelihood (4), where diagonal elements I_{11}^{-1}, I_{22}^{-1} yield standard errors for $\hat{\mu}$ and $\hat{\eta}$

$$I(\theta) = -E\left[\frac{\partial^2 \ln L}{\partial\theta_i \partial\theta_j}\right], \quad \theta = (\mu, \eta) \tag{33}$$

The constrained search space for the identification of the 3-parameter Weibull distribution requires an estimate of the parameter β. This research proposes the maximum likelihood methods-based boundary estimation of the β parameter that can be used in constrained search space for the optimization algorithm. From (4), we get:

$$\begin{aligned}\ln L(x;\mu,\eta,\beta) =& n\ln\beta - n\beta\ln\eta + (\beta-1)\sum_{i=1}^{n}\ln\left(\frac{x_i-\mu}{\eta}\right)\\ &-\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\eta}\right)^{\beta},\end{aligned} \tag{34}$$

differentiating w.r.t. η

$$\frac{\partial \ln L(x;\mu,\eta,\beta)}{\partial\eta} = -\frac{n\beta}{\eta} + \frac{n(1-\beta)}{\eta} - \frac{n(x_i-\mu)^{\beta}}{\eta^{\beta+1}} \tag{35}$$

putting $\frac{\partial \ln L(\beta,\delta)}{\partial\eta} = 0$, we get

$$\eta^{\beta} = \frac{n(x_i-\mu)^{\beta}}{1-2\beta}. \tag{36}$$

Let $f(\beta) = (1-2\beta)^{1/\beta}$, we get

$$f(\beta) = \frac{x_i-\mu}{\eta}, \tag{37}$$

from (15) and (16)

$$f(\beta_{min}) = \frac{x_i-\mu_{min}}{\eta_{min}}, \tag{38}$$

and

$$f(\beta_{max}) = \frac{x_i-\mu_{max}}{\eta_{max}}, \tag{39}$$

since $f(\beta)$ is explicit where $\beta > 0$, the upper and lower bounds of shape parameter can be found using (38) and (39),

$$\hat{\beta}_{\min} < \beta < \hat{\beta}_{\max}. \tag{40}$$

The estimates of the upper and lower bounds of the parameters provided in the proposed method are explicit and asymptotically equivalent to the maximum likelihood method, making them an efficient and reliable option for constructing the constrained search space to find unknown parameters. The local optima problem of swarm optimization algorithms is proposed to be solved by introducing a randomness operator in the optimization process. The following will describe the constrained search space partitioning with the randomness operator using the estimates of β, η, and μ.

Constrained Search Space Partitioning with Randomness Operator

Heuristic algorithms are effective in solving optimization problems, but their performance is deteriorated by local optima. The randomness operator can provide safety net for g_{best}-PSO, which remains prevalent in Weibull applications. Future work will integrate CSSP with local-best topologies. The randomness operator ρ is first initialized for each subspace of the constrained search space partition, which is used to initialize the particle positions. For a swarm comprising pn particles that navigate in $D \in \mathbb{R}^{1\times nd}$ dimensions for an optimization problem as specified in (4), each dimension is segmented into nd parts to incorporate randomness across the search space. nd can be chosen based on the size of the search space, number of swarm particles and amount of particle redistribution required. If the chosen nd is too high, it may increase computational complexity, whereas a smaller value can cause local minima problems. In this research, nd is taken to be equal to the total number of parameters to be estimated. For ith particle, where $i \in pn$, the randomness operator of the parameter η at time step k is defined as:

$$\Omega_k^{i(\eta)} = \frac{\eta_r}{nd} r_{(k)} + \eta_{\min} + \frac{\eta_r}{nd} \rho_{i(\eta)}, \tag{41}$$

where η_r and μ_r are the range of the scale and location parameter from (14) and (17), given as

$$\eta_r = 2q_{(\alpha/2)}\, \mathrm{se}(\hat{\eta}), \tag{42}$$

$$\mu_r = 2q_{(\alpha/2)}\, \mathrm{se}(\hat{\mu}), \tag{43}$$

$r_{(k)} \in R$ is the random value between 0∼1. At each time step k, the parameter η with range η_r the quadrant number $\rho_{i(\eta)}$ is given as $\rho_{i(\eta)} \subseteq Q$, where, $\rho_{i(\eta)} = \left\{\rho_{i(\eta)} \in N \mid 0 \leq \rho_{i(\eta)} < n\right\}$ and Q is $n \times m$ matrix defined as:

$$Q = \begin{bmatrix} \rho_{11} & \cdots & \rho_{1m} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \cdots & \rho_{nm} \end{bmatrix}. \tag{44}$$

For the iteration number $k = k_r$ such that $k_r \in \{k \in N \mid k_r < k_{max}\}$, if the value of G_{best} has not changed, the randomness operator comes into play, and the matrix Q is generated. The parameter k_r can be selected based on the nature of the search space, for it may not be necessary to frequently apply the regular-shaped search space randomness operator. For the smaller value of k_r, the randomness operator may be applied more frequently, resulting in an increase in computational complexity. The swarm particles are assigned new values by the rules defined in the following equations:

$$\Omega_k^{i(\beta)} = \left[r_{(k)} + \rho_{i(\beta)}\right] \frac{\beta_r}{nd} + \beta_{\min} \tag{45}$$

$$\Omega_k^{i(\mu)} = \left[r_{(k)} + \rho_{i(\mu)}\right] \frac{\mu_r}{nd} + \mu_{\min}. \tag{46}$$

In general form

$$\Omega_k^{i} = \left[r_{(k)} + Q\right] \frac{Ss}{nd} + Ss_{\min} \tag{47}$$

$$\Omega_k^{i} = \left\{\Omega_k^{i(\beta)}, \Omega_k^{i(\eta)}, \Omega_k^{i(\mu)}\right\} \tag{48}$$

where Ss is the search space defined by confidence intervals of parameters and Ω_k^i is the new position of kth particle at ith iteration where $\Omega_k^{i(\beta)}$, $\Omega_k^{i(\eta)}$, and $\Omega_k^{i(\mu)}$ are the randomness positions of β, η, and μ parameters, respectively.

$$Ss = [Ss_{\max} - Ss_{\min}]^T = \begin{bmatrix} \hat{\mu}_{\min} \\ \hat{\eta}_{\min} \\ \hat{\beta}_{\min} \end{bmatrix} - \begin{bmatrix} \hat{\mu}_{\max} \\ \hat{\eta}_{\max} \\ \hat{\beta}_{\max} \end{bmatrix} \quad (49)$$

After constructing the constrained search space partitioning and defining the randomness operator, the particle swarm optimization method is used to find the Weibull distribution parameters using failure data collected over time. The flowchart of the proposed CSSP method is presented in Figure 2. The following describes the pseudocode of the proposed scheme for parameter estimation of the Weibull distribution.

Proposed Algorithm:

1. Define max iterations, pn particles (6) and (7), and D dimensions based on number of parameters to be optimized.
2. Determine the particle nd, r_2, (11), and (12).
3. Initialize $D \times pn$ positions using (44) and velocity using (10) for swarm particles.
4. Define CSSP according to (15), (16), (17), (38), and (39).
5. Determine the randomness operator iteration number $k_r = \{k \mid k = 0.2 \cdot \text{ max iteration}, k \in \mathbb{N}\}$.
6. Compute P_{best} and G_{best} based on (8) and (9) by maximizing optimal function (4).
7. If G_{best} does not change after k_r iterations the introduce randomness operator based on (44) and (49)
8. If termination criterion reached, print results.

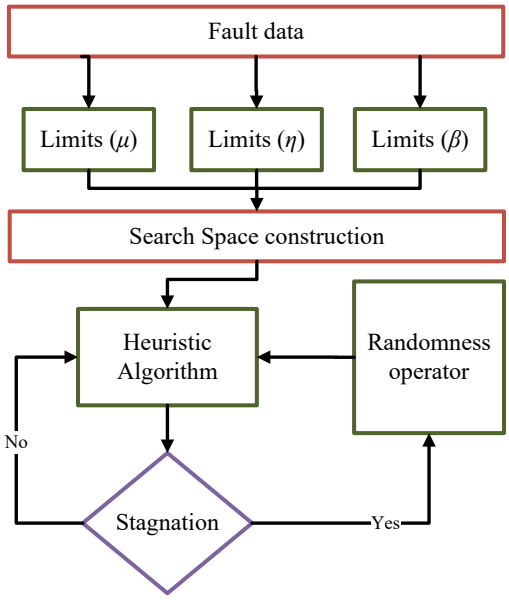


Figure 2. Flow chart of CSSP method.

A two-stage verification process is proposed to validate the CSSP framework. First, Monte Carlo simulations (Section 5.2) will assess robustness across 500 trials with varying sample sizes (10–200) and swarm configurations (10–100 particles), benchmarking against conventional PSO and state-of-the-art methods presented in the literature. After that, employing elevator fault data collected over time, Table 2, will demonstrate practical efficacy, where Weibull parameters for critical components will be estimated. The obtained results will be compared with methods presented in the literature using error metrics (MEAN, MSE, DEF). This dual approach quantitatively links CSSP's theoretical advantages (Equations (45)–(49)) to empirical performance gains.

Table 2. Failure record of elevator under study.

Part Name	Test Duration (Days)	Number of Failure	Failure Rate
Brake	600	109	18.167
Traction Wheel	600	112	18.667
Rope	600	77	12.833
Motor	600	61	10.167
Door	600	47	7.833

The results obtained using the proposed method to find the Weibull parameters using elevator fault data is described in the following section.

5. Results and Discussion

5.1. Experimental Results

This research analyzes the elevator maintenance data obtained from "Hangzhou Aolida Elevator Co. Ltd." (Hangzhou, China) from July 2021 to February 2023 to validate the proposed method. The events of the failure of different elevator parts were recorded; the details of failure of elevator parts are shown in Table 2.

For the proposed analysis, MATLAB R2023a with Global Optimization Toolbox on an Intel i9-13900K/64 GB RAM workstation was used. It is clear from Table 2 that the failure frequency of elevator parts is high. The traction wheel and elevator break have the highest failure rates, of 18.6%, and 18.16%, respectively, whereas the elevator door has the fewest failures during the testing period, with a 7.83% failure frequency. The failure distribution of each part is shown in Figure 3, which illustrates the failure rate over time.

As evident from Figure 3, each part has a different failure distribution; thus, each part will have a different failure distribution function. For example, the elevator break and traction wheel reach peak failure frequencies of around 250 days and 280 days, respectively. The elevator door reaches its peak failure after 400 days, and the rope failure rate reaches its peak at around 200 days. The failure rate data obtained experimentally is used to find the Weibull distribution parameters of each part, as explained in Section 4.

5.2. Parameter Estimation Using CSSP

The CSSP method has the advantage that it acts on each subspace separately, and so the search space can be modified in any shape depending on the upper and lower bounds of the Weibull distribution parameter, which is much more efficient than a spherical or regular-shaped search space. In addition, the local optima problem is handled by introducing the randomness operator in case the global optima does not change for some time.

The performance of the proposed parameter identification method was evaluated using Monte Carlo simulation. The proposed method is compared with the particle swarm optimization-based method presented in [24], differential evolution (DE) [26], and conventional particle swarm optimization methods. Since the traction wheel failure rate and number of failures are the highest in the historical fault data of elevator parts, from this point forward, the method presented in [24] will be referred to as Acitas et al., and simulations are conducted on the traction wheel fault data. The swarm optimization parameters, such as ω, c_1, r_1, c_2, and r_2 from (10)–(12) are kept the same for fair comparison. Different swarm sizes, i.e., 10, 25, and 50, were selected for thorough analysis, and the maximum iteration was set to 1000. The model identification analysis was performed on different numbers of samples, i.e., $10, 20, 40, 60, 80$, and 100 samples.

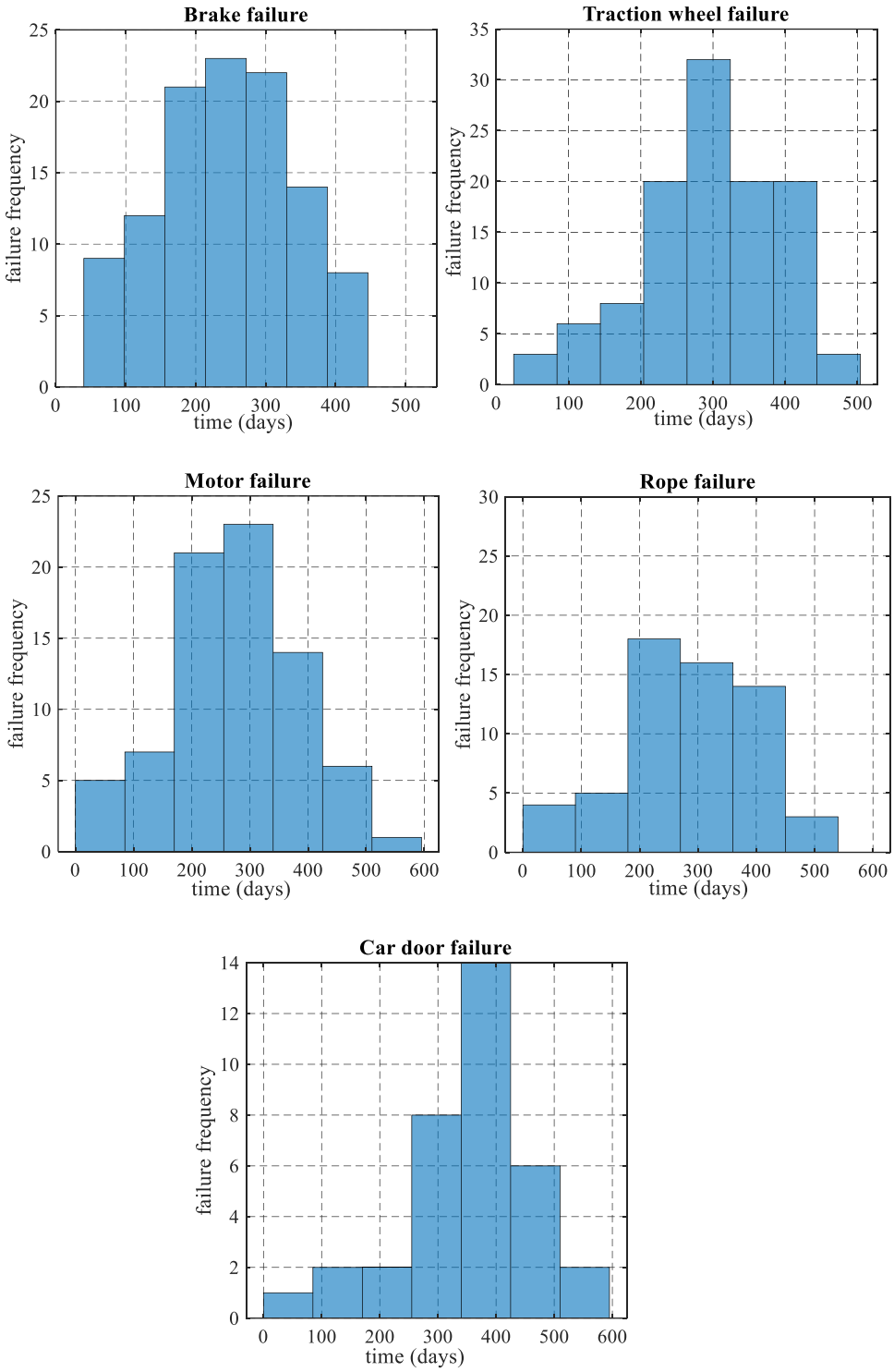


Figure 3. Comparison of failure distribution of different elevator parts.

Each simulation is repeated 500 times for Monte Carlo analysis; in total, 9000 simulations were run. The mean error, mean square error (MSE), and total deficiency (DEF) for each parameter are shown in Tables 3–5. DEF is the cumulative MSE for each parameter.

Table 3. Comparison of Errors (Mean, MSE, DEF) in Weibull Parameter Estimation for Elevator Parts (population size = 25).

n	Algorithm	Error Type	μ	η	β
10	Proposed method	MEAN	1.8989	2.6265	4.0069
		MSE	3.7651	3.905	5.198
		DEF		12.8681	
	Acitas et al.	MEAN	2.3393	2.4442	3.6064
		MSE	2.04922	1.7834	10.7942
		DEF		14.6269	
	DE	MEAN	2.164746	2.915415	4.327452
		MSE	3.915704	4.0612	5.56186
		DEF		13.538764	
	PSO	MEAN	58.8706	44.4998	39.991
		MSE	4639.5832	2887.5192	2435.5931
		DEF		9962.6957	
20	Proposed method	MEAN	2.1401	2.3544	3.0397
		MSE	1.3195	1.3544	3.8963
		DEF		6.5702	
	Acitas et al.	MEAN	2.7564	1.9267	1.3273
		MSE	1.4991	1.1385	6.8622
		DEF		9.4992	
	DE	MEAN	2.418313	2.684016	3.434861
		MSE	1.50423	1.787808	4.246967
		DEF		7.539005	
	PSO	MEAN	56.4574	43.3023	38.279
		MSE	4416.6667	2899.3836	2529.4298
		DEF		9845.4802	
40	Proposed method	MEAN	2.15926	2.4683	2.321
		MSE	1.0846	0.7658	3.2055
		DEF		5.0559	
	Acitas et al.	MEAN	2.4119	1.6082	2.4794
		MSE	1.1001	1.3097	4.9838
		DEF		7.3937	
	DE	MEAN	2.4399638	2.838545	2.71557
		MSE	1.258136	1.010856	3.59016
		DEF		5.859152	
	PSO	MEAN	58.4164	41.0697	40.4069
		MSE	4491.6681	2727.8891	2498.151
		DEF		9717.7067	
60	Proposed method	MEAN	2.2205	2.3291	2.8186
		MSE	0.5595	0.367	1.1606
		DEF		2.0871	
	Acitas et al.	MEAN	2.5751	1.3057	0.9266
		MSE	1.1606	1.1584	2.9552
		DEF		5.2743	
	DE	MEAN	2.5335905	2.678465	3.297762
		MSE	0.7726695	0.567015	1.56681
		DEF		2.9064945	
	PSO	MEAN	57.5789	42.1976	38.8488
		MSE	4437.5001	2758.0893	2587.6732
		DEF		9783.2626	

Table 3. *Cont.*

n	Algorithm	Error Type	μ	η	β
80	Proposed method	MEAN	2.0673	2.6265	2.2383
		MSE	0.297	0.1518	0.1105
		DEF		0.5593	
	Acitas et al.	MEAN	2.5751	1.5167	0.6511
		MSE	1.1848	1.1226	3.4812
		DEF		5.7886	
	DE	MEAN	2.3670585	3.598305	2.999322
		MSE	0.380457	0.249711	0.182325
		DEF		0.812493	
	PSO	MEAN	57.9621	44.8488	37.5116
		MSE	4522.9166	3096.7642	2551.0015
		DEF		10,170.6824	
100	Proposed method	MEAN	2.2013	2.3544	1.934
		MSE	0.2625	0.03797	0.0552
		DEF		0.3556	
	Acitas et al.	MEAN	2.5509	1.2778	0.4007
		MSE	0.8341	0.7802	2.3542
		DEF		3.9686	
	DE	MEAN	2.5204885	3.225528	2.59156
		MSE	0.4675125	0.07385165	0.14628
		DEF		0.68764415	
	PSO	MEAN	61.3691	43.6046	38.8604
		MSE	4834.3749	2824.9614	2428.0431
		DEF		10,087.37959	

Table 4. Comparison of Errors (Mean, MSE, DEF) in Weibull Parameter Estimation for Elevator Parts (population size = 50).

n	Algorithm	Error Type	μ	η	β
10	Proposed method	MEAN	2.373625	3.283125	4.648004
		MSE	4.555771	4.41265	5.40592
		DEF		14.374341	
	Acitas et al.	MEAN	2.830553	3.030808	4.544064
		MSE	2.7049704	2.443258	15.11188
		DEF		20.2601084	
	DE	MEAN	2.79252234	3.7317312	5.23621692
		MSE	5.023848232	4.914052	6.8967064
		DEF		16.83460663	
	PSO	MEAN	77.120486	53.844758	50.828561
		MSE	5767.001918	3667.149384	3134.60832
		DEF		12,568.75962	
20	Proposed method	MEAN	2.675125	2.943	3.526052
		MSE	1.596595	1.530472	4.052152
		DEF		7.179219	
	Acitas et al.	MEAN	3.335244	2.389108	1.672398
		MSE	1.978812	1.559745	9.60708
		DEF		13.145637	
	DE	MEAN	3.11962377	3.43554048	4.15618181
		MSE	1.92992709	2.16324768	5.26623908
		DEF		9.35941385	
	PSO	MEAN	73.959194	52.395783	48.652609
		MSE	5489.916708	3682.217172	3255.376153
		DEF		12,427.51003	

Table 4. *Cont.*

n	Algorithm	Error Type	μ	η	β
40	Proposed method	MEAN	2.699075	3.085375	2.69236
		MSE	1.312366	0.865354	3.33372
		DEF		5.51144	
	Acitas et al.	MEAN	2.918399	1.994168	3.124044
		MSE	1.452132	1.794289	6.97732
		DEF		10.223741	
	DE	MEAN	3.147553302	3.6333376	3.2858397
		MSE	1.614188488	1.22313576	4.4517984
		DEF		7.289122648	
	PSO	MEAN	76.525484	49.694337	51.3571699
		MSE	5583.143448	3464.419157	3215.120337
		DEF		12,262.68294	
60	Proposed method	MEAN	2.775625	2.911375	3.269576
		MSE	0.676995	0.41471	1.207024
		DEF		2.298729	
	Acitas et al.	MEAN	3.115871	1.619068	1.167516
		MSE	1.531992	1.587008	4.13728
		DEF		7.25628	
	DE	MEAN	3.268331745	3.4284352	3.99029202
		MSE	0.991334969	0.68608815	1.9428444
		DEF		3.620267519	
	PSO	MEAN	75.428359	51.059096	49.3768248
		MSE	5515.812624	3502.773411	3330.335408
		DEF		12,348.92144	
80	Proposed method	MEAN	2.584125	3.283125	2.596428
		MSE	0.35937	0.171534	0.11492
		DEF		0.645824	
	Acitas et al.	MEAN	3.115871	1.880708	0.820386
		MSE	1.563936	1.537962	4.87368
		DEF		7.975578	
	DE	MEAN	3.053505465	4.6058304	3.62917962
		MSE	0.488126331	0.30215031	0.226083
		DEF		1.016359641	
	PSO	MEAN	75.930351	54.267048	47.6772436
		MSE	5621.985334	3932.890534	3283.138931
		DEF		12,838.0148	
100	Proposed method	MEAN	2.751625	2.943	2.24344
		MSE	0.317625	0.0429061	0.057408
		DEF		0.4179391	
	Acitas et al.	MEAN	3.086589	1.584472	0.504882
		MSE	1.101012	1.068874	3.29588
		DEF		5.465766	
	DE	MEAN	3.251430165	4.12867584	3.1357876
		MSE	0.599818538	0.089360497	0.1813872
		DEF		0.870566234	
	PSO	MEAN	80.393521	52.761566	49.3915684
		MSE	6009.128001	3587.700978	3124.89147
		DEF		12,721.72045	

Table 5. Comparison of Errors (Mean, MSE, DEF) in Weibull Parameter Estimation for Elevator Parts (population size = 100).

n	Algorithm	Error Type	μ	η	β
10	Proposed method	MEAN	1.9402	2.0013	3.4992
		MSE	2.5944	2.8249	2.7409
		DEF		8.1603	
	Acitas et al.	MEAN	2.5087	2.6498	3.8317
		MSE	2.0601	1.944	11.7757
		DEF		15.7798	
	DE	MEAN	2.211828	2.221443	3.779136
		MSE	3.735936	3.502876	3.206853
		DEF		10.445665	
	PSO	MEAN	60.6616	85.5185	93.1852
		MSE	4592.5037	4639.991	5140.7553
		DEF		14,373.2491	
20	Proposed method	MEAN	2.0284	1.9492	4.8341
		MSE	0.7004	0.8568	2.1958
		DEF		3.7531	
	Acitas et al.	MEAN	3.0071	2.3225	2.7476
		MSE	2.0601	1.4826	9.4018
		DEF		12.9446	
	DE	MEAN	2.292092	2.222088	5.462533
		MSE	0.868496	1.130976	2.832582
		DEF		4.832054	
	PSO	MEAN	58.7866	92.2604	86.037
		MSE	4418.5906	3820.2226	4361.1613
		DEF	12.9446	12.9446	
40	Proposed method	MEAN	2.1044	1.8785	2.4723
		MSE	0.2165	0.3375	1.7614
		DEF		2.3155	
	Acitas et al.	MEAN	3.1843	2.0031	2.4672
		MSE	1.4287	1.317	8.3177
		DEF		11.0635	
	DE	MEAN	2.377972	2.160275	2.892591
		MSE	0.27279	0.4455	1.972768
		DEF		2.691058	
	PSO	MEAN	59.4133	88.2604	83.7037
		MSE	4483.3583	4605.1063	5182.0896
		DEF		14,270.5543	
60	Proposed method	MEAN	2.1187	1.8264	1.6113
		MSE	0.0506	0.0571	1.2796
		DEF		1.3874	
	Acitas et al.	MEAN	3.4113	1.8257	3.271
		MSE	1.4509	1.3188	6.8785
		DEF		9.6483	
	DE	MEAN	2.4174367	2.10036	1.885221
		MSE	0.0698786	0.0882195	1.72746
		DEF		1.8855581	
	PSO	MEAN	60.1198	82.5938	86.7407
		MSE	4686.0569	4888.5365	5422.4748
		DEF		14,997.0683	

Table 5. *Cont.*

n	Algorithm	Error Type	μ	η	β
80	Proposed method	MEAN	2.1107	1.7955	1.0347
		MSE	0.0092	0.0311	3.1437
		DEF		3.1841	
	Acitas et al.	MEAN	3.4501	1.7074	4.0186
		MSE	1.5783	1.037	7.7383
		DEF		10.3537	
	DE	MEAN	2.4167515	2.459835	1.386498
		MSE	0.0117852	0.0511595	5.187105
		DEF		5.2500497	
	PSO	MEAN	57.16	93.5185	93.074
		MSE	4714.8425	5019.3505	5596.274
		DEF		15,330.46716	
100	Proposed method	MEAN	2.0009	1.8887	0.387
		MSE	0.0276	0.0103	0.5371
		DEF		0.5751	
	Acitas et al.	MEAN	3.5941	1.873	5.0841
		MSE	1.8607	1.4747	8.9158
		DEF		12.2514	
	DE	MEAN	2.2910305	2.587519	0.51858
		MSE	0.0491556	0.0200335	1.423315
		DEF		1.4925041	
	PSO	MEAN	61.8933	96.037	87.8148
		MSE	4837.1814	4757.7226	5315.5501
		DEF		14,910.454	

It can be observed from the results in Tables 3–5 that the proposed method can identify the Weibull distribution parameters effectively. The MEAN error, MSE, and DEF for the proposed method are much lower than those of Acitas et al., differential evolution, and the conventional particle swarm optimization method. This is mainly because the search space created by the proposed scheme is much smaller, which makes it easier and quicker to find global optima in case of complex optimization problems, such as the Weibull distribution problem under investigation in this research. Secondly, the randomness operator in the proposed scheme can avoid the local optima problem. Conversely, the method proposed in Acitas et al. and conventional particle swarm optimization tend to become stuck in local optima, which affects the overall optimization process, leading to higher error values. Although the Acitas et al. method performs better than the conventional particle swarm optimization method, both of these schemes tend to become stuck in local optima.

During the optimization process, when a local optima case occurs, i.e., if the G_{best} from (9) is not changed for k_r iterations, the randomness operator is introduced, which redistributes the swarm particles over the entire search range. This helps the optimization scheme to effectively find the best possible parameter values for the Weibull distribution. Figure 4 shows the position of swarm particles in search space. The positions of the swarm particles at the event of local optima are shown in Figure 4a–c. It can be seen that the swarm particles are concentrated around the local optima. At the point when the randomness operator is introduced in the optimization process, the particles are distributed all over the search space, as shown in Figure 4d–f.

The parameters of the Weibull distribution for different elevator parts using the proposed method are shown in Table 6. Based on the Monte Carlo analysis, the sample size $n = 100$, swarm size $pn = 50$ are chosen, and the rest of the parameters are kept the same.

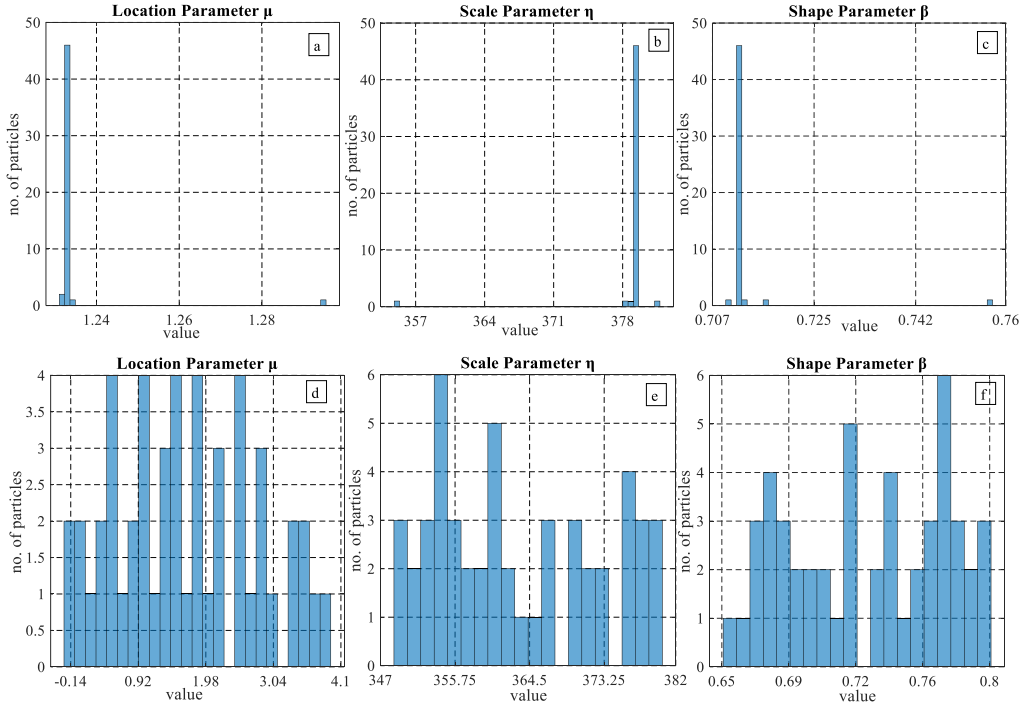


Figure 4. Randomness operator's effect on search space, before (**a**–**c**) and after (**d**–**f**) application, for parameters μ, η and β.

Table 6. Weibull parameter estimates for elevator components.

Part Name	Location μ	Scale η	Shape β
Brake	3.702	401.18	1.054
Traction Wheel	3.3701	374.461	0.7625
Rope	2.651	291.471	0.8133
Motor	2.518	316.382	0.9167
Car Door	2.831	297.641	0.839

One of the advantage of CSSP method is that the simulation time to reach the optimum solution is reduced. Figure 5 shows the convergence graph for the proposed method and the convergence graph presented in Acitas et al. It can be seen that the optimum value is achieved much more quickly in the case of the proposed method—68 iterations—whereas Acitas et al. took more time to find the global optima, i.e., 184 iterations. The main reason is that the search space is more optimized in the proposed method than in Acitas et al.; therefore, only a few iterations are needed to find the global optima. Since the conventional particle swarm optimization has a much larger search space, here, the proposed method is compared only with Acitas et al.

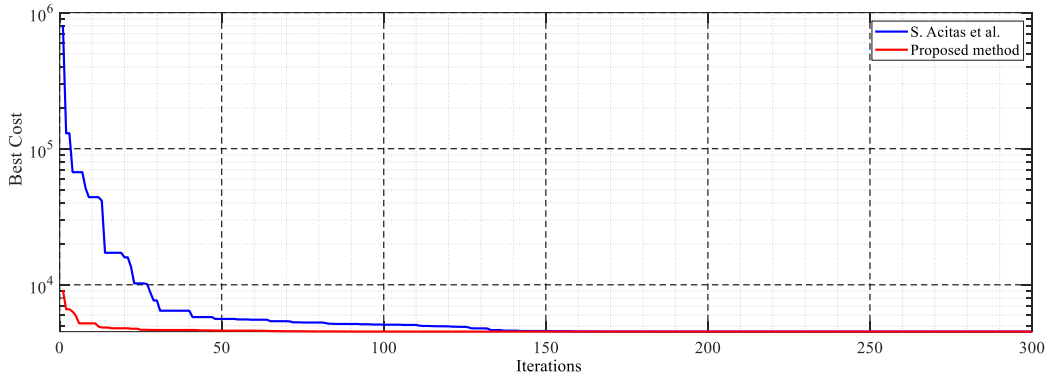


Figure 5. Error convergence analysis.

6. Conclusions

This paper presented a heuristic algorithm-based scheme for estimating the parameters of the Weibull distribution. The parameter estimation problem was first reformulated as an optimization problem. To address this, we proposed the constrained search space partitioning (CSSP) method, which constructs a narrowed search space for efficient parameter estimation. The upper and lower bounds of the Weibull distribution parameters were derived to define the constrained search space. Each parameter's search dimension was partitioned into separate subspaces. For the location and scale parameters, the bounds were determined using confidence intervals from a modified maximum likelihood method. For the shape parameter, a novel method was introduced to compute extremal values using likelihood estimates and historical fault data. By leveraging these extremal values, the search space was "squeezed," increasing the likelihood of quickly and accurately locating the global optimum compared to conventional heuristic search spaces. Since the CSSP method is data-dependent, it is more likely to converge to the true parameters than arbitrary search spaces. To mitigate the local optima problem inherent in heuristic algorithms, a randomness operator was incorporated. This operator redistributes swarm particles across the entire search space whenever stagnation is detected, ensuring continued exploration. The optimization problem was solved using the particle swarm optimization (PSO) algorithm applied to the partitioned search space. Extensive Monte Carlo simulations demonstrated that the proposed method outperforms existing techniques in both parameter identification error and computational time. To validate real-world applicability, the method was tested on elevator fault data, where it achieved superior accuracy and faster estimation compared to state-of-the-art approaches.

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