

Article

Study on the Load-Bearing Characteristics Analysis Model of Non-Pneumatic Tire with Composite Spokes

Muyang Sun , Weidong Liu, Qiushi Zhang , Yuxi Chen, Jianshan Jiang and Xiaotong Liu

State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun 130025, China; sunmy1521@mails.jlu.edu.cn (M.S.); wdlju@jlu.edu.cn (W.L.); chenyx1521@mails.jlu.edu.cn (Y.C.); jiangjs1521@mails.jlu.edu.cn (J.J.); liuxt1521@mails.jlu.edu.cn (X.L.)

* Correspondence: qszhang@jlu.edu.cn

Abstract: This study aims to analyze the load-bearing characteristics of non-pneumatic tires with composite spokes using experimental and finite element simulation methods and to establish a mechanical analysis model based on the Timoshenko beam theory. Subsequently, experiments were conducted on carbon fiber-reinforced plastics and rubbers to establish the corresponding constitutive model. A finite element model of the non-pneumatic tires with composite spokes was also developed. The main structural and material parameters were selected, and their correlation with the vertical stiffness of the non-pneumatic tires with composite spokes was studied using response surface methodology. The stiffness characteristics of the composite spokes were simplified, and a load-bearing characteristic analysis model was established. The results indicated that among the parameters of the reinforcement plate structure and rubber, the constitutive parameter C_{10} of the rubber in the spokes had the greatest impact, with a comprehensive influence value of 319.83 N/mm. Under a load of 5000 N, the load-bearing characteristic analysis model results were consistent with those of the finite element simulation, with a maximum relative error of 7.49%. The proposed load-bearing characteristic analysis model can assist in the rapid design and performance prediction of non-pneumatic tires with composite spokes.

Keywords: non-pneumatic tire; composite spoke; finite element simulation; load-bearing characteristics analysis model; Timoshenko beam



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1. Introduction

As the demand for tire safety and durability in automobiles continues to increase, emerging non-pneumatic tires (NPTs) have garnered significant attention. In 2006, Michelin pioneered the concept of the NPT and introduced the Tweel NPT [1,2]. Resilient technologies have been developed for honeycomb-spoked NPTs [3]. In 2012, Zhao et al. designed a mechanical elastic wheel with spokes composed of mechanical hinge structures [4]. In 2010, Kim et al. designed an NPT with Auxetic Honeycomb Spoke [5]. In 2019, Michelin introduced Uptis NPT and incorporated composite materials into its structure [6].

In recent years, researchers have primarily studied the mechanical properties of NPT using curved-beam models. In 2010, Renuka et al. analyzed the mechanical characteristics of Michelin's Lunar NPT shear band structure under compression between two frictionless rigid planes [7]. In 2011, Gasmi et al. applied the curved beam theory to examine the contact between a ring and flat, rigid ground and derived the governing differential equations through the principle of virtual work [8]. In 2012, they solved the problem of the frictionless contact of a compressed ring and established an analytical model for NPT on frictionless, rigid ground, enabling a comprehensive parameter analysis of tire-related quantities [9,10]. In 2021, Liang et al. developed an analytical model for NPTs considering the nonlinearity of spoke stiffness and validated the accuracy of the model using finite element software [11].

In 2018, Rugsaj et al. fitted the stress-strain relationship of elastic materials to select an appropriate constitutive model using physical experimental results [12]. In 2019, they

analyzed the bearing stiffness and stress distribution of NPTs with four different spoke geometries using finite element analysis [13]. In 2018, Jin et al. derived the relationship between the load-bearing capacity of a honeycomb NPT and its geometric characteristics through finite element simulation analysis [14]. In 2019, Ganniari–Papageorgiou et al. investigated the influence of the geometric parameters of honeycomb tires on the stress distribution and ground pressure using finite element modeling [15]. In the same year, Marcin Żmuda et al. investigated the deformation of spokes and ground imprint pressure of the Tweel under static loading through finite element simulation and experimentation [16,17]. In 2023, Liu et al. evaluated the stiffness performance of High Load Capacity non-pneumatic tires (HC tires) using a comprehensive tire stiffness tester. They established a nonlinear finite element model (FEM) for HC tires and validated its accuracy [18].

The current generation of Michelin’s Uptis NPT has highly innovative structures. Dhrangdhariya et al. conducted a study on the relationship between the Uptis spoke design and NPT stiffness using finite element analysis. They also optimized the spoke design of Uptis spokes [19,20]. Liang et al. developed a FEM for the Uptis NPT and analyzed its bearing characteristics. According to the existing patent information, the distinctive feature of Uptis is the integration of composite reinforcement plates within its spokes [21]. Current research has largely overlooked the impact of composite material applications on the mechanical performance of NPTs [22].

When conducting research on NPTs, appropriate data analysis methods must be selected. In 2015, Kim et al. conducted finite element simulations of honeycomb NPTs with various design parameters. They constructed a response surface model for the NPT rolling resistance and optimized it [23]. In 2022, Liu et al. analyzed the influence of various parameters on the three-dimensional stiffness of an FS-NPT using a response surface model [24]. Response surface analysis is an important method for guiding NPT design.

Although numerous studies have been conducted on the stiffness characteristics of NPTs with different structural forms using experiments, simulations, and curved beam models, most have focused on NPTs with homogeneous materials in their spoke structures. Considering that the spokes embedded with composite materials can change the load-bearing characteristics and enhance the design flexibility of NPTs [21], it is crucial to investigate the influence of composite material anisotropy and develop universal mechanical models to analyze their forces and deformation.

Therefore, this study employed a typical NPT-CS (Uptis) for a case study and conducted mechanical property experiments on carbon fiber-reinforced plastics (CFRPs) and rubber materials (Section 2). Based on the experimental data, the parameters for the anisotropic constitutive and neo-Hookean models were determined. Subsequently, the FEM of the Uptis NPT, with CFRP as the spoke reinforcement structure, was established. Furthermore, a load-bearing characteristic analysis (LCA) for the NPT-CS was established based on the Timoshenko beam theory (Section 3). An experimental matrix was designed to investigate the influence of the reinforcement plate and rubber material parameters on the vertical stiffness of the NPT-CS. Simulation results were utilized to establish a response surface model (RSM), conduct a sensitivity analysis, and propose the deformation mode and load-carrying principle of Uptis NPT. We then solved the LCA model and validated its effectiveness by comparing it with FEM simulation results (Section 4). The proposed LCA model can efficiently and accurately calculate the load capacity of NPT and assist in rapidly designing the structural dimensions and material parameters.

2. Experiments

2.1. CFRP

As described in the patent, the Uptis spoke structure incorporates composite materials for reinforcement. In this study, CFRP was chosen for experimentation and applied to the FEM simulations. CFRP was subjected to quasi-static tensile and compression tests following the ASTM D3039 Standard Test Method for Tensile Properties of Polymer Matrix Composite Materials and ASTM D6641 Standard Test Method for Compressive Properties of

Polymer Matrix Composite Materials Using a Combined Loading Compression (CLC) Test Fixture. These tests were performed at 0° and 90° angles to determine material parameters such as elastic modulus and Poisson's ratio. Specimens with $\pm 45^\circ$ fiber orientation underwent off-axis tensile testing to assess the shear properties of CFRP. Experiments were conducted using a microcomputer-controlled electronic universal testing machine (WDW3100) at a constant speed of 2 mm/min. A Vic-3D non-contact full-field strain measurement system was employed to measure the material strain, as shown in Figure 1. The test samples were all rectangular, and their dimensions are listed in Table 1.

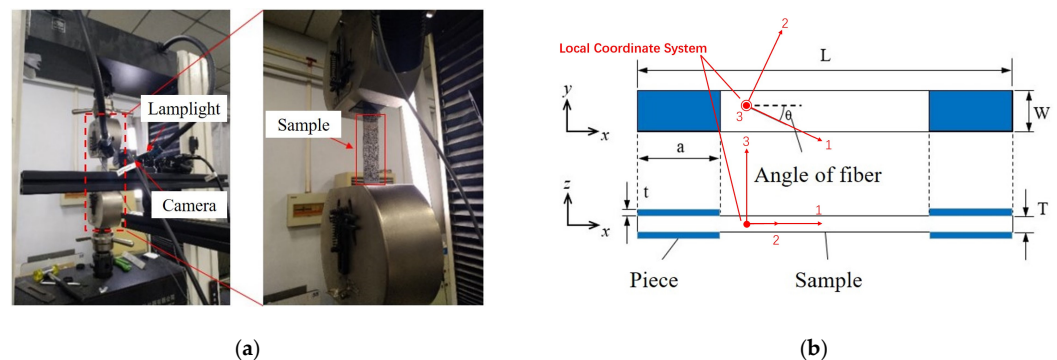


Figure 1. Experiments on CFRPs: (a) test equipment and strain measurement device and (b) test samples.

Figure 2a,b show the tensile stress–strain curves of CFRPs with fiber orientations of 0° and 90° , respectively. Figure 2c shows the off-axis tensile stress–strain curve of CFRPs with a $\pm 45^\circ$ fiber orientation.

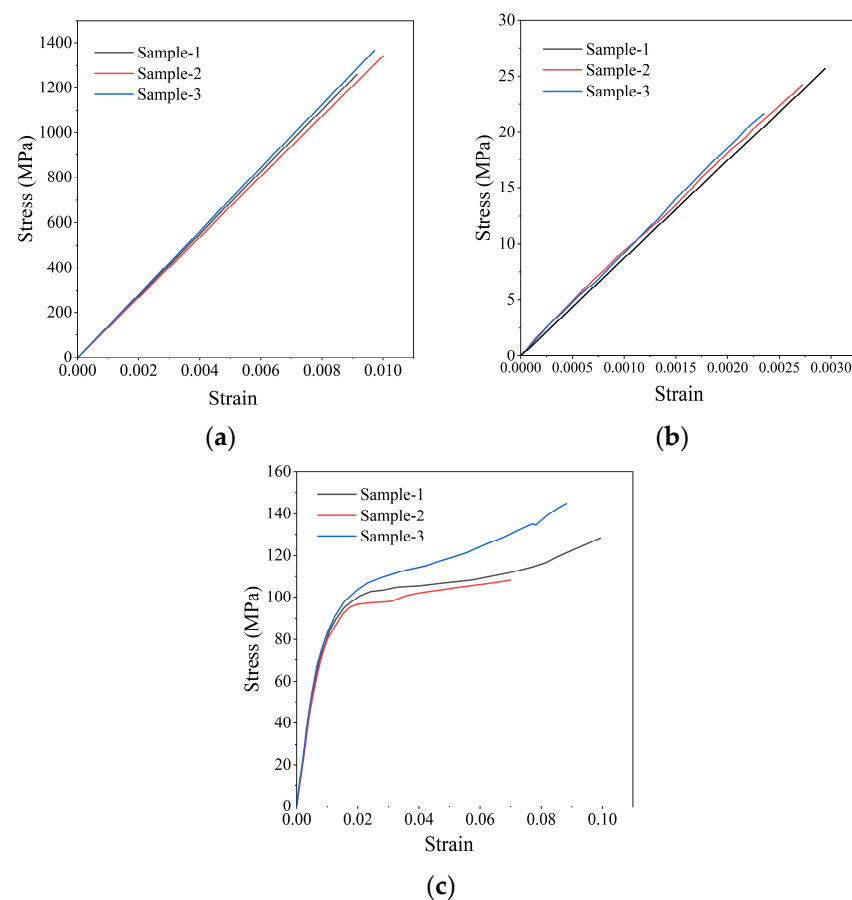


Figure 2. Quasi-static tensile stress–strain curve of CFRPs: (a) 0° tensile; (b) 90° tensile; (c) $\pm 45^\circ$ tensile.

Table 1. Dimensions of the CFRP tensile, compression, and shear specimens and reinforcement plates.

	Fiber Orientation Angle	Width (mm)	Length (mm)	Thickness (mm)	Reinforcement Piece Length (mm)	Reinforcement Piece Thickness (mm)
Tensile	0°	15	250	0.9	56	1.5
	90°	25	175	2.1	25	1.5
Compression	0°	12	150	2.4	63.5	1.5
	90°	12	150	2.4	63.5	1.5
Shear	±45°	25	250	2.4	56	1.5

2.2. Rubbers

Michelin's Uptis NPT utilizes different rubber formulations that are tailored to the performance requirements of various structural components. To precisely simulate the mechanical characteristics of the rubbers, uniaxial tensile tests were conducted on three sets of rubbers with hardness values of 75A, 88A, and 92A, as shown in Figure 3. Subsequently, the test data were used to inversely determine the parameters of the hyperelastic constitutive models. The rubber tensile specimens were obtained by cutting vulcanized flat samples with a thickness of 2 mm, gauge length of 25 mm, and width of 6 mm.

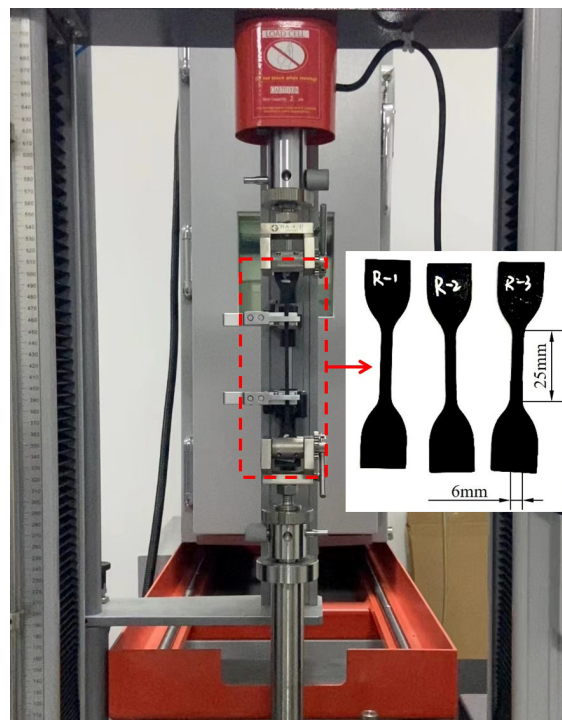
**Figure 3.** Rubber tensile test.

Figure 4 shows the tensile stress–strain curves for the three rubbers during the loading stages. These curves display nonlinear shapes in the strain range of 100%. In the following sections, the hyperelastic constitutive model for the rubbers used in Uptis NPT utilizes the experimentally acquired stress–strain curves for fitting.

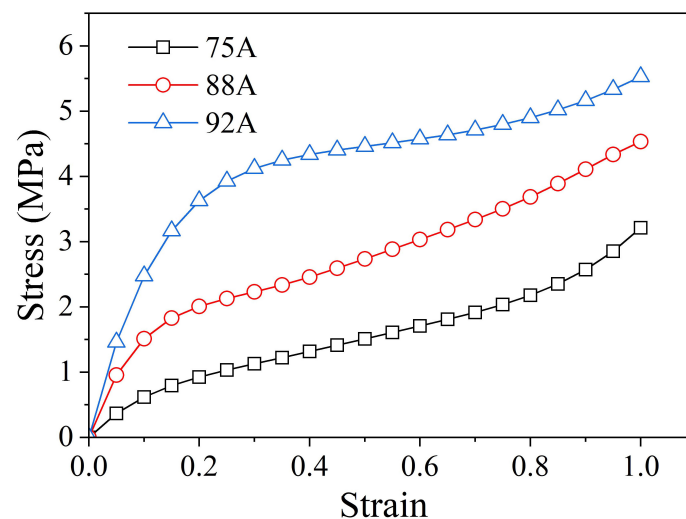


Figure 4. Stress–strain curves of the rubber tensile tests.

3. Simulation Models

3.1. Constitutive Models

The reinforcement plates employed in the NPT-CS were made of anisotropic CFRP. Given that the composite material utilized in this study comprised fibers uniformly aligned in one direction, it manifested orthotropic anisotropy. Consequently, only nine independent constants are necessary for its constitutive equation, which are summarized in Table 2 according to the experiments [25].

Table 2. Anisotropic mechanical constitutive parameters of CFRP.

E_1 (MPa)	E_2 (MPa)	E_3 (MPa)	μ_{12}	μ_{13}	μ_{23}	G_{12} (MPa)	G_{13} (MPa)	G_{23} (MPa)
128,000	8800	8800	0.357	0.357	0.35	3147	3147	2300

The definitions of directions 1, 2 and 3 are shown in Figure 2b. The constitutive model of CFRP uses a local coordinate system defined on the material, where the x-axis (direction 1) is the fiber arrangement direction in CFRP, the y-axis (direction 2) is orthogonal to the x-axis (direction 1) in the CFRP plane, and the z-axis is the normal direction of the CFRP plane, which defines the right-handed coordinate system. E_1 , E_2 , and E_3 ($E_3 = E_2$) represent the elastic moduli along the x-, y-, and z-axes, respectively. G_{12} , G_{13} ($G_{13} = G_{12}$), and G_{23} represent the shear moduli in the x-y, x-z, and y-z planes, respectively. μ_{12} , μ_{23} ($\mu_{13} = \mu_{12}$), and μ_{23} represent Poisson's ratios in the x-y, x-z, and y-z planes, respectively, of the material within the Cartesian coordinate system's 12 planes (x-y plane), 13 planes (x-z plane), and 23 planes (y-z plane).

The NPT-CS components investigated in this study, such as the spokes and tread, were all made of rubber. In the FEM, a hyperelastic constitutive model was used to simulate the behavior of rubber. The stress–strain relationship in a hyperelastic constitutive model is typically represented by the strain energy function U . The strain-energy function can take various forms. The Mooney–Revlin (MR) model, Yeoh model and neo-Hookean (N-H) model are often used and have high accuracy in small deformation conditions, but MR and Yeoh models are multi-order constitutive, including multiple material parameters. To study the influence of the material characteristic parameters on the load-carrying capacity of the NPT, a first-order polynomial N-H model was selected, which is expressed as follows:

$$U = C_{10}(\bar{I}_1 - 3) + \frac{1}{D_1}(J - 1)^2. \quad (1)$$

In Equation (1), J represents the elastic volume ratio; $\bar{I}_1$ describes the degree of material distortion; and C_{10} and D_1 describe the shear characteristics and compressibility properties of the material, respectively.

During the load-bearing process of the NPT-CS, the rubber components undergo slight deformations. The N-H model is suitable for the FEM simulation of the NPT-CS. However, when fitting the constitutive parameters, experimental data must be selected within a small strain range [26]. In this study, the stress–strain curves within a 15% strain range for the three sets of rubber samples were used to fit C_{10} . The values obtained were 1.18, 2.91, and 4.77. A comparison between the stress–strain curves obtained from the N-H constitutive model and the experimental results is shown in Figure 5. In subsequent studies, the range of constitutive parameter C_{10} for rubber materials was set from 1.0 to 5.0. Additionally, rubber materials are assumed to be incompressible by default; that is, D_1 is equal to zero.

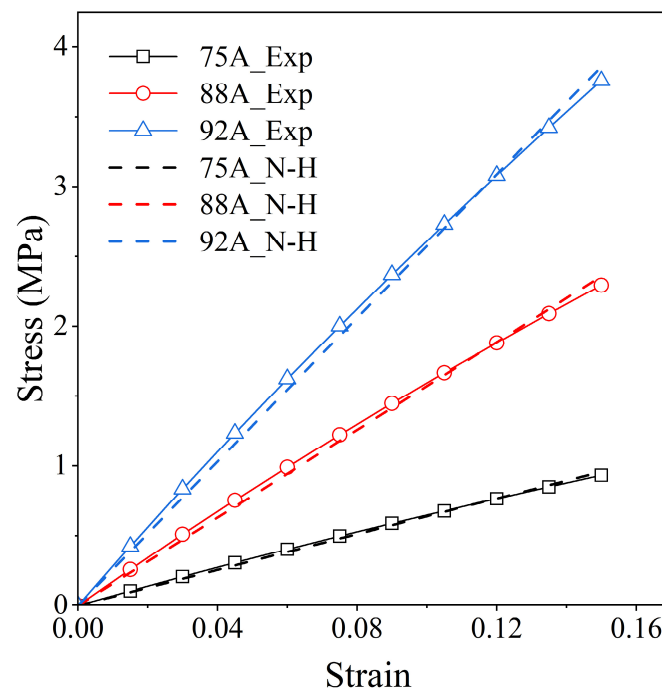


Figure 5. Comparison between the neo-Hookean constitutive fitting curves and the experimental data at 15% strain.

3.2. NPT FE Model

The NPT-CS model consist of five components: a rim, spokes, reinforcement plates, a tread, and inner and outer steel wire layers. The reinforcement plates consist of 5 mm wide CFRP plates evenly embedded within the spokes at specific intervals. A rubber cushion was designed to connect the two reinforcement plates to each spoke. The inner and outer steel wire layers were embedded within the tread to form a shear band. The overall size of the NPT-CS FE model was determined based on an inflatable tire (215/50R17) that could be replaced by an Uptis. Figure 6 and Table 3 present the overall and spoke structures of the established geometric models.

A three-dimensional FEM of the NPT-CS was constructed using the commercial finite element analysis software Abaqus. The materials for the spokes and tread in the model were characterized using a neo-Hookean constitutive model. A three-dimensional, eight-nodes/six-nodes linear brick, hybrid, constant pressure element (C3D8H/C3D6H) was selected to simulate it. Four steel wire layers within the shear band were represented using a linear elastic model with a modulus of 80 GPa and a Poisson's ratio of 0.3. Each layer was defined as a rebar layer within the shell elements, with the internal wire having a diameter of 1 mm. The spacing between adjacent wires was set to 2 mm. A four-node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane

strain element (S4R) was used to simulate it. The reinforced plate was simulated by using the composite shell section with the element type of S4R. The anisotropic material properties of the CFRP were attained by adjusting the fiber orientation and the number of layers. When establishing the local coordinate system for the anisotropic material, the X-Y plane was selected as the plane of the reinforcement plate. Within the X-Y plane, the direction perpendicular to the tire axis was assigned as the X-direction (direction 1), whereas the direction parallel to the tire axis was designated as the Y-direction (direction 2). To maintain the transverse symmetry of the NPT-CS structure, the fibers in the reinforcement plate were divided into two layers for placement. The fiber angles in the two layers were symmetrical along the X-direction of the local coordinate system. In addition, the thickness of each layer was set to half the total thickness of the reinforcement plate. The meshing details for each part of the NPT-CS are listed in Table 4.

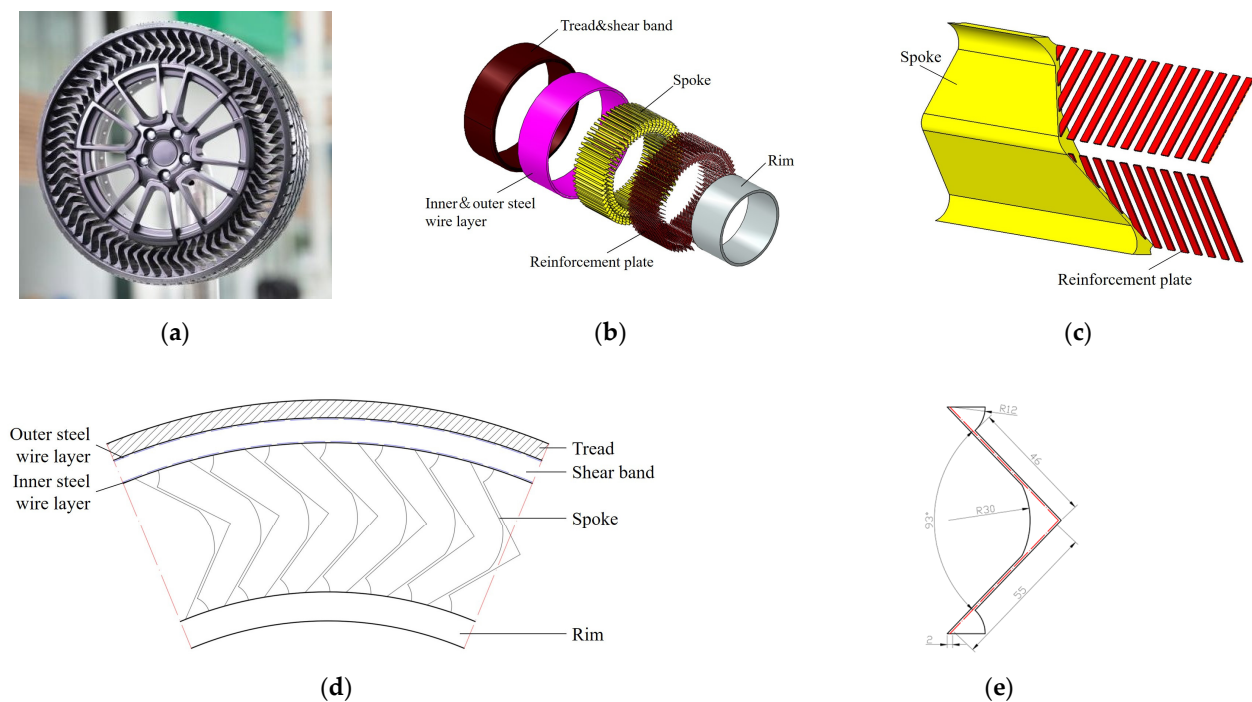


Figure 6. Uptis NPT: (a) physical representation; (b) assembly chart; (c) schematic of the spoke structure; (d) local cross-sectional view; (e) spoke geometry diagram.

Table 3. Key design parameters of the Uptis NPT.

Parameter	Outer Diameter (mm)	Tread Width (mm)	Rim Outer Diameter (mm)	Rim Width (mm)	Shear Band Thickness (mm)	Inner Steel Wire Layer Diameter (mm)	Outer Steel Wire Layer Diameter (mm)
Value	646.8	215	431.8	165.1	14	601.5	626.5

Table 4. Key design parameters of the UPTIS NPT.

Part	Number	Mesh Type	Element Number
Reinforcement plate	30 per spoke	S4R	22 per spoke
Thread and Shear band	1	C3D8H/C3D6H	33,480
Inner Steel Wire	1	S4R	4400
Outer Steel Wire	1	S4R	4400
Spoke	60	C3D8H/C3D6H	3612 per spoke

A rigid analytical plane was established to simulate the road surfaces. The coefficient of friction was set to 0.3. The values of the degrees of freedom of the nodes in the host body element were tied based on their isoparametric locations in the elements of the shear band and tread. Four rebar layers and reinforcement plates were embedded into the host elements of the shear bands and spokes. The rim was set as a rigid body. Subsequently, the rigid analytical plane was fixed, and a vertical load was applied at the center of the rim, as shown in Figure 7. We established a static, general analysis step with a duration of 1 s to perform static loading analysis of NPT. Considering that the Uptis target vehicle application (Chevrolet bolt) is about 2 tons at full load [6], the load value was set to 5000 N, and the load increased linearly with time in the step.

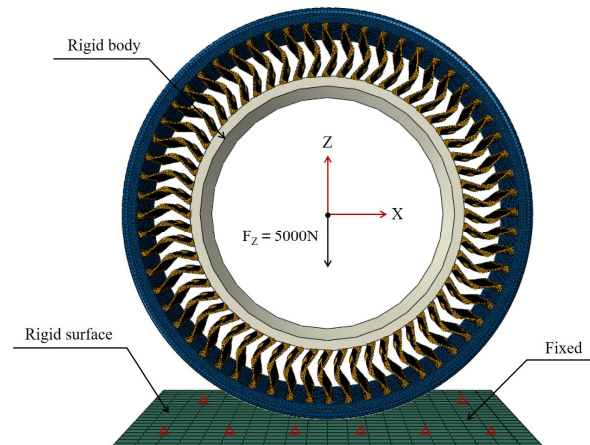


Figure 7. FEM of the vertical loading experiment for NPT.

3.3. NPT LCA Model

As shown in Figure 8, the spokes underwent tension and compression deformations during the vertical loading of the NPT. Specifically, spokes under tension suspended the load from the arch of the wheel above the contact area, and compressed spokes carried the load to the contact area directly, which are called “top loaders” and “bottom loaders”. Consequently, the shear band can be divided into contact and non-contact regions. When a vertical load is applied, the shear band in the contact region transitions from an arc to a straight line. This process leads to a uniform compression deformation of the included spokes. Therefore, the superposition principle was employed in the LCA model. Initially, the spokes and shear bands in the non-contact region were modeled and solved, followed by the superimposition of the reaction force generated by the compression spokes in the contact region. A polar coordinate system with a fixed rim center as the origin was defined in the LCA model.

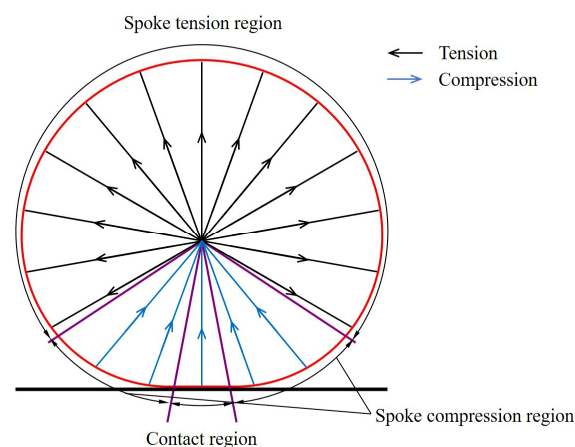


Figure 8. Nomenclature and illustration of the loaded Uptis NPT.

3.3.1. Solution Procedure

In this paper, the lateral force of the vertical quasi-static loading condition is not considered, and the non-pneumatic tire is simplified into a two-dimensional LCA model for calculation. In the LCA model, the shear band was simplified as a Timoshenko beam, which is the simplest thin elastic continuum model that accounts for three primary deformation mechanisms: normal extension, normal bending, and transverse shearing. When the NPT was loaded quasistatically at its hub, three regions were developed as the spokes acted under tension and compression:

- Contact region (included spokes are independently calculated compression forces and do not apply a load to the circular beam)
- Spoke compression region (region between the edge of contact and the angle where spoke tension, including spokes, is compressed and load is applied to the circular beam)
- Spokes tension region (spokes are engaged in tension)

In the coordinate system, the angle corresponding to the intersection between the contact region and the spoke compression region, with respect to the center of the NPT-ground contact, is referred to as the contact angle θ_c . Similarly, the angle corresponding to the junction between the spokes compression region and the spokes tension region, with respect to the center of the NPT-ground contact, is called the transition angle θ_t . At the transition angle, the radial deformation of the shear band was zero.

Figure 9 shows the deformation modes of the Timoshenko beam theory, and the tangential, radial, and twisting deformations of each cross-section along the beam are represented by $u_{\theta i}$, u_{ri} , and Φ_i respectively (where $i = 1, 2$, and 3 correspond to the three regions). At the cross section of the shear band, three resultant forces are generated: shear force V , radial force N , and bending moment M . The equilibrium equations are as follows [8]:

$$\begin{aligned} N &= \frac{EA}{R} \left(\frac{du_{\theta}}{d\theta} + u_r \right), \\ V &= \frac{GA}{R} \left(\frac{du_r}{d\theta} - u_{\theta} + R \times \Phi \right), \\ M &= \frac{EI}{R} \frac{d\Phi}{d\theta}, \end{aligned} \quad (2)$$

where E is the elastic modulus of the shear band along the tangential direction; G is the shear modulus of the shear band material; A is the cross-sectional area of the shear band in the radial direction; I is the moment of inertia of the shear band in the radial direction; and R is the mean radius.

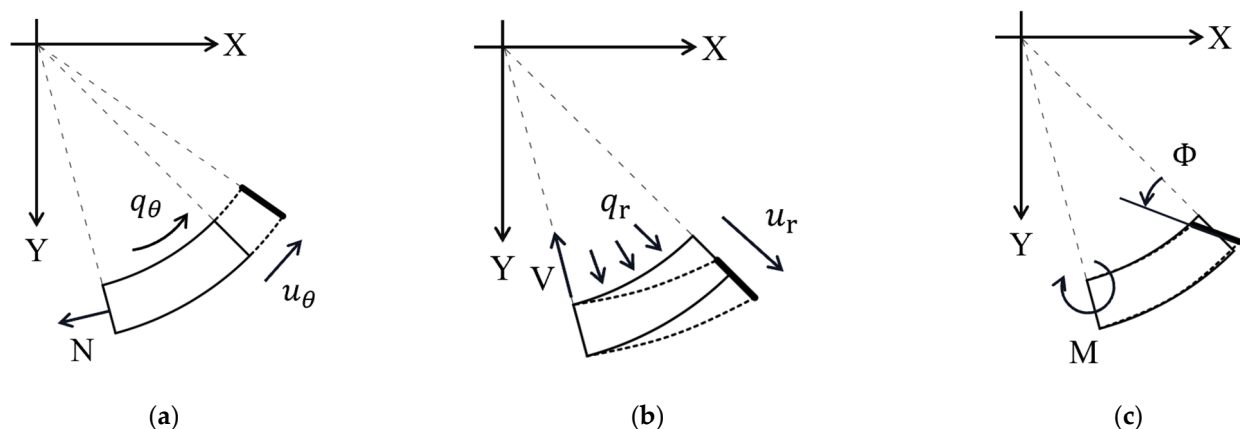


Figure 9. Illustration of shear band deformation modes: (a) tangential deformation; (b) radial deformation; (c) twisting deformation.

- Contact region

In this region, the shear band is subject only to ground reaction forces. The ground reaction force per unit length of the shear band is defined as q . The approximate governing differential equation for a uniformly curved extensional Timoshenko beam is as follows [10]:

$$\begin{aligned} A \frac{d^2 u_{\theta 1}}{d\theta^2} - GA \times u_{\theta 1} + (EA + GA) \frac{du_{r1}}{d\theta} + RGA \times \Phi_1 &= R^2 bq \sin \theta, \\ -GA \frac{d^2 u_{r1}}{d\theta^2} + EA \times u_{r1} + (EA + GA) \frac{du_{\theta 1}}{d\theta} - RGA \frac{d\Phi_1}{d\theta} &= R^2 bq \cos \theta, \\ EI \frac{d^2 \Phi_1}{d\theta^2} - R^2 GA \times \Phi_1 - RGA \frac{du_{r1}}{d\theta} + RGA \times u_{\theta 1} &= 0, \end{aligned} \quad (3)$$

where b is the width of the shear band.

- Spokes compression/tension regions

In these two regions, the external forces acting on the shear band were generated only by the deformation of the spokes. Owing to the difference in stiffness of the spokes during tension and compression, when modeling the NPT, the discrete forces from the spokes are simplified to uniformly distributed loads. The expressions for compression stiffness j and tension stiffness k per unit length of the spoke on the shear band are defined as follows:

$$\begin{aligned} j &= \frac{k_c \times N}{2\pi R}, \\ k &= \frac{k_t \times N}{2\pi R}, \end{aligned} \quad (4)$$

where k_c denotes the compression stiffness per spoke; k_t is the tension stiffness per spoke; and N is the number of spokes.

Therefore, the approximate governing differential equations for the shear band in the compression and tension regions of the spokes are as follows:

- Spokes Compression Region

$$\begin{aligned} EA \frac{d^2 u_{\theta 2}}{d\theta^2} - GA \times u_{\theta 2} + (EA + GA) \frac{du_{r2}}{d\theta} + RGA \times \Phi_2 &= 0, \\ -GA \frac{d^2 u_{r2}}{d\theta^2} + EA \times u_{r2} + (EA + GA) \frac{du_{\theta 2}}{d\theta} - RGA \frac{d\Phi_2}{d\theta} &= -R^2 j \times u_{r2}, \\ EI \frac{d^2 \Phi_2}{d\theta^2} - R^2 GA \times \Phi_2 - RGA \frac{du_{r2}}{d\theta} + RGA \times u_{\theta 2} &= 0, \end{aligned} \quad (5)$$

- Spokes Tension Region

$$\begin{aligned} EA \frac{d^2 u_{\theta 3}}{d\theta^2} - GA \times u_{\theta 3} + (EA + GA) \frac{du_{r3}}{d\theta} + RGA \times \Phi_3 &= 0, \\ -GA \frac{d^2 u_{r3}}{d\theta^2} + EA \times u_{r3} + (EA + GA) \frac{du_{\theta 3}}{d\theta} - RGA \frac{d\Phi_3}{d\theta} &= -R^2 k \times u_{r3}, \\ EI \frac{d^2 \Phi_3}{d\theta^2} - R^2 GA \times \Phi_3 - RGA \frac{du_{r3}}{d\theta} + RGA \times u_{\theta 3} &= 0, \end{aligned} \quad (6)$$

To solve the set of governing differential equations given using Equations (3), (5) and (6), boundary conditions were added to the equations for the contact region and spokes tension region based on the deformation characteristics of the NPT under load. Considering the sinking of the NPT by a certain amount δ under load, the deformation of the shear band

is flattened into a straight beam in the contact region, and its deformation satisfies the following relationship [10]:

$$R - \delta - R \cos \theta = u_{r1} \cos \theta - u_{\theta 1} \sin \theta, \quad (7)$$

For the spokes tension region, the boundary conditions are defined as follows:

$$u_{\theta 3}(\pi) = 0, u_{r3}'(\pi) = 0, \Phi_3(\pi) = 0, \quad (8)$$

The deformation functions of the shear band in different regions with the corresponding unknown parameters were solved as follows:

$$\begin{aligned} u_{\theta 1} &= f_1(\theta, C_1, C_2, C_3, q_0, q_1, q_2, q_3), \\ u_{r1} &= f_2(\theta, C_1, C_2, C_3, q_0, q_1, q_2, q_3), \\ \Phi_1 &= f_3(\theta, C_1, C_2, C_3, q_0, q_1, q_2, q_3), \\ u_{\theta 2} &= f_4(\theta, C_4, C_5, C_6, C_7, C_8, C_9), \\ u_{r2} &= f_5(\theta, C_4, C_5, C_6, C_7, C_8, C_9), \\ \Phi_2 &= f_6(\theta, C_4, C_5, C_6, C_7, C_8, C_9), \\ u_{\theta 3} &= f_7(\theta, C_{10}, C_{11}, C_{12}), \\ u_{r3} &= f_8(\theta, C_{10}, C_{11}, C_{12}), \\ \Phi_3 &= f_9(\theta, C_{10}, C_{11}, C_{12}), \end{aligned} \quad (9)$$

where q_0, q_1, q_2 and q_3 are the undetermined constants. Using symmetry, the contact pressure q is an even function, and can therefore be expressed in terms of the following series of cosines:

$$q = q_0 + q_1 \cos \theta + q_2 \cos 2\theta + q_3 \cos 3\theta, \quad (10)$$

According to the above equations, there are a total of 19 coefficients, including three deformation coefficients in the contact region, four ground reaction forces on the shear band, six deformation coefficients in the spoke compression region, and three deformation coefficients in the spoke tension region. Additionally, δ, θ_c and θ_t are also unknown. These constants are determined by equating all the essential and natural quantities at the contact edge, which is the boundary between the three regions. The 12 continuity conditions are as follows:

$$\begin{aligned} M_1(\theta_c) &= M_2(\theta_c), N_1(\theta_c) = N_2(\theta_c), V_1(\theta_c) = V_2(\theta_c), \\ u_{\theta 1}(\theta_c) &= u_{\theta 2}(\theta_c), u_{r1}(\theta_c) = u_{r2}(\theta_c), \Phi_1(\theta_c) = \Phi_2(\theta_c), \\ M_2(\theta_t) &= M_3(\theta_t), N_2(\theta_t) = N_3(\theta_t), V_2(\theta_t) = V_3(\theta_t), \\ u_{\theta 2}(\theta_t) &= u_{\theta 3}(\theta_t), u_{r2}(\theta_t) = u_{r3}(\theta_t), \Phi_2(\theta_t) = \Phi_3(\theta_t), \end{aligned} \quad (11)$$

To ensure accuracy, the trigonometric and hyperbolic functions in Equation (7) are expanded into a series of expressions composed of $\cos \theta$ through the Fourier transform. This is expressed as follows:

$$\sum_{n=1}^5 C_{0n} \cos^n \theta = 0, n = 1, 2, 3, 4, 5, \quad (12)$$

In Equation (12), C_{0n} represents the coefficients of each order of $\cos \theta$ and n is taken up to the 5th order for sufficient computational accuracy. This equation holds true within the

contact region; thus, all Co_n terms should be zero. Solving the above system of equations simultaneously resulted in 19 unknowns and 17 equations. Finally, a functional relationship in terms of θ_c and θ_t is derived.

The force transmitted from the spoke compression and spoke tension regions of the NPT to the ground during loading is denoted as F_{nc} and is calculated as follows:

$$F_{nc} = \int_0^{\theta_c} 2qd\theta, \quad (13)$$

The force generated by the compression deformation of the spokes in the contact region is represented by F_c and is calculated as follows:

$$F_c = 2Rj \times \delta \times \theta_c, \quad (14)$$

The total reaction force F of the NPT under a load is equal to the sum of Equations (13) and (14).

$$F = F_{nc} + F_c, \quad (15)$$

3.3.2. Calculation of θ_t

When the NPT is loaded, its deformation function should be a single-variable function related to θ_c . Therefore, the determination of θ_t under a specific θ_c is required. Comparing the governing differential equations of the spoke compression and tension regions shows that the only difference is in the stiffness of the spoke. First, assuming $\theta_t = \theta_c$, the root of the radial deformation function for the spoke tension region is solved as follows:

$$u_{r3}(\theta_{t1}') = 0, \quad (16)$$

The angle corresponding to this value is the approximate transitional angle θ_{t1}' . Using this θ_{t1}' as the initial value, let $\theta_t = \theta_{t1}'$ and apply the same method again for the solution, obtaining a new approximate transitional angle θ_{t2}' . This process is iterated multiple times until the difference between the approximate transition angles obtained in two consecutive iterations is less than the precision requirement. The precision requirement is denoted by α . This criterion is expressed as follows:

$$|\theta_{tn}' - \theta_{t(n-1)}'| < \alpha, \quad (17)$$

In this case, θ_{tn}' can be considered as the true value of the θ_t for the given θ_c . This study sets the precision requirement α to be 0.001 mm, and subsequent calculations are conducted based on this standard. Through this method, the deformation model and stiffness characteristics of the NPT under different θ_c can be accurately calculated using the Timoshenko beam load-bearing characteristics analysis model, referred to as the LCA model. According to the above Equations (5) and (6), it can be seen that when the spoke tension and compression deformation exhibit approximate linear characteristics without tangential force, non-pneumatic tires with the spoke of other structures can also be calculated by the LCA model.

3.3.3. Main Parameters

As mentioned above, in modeling the shear band of the NPT-CS for the LCA model, five main parameters are required: EA , GA , EI and the compression and tension stiffness of the spokes. As the shear band contains embedded steel wire layers, the tangential modulus of elasticity E can be calculated from the elastic moduli of the steel wire and rubber using the following formula:

$$E = \frac{(E_{steel}A_{steel} + E_{rub}A_{rub})}{A}, \quad (18)$$

where E_{steel} is the elastic modulus of the steel wire; E_{rub} is the elastic modulus of the shear band rubber; A_{steel} is the total cross-sectional area of the steel wire; and A_{rub} is the total cross-sectional area of the shear band. The elastic modulus E_{rub} can be approximately expressed as [27]:

$$E_{rub} = 6 \times C_{10}, \quad (19)$$

The shear modulus G of the shear band is the shear modulus of rubber, which can be approximated using the following empirical formula:

$$G = 2 \times C_{10}. \quad (20)$$

The cross-sectional area A and moment of inertia I of the shear band can be obtained from the geometric characteristics of the NPT-CS. When constructing the LCA model of the Uptis NPT, the shear band's cross-sectional area A is 2687.5 mm^2 , the moment of inertia I is $1.035 \times 10^7 \text{ mm}^4$, the radius R is 303.875 mm , and the width b is 215 mm . In addition, the stiffness of the spokes is an intrinsic parameter calculated from the FEM simulations in the following section.

4. Results and Discussion

4.1. FEM Results

Figure 10a compares the deformation modes of the spokes at various positions between the FEM results and the Uptis NPT during loading [28]. As mentioned in Section 3.3.1, the deformation of the spokes and shear band in the simulation results can be divided into three different regions (contact region, spoke compression region, and spoke tension region), which align well with real-world scenarios. When the load is 5000 N , it can be observed that the stress is primarily distributed on the reinforcing plate, and the maximum stress value is 89.5 Mpa , which is significantly lower than the strength limit of CFRP. The strain mainly occurs on the rubber block of the spoke. Therefore, the deformation modes of the spokes can be summarized as follows:

- One mixed bending–compression deformation occurred in the contact and spoke compression regions, where the reinforcement plates were bent while the rubber blocks were compressed.
- Another mixed bending–tension deformation occurred in the spokes tension region, where the reinforcement plates were bent while the rubber blocks were under tension.

The reaction force generated by the deformation of the spokes outside the contact region is transmitted to the ground through the shear band. The vertical force-displacement curve of the FEM is shown in Figure 10b.

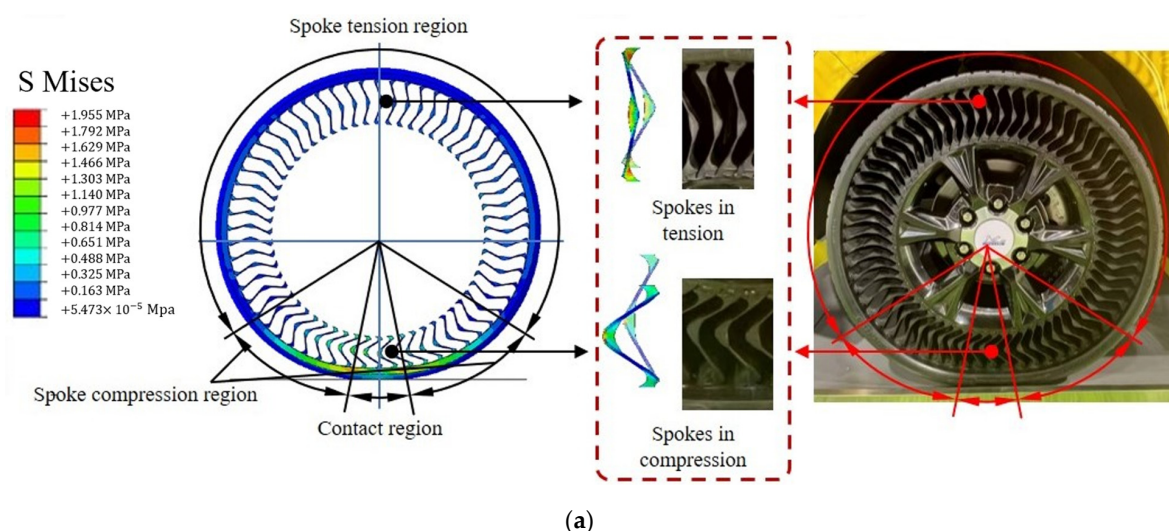


Figure 10. Cont.

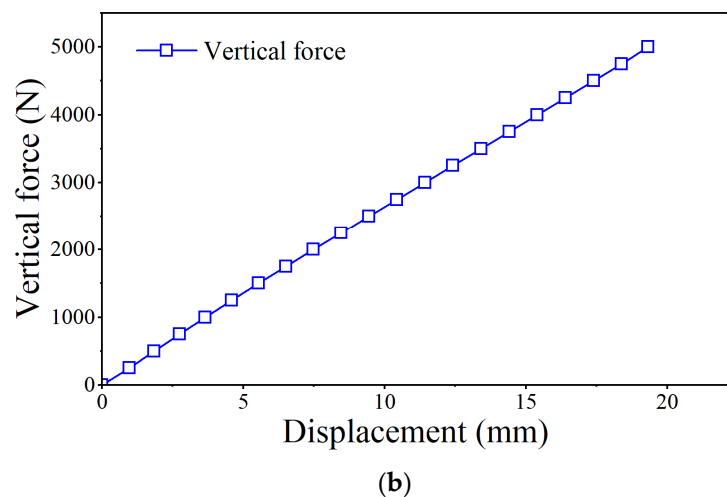


Figure 10. Comparison of the FEM results and Uptis NPT: (a) deformation of spokes and the shear band and (b) vertical load and displacement in the FEM.

4.2. RSM Analysis

Considering that the bearing capacity is determined by the choice of spokes and shear bands, the effects of anisotropic CFRP reinforcement plates and rubbers on the vertical stiffness of the Uptis NPT were analyzed using the response surface.

4.2.1. CFRP Reinforcement Plate Parameters

A Design of Experiment (DOE) matrix based on the spacing (S), fiber orientation angle (A), and thickness (T) of the reinforcement plates was proposed to investigate the relationship between the vertical force and displacement. Here, S was set to three levels: 10, 15, and 20 mm. A was set to seven levels of 0°, 15°, 30°, 45°, 60°, 75°, and 90°. T was set at three levels: 1.2, 1.8, and 2.4 mm. A full factorial experimental design involving three parameters was conducted, resulting in the establishment and calculation of 63 sets of simulation models. The fiducial values of S, A, and T were set to 15 mm, 45°, and 1.8 mm, respectively. Particularly, the upper and lower layers showed approximate isotropy when the fiber angle was 45°. The obtained parameters and vertical stiffness values are listed in Table 5.

To analyze the influence of different design parameters of the CFRP reinforcement plate on the vertical stiffness of the Uptis NPT, S, A, and T were considered as independent variables, while the vertical stiffness was considered the dependent variable. A third-order response surface was fitted using FEM simulation results. The polynomial mathematical model ($R^2 = 0.99$) was obtained using Equation (21); the coefficients are listed in Table 6. Figure 11 shows the relationship between the vertical stiffness and any two of the three independent variables. It can be seen that the vertical stiffness value has no significant fluctuation, and the dependent variable has no obvious correlation with the three independent variables.

$$f_{RR}(x, y, z) = C_0 + C_1x + C_2y + C_3z + C_4x^2 + C_5y^2 + C_6z^2 + C_7xy + C_8xz + C_9yz + C_{10}y^3, \quad (21)$$

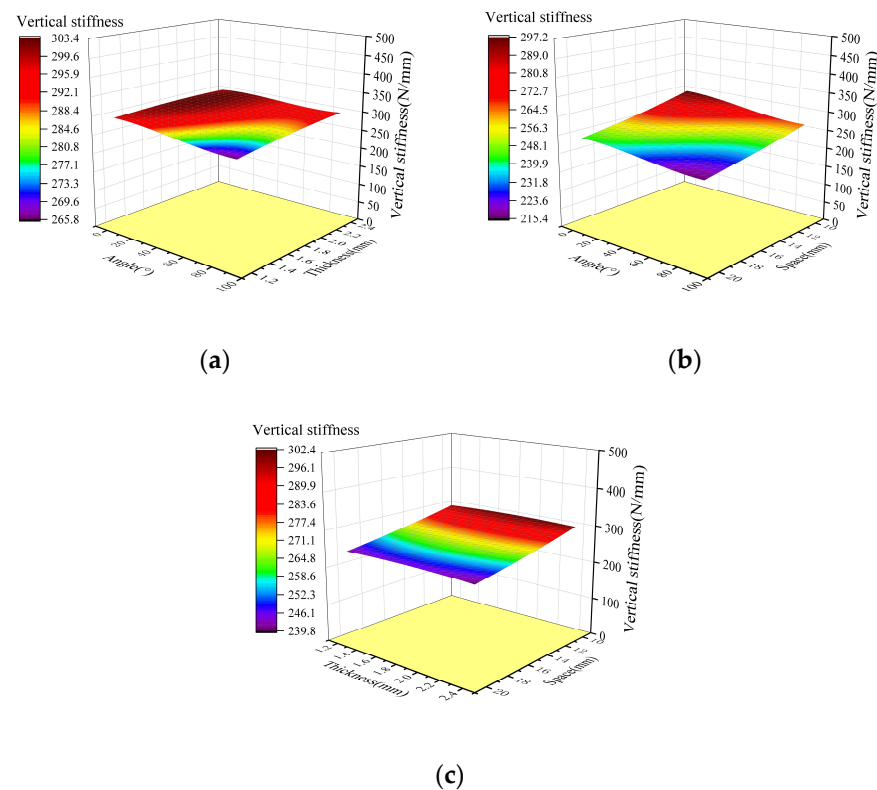
where x represents the thickness, y represents the fiber orientation angle, and z represents the spacing.

Table 5. FEM simulation results of NPTs with different reinforcement plate parameters.

No.	S (mm)	T (mm)	A (°)	Stiffness (N/mm)	No.	S (mm)	T (mm)	A (°)	Stiffness (N/mm)	No.	S (mm)	T (mm)	A (°)	Stiffness (N/mm)
1	10.00	1.20	0.00	297.81	22	15.00	1.20	0.00	266.61	43	20.00	1.20	0.00	241.27
2	10.00	1.20	15.00	296.61	23	15.00	1.20	15.00	265.51	44	20.00	1.20	15.00	240.31
3	10.00	1.20	30.00	293.63	24	15.00	1.20	30.00	262.83	45	20.00	1.20	30.00	237.96
4	10.00	1.20	45.00	283.46	25	15.00	1.20	45.00	253.80	46	20.00	1.20	45.00	230.20
5	10.00	1.20	60.00	267.83	26	15.00	1.20	60.00	240.47	47	20.00	1.20	60.00	219.05
6	10.00	1.20	75.00	264.51	27	15.00	1.20	75.00	237.70	48	20.00	1.20	75.00	216.80
7	10.00	1.20	90.00	264.99	28	15.00	1.20	90.00	238.11	49	20.00	1.20	90.00	217.14
8	10.00	1.80	0.00	300.21	29	15.00	1.80	0.00	268.78	50	20.00	1.80	0.00	243.15
9	10.00	1.80	15.00	299.93	30	15.00	1.80	15.00	268.51	51	20.00	1.80	15.00	242.92
10	10.00	1.80	30.00	298.83	31	15.00	1.80	30.00	267.49	52	20.00	1.80	30.00	242.03
11	10.00	1.80	45.00	294.67	32	15.00	1.80	45.00	263.73	53	20.00	1.80	45.00	238.72
12	10.00	1.80	60.00	288.93	33	15.00	1.80	60.00	258.58	54	20.00	1.80	60.00	234.27
13	10.00	1.80	75.00	288.06	34	15.00	1.80	75.00	257.80	55	20.00	1.80	75.00	233.61
14	10.00	1.80	90.00	288.33	35	15.00	1.80	90.00	258.05	56	20.00	1.80	90.00	233.83
15	10.00	2.40	0.00	300.85	36	15.00	2.40	0.00	269.35	57	20.00	2.40	0.00	243.66
16	10.00	2.40	15.00	300.74	37	15.00	2.40	15.00	269.24	58	20.00	2.40	15.00	243.56
17	10.00	2.40	30.00	300.19	38	15.00	2.40	30.00	268.74	59	20.00	2.40	30.00	243.11
18	10.00	2.40	45.00	298.07	39	15.00	2.40	45.00	266.77	60	20.00	2.40	45.00	241.40
19	10.00	2.40	60.00	295.59	40	15.00	2.40	60.00	264.55	61	20.00	2.40	60.00	239.43
20	10.00	2.40	75.00	295.38	41	15.00	2.40	75.00	264.35	62	20.00	2.40	75.00	239.27
21	10.00	2.40	90.00	295.53	42	15.00	2.40	90.00	264.49	63	20.00	2.40	90.00	239.39

Table 6. Coefficient list of the fitting equation for the relationship between the vertical stiffness and reinforcement plate parameters.

C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
338.85	43.79	−0.34	−8.64	−10.62	−0.01	0.11	0.26	−0.36	0.01	9.52

**Figure 11.** Response surface plots of the relationship between the vertical stiffness and reinforcement plate parameters: (a) vertical stiffness vs. fiber angle and thickness; (b) vertical stiffness vs. fiber angle and spacing; (c) vertical stiffness vs. thickness and spacing.

4.2.2. Rubber Parameters

The DOE matrix was created by considering the constitutive parameter C_{10} of the shear band rubber (T-C) and spoke rubber (S-C) as the influencing factors. The parameters of the reinforcement plates were set to fiducial values. To ensure that the shear band had sufficient stiffness, the values of T-C were set to three higher levels: 4.0, 4.5, and 5.0. S-C was set to five levels: 1.0, 2.0, 3.0, 4.0, and 5.0. A full factorial experimental design was conducted for the two parameters, resulting in the establishment and calculation of 15 sets of simulation models. The parameters and vertical stiffness values obtained are listed in Table 7.

Table 7. FEM simulation results of NPTs with different rubber parameters.

9	T-C	S-C	Stiffness (N/mm)	No.	T-C	S-C	Stiffness (N/mm)	No.	T-C	S-C	Stiffness (N/mm)
1	4.00	1.00	237.23	6	4.50	1.00	248.32	11	5.00	1.00	258.92
2	4.00	2.00	336.85	7	4.50	2.00	352.68	12	5.00	2.00	367.84
3	4.00	3.00	415.30	8	4.50	3.00	434.94	13	5.00	3.00	455.43
4	4.00	4.00	480.74	9	4.50	4.00	502.99	14	5.00	4.00	528.43
5	4.00	5.00	536.71	10	4.50	5.00	561.66	15	5.00	5.00	591.18

A third-order response surface was fitted between the vertical stiffness with T-C and S-C using the FEM simulation results. The polynomial mathematical model ($R^2 = 0.99$) was obtained using Equation (22); the coefficients are listed in Table 8. Figure 12 shows the relationship between vertical stiffness and rubber parameters. It can be seen that there is no significant correlation between vertical stiffness and T-C, but it increases significantly with the increase in S-C.

$$f_{RR}(x, y) = C_0 + C_1x + C_2y + C_3x^2 + C_4y^2 + C_5xy + C_6y^3, \quad (22)$$

where x represents the shear band rubber material constitutive parameter C_{10} , and y represents the spoke rubber material constitutive parameter C_{10} .

Table 8. Coefficient list of the fitted equation for the relationship between the vertical stiffness and rubber parameters.

C_0	C_1	C_2	C_3	C_4	C_5	C_6
112.96	−12.55	109.10	2.98	−16.32	8.22	0.98

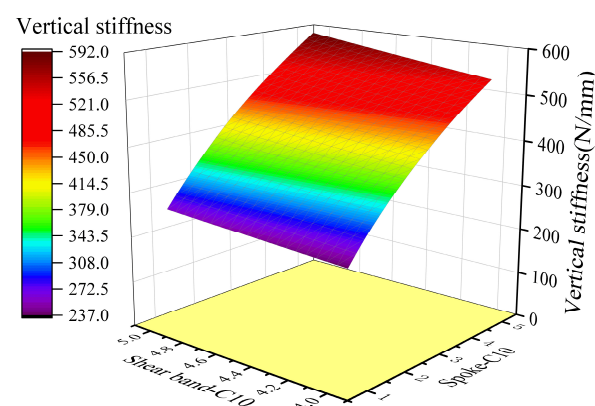


Figure 12. Response surface plot of the relationship between the vertical stiffness and rubber parameters.

4.2.3. Load-Bearing Characteristics and Sensitivity Analysis

The average gradient of the partial derivatives with respect to any independent variable within the domain of definition was calculated based on Equations (21) and (22). These average gradients reflect the influence of the independent variables on the vertical stiffness. Because the range of each variable differed slightly, a comprehensive impact value was proposed. It was defined as the product of the average gradient and its corresponding variable interval. The calculated values are listed in Table 9. Notably, the rubber constitutive parameter (S-C) of the spokes has a much greater impact on the vertical stiffness of the Uptis NPT than the other independent variables.

Table 9. Sensitivity analysis results.

	Space	Thickness	Angle	Tread- C_{10}	Spoke- C_{10}
Gradients	5.49	13.30	0.24	38.60	79.95
Range	10	1.2	90	1	4
Comprehensive Impact Value (N/mm)	54.93	15.97	21.91	38.60	319.83

As mentioned in Section 4.1, there are two deformation modes for spokes that can be regarded as series spring structures. Therefore, the overall deformation of the spoke depends on the part with the lower stiffness. Due to the significantly lower modulus of rubber compared to that of CFRP, the rubber block has a greater impact on the deformation of the spokes, which consequently affects the vertical stiffness of the NPT-CS.

4.3. LCA Model Results

4.3.1. Determination of Spoke Stiffness

In the LCA model, the initial conditions required by the model include spoke tensile stiffness k and spoke compression stiffness j . Considering that the stiffness of the spokes was primarily influenced by the spoke-rubber constitutive parameter C_{10} , FEM simulations were conducted separately for individual spokes with different C_{10} values to analyze their behavior under tension and compression. In these simulations, the parameters of the reinforcement plate were set to the fiducial values. The spoke tension range was set from 0 to 4 mm, and the compression range was set from 0 to 20 mm. Figure 13 shows the relationship curves between the tension and compression forces and the loading displacement. Linear regression was performed through the origin of each tension and compression curve, and the slopes were defined as the tensile and compressive stiffness values of a single spoke under the rubber constitutive parameter. By using Equation (4), the corresponding values of k and j for each spoke were determined. The specific data is presented in Table 10.

Based on the analysis results shown in Section 4.2, this study selected the vertical stiffness measured at a vertical displacement of 25 mm and the lateral and longitudinal stiffnesses at a vertical load of 4000 N for further analysis and calculation.

Table 10. Tensile and compression stiffness values for spokes with different parameters.

	$C_{10} = 1$	$C_{10} = 2$	$C_{10} = 3$	$C_{10} = 4$	$C_{10} = 5$
Tension Stiffness (N/mm)	30.2	58.4	84.7	109.	132.57
R^2	0.9949	0.9948	0.9948	0.9947	0.9946
Compression Stiffness (N/mm)	13.26	25.36	36.38	46.58	56.04
R^2	0.9882	0.9877	0.9873	0.9869	0.9867
k (N/mm ²)	0.95	1.83	2.66	3.43	4.16
j (N/mm ²)	0.41	0.79	1.14	1.46	1.76

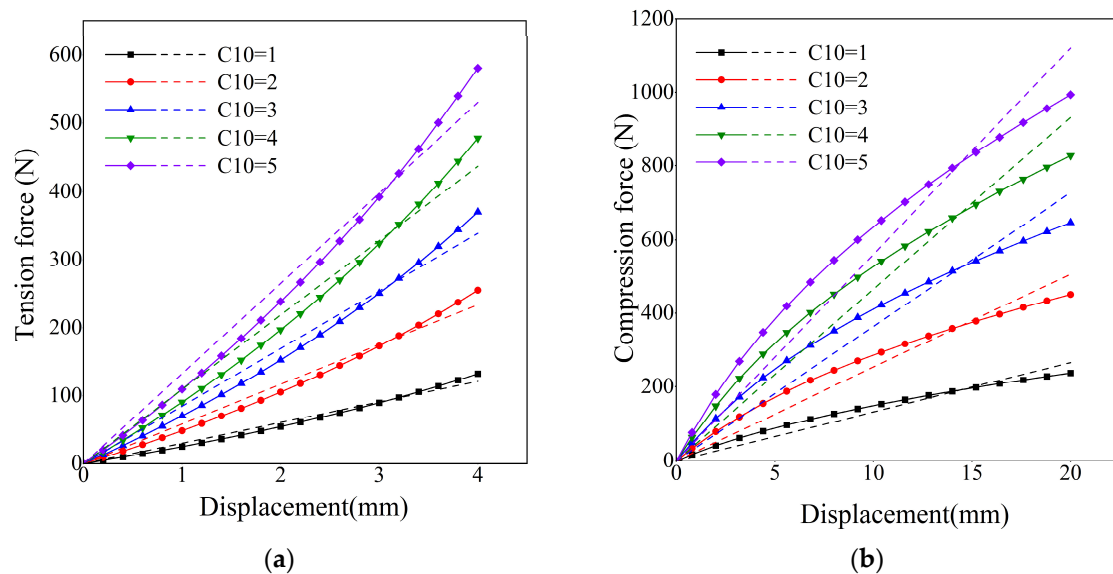


Figure 13. Spoke stiffness curves: (a) spoke tensile force–displacement curve; (b) spoke compression force–displacement curve.

4.3.2. Validation

Using Equation (7), the deflection of the NPT can be determined to increase monotonically with θ_c . This allows for the calculation of the F and δ of the NPT at different θ_c . Setting F to 5000 N, the shear band rubber C_{10} to 5.0, and the reinforcement plate parameters to their fiducial values, the δ , θ_c , and θ_t of the NPT-CS at different values of the spoke rubber constitutive parameter C_{10} were recorded. Subsequently, their vertical stiffnesses were calculated and compared with the results from the corresponding FEM (Table 11). The maximum relative error between the LCA model and the FEM is only 7.49%, showing good consistency.

Table 11. Comparison of vertical stiffness between the LCA model and the FEM simulation with different spoke parameters.

S-C	1	2	3	4	5
Stiffness of FEM (N/mm)	258.92	367.84	455.43	528.43	591.17
θ_c (°)	4.585	4.394	4.276	4.192	4.125
θ_t (°)	56.198	48.906	44.936	42.272	40.314
Force of LCA (N)	5000	5000	5000	5000	5000
Displacement of LCA (mm)	20.87	14.32	11.62	10.09	9.07
Stiffness of LCA (N/mm)	239.52	349.14	429.97	495.40	551.10
Relative Error	7.49%	5.08%	5.59%	6.25%	6.78%

Figure 14 compares the neutral-layer curve of the shear band from the LCA results with the corresponding elements of the shear band deformation from the FEM. The overall deformation patterns of the shear bands in both models exhibited good agreement at each point.

To verify the accuracy of the LCA under different load conditions, FEM and LCA were performed for the NPT-CS with different spoke rubber constitutive parameters (C_{10}). The ground reaction forces were set to 3000, 4000, 5000, and 6000 N, and the deflection at the center of the wheel rim was recorded for all models. A comparison is shown in Figure 15.

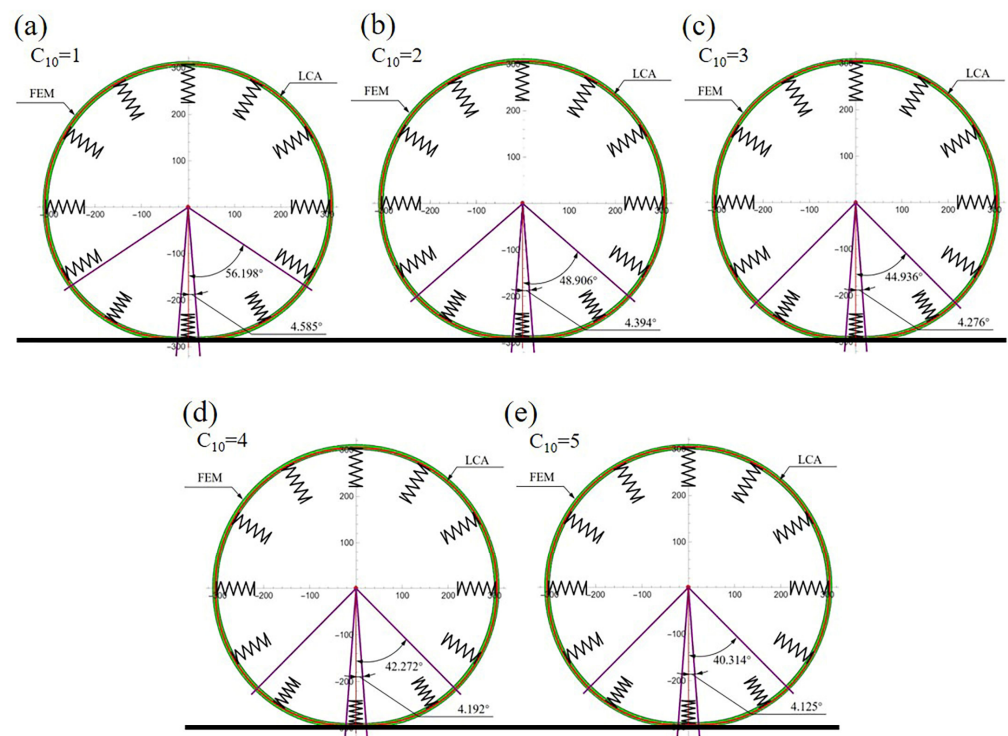


Figure 14. Comparison between the LCA model and the FEM results: (a) $C_{10} = 1$; (b) $C_{10} = 2$; (c) $C_{10} = 3$; (d) $C_{10} = 4$; (e) $C_{10} = 5$.

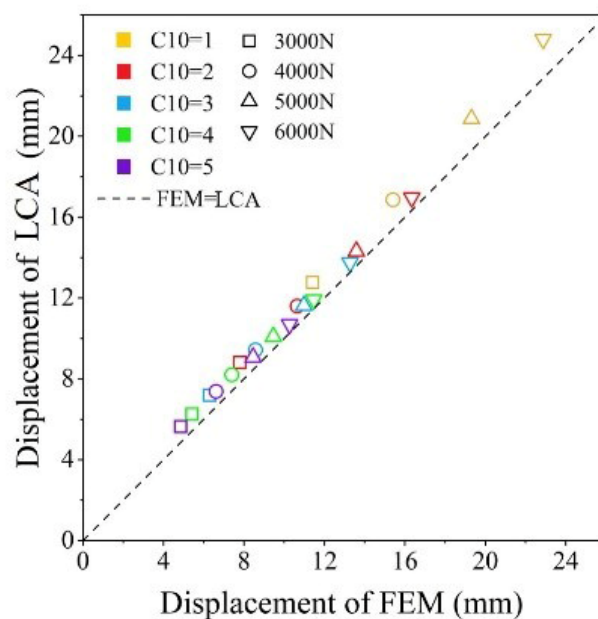


Figure 15. Comparison of deflection between the LCA and FEM under different loads.

The absolute errors of LCA under different force conditions were all less than 2 mm. When the load was 3000 N, the maximum relative error was 16%; at 4000 N, it was 11.63%; at 5000 N, it was 7.49%; and at 6000 N, it was 8.44%. Based on the fitting of the spoke stiffness in the previous section, the stiffness of the spokes in the Uptis NPT structure exhibits a certain degree of non-linear variation. In the linear fitting process, a significant fitting error occurred during the middle deformation stage of the spokes. Therefore, the calculation accuracy of the LCA model decreased slightly under the 3000 and 4000 N conditions.

In summary, the LCA model demonstrated a high level of accuracy in analyzing the vertical performance of NPT-CSs. The FEM requires approximately 1 h for computation,

whereas the time required for LCA computation using mathematical solving software is less than 5 s. This indicated that the efficiency of the LCA calculation was significantly higher than that of the FEM. It can significantly improve the design efficiency and has guiding significance for the early-stage design of NPTs.

5. Conclusions

To analyze the load-bearing characteristics of NPT-CSs and their correlation with structural and material parameters, response surface analysis was conducted on the vertical stiffness of a typical NPT-CS, Uptis, using experimental and FEM simulation studies of five design variables: spacing, fiber orientation angle, thickness of the reinforcement plate, shear band, and spoke rubber constitutive parameter C_{10} . An LCA model based on the Timoshenko beam was established for the NPT. The main findings of this study are as follows:

1. Based on the experimental data, the parameters of the anisotropic constitutive model for CFRP and the neo-Hookean constitutive model for rubber were determined. A FEM for the Uptis NPT was established, proposing a deformation mechanism for the NPT-CS load-bearing characteristics composed of two deformation modes: bending–compression and bending–tension of the spokes;
2. Using a response surface methodology to analyze the impact of the structural and material parameters of the NPT-CS on vertical stiffness, the comprehensive impact value of the spoke rubber constitutive parameter C_{10} was determined to be 319.83 N/mm based on the sensitivity analysis results, making it the most significant influencing factor;
3. Building on the Timoshenko beam theory, approximate governing differential equations for the contact region, spoke compression region, and spoke tension region were proposed and solved. Through iterative methods, θ_t was determined, simplifying the compression and tension stiffness of the spokes. Finally, an LCA model for the Uptis NPT was established;
4. The maximum errors of the LCA model compared with the FEM simulation results for the vertical stiffness values under loads of 3000, 4000, 5000, and 6000 N were 16%, 11.63%, 7.49%, and 8.44%, respectively. The deformation patterns of the spokes and shear bands were consistent with the FEM simulation results.

LCA has higher computational efficiency than FEM, allowing for the preliminary design of structural dimensions and material parameters according to the load capacity requirements during the conceptual design phase of the NPT. Additionally, it can be used for the rapid assessment of the load-bearing characteristics of previously designed NPT structures.

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