*Article*

# The Influence of the Setup on the Result of Measuring the Roundness of an Anuloid Surface

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**Abstract:** In engineering practice, anuloid surfaces are produced that are subject to strict requirements in terms of accuracy (e.g., in the bearing industry). Roundness is also often measured. The accuracy of the roundness measurement does not only depend on the accuracy of a particular metrological system. The setting of the component on the measuring device can also have a significant influence on the accuracy of the measurement. The axis of the measured surface must be perpendicular to the section in which the roundness is measured. Tilting the measured surface causes a systematic measurement error, which depends on the size of the tilt angle and the parameters of the measured surface. This study presents mathematical relations for the calculation of this error when measuring roundness on an anuloid surface. Mathematical relations for the internal and external surfaces formed by the inner and outer parts of the anuloid are derived herein. There is also a theoretical analysis of the influence of inclination and the parameters of the measured surface on the size of this error. Theoretically calculated values were compared with practically measured values of roundness. The error had a negative effect on the accuracy of the roundness measurement. It affected the roundness value, the second harmonic component, and also the roundness profile itself.

**Keywords:** roundness; systematic error; anuloid; roundness profile; harmonic analysis

## 1. Introduction

Measurement accuracy refers to how closely a measured quantity aligns with its true value, often influenced by calibration errors. In their discussion, the authors of [1] emphasize the significance of precision in both process control and product quality. They assert that precision holds the utmost importance in process control measurement as neglecting accuracy can profoundly affect a manufacturer's ability to effectively regulate processes. The metalworking industry places significant importance on acquiring tools that enhance measurement accuracy as they aid quality control, bolster reliability, and promote the goal of achieving correct results from the outset.

In the evolving landscape of global business, the necessity for measurement accuracy has heightened due to the potential time and cost savings it offers. Cylindrical components play a crucial role in engineering products, with roundness serving as a pivotal form of tolerance [2,3].

Imperfections in manufacturing can lead to varying roundness tolerances across parts, making the precision and reliability of roundness measurement essential, particularly in precision and ultraprecision scenarios [4].

In reference [5], a correlation calculus was employed to measure and comparatively evaluate the roundness profiles and roundness deviation values of the bearing journals on the main crankshaft of a medium-speed marine engine. The proposed system offered the capability of assessing the form deviations and axis positions of cylindrical surfaces, including straight shafts and crankshafts.

According to reference [6], non-contact measuring instruments are preferred when determining the size and geometry errors of high-precision components. This preference arises from the fact that mechanical measuring instruments can induce deformations, thereby compromising the accuracy of the results. Consequently, optical, optoelectronic, laser, and similar devices are extensively utilized for this purpose, as noted in references [7–10].

Many researchers have investigated bearing misalignment, its causes, and its impact on durability and performance. For example, Prabhu [11] performed an experimental investigation on misalignment effects on journal (radial) bearings. Bouyer and Fillon [12] provided an experimental analysis of misalignment effects on hydrodynamic plain journal bearing performance. Howard [13] conducted an experimental study on misalignment in gas foil journal bearings. Shi, Cao, and Chen [14] investigated the effect of angular misalignment on the dynamic characteristics of externally pressurized air journal bearings. Ma et al. [15] modelled a non-Gaussian surface and misalignment for the condition monitoring of journal bearings. Oktaviana, Tong, and Hong [16] used a skidding analysis of an angular contact ball subjected to a radial load and angular misalignment. Yang, Zhangs, and Li [17] provided a wear analysis of an angular contact ball bearing in a multiple-bearing spindle system subjected to uncertain initial angular misalignment. Ye, Wang, Gu, and Zhang [18] evaluated the effects of tilted misalignment on the loading characteristics of cylindrical roller bearings. Tong and Hong [19] investigated the fatigue life of a tapered roller bearing subject to angular misalignment. Zhao, Wang, and Hou [20] obtained measurements of the roller tilt angle in a double-row tapered roller bearing with strain gauges. Šimunović and Baršić evaluated the spindle error of a roundness measurement device [21]. The measurement of small cylinders with high accuracy was investigated by Li et al. [22]. Tiainen and Viitala studied the effect of positional errors on the accuracy of roundness measurements [23].

The above-mentioned authors concluded that misalignment in bearings could cause significant issues in terms of the performance and durability of bearings. To prevent misalignment and improper assembly, ascertaining the correct dimensions of every bearing component is necessary. However, if the measuring process is not correct (e.g., incorrect setup), a suitable component could be excluded and an improper component could be included in the assembly process, causing misalignment and improper assembly.

From the reviewed articles and also from our own experience, it is clear that high accuracy when measuring some components is essential for their proper functioning. One such component is a ball bearing, where (due to its functionality and service life) high demands are placed on the anuloid surface of the inner and outer rings. For its optimal measurement, its perfect setting is necessary. However, perfection is unattainable, and an incorrect setting can lead to incorrect measurements that could lead to the discarding of the correct piece. This imperfect setting is reflected in the inclination of the part. Therefore, this study aimed to predict and quantify the inaccuracies created by this inclination. For this purpose, mathematical equations were derived to calculate the error rate on the inner and outer anuloid surfaces.

## 2. Materials and Methods

### *Defining the Problem*

In practice, anuloid surfaces are also produced on the components (Figure 1). These surfaces can be internal or external. However, which part of the anuloid they are made of—the inner or outer part—is also important. These surfaces are often subject to strict requirements in terms of accuracy (e.g., in the bearing industry). Roundness is also often measured. The accuracy of roundness measurements does not only depend on the accuracy of a particular metrological system. The setting of the component on the measuring device can also have a significant influence on the accuracy of the measurement.
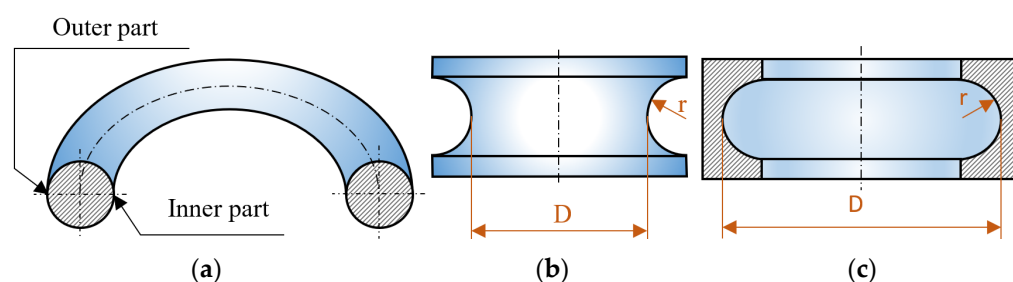
**Figure 1.** Anuloid surfaces: (**a**) anuloid; (**b**) outer surface formed by the inner part of the anuloid; (**c**) inner surface formed by the outer part of the anuloid. D: diameter of the anuloid surface; r: radius of the forming circle.

The correct setting of the measured surface in a metrological system is an important condition when measuring roundness. The measured roundness profile must lie in a plane that is perpendicular to the axis of the measured surface. In this case, the measured profile corresponds with the actual profile. Even the measured roundness value corresponds with the actual roundness. However, when the measured part is mechanically clamped (set up) to a round gauge, the measured surface may be incorrectly set. Undesirable tilting of the measured surface may occur; thus, the axis of the measurement is not identical to the axis of the part.

An incorrect setting, or tilting the measured surface, changes the measured roundness profile. On an ideal anuloid surface, the roundness profile has the shape of a circle (with the correct setting of the measured surface, where the axis of the surface is perpendicular to the measuring plane). After tilting the measured surface, a roundness profile is created, which has the shape of an ellipse (Figure 2). This also changes the measured roundness result. We concluded that there would be an error that was mathematically dependent on the size of the tilt of the measured surface (the larger the tilt angle, the larger the error). The error would have the characteristic of a systematic error and, therefore, we could characterize it as a systematic error of setting—an error caused by the inaccurate setting of the measured surface on the measuring device.
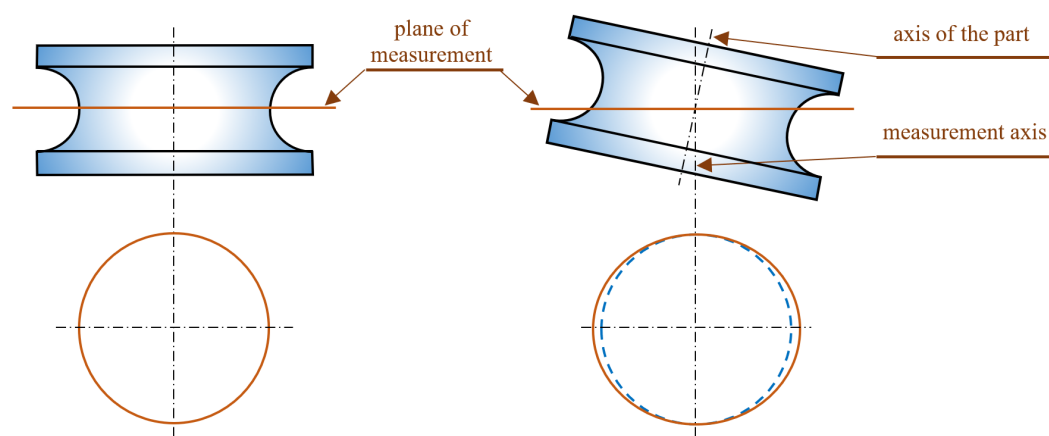


**Figure 2.** Roundness profile when measuring correctly and incorrectly clamped anuloid surfaces.

## 3. Results

### 3.1. Calculation of the Systematic Error of the Clamped Area Formed by the Inner Part of the Anuloid

The systematic error of setup for an ideal anuloid surface can be mathematically expressed as the difference between the semi-major and semi-minor axes of the ellipse, as seen in Figure 3.
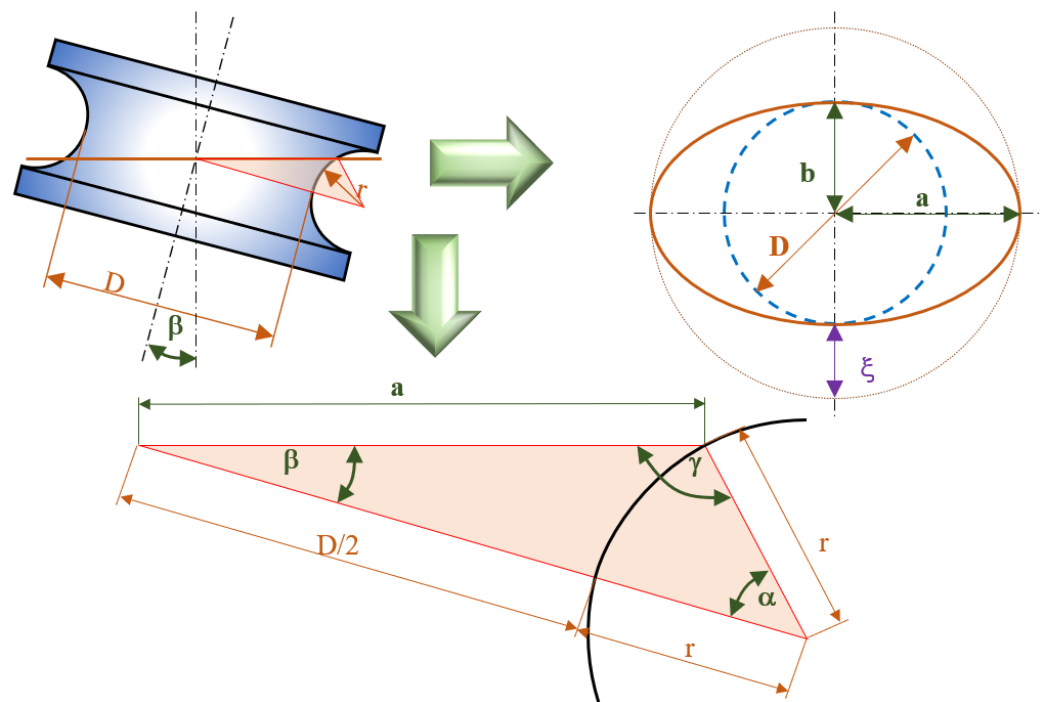
$$\xi = a - b \tag{1}$$



**Figure 3.** Scheme to determine the systematic error of a clamped area formed by the inner part of an anuloid. D: diameter of the measured anuloid surface; r: radius of the forming circle; a: semi-major axis of the ellipse (roundness profile); b: semi-minor axis of the ellipse (roundness profile); ξ: systematic error of the setting.

The value of the semi-minor axis of the ellipse is equal to the radius of the measured anuloid surface:

$$b = \frac{D}{2} \tag{2}$$

When calculating the semi-major axis of the ellipse, it is necessary to start from the triangle, which is created during tilting (Figure 3). We used the sine theorem:

$$\frac{a}{\sin \alpha} = \frac{r}{\sin \beta} = \frac{\frac{D}{2} + r}{\sin \gamma} \tag{3}$$

Using gradual modifications, we obtained:

$$\gamma = arc\sin\left(\frac{D + 2r}{2r}\sin\beta\right) \tag{4}$$

Considering that angle $\gamma$ is an obtuse angle and $\sin(\gamma) = \sin(180 - \gamma)$, we could modify this:

$$\gamma = 180 - arc\sin\left(\frac{D + 2r}{2r}\sin\beta\right) \tag{5}$$

The sum of the angles in a triangle is 180°, so:

$$\alpha = 180 - \beta - \gamma \tag{6}$$

After editing, this became:

$$\alpha = arc\sin\left(\frac{D + 2r}{2r}\sin\beta\right) - \beta \tag{7}$$

From sine theorem (3), we also obtained:

$$a = \frac{r \sin \alpha}{\sin \beta} \tag{8}$$

By substituting relation (7) into relation (8), we obtained:

$$a = \frac{r \sin \left( arc \sin \left( \frac{D + 2r}{2r} \sin \beta \right) - \beta \right)}{\sin \beta} \tag{9}$$

The relation for the systematic error of setting the area formed by the inner part of the anuloid was obtained by substituting relation (9) and (2) into relation (1):

$$\xi = \frac{r \sin \left( arc \sin \left( \frac{D + 2r}{2r} \sin \beta \right) - \beta \right)}{\sin \beta} - \frac{D}{2} \tag{10}$$

Relation (9) applies to the calculation of the systematic error of setting the outer and inner surface, which is formed by the inner part of the anuloid. This systematic error is dependent on the diameter of the anuloid surface "*D*", the radius of the forming circle "*r*", and the angle of inclination "*β*" of the measured surface. Figure 4 graphically presents the dependence of the calculated systematic error of setting the anuloid surface (formed by the inner part of the anuloid) on the diameter of the surface. A radius of the forming circle of *r* = 5 mm was considered here. As the surface diameter increased, the error caused also increased.
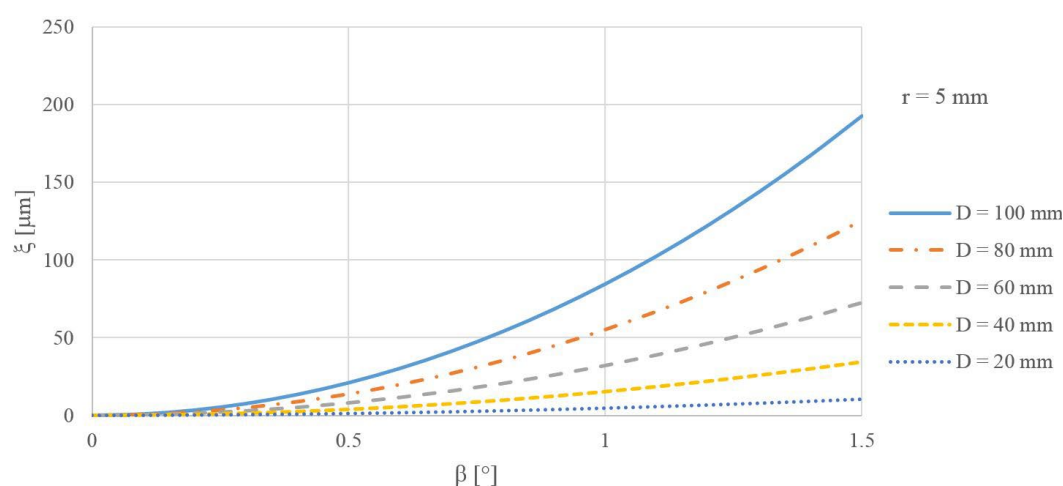


**Figure 4.** The size of the error *ξ* for different tilts *β* and diameters *D* of the area formed by the inner part of the anuloid.

The effect of the radius of the forming circle on the caused error is presented in Figure 5. A diameter of the anuloid surface of *D* = 50 mm was considered here. As the radius of the forming circle decreased, the error caused increased.
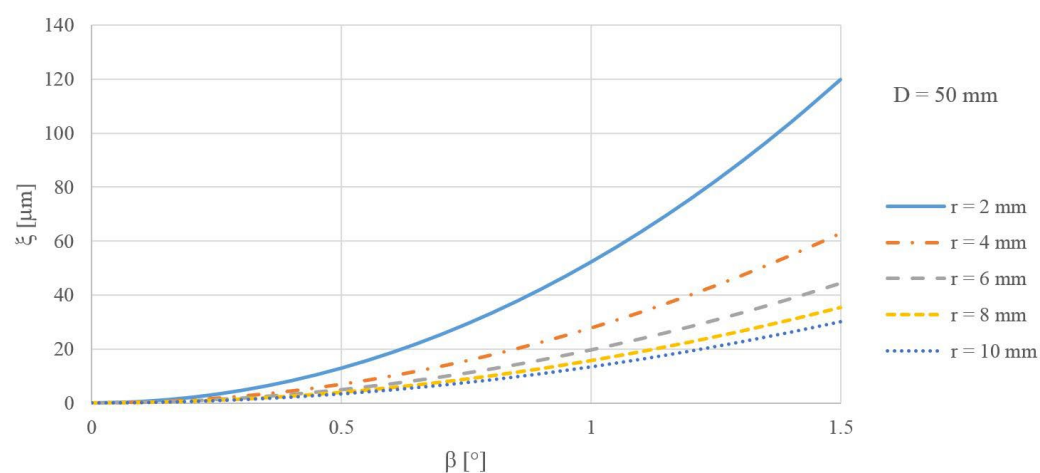
**Figure 5.** The size of the error $\xi$ for different tilts $\beta$ and radii of the forming circle $r$ of the surface formed by the inner part of the anuloid.

### 3.2. Calculation of the Systematic Error of the Clamped Area Formed by the Outer Part of the Anuloid

The systematic error of setup for an ideal anuloid surface can be mathematically expressed as the difference between the semi-major and semi-minor axes of the ellipse, as seen in Figure 6.
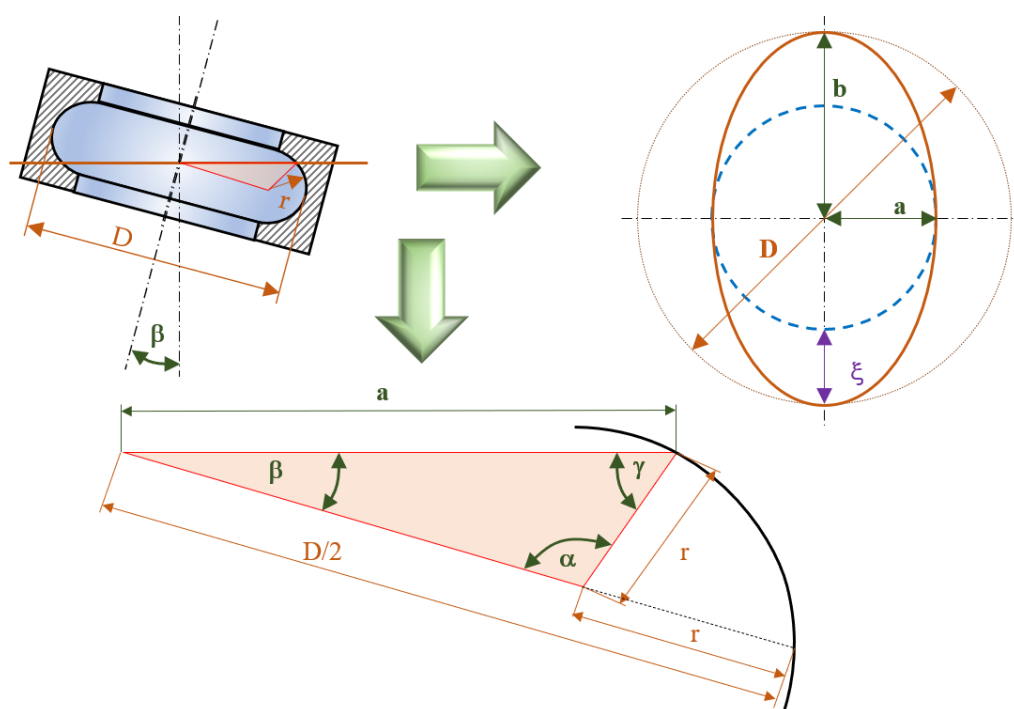
$$\xi = b - a \tag{11}$$



**Figure 6.** Scheme to determine the systematic error of a clamped area formed by the outer part of an anuloid. D: diameter of the measured anuloid surface; r: radius of the forming circle; a: semi-minor axis of the ellipse (roundness profile); b: semi-major axis of the ellipse (roundness profile); ξ: systematic error of the setting.

The value of the semi-major axis of the ellipse is equal to the radius of the measured anuloid surface:

$$b = \frac{D}{2} \tag{12}$$

When calculating the semi-minor axis of the ellipse, it is necessary to start from the triangle, which is created during tilting (Figure 6). Similar to the previous case, we used the sine theorem. Using gradual modifications, we obtained:

$$\gamma = arc\sin\left(\frac{D - 2r}{2r}\sin\beta\right) \tag{13}$$

The sum of the angles in a triangle is 180°, so:

$$\alpha = 180 - \beta - \gamma = 180 - \beta - arc\sin\left(\frac{D - 2r}{2r}\sin\beta\right) \tag{14}$$

From the sine theorem we also obtained:

$$a = \frac{r\sin\alpha}{\sin\beta} \tag{15}$$

By substituting relation (14) into relation (15), we obtained:

$$a = \frac{r\sin\left(180 - \beta - arc\sin\left(\frac{D - 2r}{2r}\sin\beta\right)\right)}{\sin\beta} \tag{16}$$

The relation for the systematic error of setting the area formed by the outer part of the anuloid was obtained by substituting relation (16) and (12) into relation (11):

$$\xi = \frac{D}{2} - \frac{r\sin\left(180 - \beta - arc\sin\left(\frac{D - 2r}{2r}\sin\beta\right)\right)}{\sin\beta} \tag{17}$$

Relation (17) applies to the calculation of the systematic error of setting the outer and inner surface, which is formed by the outer part of the anuloid. This systematic error is dependent on the diameter of the anuloid surface "$D$", the radius of the forming circle "$r$", and the angle of inclination "$\beta$" of the measured surface. Figure 7 graphically presents the dependence of the calculated systematic error of setting the anuloid surface (formed by the outer part of the anuloid) on the diameter of the surface. A radius of the forming circle of $r = 5$ mm was considered here. As the surface diameter increased, the error caused also increased.
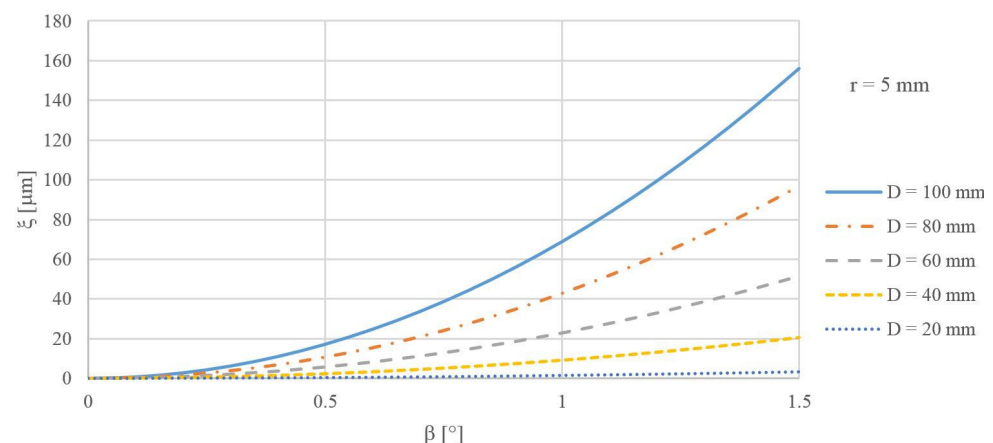


**Figure 7.** The size of the error $\xi$ for different tilts $\beta$ and diameters $D$ of the area formed by the outer part of the anuloid.

The effect of the radius of the forming circle on the caused error is presented in Figure 8. A diameter of the anuloid surface of $D = 50$ mm was considered here. As the radius of the forming circle decreased, the error caused increased.
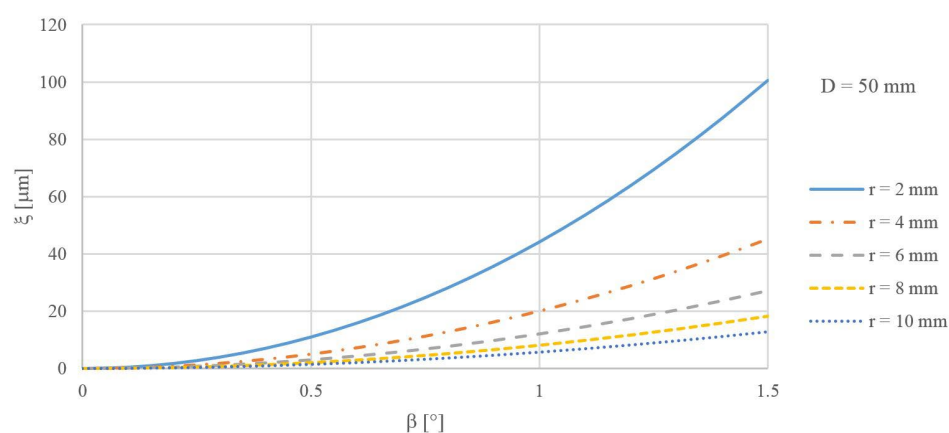
**Figure 8.** The size of the error $\xi$ for different tilts $\beta$ and radii of the forming circle $r$ of the surface formed by the outer part of the anuloid.

## 4. Discussion

*Comparison of Calculated and Measured Values*

For a practical check of the derived mathematical relations, roundness was measured using real anuloid surfaces. The components were the inner rings of a ball bearing (Figure 9a), which had outer surfaces formed by the inner part of the anuloid. The measurements were obtained using a Rondcom 60 A round gauge (Accretech, Tokyo, Japan; Figure 9b). A component was clamped onto the measuring table of the device. The measured surface was centered and leveled in automatic mode so that the measurement plane was perpendicular to the axis of the part. The roundness was then measured. The parameters for the roundness measurement and evaluation were as follows:

- Rotation precision: $(0.02 + 6 \, H/10{,}000)$ μm ($H$ is the height from the table top to the measuring point in mm).
- Number of measured points for the roundness measurement: 3600.
- Method of roundness evaluation: MZC (the minimum zone circle).
- The filter used to evaluate the roundness was a Gauss low 150 UPR (undulations per revolution).
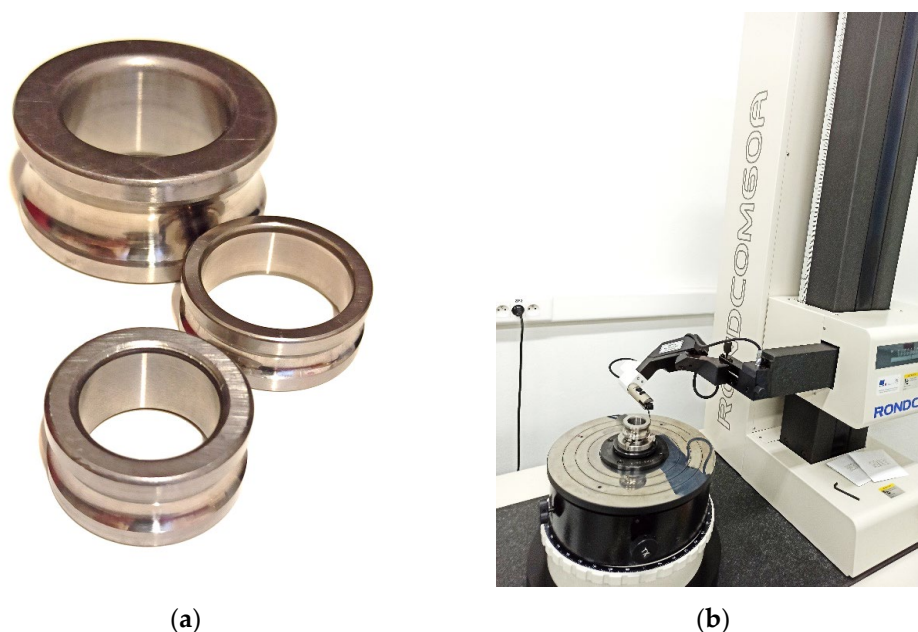- Measurement speed (rotation): 4.0 min$^{-1}$.



(**a**)                                                                 (**b**)

**Figure 9.** Experimental setup: (**a**) measured parts; (**b**) round gauge Rondcom 60 A.

The round gauge enabled the tilting of the measuring table. This allowed the part to be tilted. The measurement plane was thus inclined and was not perpendicular to the axis of the part. The size of the angle of inclination was measured using the touch probe of the round gauge. The part was re-centered and the roundness of the inclined surface was measured. The change in roundness was observed as follows:

$$Measured\ error = roundness\ {}_{of\ the\ inclined\ surface} - roundness\ {}_{of\ the\ leveled\ surface} \tag{18}$$

A comparison of the measured error with calculated systematic error (calculated error) is presented in Table 1. A harmonic analysis was also performed on the roundness profiles. This is a tool that decomposes a roundness profile into simple harmonic functions. The application of the harmonic analysis (Fourier series) to roundness is described in [24]. As an ellipse is created when an anuloid surface is tilted, it can be assumed that the systematic error of the tilt is manifested by a significant increase in the value of the second harmonic component. With such a profile, the roundness would be equal to twice the amplitude. Therefore, in the calculations, a double value of the second harmonical component (2nd HC) must be considered. We assumed that twice the value of the increase in the second harmonic component would be approximately equal to the systematic tilt error. This inclination should not have such a significant effect on the other components.

$$Difference\ 2^{nd}\ HC = 2 \cdot 2^{nd}\ HC\ {}_{of\ the\ inclined\ surface} - 2 \cdot 2^{nd}\ HC\ {}_{of\ the\ leveled\ surface} \tag{19}$$

The measurement was carried out on three surfaces:

- Anuloid 1: surface diameter D = 29.3 mm and radius of the forming circle r = 3.5 mm; the measured profiles are presented in Figure 10a,d.
- Anuloid 2: surface diameter D = 26.5 mm and radius of the forming circle r = 5 mm; the measured profiles are presented in Figure 10b,e.
- Anuloid 3: surface diameter D = 42.5 mm and radius of the forming circle r = 9 mm; the measured profiles are presented in Figure 10c,f.

**Table 1.** Measured and calculated values.

| Surface | Tilt β (°) | Roundness (μm) | 2nd HC (μm) | Calculated Error (μm) | Measured Error (μm) | Differ. 2nd HC (μm) |
|---|---|---|---|---|---|---|
| Anuloid 1 | 0 | 1.396 | 0.278 | - | - | - |
| Anuloid 1 | 0.528 | 5.207 | 2.368 | 3.034 | 3.811 | 4.180 |
| Anuloid 1 | 1.034 | 15.357 | 7.510 | 11.644 | 13.961 | 14.464 |
| Anuloid 2 | 0 | 1.437 | 0.193 | - | - | - |
| Anuloid 2 | 0.566 | 6.243 | 2.781 | 3.710 | 4.806 | 5.176 |
| Anuloid 2 | 0.802 | 11.336 | 5.580 | 7.454 | 9.899 | 10.774 |
| Anuloid 2 | 1.100 | 20.366 | 10.215 | 14.042 | 18.929 | 20.044 |
| Anuloid 3 | 0 | 0.328 | 0.056 | - | - | - |
| Anuloid 3 | 0.479 | 2.777 | 1.333 | 1.691 | 2.449 | 2.554 |
| Anuloid 3 | 0.745 | 6.305 | 3.153 | 4.091 | 5.977 | 6.194 |
| Anuloid 3 | 1.109 | 13.139 | 6.549 | 9.073 | 12.811 | 12.986 |

(**a**) Anuloid 1, β = 0°     (**b**) Anuloid 2, β = 0°     (**c**) Anuloid 3, β = 0°

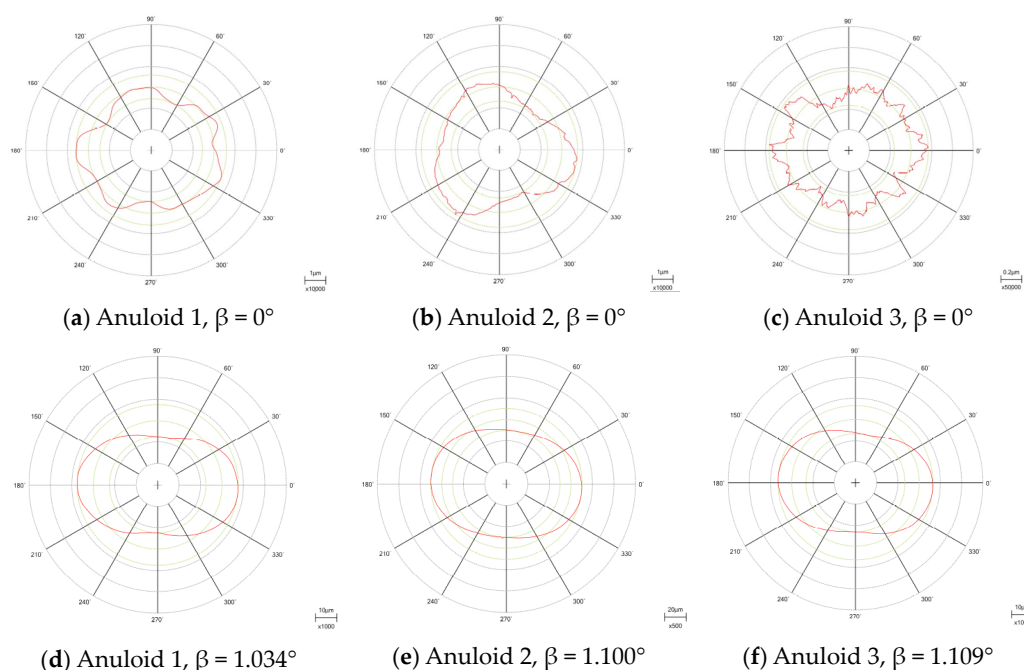(**d**) Anuloid 1, β = 1.034°     (**e**) Anuloid 2, β = 1.100°     (**f**) Anuloid 3, β = 1.109°

**Figure 10.** Roundness profiles measured on the surface formed by the inner anuloid (β: inclination angle): (**a**) Anuloid 1, β = 0°; (**b**) Anuloid 2, β = 0°; (**c**) Anuloid 3, β = 0°; (**d**) Anuloid 1, β = 1.034°; (**e**) Anuloid 2, β = 1.100°; (**f**) Anuloid 3, β = 1.109°.

Figure 11 shows a graphical comparison of the calculated error, measured error (difference between real measured roundness), and 2nd HC difference. The calculated error values corresponded with the actual differences between the values before tilting and after tilting. The real values of the measured error and 2nd HC difference were greater than the theoretical values. This was because the theoretical error values were calculated for an ideal anuloid surface, or a surface with absolute roundness (roundness = 0 μm). However, this cannot be achieved on a real surface; thus, the real results could only approximate the calculated values.
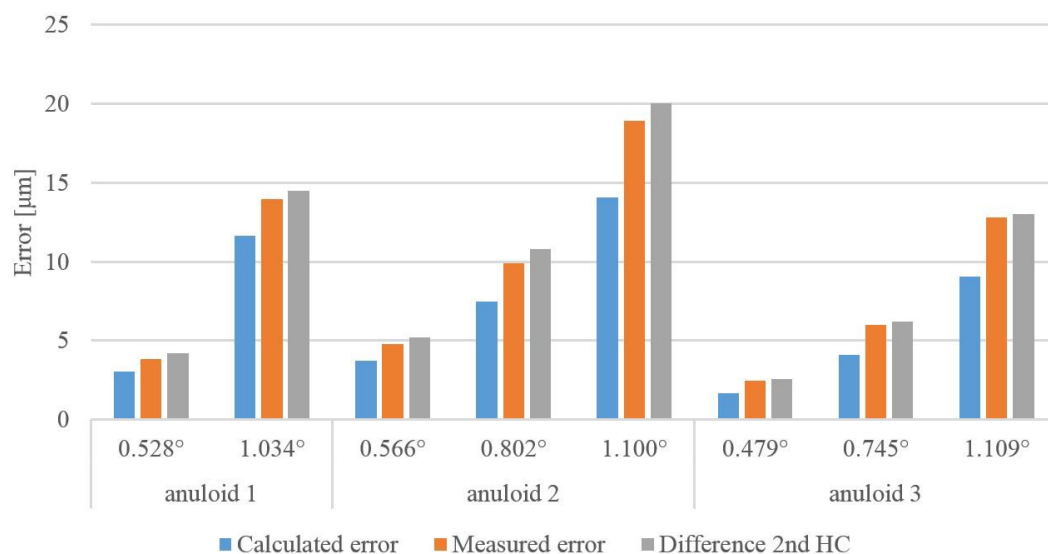


**Figure 11.** The comparison of calculated and measured values.

### 5. Conclusions

In this article, mathematical relations were derived for the calculation of a systematic error caused by the inappropriate setting of a measured area on a round gauge. Mathematical relations were applied to both the external and internal surfaces created by the external or internal segment of the anuloid. This was the main goal of the article. The validity of the relationships was also confirmed by the results from the practical measurement. We concluded that the theoretically derived mathematical relations for anuloid surfaces corresponded with the real measured values. We can also make the following statements:

- The value of the setup error ($\xi$) depended on:
    - the inclination of the measured anuloid surface ($\beta$);
    - the diameter of the anuloid surface (D);
    - the radius of the forming circle (r).
- As the angle of inclination of the measured surface ($\beta$) increased, the calculated error also increased.
- As the diameter of the measured area (D) increased, the calculated error also increased.
- As the radius of the forming circle (r) increased, the calculated error decreased.

The tilt of the measured surface negatively affected the accuracy of the roundness measurement. After tilting, the roundness profile was elliptical; tilting also affected the measured roundness profile. However, if the roundness profile was elliptical and the inclination was in the direction of the semi-minor axis of the ellipse, then the inclination of such a surface could reduce the measured roundness. Then, the accuracy of the measurement was affected in the opposite way; a smaller roundness than was actually measured on the surface was calculated. Such a case is analyzed in [25].

Modern round gauges automatically determine the angle of inclination and level the position of the measured surface, even before the actual measurement; thus, they measure the correct profile. Older round gauges cannot detect this angle, so the operator must detect this angle and make the necessary adjustments to the position of the measured surface. Most round gauge software is not open-source, so the implementation of derived formulas serves to manually derive the inclination of older round gauges. The verification of inclination is determined by the software of modern round gauges.

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**Data Availability Statement:** The data presented in this study are available on request from the corresponding author. The data are not publicly available due to the large data capacity.

**Conflicts of Interest:** The authors declare no conflicts of interest.

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