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Distributed Adaptive Consensus Tracking Control for Second-Order Nonlinear Heterogeneous Multi-Agent Systems with Input Quantization

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Abstract: In this paper, the problem of distributed adaptive consensus tracking control for second-order nonlinear heterogeneous multi-agent systems (MASs) with input quantization is considered. A distributed output feedback control scheme based on a K-filter is developed to suppress the influences of unknown disturbances and input quantization. In contrast to existing approaches, an additional design parameter is introduced into the controller design to ensure that the subsystem tracking error converges to an arbitrarily small residual set. Through Lyapunov stability analysis, it can be proved that the proposed control scheme can achieve distributed consensus tracking control of second-order nonlinear heterogeneous MASs. In addition, all signals in the closed-loop system are shown to be globally uniformly bounded. Finally, a practical example demonstrates the effectiveness of the proposed control method.

Keywords: multi-agent system (MAS); consensus tracking control; distributed output feedback; input quantization

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1. Introduction

In recent years, cooperative control of multi-agent systems (MASs) has attracted a lot of attention, such as for unmanned aerial vehicle formation [1], wireless sensor networks [2], and multi-robot manipulator collaboration [3]. Compared with a single agent, MASs can accomplish complex control tasks through division of labor and cooperation among agents. Researchers have developed many effective cooperative control schemes for MASs.

The consensus problem of MASs has been a hot research topic in the field of cooperative control, and many outstanding research results have been presented, such as the mean square consensus problem [4], the consensus optimization problem [5], the robust consensus problem [6], and the adaptive consensus problem [7]. Different from the general consensus control problem, consensus tracking control requires each agent to track a dynamic desired trajectory and, thus, has broader application prospects. In [8], a consensus tracking control scheme was developed for linear leader-follower networks by designing a class of distributed reference observers. In [9], an extra estimator was designed for each agent to solve the consensus tracking control for nonlinear high-order MASs with unknown parameters. In [10], an event-triggered consensus control scheme was proposed for switched stochastic nonlinear systems to reduce the communication traffic. The above research results assume that all agents have the same dynamical model. In some practical applications, different agents need to be equipped with different devices to accomplish complex control tasks—the corresponding systems are called heterogeneous MASs. In [11,12], the consensus problem and the mean-square consensus problem of heterogeneous MASs were studied, and the conditions for the system to achieve consensus were given. In [13], consensus protocols were proposed for second-order heterogeneous dynamic agents, and the sufficient conditions for all agents to reach consensus were given. However, in the above literature, it is assumed that the states of agents can be observed, which may not be satisfied in practical applications.

In the past decade, distributed control schemes for MASs using only system outputs have been widely studied. For example, in [14], a distributed cyclic small gain output feedback control scheme was developed for nonlinear MASs. In [15], a distributed observer was designed to solve the output regulation problem when the follower cannot directly obtain the external system state. In [16], the bipartite consensus problem for continuous-time MASs was studied, and a dynamic output feedback method was proposed to design bipartite consensus controllers. In [17], considering switched directed networks, a formation tracking control scheme using the output information of agents was designed. However, the dynamics of all agents considered in the above literature are the same. According to the above analysis, it is more practical to study the distributed output feedback consensus tracking control of heterogeneous MASs, which is the first motivation for this paper.

Quantitative control has been widely used in industrial fields, such as power systems and network control systems. For example, in order to save limited bandwidth resources in wireless communication networks, quantization techniques are needed to reduce the communication rate during information transmission. Information transmission between agents in MASs generally needs to be quantified due to network bandwidth limitations, and information quantization will affect the performance and stability of the system. Therefore, it is necessary to study the influence of signal quantization on the cooperative control of MASs. In order to avoid chattering, hysteresis quantizers have been intensively studied. In [18], the consensus tracking control of nonlinear MASs with quantized input was solved using a new quantizer decomposition method and command filtering neural control. In [19], based on a prescribed performance function, an adaptive fuzzy event-triggered control strategy was designed for MASs with input quantization. In [20], a consensus tracking control strategy was designed for MASs with more general nonlinearities, in which some online estimators were introduced to reduce the effect of input quantization. The existing approaches have studied the distributed input quantization consensus tracking control problem of MASs with the same dynamic model of agents. However, the research on distributed control of heterogeneous MASs with input quantization is still limited, which is the second motivation for this paper.

In this paper, a distributed consensus tracking control scheme is designed for second-order nonlinear heterogeneous MASs. The novelty of the proposed control scheme is highlighted as follows:

1. Compared with the existing results for distributed consensus tracking control of MASs with input quantization, the MASs considered in this paper use a more general dynamic model.
2. Different from the general K-filters in [21–23], an additional design parameter is introduced into the proposed K-filter, and this design parameter can improve the estimation performance of the filter.
3. In this paper, the consensus tracking errors of MASs can converge to an arbitrarily small set by adjusting only one controller parameter. Compared with the results in [24], the proposed method has a wider range of parameter selections.

This paper is organized as follows. In Section 2, some basic knowledge and preliminary descriptions are given. In Section 3, a distributed output feedback consensus tracking control scheme is designed. In Section 4, the effectiveness of the developed scheme is verified. Finally, Section 5 concludes the paper.

2. Preliminaries and Problem Statement

In this section, some basic information is presented. Then, the distributed consensus tracking control problem for nonlinear MASs with input quantization is formulated.

2.1. Notations and Algebraic Graph Basics

For matrices X and Y, $X \otimes Y$ represents their Kronecker product. $\lambda_{min}(M)$ and $\lambda_{max}(M)$ represent the minimum eigenvalue and maximum eigenvalue of a matrix M, respectively. $\|\cdot\|$ denotes the Euclidean norm of a vector or the induced 2-norm of a matrix.

Consider a MAS with N agents. If each agent is regarded as a vertex, the communication topology among agents can be described by a directed graph $\mathcal{G} = (\mathbb{V}, \mathbb{E})$, where $\mathbb{V} \triangleq \{1, 2, \cdots, N\}$ represents the set of vertices, and $\mathbb{E} \triangleq \{(i, j) : i \in \mathbb{V}, j \in \mathbb{N}_i\}$ represents the set of edges. For each agent, the neighbor set is defined as $\mathbb{N}_i \triangleq \{j \in \mathbb{V}$: agent i can receive information from agent $j\}$. A weight $\mathbf{a}_{ij}$ is assigned to each edge $(i, j) \in \mathbb{E}$, $\mathbf{a}_{ij} = 1$ if $j \in \mathbb{N}_i$ and $\mathbf{a}_{ij} = 0$ otherwise. Then, the Laplacian matrix associated with $\mathcal{G}$ is given as $\mathcal{L} = [\mathbf{r}_{ij}] \in \mathbb{R}^{N \times N}$, where $\mathbf{r}_{ii} = \sum_{j=1, j\neq i}^{N} \mathbf{a}_{ij}$ and $\mathbf{r}_{ij} = -\mathbf{a}_{ij}$ $(i \neq j)$. The digraph $\mathcal{G}$ contains a directed spanning tree if at least one node has a directed path to all the other nodes, and this node is called the root node. In addition, the adjacency matrix is defined as $\mathcal{H} = diag\{\mathbf{h}_1, \cdots, \mathbf{h}_N\}$, and $\mathbf{h}_i > 0$ if the desired trajectory can be obtained directly by agent i and $\mathbf{h}_i = 0$ otherwise.

2.2. Problem Formulation

Consider a second-order nonlinear heterogeneous MAS, the dynamics of each agent are as follows:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(y_i)\theta_i + \omega_{i,1} \\ \dot{x}_{i,2} = \phi_i(y_i)\mathbf{Q}_i(u_i) + f_{i,2}(y_i)\theta_i + \omega_{i,2} \\ y_i = x_{i,1}, \quad i = 1, \cdots, N \end{cases} \tag{1}$$

where $x_i = [x_{i,1}, x_{i,2}]^T \in \mathbb{R}^2$, $\mathbf{Q}_i(u_i) \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the system states, quantified control input, and output of the ith agent, respectively; $\theta_i \in \mathbb{R}^s$ is an unknown constant parameter vector; $f_{i,1}(y_i), f_{i,2}(y_i) \in \mathbb{R}^{1\times s}$ and $\phi_i(y_i) \in \mathbb{R}$ with $\phi_i(y_i) \neq 0$ are smooth nonlinear functions; and $\omega_i = [\omega_{i,1}, \omega_{i,2}]^T \in \mathbb{R}^2$ are unknown time-varying disturbances.

Remark 1. *Note that system (1) can be used to describe many practical application systems, such as single-link robot manipulator systems [25] and ship formation [26]. In addition, the model parameters in system (1) can be unknown. Compared with the results in [27–29], the MAS considered in this paper is more general.*

Assumption 1. *The communication topology $\mathcal{G}$ among agents contains a directed spanning tree. In addition, the root node has direct access to the desired trajectory.*

Assumption 2. *The desired trajectory $(y_r(t), \dot{y}_r(t), \ddot{y}_r(t))$ is piecewise continuous and bounded.*

Assumption 3. *The unknown disturbances ω_i are bounded, and there exists an unknown positive constant $\bar{\omega}$ such that $\|\omega_i\| \leq \bar{\omega}$.*

Remark 2. *Assumptions 1–3 are standard requirements in dealing with the distributed consensus tracking control problem of MASs. Assumption 2 is more relaxed than the existing ones in [30–32], in which the desired trajectory needs to be linearly parameterized.*

This paper considers the hysteresis quantizer, which is modeled as

$$\mathbf{Q}(u) = \begin{cases} q_l, & \text{if } \frac{q_l}{1+\delta} < u \le q_l,\ \mathbf{Q}^- \ge q_l \text{ or} \\ & \quad q_l \le u < \frac{q_l}{1-\delta},\ \mathbf{Q}^- \le q_l; \\ \bar{q}_l, & \text{if } q_l < u \le \frac{q_l}{1-\delta},\ \mathbf{Q}^- \ge \bar{q}_l \text{ or} \\ & \quad \frac{q_l}{1-\delta} \le u < q_{l+1},\ \mathbf{Q}^- \le \bar{q}_l; \\ 0, & \text{if } 0 \le u \le \frac{q_1}{1+\delta}, \text{ or} \\ & \quad \frac{q_1}{1+\delta} < u < q_1,\ \mathbf{Q}^- = 0; \\ -\mathbf{Q}(-u), & \text{if } u < 0, \quad l = 1, 2, 3, \cdots \end{cases} \tag{2}$$

where $q_l = \rho\epsilon^{1-l}$, $\bar{q}_l = (1+\delta)q_l$, and $\delta = (1-\epsilon)/(1+\epsilon)$. The parameters $0 < \epsilon < 1$ and $\rho > 0$ determine the quantization density of the hysteresis quantizer (2). $\mathbf{Q}^-$ represents the status prior to $\mathbf{Q}(u)$, and $\mathbf{Q}(u)$ is in the set $\mathbb{U} = \{0, \pm q_l, \pm(1+\delta)q_l\}$. The map of the hysteresis quantizer (2) is plotted in Figure 1.

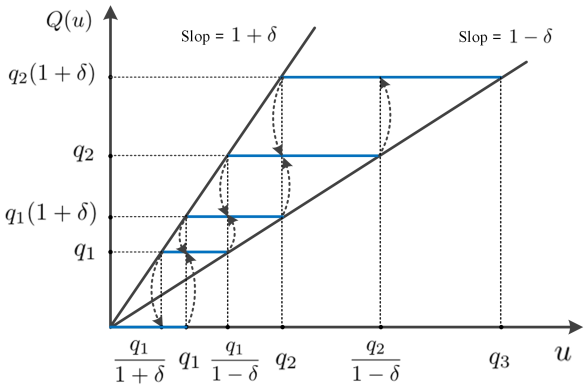

Figure 1. Hysteretic quantizer.

Remark 3. *In contrast to the general quantizer, the hysteresis quantizer (2) can enhance the ability to reduce chattering. In addition,* $\mathbf{Q}(u)$ *can be rewritten as* $\mathbf{Q}(u) = \Psi_1(t)u(t) + \Psi_2(t)$*, and*

$$\begin{cases} \Psi_1(t) = \frac{\mathbf{Q}(u)}{u(t)}, \quad \Psi_2(t) = 0, & \text{if} \quad |u(t)| \ge \rho; \\ \Psi_1(t) = 1, \quad \Psi_2(t) = \mathbf{Q}(u) - u(t), & \text{if} \quad |u(t)| < \rho. \end{cases}$$

In view of Figure 1, one has

$$\Psi_1(t) \ge \lambda, \quad |\Psi_2(t)| \le \rho, \quad \forall t \ge 0$$

where $\lambda = 2\epsilon/(1+\epsilon)$.

The control objective is to design a distributed consensus tracking control scheme for the second-order nonlinear heterogeneous MAS (1) such that: (i) all signals of the considered MAS are globally uniformly bounded; (ii) the output of each agent can track the desired trajectory.

3. Distributed Adaptive Controller Design and Stability Analysis

In this section, a distributed output feedback control scheme is presented for second-order nonlinear heterogeneous MASs, and it is proved that the proposed distributed control scheme can ensure the stability of second-order nonlinear MASs.

3.1. State Estimation

For the ith agent, a K-filter is designed to estimate the unmeasured states

$$\begin{cases} \dot{\zeta}_i = A_i\zeta_i + G_i y_i \\ \dot{\Xi}_i = A_i\Xi_i + f_i(y_i) \\ \dot{\eta}_i = A_i\eta_i + E_2\phi_i(y_i)\mathbf{Q}_i(u_i) \end{cases} \tag{3}$$

where $\zeta_i \in \mathbb{R}^2$, $\Xi_i \in \mathbb{R}^{2\times s}$, and $\eta_i \in \mathbb{R}^2$ are the filter states; $A_i = \begin{bmatrix} -\tau_i g_{i,1} & 1 \\ -\tau_i^2 g_{i,2} & 0 \end{bmatrix}$, $G_i = [\tau_i g_{i,1}, \tau_i^2 g_{i,2}]^T$, and $f_i(y_i) = [f_{i,1}(y_i), f_{i,2}(y_i)]^T$; and $\tau_i \geq 1$ is a design parameter; $g_{i,1}$ and $g_{i,2}$ are chosen such that the polynomial $s^2 + g_{i,1}s + g_{i,2}$ is Hurwitz; and $E_2 = [0\ 1]^T$.

Then, the state estimation can be expressed as

$$\hat{x}_i = \zeta_i + \Xi_i\theta_i + \eta_i. \tag{4}$$

From (1) and (4), the estimation error $\tilde{x}_i = x_i - \hat{x}_i$ satisfies

$$\dot{\tilde{x}}_i = A_i\tilde{x}_i + \omega_i. \tag{5}$$

Further, by applying the transformation

$$\varepsilon_i = \Delta_i\tilde{x}_i, \quad \Delta_i = \begin{bmatrix} 1 & 0 \\ 0 & \tau_i^{-1} \end{bmatrix}. \tag{6}$$

Then, the following error system can be obtained

$$\begin{aligned} \dot{\varepsilon}_i &= \tau_i A_{i,0}\Delta_i\tilde{x}_i + \Delta_i\omega_i \\ &= \tau_i A_{i,0}\varepsilon_i + \Delta_i\omega_i \end{aligned} \tag{7}$$

where $A_{i,0} = \begin{bmatrix} -g_{i,1} & 1 \\ -g_{i,2} & 0 \end{bmatrix}$ is Hurwitz.

Consider the following Lyapunov function

$$V_{i,0} = \varepsilon_i^T P_i \varepsilon_i \tag{8}$$

where the matrix $P_i > 0$ is the solution of $A_{i,0}^T P_i + P_i A_{i,0} = -3(N+1)I_2$.

From Assumption 3 and $\tau_i \geq 1$, the derivative of $V_{i,0}$ is obtained as

$$\begin{aligned} \dot{V}_{i,0} &= -3(N+1)\tau_i\varepsilon_i^T\varepsilon_i + 2\varepsilon_i^T P_i\Delta_i\omega_i \\ &\leq -3(N+1)\tau_i\varepsilon_i^T\varepsilon_i + \varepsilon_i^T\varepsilon_i + \|\Delta_i\|^2\|P_i\|^2\bar{\omega}^2 \\ &\leq -(3N+2)\tau_i\varepsilon_i^T\varepsilon_i + \|P_i\|^2\bar{\omega}^2. \end{aligned} \tag{9}$$

Remark 4. *Different from the K-filters in [21–23], an additional design parameter τ_i is introduced into the proposed K-filter (3). This design parameter can improve the estimation performance of the filter in the face of unknown disturbances and quantization errors. After the error transformation, τ_i appears in the negative term of (9), which will be useful for the tracking performance analysis in the next section.*

3.2. Backstepping Design Procedure

Now, the distributed backstepping controller is designed. From (1) and (6), the derivative of y_i satisfies

$$\begin{aligned} \dot{y}_i &= \hat{x}_{i,2} + f_{i,1}\theta_i + \tau_i\varepsilon_{i,2} + \omega_{i,1} \\ &= \eta_{i,2} + \zeta_{i,2} + (\Xi_{i,2} + f_{i,1})\theta_i + \tau_i\varepsilon_{i,2} + \omega_{i,1}. \end{aligned} \tag{10}$$

For each agent, some positive scalars $k_{i,1}$, $k_{i,2}$, $\gamma_{i,1}$, $\gamma_{i,2}$, $\sigma_{i,1}$, $\sigma_{i,2}$, $\sigma_{i,3}$, and ϱ_i are introduced as design parameters and define:

$$z_{i,1} = \sum_{j=1}^{N} \mathbf{a}_{ij}(y_i - y_j) + \mathbf{h}_i(y_i - y_r) \tag{11}$$

$$z_{i,2} = \eta_{i,2} - \alpha_{i,1} \tag{12}$$

where $\alpha_{i,1}$ is a virtual control function. Next, consider the following design steps:

Step 1: From (11), the derivative of $z_{i,1}$ satisfies

$$\begin{aligned}\dot{z}_{i,1} &= \mathbf{c}_i\dot{y}_i - \sum_{j=1}^{N} \mathbf{a}_{ij}\dot{y}_j - \mathbf{h}_i\dot{y}_r \\ &= \mathbf{c}_i(\eta_{i,2} + \zeta_{i,2} + \theta_i^T\varpi_{i,1} + \tau_i\varepsilon_{i,2} + \omega_{i,1}) - \mathbf{h}_i\dot{y}_r \\ &\quad - \sum_{j=1}^{N} \mathbf{a}_{ij}(\eta_{j,2} + \zeta_{j,2} + \theta_j^T\varpi_{i,j,1} + \tau_j\varepsilon_{j,2} + \omega_{j,1})\end{aligned} \tag{13}$$

where $\mathbf{c}_i = \sum_{j=1}^{N} \mathbf{a}_{ij} + \mathbf{h}_i$, $\varpi_{i,1} = (\Xi_{i,2} + f_{i,1})^T$, and $\varpi_{i,j,1} = (\Xi_{j,2} + f_{j,1})^T$.

Consider the following function

$$V_1 = \sum_{i=1}^{N}(\frac{1}{2}z_{i,1}^2 + \frac{1}{2\tau_i}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i + \sum_{j=1}^{N}\frac{\mathbf{a}_{ij}}{2\tau_i}\tilde{\theta}_{i,j}^T\Gamma_{i,j}^{-1}\tilde{\theta}_{i,j}) \tag{14}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ and $\tilde{\theta}_{i,j} = \hat{\theta}_{i,j} - \theta_j$. Moreover, $\hat{\theta}_i$ and $\hat{\theta}_{i,j}$ are the estimations of θ_i and θ_j, respectively; Γ_i and $\Gamma_{i,j}$ are positive definite matrices.

In view of (13) and (14), the derivative of V_1 satisfies

$$\begin{aligned}\dot{V}_1 = \sum_{i=1}^{N}[&\mathbf{c}_i z_{i,1}(z_{i,2} + \alpha_{i,1} + \zeta_{i,2} + \theta_i^T\varpi_{i,1} + \tau_i\varepsilon_{i,2} + \omega_{i,1}) \\ &- z_{i,1}\sum_{j=1}^{N}\mathbf{a}_{ij}(\eta_{j,2} + \zeta_{j,2} + \theta_j^T\varpi_{i,j,1} + \tau_j\varepsilon_{j,2} + \omega_{j,1}) \\ &- \mathbf{h}_i z_{i,1}\dot{y}_r + \frac{1}{\tau_i}\tilde{\theta}_i^T\Gamma_i^{-1}\dot{\hat{\theta}}_i + \sum_{j=1}^{N}\frac{\mathbf{a}_{ij}}{\tau_i}\tilde{\theta}_{i,j}^T\Gamma_{i,j}^{-1}\dot{\hat{\theta}}_{i,j}].\end{aligned} \tag{15}$$

According to Assumption 3, the following inequalities can be obtained

$$\begin{aligned}\mathbf{c}_i z_{i,1}(\tau_i\varepsilon_{i,2} + \omega_{i,1}) &\leq \mathbf{c}_i\frac{\tau_i + 1}{4}z_{i,1}^2 + \mathbf{c}_i\tau_i\varepsilon_i^T\varepsilon_i + \mathbf{c}_i\bar{\omega}^2 \\ &\leq \mathbf{c}_i\frac{\tau_i + 1}{4}z_{i,1}^2 + N\tau_i\varepsilon_i^T\varepsilon_i + N\bar{\omega}^2\end{aligned} \tag{16}$$

$$\begin{aligned}-z_{i,1}\sum_{j=1}^{N}\mathbf{a}_{ij}(\tau_j\varepsilon_{j,2} + \omega_{j,1}) &\leq \sum_{j=1}^{N}\mathbf{a}_{ij}\frac{\tau_j + 1}{4}z_{i,1}^2 + \sum_{j=1}^{N}\mathbf{a}_{ij}\tau_j\varepsilon_j^T\varepsilon_j + \sum_{j=1}^{N}\bar{\omega}^2 \\ &\leq \sum_{j=1}^{N}\mathbf{a}_{ij}\frac{\tau_j + 1}{4}z_{i,1}^2 + \sum_{j=1}^{N}\mathbf{a}_{ij}\tau_j\varepsilon_j^T\varepsilon_j + N\bar{\omega}^2.\end{aligned} \tag{17}$$

Choose the first virtual control function

$$\begin{aligned}\alpha_{i,1} =& -\tau_i k_{i,1}z_{i,1} - \zeta_{i,2} - \hat{\theta}_i^T\varpi_{i,1} - \frac{\tau_i + 1}{4}z_{i,1} + \frac{\mathbf{h}_i}{\mathbf{c}_i}\dot{y}_r \\ &+ \frac{1}{\mathbf{c}_i}\sum_{j=1}^{N}\mathbf{a}_{ij}(\eta_{j,2} + \zeta_{j,2} + \hat{\theta}_{i,j}^T\varpi_{i,j,1} - \frac{\tau_j + 1}{4}z_{i,1}).\end{aligned} \tag{18}$$

Define the following tuning functions

$$\mathcal{T}_{i,1} = \mathbf{c}_i \tau_i \Gamma_i \varpi_{i,1} z_{i,1} - \tau_i \sigma_{i,1} \Gamma_i \hat{\theta}_i \tag{19}$$

$$\mathcal{T}_{i,j,1} = -\tau_i \Gamma_{i,j} \varpi_{i,j,1} z_{i,1} - \tau_i \sigma_{i,2} \Gamma_{i,j} \hat{\theta}_{i,j}. \tag{20}$$

Substituting (16)–(20) into (15), and noting $1 \le \mathbf{c}_i \le N$, it follows that

$$\begin{aligned} \dot{V}_1 \le \sum_{i=1}^{N} [& -\tau_i k_{i,1} z_{i,1}^2 + \mathbf{c}_i z_{i,1} z_{i,2} + N\tau_i \varepsilon_i^T \varepsilon_i + \sum_{j=1}^{N} \mathbf{a}_{ij} \tau_j \varepsilon_j^T \varepsilon_j \\ & + \frac{1}{\tau_i} \tilde{\Theta}_i^T \Gamma_i^{-1} (\dot{\hat{\Theta}}_i - \mathcal{T}_{i,1}) + \sum_{j=1}^{N} \frac{\mathbf{a}_{i,j}}{\tau_i} \tilde{\Theta}_{i,j}^T \Gamma_{i,j}^{-1} (\dot{\hat{\Theta}}_{i,j} - \mathcal{T}_{i,j,1}) \\ & + 2N\bar{\omega}^2 - \sigma_{i,1} \tilde{\theta}_i^T \hat{\theta}_i - \sigma_{i,2} \sum_{j=1}^{N} \mathbf{a}_{ij} \tilde{\theta}_{i,j}^T \hat{\theta}_{i,j}]. \end{aligned} \tag{21}$$

Step 2: Define $\chi_i = [\zeta_{i,2}, \Xi_{i,2}^T, \eta_{i,2}]^T$. The derivative of $z_{i,2}$ satisfies

$$\begin{aligned} \dot{z}_{i,2} = & \phi_i(y_i) \mathbf{Q}_i(u_i) + \beta_i - \theta_i^T \varpi_{i,2} - \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i - \frac{\partial \alpha_{i,1}}{\partial y_i} (\tau_i \varepsilon_{i,2} + \omega_{i,1}) \\ & - \sum_{j=1}^{N} \mathbf{a}_{i,j} \theta_j^T \varpi_{i,j,2} - \sum_{j=1}^{N} \mathbf{a}_{i,j} \frac{\partial \alpha_{i,1}}{\partial y_j} (\tau_j \varepsilon_{j,2} + \omega_{j,1}) - \sum_{j=1}^{N} \mathbf{a}_{i,j} \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_{i,j}} \dot{\hat{\theta}}_{i,j} \\ & - \mathbf{h}_i \sum_{l=1}^{2} \frac{\partial \alpha_{i,1}}{\partial y_r^{(l-1)}} y_r^{(l)} \end{aligned} \tag{22}$$

where $\beta_i = -\tau_i^2 g_{i,2} \eta_{i,1} - \frac{\partial \alpha_{i,1}}{\partial y_i} \zeta_{i,2} - \frac{\partial \alpha_{i,1}}{\partial y_i} \eta_{i,2} - \frac{\partial \alpha_{i,1}}{\partial \chi_i} \dot{\chi}_i - \sum_{j=1}^{N} a_{i,j} (\frac{\partial \alpha_{i,1}}{\partial y_j} \zeta_{j,2} + \frac{\partial \alpha_{i,1}}{\partial y_j} \eta_{j,2} + \frac{\partial \alpha_{i,1}}{\partial \chi_j} \dot{\chi}_j)$, and $\varpi_{i,2} = \frac{\partial \alpha_{i,1}}{\partial y_i} (\Xi_{i,2} + f_{i,1})^T$, $\varpi_{i,j,2} = \frac{\partial \alpha_{i,1}}{\partial y_j} (\Xi_{j,2} + f_{j,1})^T$.

Consider the second Lyapunov function

$$V_2 = V_1 + \sum_{i=1}^{N} (V_{i,0} + \frac{1}{2} z_{i,2}^2 + \frac{\lambda_i}{2\tau_i \gamma_i} \tilde{\mu}_i^2) \tag{23}$$

where $\tilde{\mu}_i = \hat{\mu}_i - \mu_i$, and $\hat{\mu}_i$ is the estimation of $\mu_i = 1/\lambda_i$.

From Remark 2, the control input $\mathbf{Q}_i(u_i)$ of each agent can be rewritten as

$$\begin{aligned} & \mathbf{Q}_i(u_i) = \Psi_{i,1}(t) u_i + \Psi_{i,2}(t) \\ & \Psi_{i,1}(t) \ge \lambda_i, \quad |\Psi_{i,2}(t)| \le \rho_i \end{aligned} \tag{24}$$

where $\lambda_i = 2\epsilon_i / (1 + \epsilon_i)$ and $0 < \epsilon_i < 1$, $\rho_i > 0$ are quantizer parameters.

According to Assumption 3 and (24), the following inequalities can be obtained

$$-z_{i,2} \frac{\partial \alpha_{i,1}}{\partial y_i} (\tau_i \varepsilon_{i,2} + \omega_{i,1}) \le \frac{\tau_i + 1}{4} (\frac{\partial \alpha_{i,1}}{\partial y_i})^2 z_{i,2}^2 + \tau_i \varepsilon_i^T \varepsilon_i + \bar{\omega}^2 \tag{25}$$

$$\begin{aligned} & - z_{i,2} \sum_{j=1}^{N} \mathbf{a}_{ij} \frac{\partial \alpha_{i,1}}{\partial y_j} (\tau_j \varepsilon_{j,2} + \omega_{j,1}) \\ & \qquad \le \sum_{j=1}^{N} \mathbf{a}_{ij} \frac{\tau_j + 1}{4} (\frac{\partial \alpha_{i,1}}{\partial y_j})^2 z_{i,2}^2 + \sum_{j=1}^{N} \mathbf{a}_{ij} \tau_j \varepsilon_j^T \varepsilon_j + N\bar{\omega}^2 \end{aligned} \tag{26}$$

$$z_{i,2} \phi_i(y_i) \Psi_{i,2}(t) \le z_{i,2}^2 \phi_i^2(y_i) + \frac{1}{4} \rho_i^2. \tag{27}$$

Choose the second virtual control function

$$\begin{aligned}\alpha_{i,2} =&\tau_i k_{i,2} z_{i,2} + \mathbf{c}_i z_{i,1} + \phi_i^2(y_i) z_{i,2} + \beta_i - \hat{\theta}_i^T \varpi_{i,2} - \sum_{j=1}^{N} \mathbf{a}_{i,j} \hat{\theta}_{i,j}^T \varpi_{i,j,2} \\ &+ \frac{\tau_i + 1}{4} (\frac{\partial \alpha_{i,1}}{\partial y_i})^2 z_{i,2} + \sum_{j=1}^{N} \mathbf{a}_{i,j} \frac{\tau_j + 1}{4} (\frac{\partial \alpha_{i,1}}{\partial y_j})^2 z_{i,2} - \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \mathcal{T}_{i,2} \\ &- \sum_{j=1}^{N} \mathbf{a}_{i,j} \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_{i,j}} \mathcal{T}_{i,j,2} - \mathbf{h}_i \sum_{l=1}^{2} \frac{\partial \alpha_{i,1}}{\partial y_r^{(l-1)}} y_r^{(l)}. \end{aligned} \tag{28}$$

Define the following tuning functions

$$\begin{aligned}\mathcal{T}_{i,2} &= \mathcal{T}_{i,1} - \tau_i \Gamma_i \varpi_{i,2} z_{i,2} \\ \mathcal{T}_{i,j,2} &= \mathcal{T}_{i,j,1} - \tau_i \Gamma_{i,j} \varpi_{i,j,2} z_{i,2}. \end{aligned} \tag{29}$$

In addition, the adaptive laws $\hat{\theta}_i$ and $\hat{\theta}_{i,j}$ are designed as

$$\dot{\hat{\theta}}_i = \mathcal{T}_{i,2}, \quad \dot{\hat{\theta}}_{i,j} = \mathcal{T}_{i,j,2}. \tag{30}$$

From (23)–(30), one has

$$\begin{aligned}\dot{V}_2 \leq& \sum_{i=1}^{N} [-\sum_{l=1}^{2} \tau_i k_{i,l} z_{i,l}^2 + z_{i,2}\alpha_{i,2} + z_{i,2}\phi_i(y_i)\Psi_{i,1}(t)u_i + \frac{\lambda_i}{\tau_i \gamma_i} \tilde{\mu}_i \dot{\hat{\mu}}_i \\ &- (1+2N)\tau_i \varepsilon_i^T \varepsilon_i + 2\sum_{j=1}^{N} \mathbf{a}_{i,j} \tau_j \varepsilon_j^T \varepsilon_j - \sigma_{i,1} \tilde{\theta}_i^T \hat{\theta}_i - \sigma_{i,2} \sum_{j=1}^{N} \mathbf{a}_{ij} \tilde{\theta}_{i,j}^T \hat{\theta}_{i,j} \\ &+ \frac{1}{4}\rho_i^2 + (3N + 1 + \|P_i\|^2)\bar{\omega}^2]. \end{aligned} \tag{31}$$

The control law is designed as

$$u_i = -\frac{z_{i,2}\hat{\mu}_i^2 \alpha_{i,2}^2}{\phi_i(y_i)(|z_{i,2}\hat{\mu}_i \alpha_{i,2}| + \varrho_i)} \tag{32}$$

where the adaptive law $\hat{\mu}_i$ is updated by

$$\dot{\hat{\mu}}_i = \tau_i \gamma_i z_{i,2} \alpha_{i,2} - \tau_i \gamma_i \sigma_{i,3} \hat{\mu}_i. \tag{33}$$

By considering the inequality $0 \leq \frac{xy}{x+y} < y, \forall x \geq 0, y > 0$, it can be obtained that

$$\begin{aligned}z_{i,2}\phi_i(y_i)\Psi_{i,1}(t)u_i \leq& -\frac{\lambda_i z_{i,2}^2 \hat{\mu}_i^2 \alpha_{i,2}^2}{|z_{i,2}\hat{\mu}_i\alpha_{i,2}| + \varrho_i} \\ \leq& -\frac{\lambda_i |z_{i,2}\hat{\mu}_i\alpha_{i,2}|(|z_{i,2}\hat{\mu}_i\alpha_{i,2}| + \varrho_i)}{|z_{i,2}\hat{\mu}_i\alpha_{i,2}| + \varrho_i} + \frac{\lambda_i |z_{i,2}\hat{\mu}_i\alpha_{i,2}|\varrho_i}{|z_{i,2}\hat{\mu}_i\alpha_{i,2}| + \varrho_i} \\ \leq& -\lambda_i |z_{i,2}\hat{\mu}_i\alpha_{i,2}| + \frac{\lambda_i |z_{i,2}\hat{\mu}_i\alpha_{i,2}|\varrho_i}{|z_{i,2}\hat{\mu}_i\alpha_{i,2}| + \varrho_i} \\ \leq& -\lambda_i z_{i,2}\hat{\mu}_i\alpha_{i,2} + \lambda_i \varrho_i. \end{aligned} \tag{34}$$

Substituting (33) and (34) into (31) and noting $\mu_i = 1/\lambda_i$, it follows that

$$\dot{V}_2 \leq \sum_{i=1}^{N}[-\sum_{l=1}^{2}\tau_i k_{i,l} z_{i,l}^2 - \sigma_{i,1}\tilde{\theta}_i^T\hat{\theta}_i - \sigma_{i,2}\sum_{j=1}^{N}\mathbf{a}_{ij}\tilde{\theta}_{i,j}^T\hat{\theta}_{i,j}$$
$$- \lambda_i\sigma_{i,3}\tilde{\mu}_i\hat{\mu}_i - (1+2N)\tau_i\varepsilon_i^T\varepsilon_i + 2\sum_{j=1}^{N}\mathbf{a}_{i,j}\tau_j\varepsilon_j^T\varepsilon_j$$
$$+ \frac{1}{4}\rho_i^2 + \lambda_i\varrho_i + (3N+1+\|P_i\|^2)\bar{\omega}^2]. \tag{35}$$

Remark 5. *Note that the designed control scheme is fully distributed. To reduce the information interaction between agents, an adaptive law $\hat{\theta}_{i,j}$ is introduced for each agent to estimate the uncertain parameter vector θ_j of its neighbors. In addition, an adaptive law $\hat{\mu}_i$ is introduced in the controller to compensate for the influence of the hysteresis quantizer.*

3.3. Stability Analysis

The main results are summarized as follows.

Theorem 1. *Consider the second-order nonlinear heterogeneous MAS (1), the hysteresis quantizer (2), the K-filter (3), the adaptive laws (30), (33), and the control law (32). All signals of the second-order nonlinear heterogeneous MAS are globally bounded, and the tracking error of each agent can converge to an arbitrary small set.*

Proof. Considering $-\tilde{\theta}_i^T\hat{\theta}_i \leq -\frac{1}{2}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\theta_i^T\theta_i$, $-\tilde{\theta}_{i,j}^T\hat{\theta}_{i,j} \leq -\frac{1}{2}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} + \frac{1}{2}\theta_{i,j}^T\theta_{i,j}$, $-\tilde{\mu}_i\hat{\mu}_i \leq -\frac{1}{2}\tilde{\mu}_i^2 + \frac{1}{2}\mu_i^2$, and

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{a}_{ij}\tau_j\varepsilon_j^T\varepsilon_j = \sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{a}_{ji}\tau_i\varepsilon_i^T\varepsilon_i \leq N\sum_{i=1}^{N}\tau_i\varepsilon_i^T\varepsilon_i. \tag{36}$$

Then, the inequality (35) can be rewritten as

$$\dot{V}_2 \leq \sum_{i=1}^{N}[-\sum_{l=1}^{2}\tau_i k_{i,l} z_{i,l}^2 - \frac{\sigma_{i,1}}{2}\tilde{\theta}_i^T\tilde{\theta}_i - \frac{\sigma_{i,2}}{2}\sum_{j=1}^{N}\mathbf{a}_{i,j}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j}$$
$$- \frac{\lambda_i\sigma_{i,3}}{2}\tilde{\mu}_i^2 - \tau_i\varepsilon_i^T\varepsilon_i] + C$$
$$\leq -\varsigma V_2 + C \tag{37}$$

where

$$\varsigma = \min_{1\leq i\leq N}\tau_i\{\frac{1}{\lambda_{max}(P_i)}, 2k_{i,1}, 2k_{i,2}, \sigma_{i,1}\lambda_{min}(\Gamma_i),$$
$$\sigma_{i,2}\lambda_{min}(\Gamma_{i,j}), \gamma_i\sigma_{i,3}\}$$
$$C = \frac{1}{4}\rho_i^2 + \lambda_i\varrho_i + \frac{\sigma_{i,1}}{2}\theta_i^T\theta_i + \sum_{j=1}^{N}\frac{\sigma_{i,2}\mathbf{a}_{i,j}}{2}\theta_{i,j}^T\theta_{i,j} + \frac{\lambda_i\sigma_{i,3}}{2}\mu_i^2.$$

It follows from (37) that

$$0 \leq V_2(t) \leq \frac{C}{\varsigma} + [V_2(0) - \frac{C}{\varsigma}]e^{-\varsigma t}. \tag{38}$$

As a result

$$\lim_{t\to+\infty} V_2(t) \leq \frac{C}{\varsigma}. \tag{39}$$

It follows that $\varepsilon_i, z_{i,1}, z_{i,2}, \hat{\theta}_i, \hat{\theta}_{i,j}, \hat{\mu}_i$ are bounded. From (3) and (11), together with the boundedness of y_r, it is known that y_i, ζ_i and Ξ_i are bounded. Then, Ξ_i, η_i, $\alpha_{i,1}$, $\alpha_{i,2}$, u_i, and x_i are bounded. Thus, all signals are globally bounded. In addition, it can be seen from (38) that the tracking errors can converge to an arbitrary small set by increasing ς. Since ς increases monotonically with increase in $\min_{1\leq i\leq N}\{\tau_i\}$, by adjusting τ_i, the tracking errors can converge to an arbitrary small set. This completes the proof. □

Remark 6. *By adjusting the parameter τ_i, the tracking errors of MASs can converge to an arbitrary small set without further adjusting other parameters as required in [24]. Therefore, the proposed method can be more convenient for adjusting the consensus tracking control performance of MASs in practical applications. Although the consensus tracking performance of MASs can be improved by increasing τ_i, it can be seen from the distributed control law (32) that too large τ_i may cause the high gain problem of the controller. Therefore, in practical applications, the selection of τ_i should not be too large.*

4. An Illustrative Example

Consider an MAS containing four agents, where the dynamics of each agent are as follows:

$$\ddot{y}_i + \Omega\dot{y}_i + My_i^3 + ly_i = \mathbf{Q}_i(u_i), \quad i = 1,2,3,4, \tag{40}$$

where y_i is the course angular velocity; u_i is the control input; Ω, M and l are unknown constant parameters.

By defining $x_{i,1} = y_i$, $x_{i,2} = \dot{y}_i + \Omega y_i$, $\theta_i = [\Omega, M, l]^T$, $\phi_i(y_i) = 1$, $f_{i,1}(y_i) = \begin{bmatrix} -y_i & 0 & 0 \end{bmatrix}$, and $f_{i,2}(y_i) = \begin{bmatrix} 0 & -y_i^3 & -y_i \end{bmatrix}$. Then, the system (40) can be rewritten as follows

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(y_i)\theta_i + \omega_{i,1} \\ \dot{x}_{i,2} = \phi_i(y_i)\mathbf{Q}_i(u_i) + f_{i,2}(y_i)\theta_i + \omega_{i,2} \end{cases} \tag{41}$$

In the simulation, the communication topology of the MAS is shown in Figure 2. The parameters of the MAS are selected as $[\Omega_1, \Omega_2, \Omega_3, \Omega_4] = [0.21, 0.2, 0.23, 0.21]$, $[M_1, M_2, M_3, M_4] = [0.08, 0.1, 0.12, 0.15]$, $[l_1, l_2, l_3, l_4] = [0.28, 0.3, 0.35, 0.25]$, and the desired trajectory $y_r(t)$ is generated by $y_r = \sin(t)\cos(2t)$. The time-varying disturbances are set as $\omega_i(t) = [0, 0.1i\sin(it)]^T$.

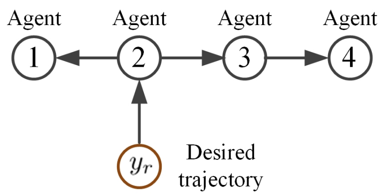

Figure 2. Communication topology graph $\mathcal{G}$.

The initial state of each agent is set to $y_i(0) = 0.1i$, and all other initial conditions are zero. The parameters of the hysteresis quantizer (2) are chosen as $\rho_i = 0.2$ and $\epsilon_i = 0.6$. The design parameters are chosen as $\tau_i = 6$, $g_{i,1} = 4$, $g_{i,2} = 4$, $k_{i,1} = k_{i,1} = 2$, $\Gamma_i = I_2$, $\Gamma_{i,j} = I_2$, $\gamma_{i,1} = 1$, $\gamma_{i,2} = 1$, $\sigma_{i,1} = 0.1$, $\sigma_{i,2} = 0.1$, $\sigma_{i,3} = 0.1$, and $\varrho_i = 0.1$.

Applying the proposed distributed control method, the output and tracking errors of each agent are shown in Figure 3. The quantized control inputs of the MAS are shown in Figure 4. It can be seen that the distributed consensus control of second-order nonlinear heterogeneous MASs with input quantization has been implemented. In addition, the proposed distributed control method is robust to unknown disturbances.

Next, we demonstrate through simulation that the tracking error can be reduced by adjusting τ_i. In the simulation, τ_i is adjusted to 10, while other parameters remain unchanged. Then, the output and tracking errors of each agent are shown in Figure 5. By comparing Figures 3 and 5, the conclusions in Remark 6 are verified. In addition, the quantized control inputs of the MAS are shown in Figure 6. As can be seen from

Figure 6, increasing τ_i will not have a significant impact on the quantization control input, but may cause the gain of the controller to become larger at the initial moment.

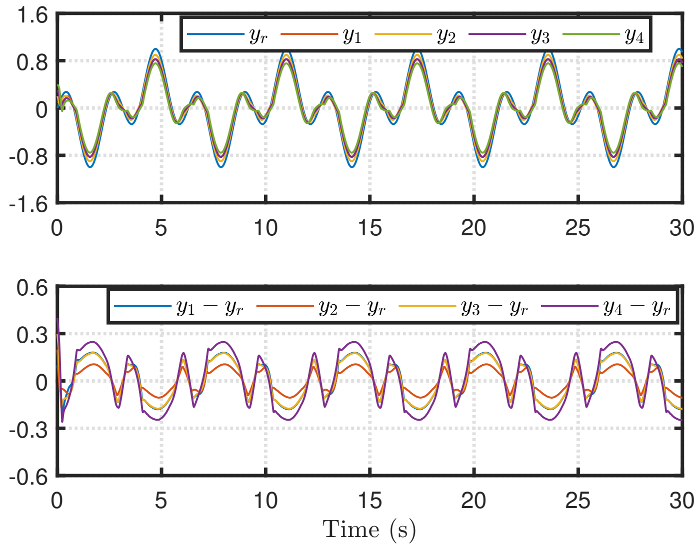

Figure 3. Agent outputs and tracking errors.

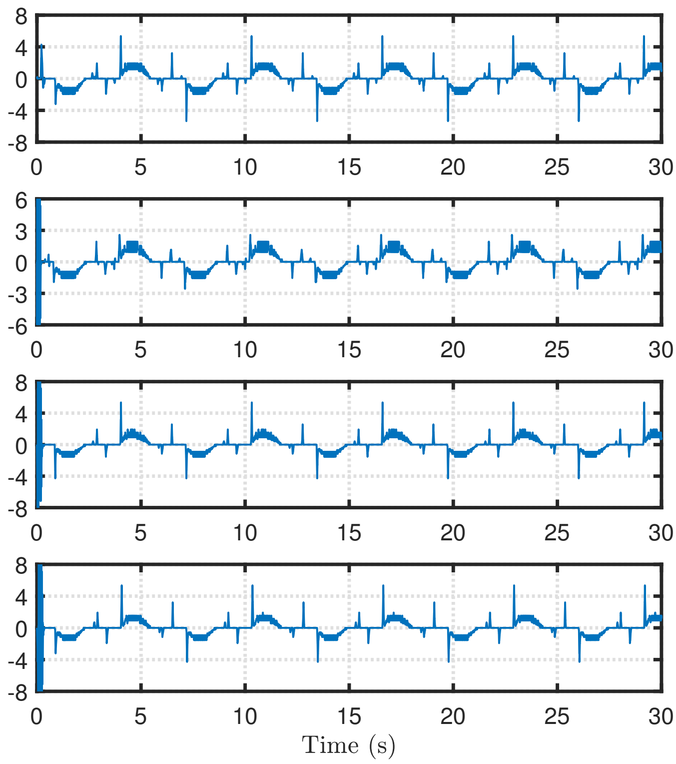

Figure 4. Quantified control inputs.

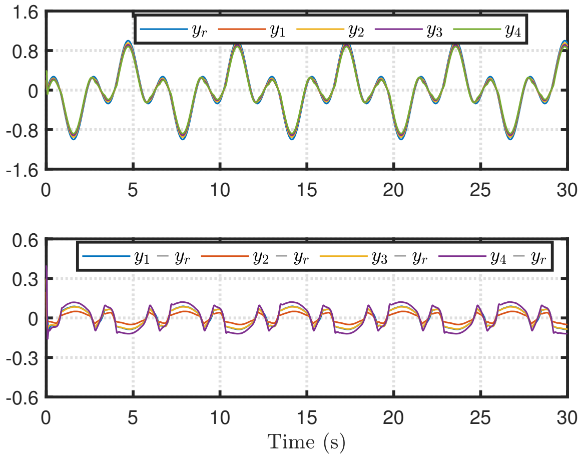

Figure 5. Agent outputs and tracking errors.

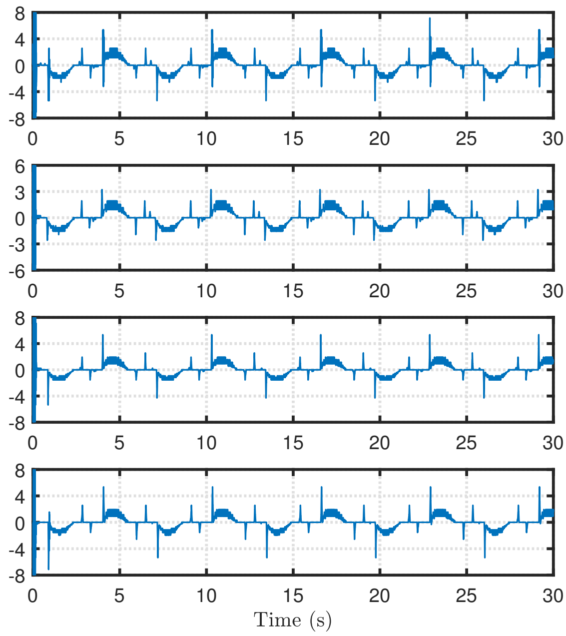

Figure 6. Quantified control inputs ($\tau_i = 10$).

5. Conclusions

In this paper, the distributed consensus tracking control problem has been addressed for second-order nonlinear heterogeneous MASs with input quantization. A distributed output feedback control scheme based on a K-filter has been proposed. Different from the results in the existing literature, an additional design parameter is introduced into the proposed controller design. By adjusting this parameter, the tracking errors of MASs can converge to an arbitrarily small residual set. A practical example verifies the effectiveness of the proposed scheme.

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