axioms

MDPI

Article

Comprehensive Criteria for the Extrema in Entropy Production Rate for Heat Transfer in the Linear Region of Extended Thermodynamics Framework

George D. Verros

Department of Chemistry, Aristotle University of Thessaloniki, P.O. Box 454 Plagiari, 57500 Epanomi, Greece; gdverros@yahoo.gr

Received: 18 September 2020; Accepted: 5 October 2020; Published: 8 October 2020



Abstract: In this work comprehensive criteria for detecting the extrema in entropy production rate for heat transfer by conduction in a uniform body under a constant volume in the linear region of Extended Thermodynamics Framework are developed. These criteria are based on calculating the time derivative of entropy production rate with the aid of well-established engineering principles, such as the local heat transfer coefficients. By using these coefficients, the temperature gradient is replaced by the difference of this quantity. It is believed that the result of this work could be used to further elucidate irreversible processes.

Keywords: entropy production; non equilibrium thermodynamics; criteria

1. Introduction

Non equilibrium thermodynamics is a rapidly growing branch of chemical physics with many applications in science and engineering, from power engineering to environmental sciences, from chaos to complex systems, and from life sciences to nanosciences. The fundamentals of non-equilibrium thermodynamics are given in several text books [1–5] and reviews [6–8]. One of the most fundamental aspects presented in these works is the calculation of entropy production rate which is strongly related to irreversible processes such as heat transfer or diffusion.

In 1912, Ehrenfest introduced the fundamental question, in relation to the non-equilibrium stationary states, on the existence of a function that achieves its extreme value as does entropy for stationary states in equilibrium thermodynamics. Today, it is evident in the literature [1–4] that entropy production rate in non-equilibrium thermodynamics plays the same role as entropy does for stationary states in equilibrium thermodynamics. Therefore, it is crucial to establish criteria for detecting the extrema (minimum or maximum) of entropy production rate. This has been the subject of intensive research in recent years as is reviewed by many workers in the field.

More specifically, Onsager [9–12], by assuming linear phenomenological relations between fluxes ($\mathbf{J_i}$) and forces ($\mathbf{X_i}$) (Linearity Axiom: $\mathbf{X_i} = \mathbf{\Sigma}\, R_{ij}\mathbf{J_i}$) based on statistical mechanics arguments, proved that the R_{ij} coefficients are symmetric (Onsager Reciprocal Relations, ORR). Moreover, Onsager introduced the least dissipation principle. This principle reduces to maximization of entropy production at fixed thermodynamic forces.

Alternatively, the Prigogine theorem [5,6] states, for systems in the linear regime (both the linear phenomenological relations and ORR hold true) that total internal entropy production reaches a minimum value at non equilibrium stationary states.

The Ziegler principle [13,14] states if the thermodynamic forces ($\mathbf{X_i}$) are preset, then true thermodynamic fluxes ($\mathbf{J_i}$) satisfying the entropy production rate equation ($\sigma = \mathbf{\Sigma}\, \mathbf{J_i}\, \mathbf{X_i}$) give the maximum value of the entropy production rate ($\sigma(\mathbf{J})$).

Many authors [15–22] have developed different approaches, but criticism [23–26] has been made of each of them. Please note that criteria for detecting the minimum or maximum [27] or alternative approaches [28] are available as a possible answer to the Ehrenfest question.

On the other hand, Extended Thermodynamics is a powerful tool for investigating physicochemical processes outside the region of local equilibrium. The aim of this work is not only to establish simple criteria for entropy production rate extrema in the linear region of Extended Thermodynamics Framework, but also to introduce in the area the well-established engineering concept of heat transfer coefficients.

2. Theoretical Part and Results

Irreversible thermodynamics close to equilibrium is based on three independent axioms above and beyond those of equilibrium Thermodynamics [1–6]:

(1) The equilibrium thermodynamic relations apply to systems that are not in equilibrium, provided that the gradients are not too large (quasi-equilibrium axiom).

(2) All the fluxes ($\mathbf{J_i}$) in the system may be written as linear relations involving all the thermodynamic forces, $\mathbf{X}_i$ (linearity axiom, $\mathbf{X}_i = \sum_{j=1}^{K} R_{ij}\mathbf{J_j}$; i = 1, 2, ..., K).

(3) In the absence of magnetic fields and assuming linearly independent fluxes or thermodynamic forces the matrix of coefficients in the flux–force relations are symmetrical. This axiom is known as the Onsager Reciprocal Relations (ORR): $R_{ij} = R_{ji}$.

The starting point of this work is the entropy (s) per unit mass balance in a local form [1–6]:

$$\rho\frac{ds}{\mathrm{d}t} + \nabla\cdot\left(\frac{\mathbf{J_q} - \sum_{i=1}^{K}\mathbf{J_i}\mu_i}{T}\right) = \sigma;\ \sigma = \mathbf{XJ} \tag{1}$$

where ρ is the mass density, symbol σ stands for the entropy production rate per unit volume, T is the absolute temperature, $\mathbf{J_q}$ represents the heat flux, t is time, μ_i stands for the chemical potential of i-th substance, $\mathbf{J_i}$ is the i-th substance molar flux defined relative to the center of molar mass and K is the total number of substances participating in the diffusion.

In the Extended Thermodynamics framework, the axiom of local equilibrium is abandoned. The entropy production term per unit volume (σ) for heat transfer by conduction in a uniform body under constant volume in the extended thermodynamics area is written as [5,6,23]:

$$\sigma = \left(-\frac{\nabla T}{T^2} - a\frac{\partial \mathbf{J_q}}{\partial t}\right)\mathbf{J_q} \tag{2}$$

The Cattaneo equation is directly derived from the above equation by further applying the linearity axiom [5,6,23]:

$$\mathbf{J_q} = L_{qq}\left(-a\frac{\partial \mathbf{J_q}}{\partial t} - \frac{\nabla T}{T^2}\right);\ \tau\frac{\partial \mathbf{J_q}}{\partial t} = -\left(\mathbf{J_q} + k\nabla T\right); k = L_{qq}/T^2; \tau = L_{qq}a; a = \tau/L_{qq} = \tau/kT^2 \tag{3}$$

All the symbols are explained in the nomenclature section. The relaxation time τ for heat transfer by conduction in a uniform body under constant volume is a positive constant [5,6,23]. A detailed review of the Cattaneo equation is given elsewhere [5,6,23].

In this work, the heat flux in the extended thermodynamics area is written in terms of the local heat transfer coefficient close to equilibrium ($\mathbf{h_{loc}}$) and the residual local heat transfer coefficient (h_{res}) is written as:

$$\mathbf{J_q} = h_{\mathrm{res}}\mathbf{h_{loc}}(T - T_0) \tag{4}$$

The residual local heat transfer coefficient may be parameterized as a positive quantity. The local heat transfer coefficient ($\mathbf{h_{loc}}$) close to equilibrium is a vector defined as:

$$-k\frac{\partial T}{\partial x_j} = h_{loc,j}(T - T_0)\ ;\ j\ =\ 1,\ 2,\ 3 \tag{5}$$

If $h_{\text{res}} = 1$, then $\tau = 0$ and the Fourier law is directly obtained from the Cattenao equation. The reference temperature T_0 is defined in such a way that $\mathbf{h_{loc}}h_{\text{res}}(T - T_0) > 0$ for positively defined heat flux.

In this way the temperature gradient is replaced by the temperature difference. This is not a new idea; the origin of this idea could be found in many textbooks [1,29] as the definition of the local heat transfer coefficients close to equilibrium ($h_{loc,j}$). These coefficients in the most general case involving an industrial process are functions not only of the process conditions, but also of the reference temperature T_0. They are calculated in terms of dimensionless groups such as the Nusselt number by using experimental correlations of other dimensionless groups such as the Prandtl number [1,29]. The main advantage of using these coefficients is that one could use these coefficients at different scales based on the similarity principle [1] without resorting to analytical or numerical solutions to calculate the heat flux.

Moreover, by assuming that the solution to the Cattaneo equation could be written as $T - T_0 = C(t)\prod_{j=1}^{3} C_{1j}\left(x_j\right)$ and by replacing this solution to Equation (5), one could directly show that the local heat transfer coefficient close to equilibrium ($\boldsymbol{h_{loc}}$) is independent of time. Furthermore, by introducing the heat transfer coefficients into the Cattaneo equation and by further taking into account that the local heat transfer coefficient close to equilibrium is independent of time, it can be directly shown that the following equation holds true:

$$\begin{gathered}\tau\frac{\mathrm{d}h_{\text{res}}(T-T_0)}{\mathrm{d}t} = (1 - h_{\text{res}})(T - T_0)\ \text{or}\ \tau\frac{\mathrm{d}h_{\text{res}}C(t)}{\mathrm{d}t} = (1 - h_{\text{res}})C(t) \\ \text{or}\ \frac{\mathrm{d}h_{\text{res}}}{\mathrm{d}t} = -h_{\text{res}}\frac{\mathrm{d}\ln C(t)}{\mathrm{d}t} + (1 - h_{\text{res}})/\tau\end{gathered} \tag{6}$$

The above equation shows that the residual local heat transfer coefficient (h_{res}) is independent of position and it is function of time.

By neglecting viscous dissipation and by assuming an absence of external forces acting on the system, the time derivative of temperature is calculated as [1–4]:

$$\left(\frac{\partial T}{\partial t}\right)_V = -\frac{1}{\rho c_{\text{v}}}\nabla\cdot\mathbf{J_q} = -\frac{1}{\rho c_{\text{v}}}\nabla\cdot\mathbf{h_{loc}}h_{\text{res}}(T - T_0) = -\frac{h_{\text{res}}}{\rho c_{\text{v}}}\nabla\cdot\mathbf{h_{loc}}(T - T_0) = \frac{kh_{\text{res}}}{\rho c_{\text{v}}}\nabla^2 T \tag{7}$$

where c_{v} represents the specific heat capacity per unit mass under a constant volume.

The above equation in terms of the solution $T - T_0 = C(t)\prod_{j=1}^{3} C_{1j}(x_j)$ is written as:

$$\begin{gathered}\left(\frac{\partial C(t)\prod_{j=1}^{3} C_{1j}\left(x_j\right)}{\partial t}\right)_V = -\frac{h_{\text{res}}C(t)}{\rho c_{\text{v}}}\nabla\cdot\mathbf{h_{loc}}\prod_{j=1}^{3} C_{1j}(x_j) \\ \text{or}\left(\frac{\partial C(t)}{\partial t}\right)_V = -\lambda h_{\text{res}}C(t)\ ;\ \frac{\mathrm{d}\ln(C(t))}{\mathrm{d}t} = -\lambda h_{\text{res}}\end{gathered} \tag{8}$$

where λ is a constant having the inverse of time as units. Since $h_{\text{res}} > 0$ the condition for a bounded solution to Equation (8) requires $\lambda > 0$. Based on the above result Equation (6) is re-written as:

$$\frac{\mathrm{d}h_{\text{res}}}{\mathrm{d}t} = \lambda h_{\text{res}}^2 + (1 - h_{\text{res}})/\tau \tag{9}$$

The above equation is a constitutive equation for the residual local heat transfer coefficient.

The entropy production P_{in} under constant volume by using the definitions of heat transfer coefficients and the Cattaneo equation is given as:

$$P_{\text{in}} = \int_V \sigma \mathrm{d}V = \int_V \mathbf{J_q} \cdot \left(\nabla 1/T - a\partial \mathbf{J_q}/\partial t\right)\mathrm{d}V = \int_V \left(h_{\text{loc}}^2 h_{\text{res}}^2 (T - T_0)^2/kT^2\right)\mathrm{d}V = \\ = \int_V \left(k h_{\text{res}}^2 (\nabla T)^2/T^2\right)\mathrm{d}V \tag{10}$$

According to the above equation the entropy production per unit volume P_{in} equals to zero for uniform temperature $(\nabla T = 0)$. This result for the linear region is in accordance with the literature (p. 41, ref [23]). However, the heat flux calculated by the Cattaneo equation can be non-zero in the case of uniform temperature $(\nabla T = 0)$. Therefore, in the most general case of the nonlinear region the entropy production per unit volume is not zero [30,31], even in the case of uniform temperature $(\nabla T = 0)$.

One can directly show that if the derivative with respect to time of entropy production P_{in} inside a non-equilibrium thermodynamic system with a constant volume is greater than zero or in other words, $\frac{\mathrm{d}P_{\text{in}}}{\mathrm{d}t} = \int_V \frac{\mathrm{d}\sigma}{\mathrm{d}t}\mathrm{d}V > 0$, then entropy production monotonically increases with time. Therefore, the achieved value for entropy production is at a maximum and vice versa for $\mathrm{d}P_{\text{in}}/\mathrm{dt} < 0$ [1–4].

The derivative of entropy production P_{in} with respect to time for heat transfer under constant volume is given as:

$$\frac{\mathrm{d}P_{\text{in}}}{\mathrm{d}t} = \int_V \frac{\mathrm{d}\sigma}{\mathrm{d}t}\mathrm{d}V = \int_V \frac{\mathrm{d}\left(\mathbf{J_q} \cdot \left(\nabla 1/T - a\partial \mathbf{J_q}/\partial t\right)\right)}{\mathrm{d}t}\mathrm{d}V = \int_V \frac{\mathrm{d}\left(\mathbf{J_q} \cdot (\nabla 1/T - (1 - h_{\text{res}})\nabla 1/T)\right)}{\mathrm{d}t}\mathrm{d}V \tag{11}$$

In the derivation of the above equation the definitions of heat transfer coefficients as well as the Cattaneo equation were used.

By using the identity $\nabla \cdot (\mathbf{A}f) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot (\nabla f)$ and by taking into account that the residual heat transfer coefficient is independent of position the above integral is written as:

$$\frac{\mathrm{d}P_{\text{in}}}{\mathrm{d}t} = \int_V \frac{\mathrm{d}\left(\mathbf{J_q} \cdot \nabla(h_{\text{res}}/T)\right)}{\mathrm{d}t}\mathrm{d}V = \int_V \frac{\mathrm{d}\left(\mathbf{J_q} h_{\text{res}} \cdot \nabla(1/T)\right)}{\mathrm{d}t}\mathrm{d}V = \\ = \int_V \left\{\frac{\mathrm{d}\left(\nabla \cdot \mathbf{J_q} h_{\text{res}}/T\right)}{\mathrm{d}t} - \frac{\mathrm{d}\left(h_{\text{res}}/T\nabla \cdot \mathbf{J_q}\right)}{\mathrm{d}t}\right\}\mathrm{d}V \tag{12}$$

By using Gauss's theorem, the first term of the above integral can be written as surface integral. Since boundary conditions are assumed to be time-independent on the boundary [4,5], this surface integral vanishes, and the above equation is written as:

$$\frac{\mathrm{d}P_{\text{in}}}{\mathrm{d}t} = -\int_V \frac{d\left(h_{\text{res}}/T\nabla \cdot \mathbf{J_q}\right)}{\mathrm{d}t}\mathrm{d}V \tag{13}$$

Moreover, the following equation is directly obtained by further using the above equation and Equation (7) as well as the chain rule for partial derivatives:

$$-\frac{\mathrm{d}\left(h_{\text{res}}/T\nabla \cdot \mathbf{J_q}\right)}{\mathrm{d}t} = -\rho c_{\text{v}}\left(\frac{\partial T}{\partial t}\right)^2\left(h_{\text{res}}/T^2\right) + \rho c_{\text{v}}(\mathrm{d}h_{\text{res}}/\mathrm{d}t)/T\left(\frac{\partial T}{\partial t}\right) \tag{14}$$

If $h_{\text{res}} = 1$ then $\tau = 0$ then the Fourier law is valid and the above equation is reduced to the well-known criterion for entropy production extrema for heat transfer close to equilibrium found in

many textbooks [1–4]. The second term on the right-hand side of the above equation by further using Equation (9) is analyzed as:

$$(\mathrm{d}h_{\mathrm{res}}/\mathrm{d}t)/T\left(\frac{\partial T}{\partial t}\right) = \frac{1}{T}\left(\lambda h_{\mathrm{res}}^2 + (1-h_{\mathrm{res}})/\tau\right)\frac{\partial T}{\partial t} \tag{15}$$

Finally, the criterion for minimum entropy production is formulated by combining Equations (14) and (15) with Equation (11) as:

$$\begin{aligned}\frac{\mathrm{d}P_{\mathrm{in}}}{\mathrm{d}t} &= \int_V \left\{-\frac{\rho c_{\mathrm{v}}}{T^2}h_{\mathrm{res}}\left(\frac{\partial T}{\partial t}\right)^2 + \frac{\rho c_{\mathrm{v}}}{T}\left(\lambda h_{\mathrm{res}}^2 + (1-h_{\mathrm{res}})/\tau\right)\frac{\partial T}{\partial t}\right\}\mathrm{d}V \\ &= h_{\mathrm{res}}\int_V \left\{-\frac{\rho c_{\mathrm{v}}}{T^2}\left(\frac{\partial T}{\partial t}\right)^2\right\}\mathrm{d}V + \rho c_{\mathrm{v}}\left(\lambda h_{\mathrm{res}}^2 + (1-h_{\mathrm{res}})/\tau\right)\int_V \left\{\frac{1}{T}\left(\frac{\partial T}{\partial t}\right)\right\}\mathrm{d}V\end{aligned} \tag{16}$$

In the case that $\left(\lambda h_{\mathrm{res}}^2 + (1-h_{\mathrm{res}})/\tau\right)\frac{\partial T}{\partial t} < 0$ for both time and space coordinates then $\frac{\mathrm{d}P_{\mathrm{in}}}{\mathrm{d}t} < 0$; one could anticipate that a minimum is reached for the entropy production of the whole system. In the opposite case $\frac{\mathrm{d}P_{\mathrm{in}}}{\mathrm{d}t} > 0$ a maximum in entropy production could be attained if $\left(\lambda h_{res}^2 + (1-h_{\mathrm{res}})/\tau\right)\frac{\partial T}{\partial t} > 0$ and $h_{\mathrm{res}}\int_V \left\{-\frac{\rho c_{\mathrm{v}}}{T^2}\left(\frac{\partial T}{\partial t}\right)^2\right\}\mathrm{d}V + \rho c_{\mathrm{v}}\left(\lambda h_{\mathrm{res}}^2 + (1-h_{\mathrm{res}})/\tau\right)\int_V \left\{\frac{1}{T}\left(\frac{\partial T}{\partial t}\right)\right\}\mathrm{d}V > 0$

3. Conclusions

Comprehensive criteria for the extremum value of entropy production in the linear region of Extended Thermodynamics Framework were developed in this work. The above criteria are based on the calculation of the entropy production rate derivative with respect to time by assuming that heat flux could be approximated by the local heat transfer coefficients. These criteria were applied for the heat transfer by conduction in a uniform body under constant volume in the Extended Thermodynamics Framework. For this purpose, the residual heat transfer coefficients are introduced in the field and the temperature gradient is replaced by the difference in temperature. It is believed that the results of this work might be used to further elucidate irreversible processes.

- Criteria for the extremum value of entropy production for heat transfer in the linear region of Extended Thermodynamics Framework were developed
- Introduction of local heat transfer coefficients in the field
- The advantages of using local heat transfer coefficients are (a) calculation of heat flux without resorting to analytical or numerical solutions and (b) temperature gradient is replaced by temperature difference.

Funding: This research received no external funding.

Acknowledgments: The author is thankful to K. Somerscales for her help in the preparation of the manuscript.

Conflicts of Interest: The author declares no conflict of interest.

Nomenclature

a	thermodynamic parameter
c_v	specific heat capacity for constant volume
$\boldsymbol{h_{loc}}$	close to equilibrium local heat transfer coefficient
h_{res}	residual local heat transfer coefficient
J	flux
k	thermal conductivity
L	phenomenological coefficients relating fluxes with thermodynamic driving forces
P_{in}	entropy production rate inside the whole system
s	entropy

T	absolute temperature
T_0	reference absolute temperature
t	time
V	volume
x_j	space coordinate
X	thermodynamic driving force

Greek Letters

λ	constant
μ_i	chemical potential of *i*-th substance
ρ	mass density
σ	entropy production rate per unit volume
τ	relaxation time

References

1. Bird, R.B.; Stewart, W.E.; Lightfoot, E.N. *Transport Phenomena*, 2nd ed.; Wiley: New York, NY, USA, 2002; pp. 764–798.
2. de Groot, S.R.; Mazur, P. *Non-Equilibrium Thermodynamics*, 2nd ed.; Dover Publications: New York, NY, USA, 1984; pp. 11–82.
3. Kuiken, G.D.C. *Thermodynamics of Irreversible Processes—Applications to Diffusion and Rheology.*, 1st ed.; Wiley: New York, NY, USA, 1994; pp. 1–135.
4. Demirel, Y. *Nonequilibrium Thermodynamics, Transport and Rate Processes in Physical, Chemical and Biological Systems*, 2nd ed.; Elsevier: Amsterdam, The Netherlands, 2007; pp. 1–144.
5. Kondepudi, D.; Prigogine, I. *Modern Thermodynamics: From Heat Engines to Dissipative Structures*, 2nd ed.; Wiley: New York, NY, USA, 2015; pp. 333–455.
6. Kondepudi, D.P.; Prigogine, I. *Thermodynamics Nonequilibrium, in Encyclopedia of Applied Physics*, 2nd ed.; Trigg, G.L., Ed.; Wiley-VCH: Weinheim, Germany, 2003.
7. Demirel, Y.A.; Sandler, S.I. Nonequilibrium Thermodynamics in Engineering and Science. *J. Phys. Chem. B* **2004**, *108*, 31–43. [CrossRef]
8. Michaelides, E.E. Transport Properties of Nanofluids. A Critical Review. *J. Non Equil. Thermodynamics* **2013**, *38*, 1–79. [CrossRef]
9. Onsager, L. Reciprocal Relations in Irreversible Processes I. *Phys. Rev.* **1931**, *37*, 405–426. [CrossRef]
10. Onsager, L. Reciprocal Relations in Irreversible Processes II. *Phys. Rev.* **1931**, *37*, 2265–2279. [CrossRef]
11. Onsager, L.; Fuoss, R.M. Irreversible Processes in Electrolytes: Diffusion, Conductance, and Viscous Flow in Arbitrary Mixtures of Strong Electrolytes. *J. Phys. Chem.* **1932**, *36*, 2659–2778. [CrossRef]
12. Onsager, L. Theories and Problems of Liquid Diffusion. *Ann. N. Y. Acad. Sci.* **1945**, *46*, 241–265. [CrossRef] [PubMed]
13. Ziegler, H. *An Introduction to Thermomechanics*; North-Holland: Amsterdam, The Netherlands, 1983.
14. Martyushev, L.M.; Seleznev, V.D. Maximum Entropy Production Principle in Physics, Chemistry and Biology. *Phys. Rep.* **2006**, *426*, 1–46. [CrossRef]
15. Truesdell, C.A. *Rational Thermodynamics*; Springer: New York, NY, USA, 1984.
16. Muschik, W.; Ehrentraut, H.; Papenfuss, C. Mesoscopic Continuum Mechanics. In *Geometry, Continua and Microstructure*; Collection Travaux en Cours 60; Maugin, G., Ed.; Herrman: Paris, France, 1999.
17. Öttinger, H.C. *Beyond Equilibrium Thermodynamics*; Wiley: Hoboken, NJ, USA, 2005.
18. Sieniutycz, S.; Farkas, H. *Variational and Extremum Principles in Macroscopic Systems*; Elsevier: Oxford, UK, 2005.
19. Bejan, A. *Entropy Generation Minimization*; CRC Press: Boca Raton, FL, USA, 1995.
20. Bejan, A. *Shape and Structure, from Engineering to Nature*; Cambridge University Press: Cambridge, UK, 2000.
21. Bejan, A.; Lorente, S. The Constructal Law of Design and Evolution in Nature. *Phil. Trans. R. Soc. B* **2010**, *365*, 1335–1347. [CrossRef] [PubMed]
22. Andresen, B. Finite-time thermodynamics. In *Finite-Time Thermodynamics and Thermoeconomics, Advances in Thermodynamics*; Sieniutycz, S., Salamon, P., Eds.; Taylor and Francis: New York, NY, USA, 1990.
23. Jou, D.; Lebon, G.; Casas-Vazquez, J. *Extended Irreversible Thermodynamics*, 3rd ed.; Springer: Berlin, Germany, 2001; pp. 39–70.

24. Grinstein, G.; Linsker, R. Comments on a Derivation and Application of the "Maximum Entropy Production" Principle. *J. Phys. A Math. Theory* **2007**, *40*, 9717–9720. [CrossRef]
25. Jaynes, E.T. The Minimum Entropy Production Principle. *Ann. Rev. Phys. Chem.* **1980**, *31*, 579–601. [CrossRef]
26. Martyuchev, L.M. Entropy and Entropy Production: Old Misconceptions and New Breakthroughs. *Entropy* **2013**, *15*, 1152–1170. [CrossRef]
27. Di Vita, A. Maximum or Minimum Entropy Production? How to Select a Necessary Criterion of Stability for a Dissipative Fluid or Plasma. *Phys. Rev. E Stat. Nonlinear Soft Matter Phys.* **2010**, *81*, 041137. [CrossRef] [PubMed]
28. Lucia, U.; Grazzini, G. The Second Law Today: Using Maximum-Minimum Entropy Generation. *Entropy* **2015**, *17*, 7786–7797. [CrossRef]
29. Slattery, J.C. *Advanced Transport Phenomena*, 1st ed.; Cambridge University Press: Cambridge, UK, 1999; pp. 283–366.
30. Li, S.-N.; Cao, B.-Y. Generalized variational principles for heat conduction models based on Laplace transform. *Int. J. Heat Mass Transf.* **2016**, *103*, 1176–1180. [CrossRef]
31. Li, S.-N.; Cao, B.-Y. On Entropic Framework Based on Standard and Fractional Phonon Boltzmann Transport Equations. *Entropy* **2019**, *21*, 204. [CrossRef]



© 2020 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).