axioms

MDPI

Editorial

Probability, Statistics and Estimation

Yolanda M. Gómez [1,*] and Inmaculada Barranco-Chamorro [2,*]

[1] Departamento de Estadística, Facultad de Ciencias, Universidad del Bío-Bío, Concepción 4081112, Chile
[2] Departamento de Estadística e Investigación Operativa, Facultad de Matemáticas, Universidad de Sevilla, 41012 Sevilla, Spain
* Correspondence: ygomez@ubiobio.cl (Y.M.G.); chamorro@us.es (I.B.-C.)

1. Introduction

This Special Issue of *Axioms*, entitled "Probability, Statistics and Estimation", comprises 15 papers that were accepted for publication after a rigorous review process. In these papers, new theoretical and computational methodologies related to current topics of interest in statistics and probability are given. Specifically, new results on the theory of distributions are presented, including different sampling plans and applications of interest in reliability; estimations of parameters from a classical and Bayesian point of view; and results on logistic regression models, analyses of correlations, time series, and processes.

2. Overview of Papers Published in This Special Issue

In Contribution 1, a preliminary test estimator for the common mean, μ, of independent and non-homogeneous normal samples is proposed. It is assumed that prior information regarding μ is available. Thus, a hypothesis test whose null hypothesis is to propose a reference value for μ can be carried out by using an initial sample. The result of this pretest is used to introduce a better estimator of μ. The proposed results are compared to other estimators in the literature. Applications to real meta-analyses are included.

The study in Contribution 2 considers the logistic regression transfer learning problem supported by differential privacy. In cases where transferable sources are known, the authors propose a two-step transfer learning algorithm. For scenarios with unknown transferable sources, a non-algorithmic, cross-validation-based transferable source detection method is introduced to mitigate the adverse effects of non-informative sources. The effectiveness of the proposed algorithm is validated through simulations and experiments with real-world data.

In Contribution 3, the Slash-Exponential-Fréchet distribution is introduced as an extended version of the Fréchet distribution. Its stochastic representation, probability distribution function, moments, and other relevant features are obtained. The authors highlight that this new model exhibits a lighter right tail than the Fréchet model, and is more flexible as for skewness and kurtosis. Results on Maximum Likelihood estimators are provided. A simulation study and two real-world applications are included.

In Contribution 4, the authors propose the conditions under which distribution functions of randomly stopped minimum, maximum, minimum of sums, and maximum of sums belong to the class of generalized subexponential distributions. In this work, the primary random variables are supposed to be independent and real-valued, but not necessarily identically distributed. The counting random variable describing the stopping moment of random structures is supposed to be non-negative, integer-valued, and not degenerate at zero. In addition, it is supposed that counting random variable, and the sequence of the

check for updates

Received: 4 January 2025
Accepted: 21 January 2025
Published: 23 January 2025

Citation: Gómez, Y.M., Barranco-Chamorro, I. Probability, Statistics and Estimation. *Axioms* **2025**, *14*, 82. https://doi.org/10.3390/axioms14020082

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

primary random variables are independent. At the end of this paper, it is shown that these results can be applied to the construction of new generalized subexponential distributions.

In Contribution 5, the decay of correlations for random dynamical systems is studied. Specifically, the uniformly C^2 piecewise expanding maps defined on the unit interval satisfying $\lambda(T'_w) = inf|T'_w| > 2$ are considered. A coupling method for analyzing the coupling time of observables with bounded variation is proposed. The results provided in this paper can be applied to random dynamical systems used to model phenomena in economics, climatology, physics, and biology, among other scientific disciplines.

The statistical properties of the m-Asymptotically Almost Negatively Associated (m-AANA) sequences are studied in Contribution 6, mainly those related to the gradual change point model. The effectiveness of the estimator proposed in limited samples is checked through several simulation experiments, as well as an actual hydrological application.

In Contribution 7, the linear processes generated by dependent sequences under a sub-linear expectation are considered. Using the Beveridge–Nelson decomposition of linear processes and inequalities, the moderate deviation principle for linear processes produced by an m-dependent sequence is established. The upper bounds of the moderate deviation principle for linear processes produced by negatively dependent sequences via different methods from m-dependent sequences are also proposed.

The Kavya–Manoharan Generalized Inverse Kumaraswamy (KM-GIKw) distribution is introduced in Contribution 8. This is an improved version of the generalized inverse Kumaraswamy distribution with three parameters. The shapes of the probability density and hazard rate functions, quantiles, moments, information, and entropy measures are discussed. An acceptance sampling plan is created when the life test is truncated at a predefined time. As for the inference, the parameters are estimated using the Maximum Likelihood, Bayesian, and maximum product of spacings methods. Simulation studies and applications to real datasets are also included.

In Contribution 9, the Kavya–Manoharan Power Topp–Leone (KMPTL) distribution is presented. This is a new two-parameter distribution, which is an extension of the Power Topp–Leone (PTL) model. Mathematical and statistical features, such as the quantile function, moments, generating function, and incomplete moments are obtained. Some measures of entropy are also investigated. To estimate the parameters of the KMPTL distribution, both the Maximum Likelihood and Bayesian methods are used under simple random sample (SRS) and ranked set sampling (RSS). The new model is satisfactorily compared to other competitive statistical distributions.

Applications of the extended Rayleigh distribution (ERD) in reliability studies are investigated in Contribution 10. Specifically, the Progressive First-Failure Censoring (PFFC) sampling method is considered, i.e., the units of life testing experiments are separated into groups consisting of k units, and only the first failure in each group is registered. Maximum Likelihood estimation method is studied, along with Bayesian techniques for the estimation of the reliability function under different loss functions and priors. The Markov Chain Monte Carlo (MCMC) and Monte Carlo approach are used in Bayesian computations.

In Contribution 11, the authors consider a $n \times p$ matrix $\boldsymbol{X_n}$ where the n rows are strictly stationary α-mixing random vectors, and each of the p columns is an independent and identically distributed random vector. It is assumed that $p = p_n$ goes to infinity as $n \to \infty$, satisfying $0 < c_1 \leq \frac{p_n}{n^\tau} \leq c_2 < \infty$ where $\tau > 0$ and $c_2 \geq c_1 > 0$. A logarithmic law for $L_n = \max_{1 \leq i < j \leq p_n} |\rho_{ij}|$ is obtained by using the Chen–Stein Poisson approximation method, where ρ_{ij} denotes the sample correlation coefficient between the i-th column and the j-th column of $\boldsymbol{X_n}$.

In Contribution 12, a nonlinear regression model with exponentiated skew-elliptical distributed errors is proposed. This model can be fitted to datasets with high levels of

asymmetry and kurtosis. Maximum Likelihood estimation procedures in finite samples are discussed, and the information matrix is deduced. A diagnosis of the influence for the nonlinear model is carried out. Different perturbation schemes are proposed to assess the sensitivity of their results, along with a residual analysis of the deviance components. Simulation studies and an application to a real dataset are included.

In Contribution 13, the inverse-power Muth power series model is introduced. This distribution is a composition of the inverse-power Muth and the class of power series distributions. The use of the Bell distribution in this context is emphasized for the first time in the literature. Probability density, survival, and hazard functions are studied, as well as their moments. Using the stochastic representation of the model, the Maximum Likelihood estimators are implemented by the use of the expectation-maximization algorithm, while standard errors are calculated using Oakes' method. Monte Carlo simulation studies are conducted to show the performance of the Maximum Likelihood estimators in finite samples. Applications to two real datasets are included, where their proposal is compared with some other models based on power series compositions.

In Contribution 14, a novel, simple, and fully flexible modified gamma model is introduced. The new distribution provides various shapes of densities, including symmetric, asymmetric, unimodal, and reversed J-shaped, as well as a bathtub-shaped failure rate, which are suitable for modeling the lifespan of patients with an increasing risk of death. The basic and dynamic properties of the model are examined. Four methods of estimation are discussed, along with simulations and real applications.

A new three-parameter Type-II Lehmann Fréchet distribution (LFD-TII), as a reparameterized version of the Kumaraswamy–Fréchet distribution, is introduced in Contribution 15. Using progressive Type-II censoring, different estimation methods of the LFD-TII parameters and its lifetime functions are considered. In a frequentist setup, both the Maximum Likelihood and product of the spacing estimators for the parameters are obtained, including asymptotic confidence intervals for unknown parametric functions. In the Bayesian paradigm via likelihood and spacing functions, and using independent gamma conjugate priors, the Bayes estimators of the unknown parameters are obtained under the squared-error and general-entropy loss functions. Since the proposed posterior distributions cannot be explicitly expressed, by combining two Markov chain Monte Carlo techniques, namely the Gibbs and Metropolis–Hastings algorithms, the Bayes point/interval estimates are approximated. To examine the performance of the proposed results, extensive simulation experiments are conducted. Moreover, based on several criteria, an optimum censoring plan is proposed.

Author Contributions: Writing, review and editing: Y.M.G and I.B.-C. Both Special Issue Editors have read and agreed to the published version of this editorial. All authors have read and agreed to the published version of the manuscript.

Funding: The research of I. Barranco-Chamorro was supported by IOAP of the University of Seville, Spain.

Acknowledgments: Both Special Issue Editors deeply thank Axioms (ISSN 2075-1680) and the Section Managing Editor, for all the support given to the Special Issue "Probability, Statistics and Estimation". Also, we are really grateful to all authors for submitting high-quality papers to this Special Issue.

Conflicts of Interest: The authors declare no conflicts of interest.

List of Contributions

1. Mphekgwana, P.M.; Kifle, Y.G.; Marange, C.S. Pretest Estimation for the Common Mean of Several Normal Distributions: In Meta-Analysis Context. *Axioms* **2024**, *13*, 648. https://doi.org/10.3390/axioms13090648.
2. Hou, Y.; Song, Y. Transfer Learning for Logistic Regression with Differential Privacy. *Axioms* **2024**, *13*, 517. https://doi.org/10.3390/axioms13080517.
3. Gómez, Y.M.; Barranco-Chamorro, I.; Castillo, J.S.; Gómez, H.W. An Extension of the Fréchet Distribution and Applications. *Axioms* **2024**, *13*, 253. https://doi.org/10.3390/axioms13040253.
4. Karasevičienė, J.; Šiaulys, J. Randomly Stopped Minimum, Maximum, Minimum of Sums and Maximum of Sums with Generalized Subexponential Distributions. *Axioms* **2024**, *13*, 85. https://doi.org/10.3390/axioms13020085.
5. Abdelkader, M. Analysis of Correlation Bounds for Uniformly Expanding Maps on [0, 1]. *Axioms* **2023**, *12*, 1072. https://doi.org/10.3390/axioms12121072.
6. Xu, T.; Wei, Y. A Least Squares Estimator for Gradual Change-Point in Time Series with m-Asymptotically Almost Negatively Associated Errors. *Axioms* **2023**, *12*, 894. https://doi.org/10.3390/axioms12090894.
7. Sun, P.; Wang, D.; Ding, X.; Tan, X.; Zhang, Y. Moderate Deviation Principle for Linear Processes Generated by Dependent Sequences under Sub-Linear Expectation. *Axioms* **2023**, *12*, 781. https://doi.org/10.3390/axioms12080781.
8. Alsadat, N.; Hassan, A.S.; Elgarhy, M.; Chesneau, C.; El-Saeed, A.R. Sampling Plan for the Kavya–Manoharan Generalized Inverted Kumaraswamy Distribution with Statistical Inference and Applications. *Axioms* **2023**, *12*, 739. https://doi.org/10.3390/axioms12080739.
9. Alotaibi, N.; Al-Moisheer, A.S.; Elbatal, I.; Shrahili, M.; Elgarhy, M.; Almetwally, E.M. Bayesian and Non-Bayesian Estimation for a New Extension of Power Topp–Leone Distribution under Ranked Set Sampling with Applications. *Axioms* **2023**, *12*, 722. https://doi.org/10.3390/axioms12080722.
10. Abu-Moussa, M.H.; Alsadat, N.; Sharawy, A. On Estimation of Reliability Functions for the Extended Rayleigh Distribution under Progressive First-Failure Censoring Model. *Axioms* **2023**, *12*, 680. https://doi.org/10.3390/axioms12070680.
11. Zhao, H.; Zhang, Y. A Strong Limit Theorem of the Largest Entries of a Sample Correlation Matrices under a Strong Mixing Assumption. *Axioms* **2023**, *12*, 657. https://doi.org/10.3390/axioms12070657.
12. Martínez-Flórez, G.; Gómez, Y.M.; Venegas, O. An Exponentiated Skew-Elliptic Nonlinear Extension to the Log–Linear Birnbaum–Saunders Model with Diagnostic and Residual Analysis. *Axioms* **2023**, *12*, 624. https://doi.org/10.3390/axioms12070624.
13. Barrios-Blanco, L.; Gallardo, D.I.; Gómez, H.J.; Bourguignon, M. A Compound Class of Inverse-Power Muth and Power Series Distributions. *Axioms* **2023**, *12*, 383. https://doi.org/10.3390/axioms12040383.
14. Alshehri, M.A.; Kayid, M. A Modified Gamma Model: Properties, Estimation, and Applications. *Axioms* **2023**, *12*, 262. https://doi.org/10.3390/axioms12030262.
15. Elshahhat, A.; Bhattacharya, R.; Mohammed, H.S. Survival Analysis of Type-II Lehmann Fréchet Parameters via Progressive Type-II Censoring with Applications. *Axioms* **2022**, *11*, 700. https://doi.org/10.3390/axioms11120700.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.