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Intuitionistic Hesitant Fuzzy Rough Aggregation Operator-Based EDAS Method and Its Application to Multi-Criteria Decision-Making Problems

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Abstract: The fundamental notions of the intuitionistic hesitant fuzzy set (IHFS) and rough set (RS) are general mathematical tools that may easily manage imprecise and uncertain information. The EDAS (Evaluation based on Distance from Average Solution) approach has an important role in decision-making (DM) problems, particularly in multi-attribute group decision-making (MAGDM) scenarios, where there are many conflicting criteria. This paper aims to introduce the IHFR-EDAS approach, which utilizes the IHF rough averaging aggregation operator. The aggregation operator is crucial for aggregating intuitionistic hesitant fuzzy numbers into a cohesive component. Additionally, we introduce the concepts of the IHF rough weighted averaging (IHFRWA) operator. For the proposed operator, a new accuracy function (AF) and score function (SF) are established. Subsequently, the suggested approach is used to show the IHFR-EDAS model for MAGDM and its stepwise procedure. In conclusion, a numerical example of the constructed model is demonstrated, and a general comparison between the investigated models and the current methods demonstrates that the investigated models are more feasible and efficient than the present methods.

Keywords: intuitionistic hesitant fuzzy set; rough set; weighted averaging operator; EDAS approach; MAGDM

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1. Introduction

The complexity of the socioeconomic environment increases the complexity of DM challenges. Therefore, under these scenarios, it is difficult for a single decision-maker to achieve a factual and appropriate result. In the actual world, combining the expertise of multiple professional experts is crucial to using DM models to produce more feasible and appropriate options. Therefore, in order to generate accurate and fulfilling DM results, MADM offers a highly possible and regulated approach to construct and estimate many competing criteria in every DM domain. A new approach to MAGDM called EDAS uses the alternatives' deviations from their average scores. The EDAS approach evaluates the options by utilizing the average solution (AS). Keshavarz Ghorabae et al. [1] developed it.

The approach takes into account two metrics for the appraisal: PDAS (Positive Distance from Average Solution) and NDAS (Negative Distance from Average Solution). When our requirements are in conflict, this strategy comes in very helpful. Finding the distance from the positive ideal solution (PIS) and negative ideal solution (NIS) gives the optimum alternative in compromise MADM approaches like TOPSIS and VIKOR: the distance between the PIS and NIS is used to determine which alternative is more desirable. In these MADM approaches, the optimal option has the highest distance between the PIS and NIS. The PDAS and NDAS are the two distances from the AS from which the ideal alternative is chosen in the EDAS technique. According to Keshavarz Ghorabae et al. [1], a superior option possesses lower NDAS and greater PDAS values. Zadeh [2] looked at the widely accepted idea of fuzzy sets (FSs), which effectively handle this kind of imprecise information, to solve that problem. A membership grade (MG), having a membership value between 0 and 1, is used to represent fuzzy set information. Since its conception, this idea has been significantly expanded in a variety of ways, encompassing theoretical and practical aspects. After that, Atanassov [3] explored the common notion of an IFS, which is defined by two functions: non-membership grade (non-MG) and MG. The sum of the MG and non-MG function values for IFSs is constrained to the interval $[0, 1]$. Since their establishment, IFSs have drawn a lot of attention from academics who have taken a diverse approach to studying its hybrid structure. By introducing the notion of an HFS, Torra [4] developed the idea of an FS. HFS theory allows for a variety of different values to be possible in situations where it is unclear whether to include an element in a set. Ref. [5] proposes the development of IHFSs, which are the result of combining IFSs with HFSs. This idea is useful for dealing with unclear situations, when certain values can represent MGs and non-MGs for a particular element at the same time. An IHFS is a sophisticated extension of IFS theory, developed to better represent real-world uncertainty in situations where decision-makers are uncertain about multiple possibilities for membership. Traditional IFSs accommodate MG and non-MG, but they assume a single, deterministic value for each. In contrast, IHFSs account for multiple potential membership values and incorporate them into the decision framework. IHFSs capture ambiguity more effectively than traditional fuzzy sets and IFSs by accommodating hesitation in membership definitions, which is especially useful in complex MADM problems. Pawlak [6] proposed the theory of rough sets (RSs), in which equivalence relations are essential for determining a subset's lower (LA) and upper (UA) approximation. Rough set theory is a granular approach to data analysis, particularly suited for handling imprecise, inconsistent, or incomplete data. By dividing data into lower and upper approximations, RSs provide a mechanism to isolate certainties (lower approximations) and possibilities (upper approximations), leaving ambiguous cases in the boundary region. It provides a framework for approximating sets when only partial or imprecise information is available. RSs are particularly effective in situations where the available data cannot be precisely described due to the absence of complete knowledge. Further exploration of these theories can be found in Yao [7], Xu and Xia [8], Wei [9], Qian et al. [10], and Pawlak [11]. Pawlak's RS has been extended in a number of studies, including the FRS and RFS proposed by Dubois and Prade [12] and contributions from other experts in the field [13–17]. Important advantages and insights that are not entirely achievable in classical settings can be gained via RS analysis in a fuzzy setting. Lower and upper bounds are used by classical rough sets to approach sets, this helps them to deal with uncertainty and insufficient knowledge. But they do not clearly distinguish between MGs and non-MGs. On the other hand, partial membership is supported by FSs, which more accurately represents many real-world issues which are unclear and ambiguous. An FS can be estimated in a pristine approximating domain; an FRS expands on the idea of an RS. This happens in situations when the decision attribute values are

fuzzy but the conditional attribute values are exact. Finding a set's LA and UA is the goal of FRs. It is important when the universe of the FS becomes rough, either by converting the equivalency relation into an appropriate fuzzy connection or by using an equivalency relation. Chinram et al. [18] introduced the IFRS as a result of the combination of FRs and IFSs. Furthermore, with the HFRS, RS methods may now be applied in an HF environment, with an emphasis on defining the LA and UA in that context. IHFRST, which expands upon RST, also incorporates the idea of IHFSs. To effectively manage ambiguity, hesitation, and uncertainty in tasks such as knowledge representation, data processing, and DM, IHFRST incorporates the ideas of HFRSs, IHFSs, and RSs. In the meantime, because people lack experience and their thinking is still unclear, professionals may waver between several assessment values. This is the case with regard to how to create a scientific and effective choice model to reduce decision chances and clearly comprehend decision outcomes, as well as how to appropriately and reasonably express the evaluation values of objects after taking expert understanding and opinions into consideration. By merging the IFRS and HFRS, we create an IHFRS and go over how it may be used for MADM problems in HF environments. The IHFRS is an improvement over traditional models that capture the ambiguity of real-world difficulties by using the LA and UA values with MGs and non-MGs from the unit interval. The IHFRS expands the information available for problems involving DM. As far as we are aware, and based on the analysis above, no report of the application of the EDAS technique with the hybrid studies of IHFSs and RSs employing IHF averaging in an IHF environment has been made thus far. The IHFR-EDAS model redefines the capabilities of decision-making frameworks by integrating the nuanced representation of IHFSs and the granular analysis of RSs into the robust EDAS structure. This innovative approach delivers precise, adaptable, and transparent decisions, addressing the limitations of traditional fuzzy- and rough set-based methods. By combining IHFSs and RSs within the EDAS framework, IHFR-EDAS offers a powerful, nuanced tool for addressing the multifaceted challenges in modern decision-making scenarios. The IHFR-EDAS model combines the strengths of IHFSs and RSs to enhance decision-making capabilities. IHFSs capture nuanced hesitancy and imprecision in stakeholder judgments and RSs refine this by categorizing uncertain data into precise approximations, reducing noise and ambiguity. The EDAS model evaluates alternatives based on positive and negative distance from the average, but integrating IHFSs and RSs allows for a more accurate reflection of uncertainty and hesitancy in input data. The efficiency of the developed IHF rough EDAS (IHFR-EDAS) technique has been shown using MAGDM, which is based on IHFR averaging operators. This drives the current study's investigation of averaging operators, using the EDAS method for MAGDM to examine an aggregation operator like IHFRWA.

The remainder of the manuscript is arranged as follows: The basic concepts that are applied in the subsequent sections are shown in Section 2. In Sections 3 and 4, IFRs, HFRs, and their important properties are defined. We introduce the notion of IHFRs with new SFs and AFs for IHFR values (IHFRV) in Section 5. Based on IHFRV, some basic operations for the suggested concept are shown. Then, the concept of average aggregation operators, such as IHFRWA, is presented, and Section 6 then goes into detail about their desirable properties. We introduce the IHFR-EDAS model for MADM based on the notions addressed in Section 7, and its sequential algorithmic is illustrated, applying the suggested approach. Section 8 concludes with a numerical illustration using the EDAS approach for choosing the best small hydro-power plant (SHPP) among Pakistan's several geographical areas. Furthermore, a general comparison of the examined models with some current approaches, comparing to the methods used in Section 9, illustrates that the model under investigation is more practical and efficient. The paper concludes finally in Section 10.

2. Preliminaries

Here, we cover the concepts of FSs, IFSs, HFSs, IHFSs, and RSs, along with their essential functions, operations, and relationships. These concepts will relate our studies to the next sections.

Definition 1 ([19]). Let Y be a universal set, then an FS within Y is given as

$$F = \langle y, h_F(y) | y \in Y \rangle, \quad (1)$$

where $h_F : Y \rightarrow [0, 1]$, and for each $y \in Y$, $h_F(y)$ is said to be the MG of y in Y .

Definition 2 ([20]). Let Y be a universal set, then an intuitionistic fuzzy set I within Y is given as

$$I = \{ \langle y, h_I(y), h'_I(y) \rangle | y \in Y \}, \quad (2)$$

where $h_I(y)$ and $h'_I(y)$ are functions from Y to $[0, 1]$, $\exists 0 \leq h_I(y) \leq 1, 0 \leq h'_I(y) \leq 1$ and $0 \leq h_I(y) + h'_I(y) \leq 1, \forall y \in Y$.

Definition 3. Let Y be a universal set, then a hesitant fuzzy set H within Y is given as

$$H = \{ \langle y, h_H(y) \rangle | y \in Y \}, \quad (3)$$

where $h_H(y)$ represents the collection of some values from $[0, 1]$, i.e., the maximum number of the MG of the element $y \in Y$ to H .

Definition 4 ([21]). In the definition of IHFSs, I_H in a universal set Y can be defined as follows:

$$I_H = \{ \langle z, \Theta_{I_H}(z), \Xi_{I_H}(z) | z \in Y \rangle \}, \quad (4)$$

where $\Theta_{I_H}(z)$ and $\Xi_{I_H}(z)$ are functions from Y to $[0, 1]$, expressing each element $z \in Y$, the probability of MGs and non-MGs in I_H , correspondingly, and for each element $z \in Y, \forall u_{I_H}(z) \in \Theta_{I_H}(z), \exists v_{I_H}(z) \in \Xi_{I_H}(z) \Rightarrow 0 \leq u_{I_H}(z) + v_{I_H}(z) \leq 1$, and $\forall v_{I_H}(z) \in \Xi_{I_H}(z), \exists u_{I_H}(z) \in \Theta_{I_H}(z) \Rightarrow 0 \leq u_{I_H}(z) + v_{I_H}(z) \leq 1$. IHFN is represented by $\tilde{u} = \langle \Theta_{\tilde{u}}, \Xi_{\tilde{u}} \rangle$ and occurs when Y has only one element $\langle y, \Theta_{I_H}(z), \Xi_{I_H}(z) \rangle$.

Some important operations of IHFSs are presently articulated as follows:

Let $a = \langle \Phi_a, \Psi_a \rangle$, $a_2 = \langle \Phi_{a_2}, \Psi_{a_2} \rangle$, and $a_3 = \langle \Phi_{a_3}, \Psi_{a_3} \rangle$ be IHFNs, then

- (i) $\zeta a = \langle \cup_{ha \in \Phi_a} \{1 - (1 - ha)^\zeta\}, \cup_{\hat{h}a \in \Psi_a} \{\hat{h}a\}^\zeta \rangle, \zeta \succ 0$;
- (ii) $a^\zeta = \langle \cup_{ha \in \Phi_a} \{ha\}^\zeta, \cup_{\hat{h}a \in \Psi_a} \{1 - (1 - \hat{h}a)^\zeta\} \rangle, \zeta \succ 0$;
- (iii) $a_2 \dot{+} a_3 = \langle \cup_{ha_2 \in \Phi_{a_2}, \hat{h}a_3 \in \Phi_{a_3}} \{ha_2 \dot{+} \hat{h}a_3 - ha_2.\hat{h}a_3\}, \cup_{\hat{h}a_2 \in \Psi_{a_2}, \hat{h}a_3 \in \Psi_{a_3}} \{\hat{h}a_2.\hat{h}a_3\} \rangle$;
- (iv) $a_2 \times a_3 = \langle \cup_{ha_2 \in \Phi_{a_2}, \hat{h}a_3 \in \Phi_{a_3}} \{ha_2.\hat{h}a_3\}, \cup_{\hat{h}a_2 \in \Psi_{a_2}, \hat{h}a_3 \in \Psi_{a_3}} \{\hat{h}a_2 \dot{+} \hat{h}a_3 - \hat{h}a_2.\hat{h}a_3\} \rangle$.

Furthermore, suppose that $\tilde{X} \neq \emptyset$ finite universe. For each $\tilde{N}, \tilde{M} \in IHFS(\tilde{X})$. The following is the specification for the $\tilde{N} \sqcup \tilde{M}$ and $\tilde{N} \sqcap \tilde{M}$ of $\tilde{N}$ and $\tilde{M}$:

$$\tilde{N} \sqcup \tilde{M} = \{ \langle \chi, \Phi_{\tilde{N}}(\chi) \sqcup \Phi_{\tilde{M}}(\chi), \Psi_{\tilde{N}}(\chi) \sqcap \Psi_{\tilde{M}}(\chi) \rangle | \chi \in \tilde{X} \}$$

and

$$\tilde{N} \sqcap \tilde{M} = \{ \langle \chi, \Phi_{\tilde{N}}(\chi) \sqcap \Phi_{\tilde{M}}(\chi), \Psi_{\tilde{N}}(\chi) \sqcup \Psi_{\tilde{M}}(\chi) \rangle | \chi \in \tilde{X} \},$$

where $\Phi_{\tilde{N}}(\chi) \sqcup \Phi_{\tilde{M}}(\chi) = \{ \nu \in (\Phi_{\tilde{N}}(\chi) \sqcup \Phi_{\tilde{M}}(\chi)) | \nu \geq \max(\Phi_{\tilde{N}}^-(\chi), \Phi_{\tilde{M}}^-(\chi)) \}$,
 $\Phi_{\tilde{N}}(\chi) \sqcap \Phi_{\tilde{M}}(\chi) = \{ \nu \in (\Phi_{\tilde{N}}(\chi) \sqcup \Phi_{\tilde{M}}(\chi)) | \nu \leq \min(\Phi_{\tilde{N}}^+(\chi), \Phi_{\tilde{M}}^+(\chi)) \}$ and

$\Psi_{\tilde{N}}(\chi) \sqcap \Psi_{\tilde{M}}(\chi) = \{ \nu \in (\Psi_{\tilde{N}}(\chi) \sqcup \Psi_{\tilde{M}}(\chi)) | \nu \leq \min(\Psi_{\tilde{N}}^+(\chi), \Psi_{\tilde{M}}^+(\chi)) \}$,
 $\Psi_{\tilde{N}}(\chi) \sqcup \Psi_{\tilde{M}}(\chi) = \{ \nu \in (\Psi_{\tilde{N}}(\chi) \sqcup \Psi_{\tilde{M}}(\chi)) | \nu \geq \max(\Psi_{\tilde{N}}^-(\chi), \Psi_{\tilde{M}}^-(\chi)) \}$.

Definition 5 ([20]). Let Y be a universal set and $\xi \in Y \times Y$ the arbitrary relation on Y , for any $I^* \subseteq Y$, the LA and UA of I^* with respect to the approximation space (AS) (Y, ξ) , defined as

$$\underline{\xi}(I^*) = \{ q^* \in Y \xi^*(q^*) \subseteq I^* \},$$

$$\bar{\xi}(I^*) = \{ q^* \in Y \xi^*(q^*) \cap I^* \neq \emptyset \}.$$

Therefore, $(\underline{\xi}(I^*), \bar{\xi}(I^*))$ is called a rough set and $\underline{\xi}(I^*), \bar{\xi}(I^*) : q^*(Y) \mapsto q^*(Y)$ are the LA and UA operators.

3. Construction of Intuitionistic Fuzzy Rough Set

This section draws a definition of IFRSs and their important properties.

Definition 6 ([21,22]). Let Y be a universal set and $R \in IFS(Y \times Y)$ be the IF relation. If $\sigma \subseteq IFS(Y)$, the UA and LA of σ with respect to IFAS (Y, R) are two IFSs, and represented by $\bar{R}(\sigma), \underline{R}(\sigma)$ and are as follows:

$$\bar{R}(\sigma) = \{ \langle f, \Theta_{I_{\bar{R}(\sigma)}}(f), \Xi_{I_{\bar{R}(\sigma)}}(f) \rangle / f \in Y \},$$

$$\underline{R}(\sigma) = \{ \langle f, \Theta_{I_{\underline{R}(\sigma)}}(f), \Xi_{I_{\underline{R}(\sigma)}}(f) \rangle / f \in Y \},$$

where

$$\Theta_{I_{\bar{R}(\sigma)}}(f) = \bigvee_{I \in U} [\Theta_{I_{\bar{R}}}(f, I) \vee \Theta_{I_{\sigma}}(I)]; \Xi_{I_{\bar{R}(\sigma)}}(\sigma) = \bigwedge_{I \in U} [\Xi_{I_{\bar{R}}}(f, I) \wedge \Xi_{I_{\sigma}}(I)]$$

and

$$\Theta_{I_{\underline{R}(\sigma)}}(f) = \bigwedge_{I \in U} [\Theta_{I_{\bar{R}}}(f, I) \wedge \Theta_{I_{\sigma}}(I)]; \Xi_{I_{\underline{R}(\sigma)}}(\sigma) = \bigvee_{I \in U} [\Xi_{I_{\bar{R}}}(f, I) \vee \Theta_{I_{\sigma}}(I)]$$

such that

$$0 \leq (\max(\Theta_{I_{\bar{R}(\sigma)}}(f))) + (\min(\Xi_{I_{\bar{R}(\sigma)}}(f))) \text{ and } 0 \leq (\max(\Theta_{I_{\underline{R}(\sigma)}}(f))) + (\min(\Xi_{I_{\underline{R}(\sigma)}}(f)))$$

Hence, $\bar{R}(\sigma) : \underline{R}(\sigma) \mapsto IFS(Y)$ are UA and LA operators, and $(\bar{R}(\sigma), \underline{R}(\sigma))$ are IFSs. The pair

$$\bar{R}(\sigma) = (\underline{R}(\sigma), \bar{R}(\sigma)) = \{ \langle \sigma, (\Theta_{I_{\bar{R}(\sigma)}}(\sigma), \Xi_{I_{\bar{R}(\sigma)}}(\sigma)), (\Theta_{I_{\underline{R}(\sigma)}}(\xi), \Xi_{I_{\underline{R}(\sigma)}}(\sigma)) \rangle | \sigma \in Y \},$$

is called the IFRS. For simplicity,

$$\bar{R}(\sigma) = \{ \langle \sigma, (\Theta_{I_{\bar{R}(\sigma)}}(\sigma), \Xi_{I_{\bar{R}(\sigma)}}(\sigma)), (\Theta_{I_{\underline{R}(\sigma)}}(\xi), \Xi_{I_{\underline{R}(\sigma)}}(\sigma)) \rangle | \sigma \in Y \},$$

is denoted as $\bar{R}(\sigma) = ((\underline{\Theta}, \underline{\Xi}), (\bar{\Xi}, \bar{\Theta}))$ and are called IFRS values.

Definition 7. The score function for IFRV $\mathcal{R}(\xi) = (\underline{\mathcal{R}}(\xi), \overline{\mathcal{R}}(\xi)) = ((\underline{\Theta}, \underline{\Xi}), (\overline{\Xi}, \overline{\Theta}))$ is given as

$$\widehat{S}(\mathcal{R}(\xi)) = \frac{1}{4}(2 + \underline{\Theta} + \overline{\Theta} - \underline{\Xi} - \overline{\Xi}), \widehat{S}(\mathcal{R}(\xi)) \in [0, 1].$$

Suppose $\mathcal{R}(\xi) = (\underline{\mathcal{R}}(\xi), \overline{\mathcal{R}}(\xi)) = ((\underline{\Theta}, \underline{\Xi}), (\overline{\Xi}, \overline{\Theta}))$ is an IFRV, then the accuracy function for $\mathcal{R}(\xi)$ is given as follows: $\widehat{a}(\mathcal{R}(\xi)) = \frac{1}{4}(2 + \underline{\Theta} + \overline{\Theta} + \underline{\Xi} + \overline{\Xi}), \widehat{a}(\mathcal{R}(\xi)) \in [0, 1]$.

4. Construction of Hesitant Fuzzy Rough Sets

Constructive and deductive approaches to HFRSs were covered by Yang et al. [23]. Even though the axiom technique describes the HFRS in an operator-oriented way, the constructive approach uses HF relations to approximate HFSs. We examine their constructive treatment of tentative HFRSs in the following.

Definition 8. Let Y be a universal set and $\widehat{\mathcal{R}}$ be an HF relation over $Y, \forall h \in HF(Y)$; the LA and UA of h are represented by $\underline{\widehat{\mathcal{R}}}(h)$ and $\overline{\widehat{\mathcal{R}}}(h)$, respectively, $\forall \tilde{p}, \tilde{q} \in Y$,

$$\underline{\widehat{\mathcal{R}}}(h)(\tilde{p}) = \bigwedge_{\tilde{q} \in Y} \{ \widehat{\mathcal{R}}'(\tilde{p}, \tilde{q}) \vee h(\tilde{q}) \},$$

$$\overline{\widehat{\mathcal{R}}}(h)(\tilde{p}) = \bigvee_{\tilde{q} \in Y} \{ \widehat{\mathcal{R}}(\tilde{p}, \tilde{q}) h(\tilde{q}) \}.$$

According to the HS relation $\widehat{\mathcal{R}}$, the pair $(\underline{\widehat{\mathcal{R}}}(h), \overline{\widehat{\mathcal{R}}}(h))$ is known as the HFRS of h ; both $\underline{\widehat{\mathcal{R}}}(h)$ and $\overline{\widehat{\mathcal{R}}}(h)$ are HFSs. Yang et al. [23] established the subsequent characteristics of HFR approximations. Regarding a specific universe set $Y, \widehat{\mathcal{R}}$ is an HF relation over $Y, \forall h \in HF(Y)$; we have

- (i) $\underline{\widehat{\mathcal{R}}}(h)^+(\tilde{p}) = \min\{\max\{\widehat{\mathcal{R}}^+(\tilde{p}, \tilde{q}), (h)^+(\tilde{q})\}; \tilde{q} \in Y\}$;
- (ii) $\underline{\widehat{\mathcal{R}}}(h)^-(\tilde{p}) = \min\{\max\{\widehat{\mathcal{R}}^-(\tilde{p}, \tilde{q}), (h)^-(\tilde{q})\}; \tilde{q} \in Y\}$;
- (iii) $\overline{\widehat{\mathcal{R}}}(h)^+(\tilde{p}) = \max\{\min\{\widehat{\mathcal{R}}^+(\tilde{p}, \tilde{q}), (h)^+(\tilde{q})\}; \tilde{q} \in Y\}$;
- (iv) $\overline{\widehat{\mathcal{R}}}(h)^-(\tilde{p}) = \max\{\min\{\widehat{\mathcal{R}}^-(\tilde{p}, \tilde{q}), (h)^-(\tilde{q})\}; \tilde{q} \in Y\}$.

5. Construction of Intuitionistic Hesitant Fuzzy Rough Set

In this section, we draw a definition of IHFRSs and present the counter example for the IHFRS concept.

Definition 9. Consider Y is the universal set and $\mathcal{R} \in IHFS(Y \times Y)$ is the IHF relation. The pair $(Y, \mathcal{R})$ is said to be an IHFAS. If $\xi \subseteq IHFS(Y)$, the UA and LA of ξ with respect to the IHFAS $(Y, \mathcal{R})$ are two IHFSs, which are represented by $\overline{\mathcal{R}}(\xi)$, $\underline{\mathcal{R}}(\xi)$ and defined as

$$\overline{\mathcal{R}}(\xi) = \left\{ \left\langle f, \Theta_{I_{\overline{\mathcal{R}}(\xi)}}(f), \Xi_{I_{\overline{\mathcal{R}}(\xi)}}(f) \right\rangle / f \in Y \right\},$$

$$\underline{\mathcal{R}}(\xi) = \left\{ \left\langle f, \Theta_{I_{\underline{\mathcal{R}}(\xi)}}(f), \Xi_{I_{\underline{\mathcal{R}}(\xi)}}(f) \right\rangle / f \in Y \right\},$$

where

$$\Theta_{I_{\overline{\mathcal{R}}(\xi)}}(f) = \bigvee_{I \in U} [\Theta_{I_{\mathcal{R}}}(f, I) \vee \Theta_{I_{\xi}}(I)]; \Xi_{I_{\overline{\mathcal{R}}(\xi)}}(\xi) = \bigwedge_{I \in U} [\Xi_{I_{\mathcal{R}}}(f, I) \wedge \Xi_{I_{\xi}}(I)]$$

and

$$\Theta_{I_{\underline{\mathcal{R}}(\xi)}}(f) = \bigwedge_{I \in U} [\Theta_{I_{\mathcal{R}}}(f, I) \wedge \Theta_{I_{\xi}}(I)]; \Xi_{I_{\underline{\mathcal{R}}(\xi)}}(\xi) = \bigvee_{I \in U} [\Xi_{I_{\mathcal{R}}}(f, I) \vee \Theta_{I_{\xi}}(I)]$$

such that

$$0 \leq (\max(\Theta_{I_{\bar{R}(\xi)}}(f))) + (\min(\Xi_{I_{\bar{R}(\xi)}}(f))) \text{ and } 0 \leq (\max(\Theta_{I_{\underline{R}(\xi)}}(f))) + (\min(\Xi_{I_{\underline{R}(\xi)}}(f)))$$

The pair

$$\underline{R}(\xi) = (\underline{R}(\xi), \bar{R}(\xi)) = \left\{ \left\langle \xi, (\Theta_{I_{\bar{R}(\xi)}}(\xi), \Xi_{I_{\bar{R}(\xi)}}(\xi)), (\Theta_{I_{\underline{R}(\xi)}}(\xi), \Xi_{I_{\underline{R}(\xi)}}(\xi)) \right\rangle \mid \xi \in Y \right\},$$

is called an IHFRS. For simplicity,

$$\underline{R}(\xi) = \left\{ \left\langle \xi, (\Theta_{I_{\bar{R}(\xi)}}(\xi), \Xi_{I_{\bar{R}(\xi)}}(\xi)), (\Theta_{I_{\underline{R}(\xi)}}(\xi), \Xi_{I_{\underline{R}(\xi)}}(\xi)) \right\rangle \mid \xi \in Y \right\},$$

is denoted as $\underline{R}(\xi) = ((\underline{\Theta}, \underline{\Xi}), (\bar{\Xi}, \bar{\Theta}))$ and are called IHFRS values.

Definition 10. The score function for the IHFRV $\underline{R}(\xi) = (\underline{R}(\xi), \bar{R}(\xi)) = ((\underline{\Theta}, \underline{\Xi}), (\bar{\Xi}, \bar{\Theta}))$ is given as

$$\hat{S}(\underline{R}(\xi)) = \frac{1}{4} \left[2 + \left(\frac{1}{\bar{L}} \sum_{j=1}^t \underline{\Theta}_j + \bar{\Theta}_j \right) - \left(\frac{1}{\bar{L}} \sum_{j=1}^t \bar{\Xi}_j + \underline{\Xi}_j \right) \right], \hat{S}(\underline{R}(\xi)) \in [0, 1].$$

Suppose $\underline{R}(\xi) = (\underline{R}(\xi), \bar{R}(\xi)) = ((\underline{\Theta}, \underline{\Xi}), (\bar{\Xi}, \bar{\Theta}))$ is an IHFRV, then the accuracy function for $\underline{R}(\xi)$ is as given below:

$$\hat{A}(\underline{R}(\xi)) = \frac{1}{4} \left[\frac{1}{\bar{L}} \sum_{j=1}^t \underline{\Theta}_j + \bar{\Theta}_j + \frac{1}{\bar{L}} \sum_{j=1}^t \bar{\Xi}_j + \underline{\Xi}_j \right], \hat{A}(\underline{R}(\xi)) \in [0, 1]$$

For comparing two or more IHFRVs, we apply a score function. Supremacy is indicated by higher IHFRV scores, and inferiority is shown by lower IHFRV values. If the score values are equal, we apply the AF.

Proposition 1. Suppose $(Y, \underline{R})$ is the AS and $\underline{R}(\xi_1) = (\underline{R}(\xi_1), \bar{R}(\xi_1))$ and $\underline{R}(\xi_2) = (\underline{R}(\xi_2), \bar{R}(\xi_2))$ are two IHFRVs over Y . The following results are satisfied:

- (i) $\neg(\neg \underline{R}(\xi_1)) = \xi_1$, where $\neg \underline{R}(\xi_1)$ is the complement of $\underline{R}(\xi_1)$.
- (ii) $\underline{R}(\xi_1) \sqcup \underline{R}(\xi_2) = \underline{R}(\xi_2) \sqcup \underline{R}(\xi_1)$ and $\underline{R}(\xi_1) \sqcap \underline{R}(\xi_2) = \underline{R}(\xi_2) \sqcap \underline{R}(\xi_1)$.
- (iii) $\neg(\underline{R}(\xi_1) \sqcap \underline{R}(\xi_2)) = (\neg \underline{R}(\xi_1)) \sqcup (\neg \underline{R}(\xi_2))$.
- (iv) $\neg(\underline{R}(\xi_1) \sqcup \underline{R}(\xi_2)) = (\neg \underline{R}(\xi_1)) \sqcap (\neg \underline{R}(\xi_2))$.
- (v) If $\xi_1 \sqsubseteq \xi_2$, then $\underline{R}(\xi_1) \sqsubseteq \underline{R}(\xi_2)$.
- (vi) $\underline{R}(\xi_1) \sqcup \xi_2 \supseteq \underline{R}(\xi_1) \sqcup \underline{R}(\xi_2)$.
- (vii) $\underline{R}(\xi_1 \sqcap \xi_2) \subseteq \underline{R}(\xi_1) \sqcap \underline{R}(\xi_2)$.

Now, we present the counterexample for a better explanation of the above concept of IHFRSs.

Example 1. Let $Y = \{q_1^*, q_2^*, q_3^*, q_4^*\}$ be an arbitrary set and $(Y, \underline{R})$ be an IHFAS with $\underline{R} \in Y \times Y$ the IHF relation presented in Table 1. The best possible normal decision object l^* , which is an IHFS, is now presented by a decision expert and is as described below:

$$l^* = \left\{ \begin{array}{l} \langle q_1^*, \{\{0.2, 0.4, 0.6\}, \{0.1, 0.3, 0.4\}\} \rangle, \\ \langle q_2^*, \{\{0.1, 0.4, 0.5\}, \{0.2, 0.4, 0.5\}\} \rangle, \\ \langle q_3^*, \{\{0.3, 0.4, 0.5\}, \{0.2, 0.3, 0.4\}\} \rangle, \\ \langle q_4^*, \{\{0.4, 0.5, 0.6\}, \{0.1, 0.2, 0.4\}\} \rangle \end{array} \right\}$$

Table 1. IHF relation from Y to Y.

ξ	q_1^*	q_2^*	q_3^*	q_4^*
q_1^*	$\langle \{0.2, 0.3, 0.5\}, \{0.1, 0.3, 0.4\} \rangle$	$\langle \{0.1, 0.3, 0.6\}, \{0.2, 0.3, 0.4\} \rangle$	$\langle \{0.1, 0.4, 0.5\}, \{0.1, 0.3, 0.5\} \rangle$	$\langle \{0.1, 0.2, 0.6\}, \{0.1, 0.3, 0.4\} \rangle$
q_2^*	$\langle \{0.1, 0.3, 0.5\}, \{0.1, 0.2, 0.5\} \rangle$	$\langle \{0.2, 0.3, 0.6\}, \{0.1, 0.2, 0.4\} \rangle$	$\langle \{0.2, 0.4, 0.5\}, \{0.1, 0.4, 0.5\} \rangle$	$\langle \{0.1, 0.3, 0.6\}, \{0.1, 0.2, 0.4\} \rangle$
q_3^*	$\langle \{0.2, 0.3, 0.6\}, \{0.1, 0.3, 0.5\} \rangle$	$\langle \{0.1, 0.2, 0.4\}, \{0.1, 0.2, 0.3\} \rangle$	$\langle \{0.2, 0.4, 0.5\}, \{0.2, 0.3, 0.5\} \rangle$	$\langle \{0.1, 0.4, 0.6\}, \{0.1, 0.3, 0.4\} \rangle$
q_4^*	$\langle \{0.1, 0.4, 0.5\}, \{0.2, 0.3, 0.4\} \rangle$	$\langle \{0.3, 0.4, 0.5\}, \{0.2, 0.4\} \rangle$	$\langle \{0.3, 0.4, 0.5\}, \{0.3, 0.4\} \rangle$	$\langle \{0.2, 0.3, 0.5\}, \{0.1, 0.3, 0.5\} \rangle$

This allows us to use Definition 6 to obtain $\underline{R}(\xi)$ and $\bar{R}(\xi)$.

$$\begin{aligned}
 \Theta_{I_{\bar{R}(\xi)}}(q_1^*) &= \bigvee_{l \in U} [\Theta_{I_{\bar{R}}}(q_1^*, l) \vee \Theta_{I_{\xi}}(l)] \\
 &= (0.2 \vee 0.2) \vee (0.4 \vee 0.3) \vee (0.6 \vee 0.5) \vee \\
 &\quad (0.1 \vee 0.1) \vee (0.4 \vee 0.3) \vee (0.5 \vee 0.6) \\
 &\quad \vee (0.3 \vee 0.1) \vee (0.4 \vee 0.4) \vee (0.5 \vee 0.5) \vee \\
 &\quad (0.4 \vee 0.1) \vee (0.5 \vee 0.2) \vee (0.6 \vee 0.6) \\
 \Theta_{I_{\bar{R}(\xi)}}(q_1^*) &= 0.6 \\
 \Xi_{I_{\bar{R}(\xi)}}(\xi) &= \bigwedge_{l \in U} [\Xi_{I_{\bar{R}}}(q^*, l) \wedge \Xi_{I_{\xi}}(l)] \\
 I^* &= \left\{ \begin{aligned} &\langle q_1^*, \{\{0.2, 0.4, 0.6\}, \{0.1, 0.3, 0.4\}\} \rangle, \\ &\langle q_2^*, \{\{0.1, 0.4, 0.5\}, \{0.2, 0.4, 0.5\}\} \rangle, \\ &\langle q_3^*, \{\{0.3, 0.4, 0.5\}, \{0.2, 0.3, 0.4\}\} \rangle, \\ &\langle q_4^*, \{\{0.4, 0.5, 0.6\}, \{0.1, 0.2, 0.4\}\} \rangle \end{aligned} \right\} \\
 &= (0.1 \wedge 0.1) \wedge (0.3 \wedge 0.3) \wedge (0.4 \wedge 0.4) \wedge \\
 &\quad (0.2 \wedge 0.2) \wedge (0.4 \wedge 0.3) \wedge (0.5 \wedge 0.6) \wedge \\
 &\quad (0.2 \wedge 0.1) \wedge (0.3 \wedge 0.3) \wedge (0.4 \wedge 0.5) \wedge \\
 &\quad (0.1 \wedge 0.1) \wedge (0.3 \wedge 0.2) \wedge (0.4 \wedge 0.4) \\
 \Xi_{I_{\bar{R}(\xi)}}(q_1^*) &= 0.1
 \end{aligned}$$

Similarly, in the same manner, we obtained the other values:

$$\Theta_{I_{\bar{R}(\xi)}}(q_1^*) = 0.6, \Theta_{I_{\bar{R}(\xi)}}(q_2^*) = 0.6, \Theta_{I_{\bar{R}(\xi)}}(q_3^*) = 0.6, \Theta_{I_{\bar{R}(\xi)}}(q_4^*) = 0.6,$$

$$\Xi_{I_{\bar{R}(\xi)}}(q_1^*) = 0.1, \Xi_{I_{\bar{R}(\xi)}}(q_2^*) = 0.1, \Xi_{I_{\bar{R}(\xi)}}(q_3^*) = 0.1, \Xi_{I_{\bar{R}(\xi)}}(q_4^*) = 0.1$$

$$\text{and } \Theta_{I_{\bar{R}(\xi)}}(q_1^*) = 0.1, \Theta_{I_{\bar{R}(\xi)}}(q_2^*) = 0.1, \Theta_{I_{\bar{R}(\xi)}}(q_3^*) = 0.1, \Theta_{I_{\bar{R}(\xi)}}(q_4^*) = 0.1$$

$$\Xi_{I_{\bar{R}(\xi)}}(q_1^*) = 0.5, \Xi_{I_{\bar{R}(\xi)}}(q_2^*) = 0.5, \Xi_{I_{\bar{R}(\xi)}}(q_3^*) = 0.5, \Xi_{I_{\bar{R}(\xi)}}(q_4^*) = 0.5$$

Thus, the UA and LA IHFAS operators are

$$\begin{aligned}
 \bar{R}(\xi) &= \{\langle q_1^*, \{0.6, 0.1\} \rangle, \langle q_2^*, \{0.6, 0.1\} \rangle, \\
 &\quad \langle q_3^*, \{0.6, 0.1\} \rangle, \langle q_4^*, \{0.6, 0.1\} \rangle\}
 \end{aligned}$$

and

$$\begin{aligned} \underline{R}(\xi) &= \{\langle q_1^*, \{0.1, 0.5\} \rangle, \langle q_2^*, \{0.1, 0.5\} \rangle, \\ &\quad \langle q_3^*, \{0.1, 0.5\} \rangle, \langle q_4^*, \{0.1, 0.5\} \rangle\} \end{aligned}$$

Therefore,

$$\begin{aligned} R(\xi) &= (\underline{R}(\xi), \overline{R}(\xi)) \\ &= \{\langle q_1^*, \{0.6, 0.1\}, \{0.1, 0.5\} \rangle, \\ &\quad \langle q_2^*, \{0.6, 0.1\}, \{0.1, 0.5\} \rangle, \\ &\quad \langle q_3^*, \{0.6, 0.1\}, \{0.1, 0.5\} \rangle, \\ &\quad \langle q_4^*, \{0.6, 0.1\}, \{0.1, 0.5\} \rangle\}. \end{aligned}$$

6. Intuitionistic Hesitant Fuzzy Rough Averaging Aggregation Operator

The notion of IHFRAOs is covered in this portion of the manuscript. They can be obtained by incorporating the concepts of RSs and IHF averaging operators, and the section further discusses the fundamental properties of the aggregation concepts of IHFRWA operators.

Intuitionistic Hesitant Fuzzy Rough Weighted Averaging Operator

A comprehensive evaluation of IHFRWA aggregation operators and their desired properties is the main objective of this subsection.

Definition 11. Let the collection $\hat{R}(\xi_j) = \{(\underline{R}(\xi_j), \overline{R}(\xi_j)); j = 1, 2, \dots, n\}$ of IHFRVs with weighted vectors $\omega = (\omega_1, \omega_2, \dots, \omega_n) \ni \sum_{j=1}^n \omega_j = 1$ and $0 \leq \omega_j \leq 1$. IHFRWA is defined as

$$\text{IHFRWA } \{R(\xi_1), R(\xi_2), \dots, R(\xi_n)\} = \bigcup \{\bigoplus_{j=1}^n \omega_j \hat{R}(\xi_j), \bigoplus_{j=1}^n \omega_j \overline{R}(\xi_j)\}$$

Theorem 1. Suppose the collection $R(\xi_j) = \{(\underline{R}(\xi_j), \overline{R}(\xi_j)); j = 1, 2, \dots, n\}$ of IHFRVs with weighted vectors $\omega = (\omega_1, \omega_2, \dots, \omega_n) \ni \sum_{j=1}^n \omega_j = 1$ and $0 \leq \omega_j \leq 1$, then the IHFRWA operator is determined as

$$\begin{aligned} \text{IHFRWA } \{R(\xi_1), R(\xi_2), \dots, R(\xi_n)\} &= \bigcup \{\bigoplus_{j=1}^n \omega_j \hat{R}(\xi_j), \bigoplus_{j=1}^n \omega_j \overline{R}(\xi_j)\} \\ &= \bigcup \left[\left(1 - \bigoplus_{j=1}^n (1 - \underline{\Theta}_j)^{\omega_j}, \bigoplus_{j=1}^n (\underline{\Xi}_j)^{\omega_j} \right), \left(1 - \bigoplus_{j=1}^n (1 - \overline{\Theta}_j)^{\omega_j}, \bigoplus_{j=1}^n (\overline{\Xi}_j)^{\omega_j} \right) \right]. \end{aligned}$$

Proof. The needed proof is obtained by using mathematical induction. $\square$

Then, by the defined operational property's laws, we obtained

$$\begin{aligned} R(\xi_1) + R(\xi_2) &= [(\hat{R}(\xi_1) + \hat{R}(\xi_2)), (\overline{R}(\xi_1) + \overline{R}(\xi_2))] \\ &= [(\underline{\Theta}_1 + \underline{\Theta}_2 - \underline{\Theta}_1 \underline{\Theta}_2, \underline{\Xi}_1 \underline{\Xi}_2), (\overline{\Theta}_1 + \overline{\Theta}_2 - \overline{\Theta}_1 \overline{\Theta}_2, \overline{\Xi}_1 \overline{\Xi}_2)] \end{aligned}$$

and

$$q\hat{R}(\xi_1) = (\hat{R}(\xi_1), \overline{R}(\xi_1))$$

$$= [(1 - (1 - \underline{\Theta}_1)^e, \underline{\Xi}_1)^e, (1 - (1 - \overline{\Theta}_1)^e, \overline{\Xi}_1)^e]$$

Let $n = 2$, then

$$\begin{aligned} IHFRWA(\mathbb{R}(\xi_1), \mathbb{R}(\xi_2)) &= \bigcup \{ \boxplus_{i=1}^2 \omega_i \underline{\mathbb{R}}(\xi_i), \boxplus_{i=1}^2 \omega_i \overline{\mathbb{R}}(\xi_i) \} \\ &= \left[\left(1 - \bigcup_{i=1}^2 (1 - \underline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^2 (\underline{\Xi}_i)^{\omega_i} \right), \left(1 - \bigcup_{i=1}^2 (1 - \overline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^2 (\overline{\Xi}_i)^{\omega_i} \right) \right]. \end{aligned}$$

The result is valid for $n = 2$.

The result is valid when $n = l$, then

$$\begin{aligned} IHFRWA\{\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \dots, \mathbb{R}(\xi_l)\} &= \bigcup \{ \boxplus_{i=1}^l \omega_i \underline{\mathbb{R}}(\xi_i), \boxplus_{i=1}^l \omega_i \overline{\mathbb{R}}(\xi_i) \}. \\ &= \left[\left(1 - \bigcup_{i=1}^l (1 - \underline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^l (\underline{\Xi}_i)^{\omega_i} \right), \left(1 - \bigcup_{i=1}^l (1 - \overline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^l (\overline{\Xi}_i)^{\omega_i} \right) \right]. \end{aligned}$$

We then establish that the result is valid for $n = l + 1$.

$$\begin{aligned} IHFRWA\{\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \dots, \mathbb{R}(\xi_{l+1})\} &= \bigcup \{ \boxplus_{i=1}^{l+1} \omega_i \underline{\mathbb{R}}(\xi_i), \boxplus_{i=1}^{l+1} \omega_i \overline{\mathbb{R}}(\xi_i) \}. \\ &= \left[\left(1 - \bigcup_{i=1}^{l+1} (1 - \underline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^{l+1} (\underline{\Xi}_i)^{\omega_i} \right), \left(1 - \bigcup_{i=1}^{l+1} (1 - \overline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^{l+1} (\overline{\Xi}_i)^{\omega_i} \right) \right] \end{aligned}$$

For $n = K + 1$, the intended outcome is thus true. Therefore, the necessary result is true. $\forall n \geq 1$.

According to the study above, the IHFRVs are $\underline{\mathbb{R}}(\xi_i)$ and $\overline{\mathbb{R}}(\xi_i)$. Thus, by definition 9, $\boxplus_{i=1}^n \omega_i \underline{\mathbb{R}}(\xi_i)$ and $\boxplus_{i=1}^n \omega_i \overline{\mathbb{R}}(\xi_i)$ are also IHFRVs. Therefore, $IHFRWA\{\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \dots, \mathbb{R}(\xi_n)\}$ is also an IHFRV under the IHF approximation space $(Y, \mathbb{R})$.

Example 2. Assume the set $\xi \sqsubseteq Y = \left\{ \begin{array}{l} \langle q_1^*, \{0.2, 0.4, 0.6\}, \{0.1, 0.3, 0.4\} \rangle, \\ \langle q_2^*, \{0.1, 0.4, 0.5\}, \{0.2, 0.4, 0.5\} \rangle, \\ \langle q_3^*, \{0.3, 0.4, 0.5\}, \{0.2, 0.3, 0.4\} \rangle \end{array} \right\}$ with weighted vector $\omega = \left(\begin{array}{l} 0.1714327166, 0.2725266523, \\ 0.2344326885, 0.3216079427 \end{array} \right)^t$

$$IHFRWA\{\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \mathbb{R}(\xi_3), \mathbb{R}(\xi_4)\} = \bigcup \{ \boxplus_{i=1}^4 \omega_i \underline{\mathbb{R}}(\xi_i), \boxplus_{i=1}^4 \omega_i \overline{\mathbb{R}}(\xi_i) \}.$$

$$= \bigcup \left[\left(1 - \bigcup_{i=1}^4 (1 - \underline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^4 (\underline{\Xi}_i)^{\omega_i} \right), \left(1 - \bigcup_{i=1}^4 (1 - \overline{\Theta}_i)^{\omega_i}, \bigcup_{i=1}^4 (\overline{\Xi}_i)^{\omega_i} \right) \right].$$

$$= \left[\begin{array}{c} \{0.1397944614, 0.1703255451, 0.2050404220, \\ 0.2297837569, 0.2571208706, \\ 0.2882040953, 0.3103589381, 0.3348362148, \\ 0.3626677080, 0.1811890393, \\ 0.2102509145, 0.2432952515, 0.2668478943, \\ 0.2928694989, 0.3224569451, \\ 0.3023861628, 0.3271464155, 0.3552996619, \\ 0.2361715249, 0.2632819190, \\ 0.2941073558, 0.3160784579, 0.3403527330, \\ 0.3679534064, 0.3492303607, \\ 0.3723279827, 0.3985907616\}, \{0.1229462035, \\ 0.1499763195, 0.1801460208, \\ 0.1890817076, 0.2140735788, 0.2419682903, \\ 0.2283895561, 0.2521699900, \\ 0.2787125541, 0.1599303402, 0.1858206338, \\ 0.2147181208, 0.2232770021, \\ 0.2472150009, 0.2256190428, 0.2609272935, \\ 0.2837049395, 0.3091282407, \\ 0.1818396478, 0.2070547136, 0.2351985442, \\ 0.2435342069, 0.2668478943, \\ 0.2928694989, 0.2802025656, \\ 0.3023861628, 0.3271464155\} \end{array} \right]$$

Theorem 2 introduces several important characteristics of the IHFRWA operator.

Theorem 2. Let the collection $R(\zeta_l) = \{(\underline{R}(\zeta_l), \bar{R}(\zeta_l)); l = 1, 2, \dots, m\}$ of IHFRVs with weighted vectors $\omega = (\omega_1, \omega_2, \dots, \omega_m)^t \ni \sum_{l=1}^m \omega_l = 1$ and $0 \leq \omega_l \leq 1$, then some of the significant characteristics of the IHFRWA operator appear accordingly.

- (i) Idempotency: If $R(\zeta_l) = \mathfrak{N}(\zeta), \forall l = 1, 2, \dots, m$, where $\mathfrak{N}(\zeta) = (\underline{\mathfrak{N}}(\zeta), \bar{\mathfrak{N}}(\zeta)) = ((\underline{r}, \underline{t}), (\bar{r}, \bar{t}))$. Then, $IHFRWA(R(\zeta_1), R(\zeta_2), \dots, R(\zeta_l)) = \mathfrak{N}(\zeta)$.
- (ii) Boundedness: Suppose $(R(\zeta_i))^- = (\min_i \underline{R}(\zeta_i), \max_i \bar{R}(\zeta_i))$ and $(R(\zeta_i))^+ = (\max_i \underline{R}(\zeta_i), \min_i \bar{R}(\zeta_i))$. Then, $(R(\zeta_i))^- \leq IHFRWA(R(\zeta_1), R(\zeta_2), \dots, R(\zeta_l)) \leq (R(\zeta_i))^+$.
- (iii) Monotonicity: Let $\mathfrak{N}(\iota_i) = \{(\underline{\mathfrak{N}}(\iota_i), \bar{\mathfrak{N}}(\iota_i)); i = 1, 2, \dots, n\}$ be another collection of IHFRVs $\ni \underline{\mathfrak{N}}(\iota_i) \leq \underline{R}(\zeta_i)$ and $\bar{\mathfrak{N}}(\iota_i) \leq \bar{R}(\zeta_i)$. Then,

$$IHFRWA(\mathfrak{N}(\iota_1), \mathfrak{N}(\iota_2), \dots, \mathfrak{N}(\iota_n)) \leq IHFRWA(R(\zeta_1), R(\zeta_2), \dots, R(\zeta_l)).$$

- (iv) Shift invariance: Consider $\mathfrak{N}(\iota) = \{(\underline{\mathfrak{N}}(\iota), \bar{\mathfrak{N}}(\iota)) = ((\underline{r}, \underline{t}), (\bar{r}, \bar{t}))$ are the IHFRVs, then IHFRWA

$$(R(\zeta_1) + \mathfrak{N}(\iota), R(\zeta_2) + \mathfrak{N}(\iota), \dots, R(\zeta_n) + \mathfrak{N}(\iota)) = \\ IHFRWA(R(\zeta_1), R(\zeta_2), \dots, R(\zeta_n) + \mathfrak{N}(\iota)).$$

- (v) Homogeneity: For any real number $\varepsilon > 0$, then $IHFRWA(\varepsilon \underline{R}(\zeta_1), \varepsilon \bar{R}(\zeta_2), \dots, \varepsilon \bar{R}(\zeta_l)) = \varepsilon IHFRWA(R(\zeta_1), R(\zeta_2), \dots, R(\zeta_n))$.

- (vi) Commutativity: If $\tilde{R}(\zeta_i) = \{(\underline{\tilde{R}}(\zeta_i), \bar{\tilde{R}}(\zeta_i)); i = 1, 2, \dots, n\}$ is the permutation of $R(\zeta_i) = \{(\underline{R}(\zeta_i), \bar{R}(\zeta_i))$, then $IHFRWA(R(\zeta_1), R(\zeta_2), \dots, R(\zeta_l)) = IHFRWA(\tilde{R}(\zeta_1), \tilde{R}(\zeta_2), \dots, \tilde{R}(\zeta_l))$.

Proof. (i) (**Idempotency**): As, $R(\xi_i) = \aleph(\xi), \forall i = 1, 2, \dots, n$, where $\aleph(\xi) = (\aleph(\xi), \bar{\aleph}(\xi)) = ((\underline{r}, \underline{t}), (\bar{r}, \bar{t}))$.

$$\begin{aligned} IHFRWA(R(\xi_1), R(\xi_2), \dots, R(\xi_l)) &= \bigcup \{ \boxplus_{i=1}^2 \omega_i R(\xi_i), \boxplus_{i=1}^2 \omega_i \bar{R}(\xi_i) \} \\ &= \left[\left(1 - \prod_{i=1}^2 (1 - \underline{\Theta}_i)^{\omega_i}, \prod_{i=1}^2 (\underline{\Xi}_i)^{\omega_i} \right), \left(1 - \prod_{i=1}^2 (1 - \bar{\Theta}_i)^{\omega_i}, \prod_{i=1}^2 (\bar{\Xi}_i)^{\omega_i} \right) \right], \\ \forall i, R(\xi_i) &= \aleph(\xi) = (\aleph(\xi), \bar{\aleph}(\xi)) = ((\underline{r}, \underline{t}), (\bar{r}, \bar{t})). \end{aligned}$$

Therefore,

$$\begin{aligned} &= \left[\left(1 - \prod_{i=1}^2 (1 - \underline{r}_i)^{\omega_i}, \prod_{i=1}^2 (\underline{t}_i)^{\omega_i} \right), \left(1 - \prod_{i=1}^2 (1 - \bar{r}_i)^{\omega_i}, \prod_{i=1}^2 (\bar{t}_i)^{\omega_i} \right) \right] \\ &= [(1 - \underline{r}), \underline{t}], (1 - (1 - \bar{r}), \bar{t})] \\ &= (\aleph(\xi), \bar{\aleph}(\xi)) = \aleph(\xi) \end{aligned}$$

Hence,

$$IHFRWA(R(\xi_1), R(\xi_2), \dots, R(\xi_l)) = \aleph(\xi)$$

(ii) (**Boundedness**): As

$$\begin{aligned} (\hat{R}(\xi_i))^- &= [(\min_i \{\underline{\Theta}_i\}, \max_i \{\underline{\Xi}_i\}), (\min_i \{\bar{\Theta}_i\}, \max_i \{\bar{\Xi}_i\})], (\hat{R}(\xi_i))^+ \\ &= [(\max_i \{\underline{\Theta}_i\}, \min_i \{\underline{\Xi}_i\}), (\max_i \{\bar{\Theta}_i\}, \min_i \{\bar{\Xi}_i\})] \end{aligned}$$

and

$R(\xi_i) = [(\underline{\Theta}_i, \underline{\Xi}_i), (\bar{\Theta}_i, \bar{\Xi}_i)]$. To show that $(R(\xi_i))^- \leq IHFRWA(R(\xi_1), R(\xi_2), \dots, R(\xi_l)) \leq (R(\xi_i))^+$.

As $\forall i = 1, 2, \dots, n$, we obtain

$$\begin{aligned} \min_i \{\underline{\Theta}_i\} &\leq \underline{\Theta}_i \leq \max_i \{\underline{\Theta}_i\} \Leftrightarrow 1 - \max_i \{\underline{\Theta}_i\} \leq 1 - \underline{\Theta}_i \leq 1 - \min_i \{\underline{\Theta}_i\} \\ &\Leftrightarrow \prod_{i=1}^n (1 - \max_i \{\underline{\Theta}_i\})^{\omega_i} \leq \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \leq \prod_{i=1}^n (1 - \min_i \{\underline{\Theta}_i\})^{\omega_i} \\ &\Leftrightarrow (1 - \max_i \{\underline{\Theta}_i\}) \leq \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \leq (1 - \min_i \{\underline{\Theta}_i\}) \\ &\Leftrightarrow 1 - (1 - \min_i \{\underline{\Theta}_i\}) \leq 1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \leq 1 - (1 - \max_i \{\underline{\Theta}_i\}). \end{aligned}$$

Hence,

$$\min_i \{\underline{\Theta}_i\} \leq 1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \leq \max_i \{\underline{\Theta}_i\} \quad (5)$$

Next, $\forall i = 1, 2, \dots, n$, we obtain

$$\begin{aligned} \min_i \{\underline{\Xi}_i\} \leq \underline{\Xi}_i \leq \max_i \{\underline{\Xi}_i\} &\Leftrightarrow 1 - \max_i \{\underline{\Xi}_i\} \leq 1 - \underline{\Xi}_i \leq 1 - \min_i \{\underline{\Xi}_i\} \\ &\Leftrightarrow \prod_{i=1}^n (1 - \max_i \{\underline{\Xi}_i\})^{\omega_i} \leq \prod_{i=1}^n (1 - \underline{\Xi}_i)^{\omega_i} \leq \prod_{i=1}^n (1 - \min_i \{\underline{\Xi}_i\})^{\omega_i} \\ &\Leftrightarrow (1 - \max_i \{\underline{\Xi}_i\}) \leq \prod_{i=1}^n (1 - \underline{\Xi}_i)^{\omega_i} \leq (1 - \min_i \{\underline{\Xi}_i\}) \\ &\Leftrightarrow 1 - (1 - \min_i \{\underline{\Xi}_i\}) \leq 1 - \prod_{i=1}^n (1 - \underline{\Xi}_i)^{\omega_i} \leq 1 - (1 - \max_i \{\underline{\Xi}_i\}). \end{aligned}$$

Hence,

$$\min_i \{\underline{\Xi}_i\} \leq 1 - \prod_{i=1}^n (1 - \underline{\Xi}_i)^{\omega_i} \leq \max_i \{\underline{\Xi}_i\} \quad (6)$$

In the same way, we can show that

$$\min_i \{\bar{\Theta}_i\} \leq 1 - \prod_{i=1}^n (1 - \bar{\Theta}_i)^{\omega_i} \leq \max_i \{\bar{\Theta}_i\} \quad (7)$$

and

$$\min_i \{\bar{\Xi}_i\} \leq 1 - \prod_{i=1}^n (1 - \bar{\Xi}_i)^{\omega_i} \leq \max_i \{\bar{\Xi}_i\} \quad (8)$$

Therefore, from Equations (5)–(8), we obtain

$$(\mathcal{R}(\xi_i))^- \leq IHFRWA(\mathcal{R}(\xi_1), \mathcal{R}(\xi_2), \dots, \mathcal{R}(\xi_l)) \leq (\mathcal{R}(\xi_i))^+. \quad (9)$$

(iii) (Monotonicity): As $\mathcal{N}(\iota_i) = \{(\mathcal{N}(\iota_i), \bar{\mathcal{N}}(\iota_i)) = ((r_i, t_i), (\bar{r}_i, \bar{t}_i))$ and $\mathcal{R}(\xi_i) = \{(\mathcal{R}(\xi_i), \bar{\mathcal{R}}(\xi_i))$. To prove that $\mathcal{N}(\iota_i) \leq \mathcal{R}(\xi_i)$ and $\bar{\mathcal{N}}(\iota_i) \leq \bar{\mathcal{R}}(\xi_i) \forall i = 1, 2, \dots, n$. So,

$$\begin{aligned} r_i &\leq \underline{\Theta}_i \Rightarrow 1 - \underline{\Theta}_i \leq 1 - r_i \Rightarrow \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \leq \prod_{i=1}^n (1 - r_i)^{\omega_i} \\ &\Rightarrow 1 - \prod_{i=1}^n (1 - r_i)^{\omega_i} \leq 1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \end{aligned}$$

Next,

$$t_i \geq \underline{\Xi}_i \Rightarrow \prod_{i=1}^n t_i^{\omega_i} \geq \prod_{i=1}^n \underline{\Xi}_i^{\omega_i} \quad (10)$$

We can also demonstrate that

$$1 - \prod_{i=1}^n (1 - \bar{r}_i)^{\omega_i} \leq 1 - \prod_{i=1}^n (1 - \bar{\Theta}_i)^{\omega_i} \quad (11)$$

Next,

$$\prod_{i=1}^n \bar{r}^{\omega_i} \geq \prod_{i=1}^n \bar{\Xi}_i^{\omega_i} \quad (12)$$

Hence, from Equations (9)–(12), we obtain $\mathcal{N}(\iota_i) \leq \mathcal{R}(\xi_i)$ and $\bar{\mathcal{N}}(\iota_i) \leq \bar{\mathcal{R}}(\xi_i)$.

Therefore, $IHFRWA(\mathcal{N}(\iota_1), \mathcal{N}(\iota_2), \dots, \mathcal{N}(\iota_n)) \leq IHFRWA(\mathcal{R}(\xi_1), \mathcal{R}(\xi_2), \dots, \mathcal{R}(\xi_l))$.

(iv) Since $\mathcal{N}(\iota) = \{(\mathcal{N}(\iota), \bar{\mathcal{N}}(\iota)) = ((r, t), (\bar{r}, \bar{t}))$ is the IHFRV and $\mathcal{R}(\xi_i) = \{(\mathcal{R}(\xi_i), \bar{\mathcal{R}}(\xi_i)) = [(\underline{\Theta}_i, \underline{\Xi}_i), (\bar{\Theta}_i, \bar{\Xi}_i)]$ is the collection of IHFRVs, so

$$\begin{aligned} \mathcal{R}(\xi_1) + \mathcal{N}(\iota) &= [\underline{\mathcal{R}}(\xi_1) + \mathcal{N}(\iota), \overline{\mathcal{R}}(\xi_1) + \overline{\mathcal{N}}(\iota)] \\ &= ((1 - (1 - \underline{\Theta}_i)(1 - r), \underline{\Xi}_1 t), (1 - (1 - \overline{\Theta}_i)(1 - \bar{r}), \overline{\Xi}_1 \bar{t})) \end{aligned}$$

Therefore,

$$\begin{aligned} (\mathcal{R}(\xi_1) + \mathcal{N}(\iota), \mathcal{R}(\xi_2) + \mathcal{N}(\iota), \dots, \mathcal{R}(\xi_n) + \mathcal{N}(\iota)) &= \left[\bigoplus_{i=1}^n \vartheta_i(\underline{\mathcal{R}}(\xi_i) + \mathcal{N}(\iota), \bigoplus_{i=1}^n \vartheta_i(\overline{\mathcal{R}}(\xi_i) + \overline{\mathcal{N}}(\iota)) \right] \\ &= \left[\left(1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} (1 - r_i)^{\omega_i}, \prod_{i=1}^n \underline{\Xi}_i^{\omega_i} t^{\omega_i} \right), \right. \\ &\quad \left. \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\omega_i} (1 - \bar{r}_i)^{\omega_i}, \prod_{i=1}^n \overline{\Xi}_i^{\omega_i} \bar{t}^{\omega_i} \right) \right] \\ &= \left[\left(1 - (1 - r_i)^{\omega_i} \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i}, t \prod_{i=1}^n \underline{\Xi}_i^{\omega_i} \right), \right. \\ &\quad \left. \left(1 - (1 - \bar{r}_i)^{\omega_i} \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\omega_i}, \bar{t} \prod_{i=1}^n \overline{\Xi}_i^{\omega_i} \right) \right] \\ &= \left[\left\{ \left(1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i}, \prod_{i=1}^n \underline{\Xi}_i^{\omega_i} \right) + (r, t) \right\}, \right. \\ &\quad \left. \left\{ \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\omega_i}, \prod_{i=1}^n \overline{\Xi}_i^{\omega_i} \right) + (\bar{r}, \bar{t}) \right\} \right] \\ &= \left[\left(\left(1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i}, \prod_{i=1}^n \underline{\Xi}_i^{\omega_i} \right), \right. \right. \\ &\quad \left. \left. \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\omega_i}, \prod_{i=1}^n \overline{\Xi}_i^{\omega_i} \right) \right) \right] + [(r, t), (\bar{r}, \bar{t})] \end{aligned}$$

Hence,

$$(\mathcal{R}(\xi_1) + \mathcal{N}(\iota), \mathcal{R}(\xi_2) + \mathcal{N}(\iota), \dots, \mathcal{R}(\xi_n) + \mathcal{N}(\iota)) = IHFRWA(\mathcal{R}(\xi_1), \mathcal{R}(\xi_2), \dots, \mathcal{R}(\xi_n)) + \mathcal{N}(\iota)$$

(v) Homogeneity: For any real number $\varepsilon > 0$ and $\mathcal{R}(\xi_i) = \{(\underline{\mathcal{R}}(\xi_i), \overline{\mathcal{R}}(\xi_i))$ are the IHFRVs.

Since

$$\varepsilon \mathcal{R}(\xi_i) = \{(\varepsilon \underline{\mathcal{R}}(\xi_i), \varepsilon \overline{\mathcal{R}}(\xi_i)) = [(1 - (1 - \underline{\Theta}_i)^{\varepsilon_i}, \underline{\Xi}_i^{\varepsilon_i}), (1 - (1 - \overline{\Theta}_i)^{\varepsilon_i}, \overline{\Xi}_i^{\varepsilon_i})]$$

Now,

$$\begin{aligned} IHFRWA(\varepsilon \mathcal{R}(\xi_1), \varepsilon \mathcal{R}(\xi_2), \dots, \varepsilon \mathcal{R}(\xi_l)) &= \left[\left(1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\varepsilon \omega_i}, t \prod_{i=1}^n \underline{\Xi}_i^{\varepsilon \omega_i} \right), \right. \\ &\quad \left. \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\varepsilon \omega_i}, \bar{t} \prod_{i=1}^n \overline{\Xi}_i^{\varepsilon \omega_i} \right) \right] \\ &= \left[\left\{ \left(1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i} \right)^\varepsilon, \left(\prod_{i=1}^n \underline{\Xi}_i^{\omega_i} \right)^\varepsilon \right\}, \right. \\ &\quad \left. \left\{ \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\omega_i} \right)^\varepsilon, \left(\prod_{i=1}^n \overline{\Xi}_i^{\omega_i} \right)^\varepsilon \right\} \right] \\ &= \varepsilon IHFRWA(\mathcal{R}(\xi_1), \mathcal{R}(\xi_2), \dots, \mathcal{R}(\xi_n)). \end{aligned}$$

Hence, $IHFRWA(\varepsilon \mathcal{R}(\xi_1), \varepsilon \mathcal{R}(\xi_2), \dots, \varepsilon \mathcal{R}(\xi_l)) = \varepsilon IHFRWA(\mathcal{R}(\xi_1), \mathcal{R}(\xi_2), \dots, \mathcal{R}(\xi_n))$.

(vi) Commutativity: Let

$$\begin{aligned} IHFRWA(\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \dots, \mathbb{R}(\xi_n)) &= \left[\bigoplus_{i=1}^n \omega_i \mathbb{R}(\xi_i), \bigoplus_{i=1}^n \omega_i \overline{\mathbb{R}}(\xi_i) \right] \\ &= \left[\left(1 - \prod_{i=1}^n (1 - \underline{\Theta}_i)^{\omega_i}, \prod_{i=1}^n (\Xi_i)^{\omega_i} \right), \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}_i)^{\omega_i}, \prod_{i=1}^n (\overline{\Xi}_i)^{\omega_i} \right) \right]. \end{aligned}$$

Since $(\tilde{\mathbb{R}}(\xi_1), \tilde{\mathbb{R}}(\xi_2), \dots, \tilde{\mathbb{R}}(\xi_l))$ is any permutation of $(\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \dots, \mathbb{R}(\xi_l))$, then

we obtain $\mathbb{R}(\xi_i) = \tilde{\mathbb{R}}(\xi_i), \forall i = 1, 2, \dots, n$.

$$\begin{aligned} &= \left[\left(1 - \prod_{i=1}^n (1 - \underline{\Theta}'_i)^{\omega_i}, \prod_{i=1}^n (\Xi'_i)^{\omega_i} \right), \left(1 - \prod_{i=1}^n (1 - \overline{\Theta}'_i)^{\omega_i}, \prod_{i=1}^n (\overline{\Xi}'_i)^{\omega_i} \right) \right] \\ &= \left[\bigoplus_{i=1}^n \omega_i \tilde{\mathbb{R}}(\xi_i), \bigoplus_{i=1}^n \omega_i \overline{\tilde{\mathbb{R}}}(\xi_i) \right] \\ &= IHFRWA(\tilde{\mathbb{R}}(\xi_1), \tilde{\mathbb{R}}(\xi_2), \dots, \tilde{\mathbb{R}}(\xi_l)) \end{aligned}$$

Hence,

$$IHFRWA(\mathbb{R}(\xi_1), \mathbb{R}(\xi_2), \dots, \mathbb{R}(\xi_l)) = IHFRWA(\tilde{\mathbb{R}}(\xi_1), \tilde{\mathbb{R}}(\xi_2), \dots, \tilde{\mathbb{R}}(\xi_l))$$

□

7. EDAS Approach for MAGDM Utilizing Rough Aggregation Operators with IHF Information

DM problems become more complex in this competitive environment as the socio-economic environment becomes more complex. Therefore, it is more difficult for an expert to make a precise and appropriate decision in this circumstance. In the real world, it is essential to combine the opinions of a number of competent experts in order to apply group decision-making models to produce more fulfilling and feasible results. As a result, MADM has the capacity and discipline to improve and evaluate a variety of competing criteria in every aspect of decision making to provide more acceptable and workable decisions. Here, we solve the MADM technique using the EDAS technique. This was based on the PDAS and NDAS from the AVS. It is thought that the appropriate choice has a lower NDAS score and a higher PDAS value. To investigate the hybrid structure of the EDAS technique with IHFRVs, we constructed the IHFR-EDAS method, in which experts supplied their evaluation results as IHFRVs. The following are the fundamental stages of construction using the suggested method under IHF rough information. Let the alternative set m^* be denoted by $q^* = \{q_1^*, q_2^*, \dots, q_{m^*}^*\}$, and the decision attribute set n^* be represented by $C^* = \{c_1^*, c_2^*, \dots, c_{g^*}^*\}$. Suppose the professional decision-maker set t^* is represented by $E^* = \{e_1^*, e_2^*, \dots, e_{t^*}^*\}$, and they compare their attributes $c_s^* (s = 1, 2, \dots, g)$ to their assessment for each alternative $q_r^* (r = 1, 2, \dots, h)$. Suppose $\xi = \{\xi_1^*, \xi_2^*, \dots, \xi_{g^*}^*\}$ are the weighted vectors for attributes c_j^* and $\omega = \{\omega_1^*, \omega_2^*, \dots, \omega_{g^*}^*\}$ are the weighted vectors for the professional decision-maker set $D_l^* (l = 1, 2, \dots, t) \ni \sum_{u=1}^n \chi_{\hat{u}} = 1, \sum_{v=1}^n \vartheta_{\hat{s}} = 1$ and $0 \leq \chi_{\hat{u}}, \vartheta_{\hat{s}} \leq 1$. Below is an analysis of the standard algorithm for the EDAS method with an IHF rough environment.

Combine the expert decision-makers' evaluations of each alternative q_i^* in relation to their attribute c_j^* , then construct the decision matrix, i.e.,

$$M^* = [R(\zeta_{ij}^l)]_{g^* \times h^*}$$

Where $R(\zeta_{ij}^l)$ indicates the IHFRVs of alternatives q_i^* in relation to their attribute c_j^* by the professional decision-maker set D_j^* .

Utilizing the proposed approach, by combining the information provided by all decision-makers against their weight vector, the aggregated decision matrix is produced:

$$M^* = [R(\zeta_{ij})]_{(g^* \times h^*)}$$

After that, the aggregated matrix is normalized.

$$M^* = [R(\zeta_{ij})]_{(g^* \times h^*)} \text{ to } M^{*R} = [R(\zeta_{ij}^R)]_{(g^* \times h^*)}, i.e.$$

$$M^{*R} = \left\{ \begin{array}{l} R(\zeta_{ij}) = ((\underline{\Theta}_{ij}, \underline{\Xi}_{ij}), (\overline{\Theta}_{ij}, \overline{\Xi}_{ij})) \text{ for benefit} \\ R(\zeta_{ij}^n) = R(\zeta_{ij})^c = ((\underline{\Xi}_i, \underline{\Theta}_i), (\overline{\Xi}_i, \overline{\Theta}_i)) \text{ for cost} \end{array} \right\}$$

Calculate the AVS value for each attribute, applying the recommended method for every selection.

$$AVS = [AVS]_{(1 \times h^*)} = \left[\frac{1}{g^*} \sum_{i=1}^{g^*} R(\zeta_{ij}^R) \right]_{(1 \times h^*)}$$

$$\Rightarrow \left[\left(1 - \prod_{i=1}^{g^*} (1 - \underline{\Theta}_{ij})^{\frac{1}{g^*}}, \prod_{i=1}^{g^*} (\underline{\Xi}_{ij})^{\frac{1}{g^*}} \right), \left(1 - \prod_{i=1}^{g^*} (1 - \overline{\Theta}_{ij})^{\frac{1}{g^*}}, \prod_{i=1}^{g^*} (\overline{\Xi}_{ij})^{\frac{1}{g^*}} \right) \right]_{(1 \times h^*)}$$

Step 5: The approach below can be used to calculate the PDAS and NDAS based on the calculated AVS.

$$PDAS_{ij} = [PDAS_{ij}]_{(g^* \times h^*)} = \frac{\max(0, S(R(\zeta_{ij}^R)) - S(AVS_j))}{S(AVS_j)},$$

$$NDAS_{ij} = [NDAS_{ij}]_{(g^* \times h^*)} = \frac{\max(0, S(AVS_j) - S(R(\zeta_{ij}^R)))}{S(AVS_j)}$$

Step 6: The positive weight distance (SP_i) and negative weight distance (SN_i) are then computed: $SP_i = \sum_{j=1}^n \xi_j PDAS_{ij}$ and $SN_i = \sum_{j=1}^n \xi_j NDAS_{ij}$

Step 7: The (SP_i) and (SN_i) are normalized by applying the formula: $\acute{N}SP_i = \frac{SP_i}{\max_i(SP_i)}$ and $\acute{N}SN_i = 1 - \frac{SN_i}{\max_i(SN_i)}$.

Step 8: Calculate the evaluation score based on $\acute{N}SP_i$ and $\acute{N}SN_i$. The appraisal score ($\bar{A}S$) value is calculated using the formula that follows:

$$\bar{A}S_i = \frac{1}{2}(\acute{N}SP_i + \acute{N}SN_i)$$

Step 9: Arrange all of the values in a specific order depending on the $\bar{A}S_i$ value. The greater the $\bar{A}S_i$ value, the more beneficial.

8. Example of Using EDAS Method

The efficacy and superiority of the analysed approach is demonstrated by presenting a real-world MADM example of a small hydro-power plant (SHPP). Consider a construction company that started a project using the four SHPPs q_1^* , q_2^* , q_3^* , and q_4^* in various locations throughout Pakistan. To determine which power plant is most suitable for building,

these locations are further assessed. These four SHPPs are assessed by the three experts $E_i (i = 1, 2, 3)$. The experts assess these four SHPPs regarding three criteria: c_1^* = constructability, c_2^* = socioeconomic climate, and c_3^* = purchasing and feed-in tariffs, with $\omega = (0.556040631, 0.1714327168, 0.2725266523)^t$. In the form of IHFRVs, the competent experts evaluate each q_i^* 's assessment report with regard to the relevant criteria. The above stepwise decision algorithm of the EDAS method is now used along with the generated IHFRWA operator approach to obtain the most suitable SHPP system.

Step 1: Construct a decision matrix $M^* = [\mathbb{R}(\xi_{ij}^l)]_{m^* \times n^*}$, which is presented in Table 2, by collecting the expert decision-makers' combined evaluation information for each alternative q_i^* with regard to their criteria c_i^* .

Table 2. IHF rough evaluation information.

ξ	c_1^*	c_2^*	c_3^*
q_1^*	$\langle (\{0.2, 0.3\}, \{0.1, 0.4\}), \{0.3, 0.5\}, \{0.2, 0.3\} \rangle$	$\langle (\{0.3, 0.4\}, \{0.1, 0.5\}), \{0.2, 0.5\}, \{0.1, 0.4\} \rangle$	$\langle (\{0.1, 0.2\}, \{0.4, 0.5\}), \{0.2, 0.5\}, \{0.4, 0.5\} \rangle$
q_2^*	$\langle (\{0.3, 0.4\}, \{0.1, 0.2\}), \{0.1, 0.6\}, \{0.1, 0.3\} \rangle$	$\langle (\{0.3, 0.5\}, \{0.1, 0.2\}), \{0.2, 0.4\}, \{0.3, 0.5\} \rangle$	$\langle (\{0.2, 0.5\}, \{0.3, 0.5\}), \{0.1, 0.4\}, \{0.3, 0.4\} \rangle$
q_3^*	$\langle (\{0.2, 0.4\}, \{0.3, 0.6\}), \{0.1, 0.5\}, \{0.1, 0.3\} \rangle$	$\langle (\{0.3, 0.4\}, \{0.1, 0.5\}), \{0.3, 0.4\}, \{0.2, 0.4\} \rangle$	$\langle (\{0.1, 0.4\}, \{0.2, 0.5\}), \{0.3, 0.4\}, \{0.2, 0.3\} \rangle$
q_4^*	$\langle (\{0.1, 0.2\}, \{0.2, 0.4\}), \{0.1, 0.4\}, \{0.1, 0.2\} \rangle$	$\langle (\{0.1, 0.2\}, \{0.3, 0.4\}), \{0.2, 0.4\}, \{0.2, 0.5\} \rangle$	$\langle (\{0.2, 0.4\}, \{0.4, 0.5\}), \{0.3, 0.4\}, \{0.4, 0.5\} \rangle$

Step 2: The aggregated decision matrix is $M^* = [\mathbb{R}(\xi_{ij})]_{(g^* \times h^*)}$, which is the result of using the IHFRWA operators to aggregate the collective data of decision-makers versus their weight vector.

Step 3: Although all of the criteria are beneficial, they must be normalized.

Step 4: Applying the suggested methodology, find the value of AVS for each alternative for every criterion stated in Table 3.

Table 3. The value of the average solution (AVS) of IHFRS.

q_1^*	$\left\langle \begin{pmatrix} 0.1926000919, \\ 0.2181052989, \\ 0.2136573611, \\ 0.2959195844, \\ 0.2503771490, \\ 0.2740572185, \\ 0.2740572185, \\ 0.2929900576 \end{pmatrix}, \begin{pmatrix} 0.1941523420, \\ 0.2332144004, \\ 0.2713971109, \\ 0.3067148640, \\ 0.3568103410, \\ 0.3879878369, \\ 0.4184634786, \\ 0.4466524463 \end{pmatrix} \right\rangle, \left\langle \begin{pmatrix} 0.2572475241, \\ 0.3465445678, \\ 0.3147464960, \\ 0.3971307547, \\ 0.3839854317, \\ 0.4580454741, \\ 0.4316731951, \\ 0.5000000001 \end{pmatrix}, \begin{pmatrix} 0.2452379293, \\ 0.2818237031, \\ 0.2959195844, \\ 0.3300486534, \\ 0.2992482541, \\ 0.3332159717, \\ 0.3463031601, \\ 0.3779899732 \end{pmatrix} \right\rangle$
q_2^*	$\left\langle \begin{pmatrix} 0.2740572185, \\ 0.3613333252, \\ 0.3147464960, \\ 0.3971307547, \\ 0.3336885811, \\ 0.4137955371, \\ 0.3710355055, \\ 0.4466524463 \end{pmatrix}, \begin{pmatrix} 0.1595773808, \\ 0.2332144004, \\ 0.1763769039, \\ 0.2485419655, \\ 0.2128547996, \\ 0.2818237031, \\ 0.2285893404, \\ 0.2961795986 \end{pmatrix} \right\rangle, \left\langle \begin{pmatrix} 0.1179904378, \\ 0.2102607332, \\ 0.2102607332, \\ 0.2482643524, \\ 0.4381175028, \\ 0.4968981172, \\ 0.4651562602, \\ 0.5211082498 \end{pmatrix}, \begin{pmatrix} 0.1950168317, \\ 0.2281338150, \\ 0.2401363430, \\ 0.2713971109, \\ 0.3000000001, \\ 0.3287979790, \\ 0.3392351781, \\ 0.3664190229 \end{pmatrix} \right\rangle$

Table 3. Cont.

q_3^*	$\left\langle \left(\begin{array}{c} 0.1926000919, \\ 0.2770651944, \\ 0.2136573611, \\ 0.2959195844, \\ 0.3119536715, \\ 0.3839327528, \\ 0.3298981581, \\ 0.4000000000 \end{array} \right), \left(\begin{array}{c} 0.2420975271, \\ 0.3332159717, \\ 0.3147464960, \\ 0.3971307547, \\ 0.4447682480, \\ 0.5115207068, \\ 0.4979901541, \\ 0.5583440359 \end{array} \right) \right\rangle$	$\left\langle \left(\begin{array}{c} 0.1950168317, \\ 0.2281338150, \\ 0.2160110716, \\ 0.2482643524, \\ 0.4194429346, \\ 0.4433270348, \\ 0.4345840639, \\ 0.4578452585 \end{array} \right), \left(\begin{array}{c} 0.1458525123, \\ 0.1763769039, \\ 0.1869555665, \\ 0.2160110716, \\ 0.2572475241, \\ 0.2837910282, \\ 0.2929900576, \\ 0.3182562425 \end{array} \right) \right\rangle$
q_4^*	$\left\langle \left(\begin{array}{c} 0.1284303801, \\ 0.1941523420, \\ 0.1458525123, \\ 0.2102607332, \\ 0.1836823195, \\ 0.2452379293, \\ 0.2000000000, \\ 0.2603251516 \end{array} \right), \left(\begin{array}{c} 0.2770651944, \\ 0.3121081970, \\ 0.2959195844, \\ 0.3300486534, \\ 0.3839327528, \\ 0.4137955371, \\ 0.4000000000, \\ 0.4290839525 \end{array} \right) \right\rangle$	$\left\langle \left(\begin{array}{c} 0.1763769039, \\ 0.2102607332, \\ 0.2160110716, \\ 0.2482643524, \\ 0.3426228233, \\ 0.3696673005, \\ 0.3742569498, \\ 0.4000000000 \end{array} \right), \left(\begin{array}{c} 0.2102607332, \\ 0.2485419655, \\ 0.2713971109, \\ 0.3067148640, \\ 0.2603251516, \\ 0.2961795986, \\ 0.3175858739, \\ 0.3506647074 \end{array} \right) \right\rangle$

Step 5: We may determine the score value of the AVS based on the derived AVS_i ($i = 1, 2, 3$), as shown in Table 3, and then calculate the $PDAS$ and $NDAS$, as shown in Tables 4 and 5.

Table 4. The results of PDAS matrix of IHFRS.

	c_1^*	c_2^*	c_3^*
q_1^*	0.03828009306	0.03828009306	0.00000000000
q_2^*	0.03836576853	0.00000000000	0.00000000000
q_3^*	0.00000000000	0.02916950736	0.004067812057
q_4^*	0.02591481497	0.00000000000	0.00000000000

Table 5. The results of NDAS matrix of IHFRS.

	c_1^*	c_2^*	c_3^*
q_1^*	0.00000000000	0.00000000000	0.09932329278
q_2^*	0.00000000000	0.009375186348	0.08098661866
q_3^*	0.008483035593	0.00000000000	0.00000000000
q_4^*	0.00000000000	0.02603023896	0.02603023896

Step 6: The SP_i and SN_i are then determined using the criteria weight vector $\omega = (0.556040631, 0.1714327168, 0.2725266523)^t$; it is displayed in Table 6.

Table 6. The results of SP_i and SN_i of IHFRS.

SP_1	0.02784774745	SN_1	0.02706824448
SP_2	0.02133292614	SN_2	0.02367822573
SP_3	0.006109195096	SN_3	0.004716912464
SP_4	0.01440969007	SN_4	0.01155636847

Step 7: The SP_i and SN_i should now be normalized as indicated below.

$$\mathbb{N}SP_1 = 1.000000000, \mathbb{N}SP_2 = 0.7660557170, \mathbb{N}SP_3 = 0.2193784293, \mathbb{N}SP_4 = 0.5174454449$$

and

$$\mathbb{N}SN_1 = 0.000000000, \mathbb{N}SN_2 = 0.8747603025, \mathbb{N}SN_3 = 0.8257399933, \mathbb{N}SN_4 = 0.5730654613$$

Step 8: The $\bar{A}S$ value is now determined using $\mathbb{N}SP_i$ and $\mathbb{N}SN_i$ as

$$\bar{A}S_1 = 0.8830278585, \bar{A}S_2 = 0.8204080100, \bar{A}S_3 = 0.5225592115, \bar{A}S_4 = 0.5452554530$$

Step 9: Table 7 displays the ranking results of the suggested models using the EDAS technique and are dependent on the calculations described above. Thus, the business should choose the best SHPP q_1^* . Furthermore, the average of the IHFR data on MGs and non-MGs is taken. In the form of IFRVs, the combined evaluation of each competent expert's q_i^* assessment report with regard to the relevant criteria c_i^* is obtained. The above stepwise decision algorithm of the EDAS method is now used along with the generated IFRWA operator approach to obtain the most suitable SHPP system.

Table 7. IHFRS results of $\bar{A}S_i$.

q_1^*	q_2^*	q_3^*	q_4^*
0.8830278585	0.8204080100	0.5225592115	0.5452554530

Step 10: Construct a decision matrix $M^* = [\mathbb{R}(\xi_{ij}^l)]_{m^* \times n^*}$, which is presented in Table 8, by collecting the combined expert decision-makers' evaluation information for each alternative q_i^* with regard to their criteria c_i^* .

Table 8. IF rough evaluation information.

ξ	c_1^*	c_2^*	c_3^*
q_1^*	$\langle (0.25, 0.25), (0.4, 0.25) \rangle$	$\langle (0.35, 0.3), (0.35, 0.25) \rangle$	$\langle (0.15, 0.45), (0.35, 0.45) \rangle$
q_2^*	$\langle (0.35, 0.15), (0.35, 0.2) \rangle$	$\langle (0.4, 0.15), (0.3, 0.4) \rangle$	$\langle (0.35, 0.4), (0.25, 0.35) \rangle$
q_3^*	$\langle (0.3, 0.45), (0.3, 0.2) \rangle$	$\langle (0.35, 0.3), (0.35, 0.3) \rangle$	$\langle (0.25, 0.35), (0.35, 0.25) \rangle$
q_4^*	$\langle (0.15, 0.3), (0.25, 0.15) \rangle$	$\langle (0.15, 0.35), (0.3, 0.35) \rangle$	$\langle (0.3, 0.45), (0.35, 0.45) \rangle$

The aggregated decision matrix $M^* = [\mathbb{R}(\xi_{ij})]_{(m^* \times n^*)}$ is the result of using the IFRWA operators to aggregate the collective data of decision-makers versus their weight vector. Although all of the criteria are beneficial, they must be normalized.

Applying the suggested methodology, find the value of AVS for each alternative for each of the criteria listed in Table 9. We may determine the score value of the AVS based on the derived AVS_i ($i = 1, 2, 3$), as shown in Table 9, and then calculate the $PDAS$ and $NDAS$, as shown in Tables 10 and 11.

Table 9. The value of the average solution (AVS) of IFRS.

c_1^*	$((0.3380418576, 0.3738952871), (0.4106029333, 0.2583140461))$
c_2^*	$((0.4004865055, 0.3530916044), (0.4084280166, 0.4106029333))$
c_3^*	$((0.3380418576, 0.4951145572), (0.4094358073, 0.4716764136))$

Table 10. The results of PDAS matrix of IFRS.

	c_1^*	c_2^*	c_3^*
q_1^*	0.01585899626	0.05123165954	0.00000000000
q_2^*	0.1103575076	0.05123165954	0.03892504292
q_3^*	0.00000000000	0.02678441164	0.1231622086
q_4^*	0.00000000000	0.00000000000	0.00000000000

Table 11. The results of NDAS matrix of IFRS.

	c_1^*	c_2^*	c_3^*
q_1^*	0.00000000000	0.00000000000	0.1014702331
q_2^*	0.00000000000	0.00000000000	0.00000000000
q_3^*	0.07863951504	0.00000000000	0.00000000000
q_4^*	0.07863951504	0.1443463237	0.01723306750

The SP_i and SN_i are then determined using the weight vector of the criteria $\omega = (0.556040631, 0.1714327168, 0.2725266523)^t$, which is displayed in Table 12.

Table 12. The results SP_i and SN_i of IFRS.

SP_1	0.01760102887	SN_1	0.02765334293
SP_2	0.08075415238	SN_2	0.00000000000
SP_3	0.03815670886	SN_3	0.04372676556
SP_4	0.00000000000	SN_4	0.07316891818

The SP_i and SN_i should now be normalized as indicated below.

$$\bar{N}SP_1 = 0.2179581898, \bar{N}SP_2 = 1.000000000, \bar{N}SP_3 = 0.4725046048, \bar{N}SP_4 = 0.000000000$$

and

$$\bar{N}SN_1 = 0.000000000, \bar{N}SN_2 = 0.6220616128, \bar{N}SN_3 = 0.4023860589, \bar{N}SN_4 = 1.000000000$$

The $\bar{A}S$ value is now determined using $\bar{N}SP_i$ and $\bar{N}SN_i$ as

$$\bar{A}S_1 = 0.1089790949, \bar{A}S_2 = 0.8110308065, \bar{A}S_3 = 0.4374453318, \bar{A}S_4 = 0.5000000000$$

Table 13 shows the ranking results of the suggested models using the EDAS approach; these are dependent on the calculations described above. Thus, the business should choose the best SHPP q_1^* .

Table 13. The IFRS results of $\bar{A}S_i$.

q_1^*	q_2^*	q_3^*	q_4^*
0.1089790949	0.8110308065	0.4374453318	0.5000000000

Table 14 provides the ranking values for the preceding discussion. The advantages of our proposed approach, as outlined in the above comparative analysis, can be summarized. The IHFS is well suited for representing uncertain or fuzzy information in MADM problems

due to the availability of two sets of MG and Non MG with various possible values, a feature not achievable by the IFS. The IHFS can also be employed for processing MADM and comparison methods based on its inherent capabilities. After minor modification of the IFS, the IHFS uses the general form of the IFS. Because it is capable of supporting many degrees of MG and Non MG, simultaneously, the IHFS is seen as superior to the IFS. This enables a fuller and more complex representation of uncertainty and hesitancy. This results in enhanced DM procedures and more accurate and flexible modeling of complex problems.

Table 14. Comparison of the IHFRS and IFRS results of $\bar{A}S_i$.

q_1^*	q_2^*	q_3^*	q_4^*
0.8830278585	0.8204080100	0.5225592115	0.5452554530
0.1089790949	0.8110308065	0.4374453318	0.5000000000

9. Comparative Analysis

The PDAS and NDAS from the AVS provide the foundation for the EDAS technique. The suitable option is thought to be the combination of a higher PDAS and a lower NDAS. Here, a comparison with various current methods in context [18] has been conducted to establish our investigated IHFR-EDAS approach's supremacy. Table 7 provides the aggregate results obtained from a comparison analysis of current methods [18,24–26] with our method, according to Table 2, with criteria weight vector $\omega = (0.556040631, 0.1714327168, 0.2725266523)^t$. The developed described example cannot be solved by applying IHF rough values using the current approaches. Nevertheless, the methods presented in [27–29] provide approximate information, but they cannot be used to solve the proposed model. It is obvious that the current methods need certain basic information and are unable to solve and assess the established instance. Existing methods like Fuzzy IF-TOPSIS, IF-EDAS, and IF-GRA often focus on single-layer uncertainty (e.g., membership degrees) while IHFR-EDAS incorporates multi-dimensional hesitancy (from IHFS) and data-driven approximations (from RS), enabling deeper analysis, and also IHFR-EDAS provides clearer justifications for rankings by combining subjective hesitancy evaluations with objective data approximations. Furthermore, existing fuzzy MADM methods like TOPSIS or AHP often lack nuanced mechanisms to manage both hesitancy and rough approximations simultaneously and IHFR-EDAS provides dual-layer uncertainty handling. Traditional fuzzy MADM models (e.g., Fuzzy TOPSIS, Fuzzy AHP) focus on membership grades, they do not adequately capture multi-valued hesitancy or distinguish between certain and uncertain classifications while IHFR-EDAS bridges this gap by incorporating both hesitant fuzzy parameters and RS-based approximations, offering a richer representation of uncertainty. Therefore, the suggested approach is more capable and efficient than the current approaches.

An Analysis Comparing the Current IHFR-EDAS Approach with IFR-EDAS

If the MG and non-MG degrees are both occupied by a single element, IFRSs can be regarded as a specific example of IHFRSs. Transferring IHFRSs to IFRSs involves finding out the average values of the MG and non-MG degrees in order to make comparison easier. Following conversion, Table 8 can display the intuitionistic information. IFR-EDAS can then be used to calculate various assessment values. The final alternative ranking is $q_1^* \geq q_2^* \geq q_4^* \geq q_3^*$ and the preferred alternative is q_1^* . It is evident that the ranking obtained from the method suggested by the suggested technique differs from IFRSs [18]. The main goal is to average the MG and non-MG degrees of the IFRSs, which could lead

to information loss and deception. To facilitate a comparison, transforming a IHFRS to a IFRS involves calculating the average values of MG and Non MG degrees. Once converted, the intuitionistic information is presented in Table 8. Subsequently, the broad assessment values can be computed using EDAS within an IF environment.

10. Conclusions

The factual information on specific facts usually remains unknown in DM problems, and this lack of clarity contributes complexity and difficulty to the DM process. RSs and IHFS are general mathematical instruments that can easily deal with ambiguous and imprecise information. The EDAS technique is crucial to the DM process when there are more conflicting criteria in MADM scenarios. We develop the IHFR-EDAS technique to investigate a hybrid structure of the EDAS technique using IHFRVs. Introducing the IHFR-EDAS approach based on the IHF rough averaging operator is the objective of this work. Furthermore, we propose the idea of IHFRWA operators. A detailed description of the developed operator's basic desirable features is provided. For the proposed operators, new accuracy and scoring functions are constructed. The suggested method is then used to illustrate the IHFR-EDAS model for MADM and its iterative algorithm. The built model is finally shown numerically, and a general comparison of the models under study with a few contemporary methods shows that the models under study are more effective and useful than the methods now in use. Using intuitionistic, hesitant, and Pythagorean fuzzy data, we will expand on the suggested approach in subsequent research to incorporate a variety of aggregation operators, including Dombi operations, Einstein operations, etc. We will also concentrate on how the suggested approach may be used in various real-world issues utilizing Pythagorean, hesitant, and intuitionistic fuzzy information. Furthermore, we will apply the developed method to other domains, including medical diagnostics, and expand it to other generalizations of FSs. Many real-world problems involve conflicting opinions, incomplete data, and hesitation. IHFR-EDAS effectively models such scenarios. The EDAS model helps decision-makers evaluate complex scenarios more reliably, reducing bias and error, and its application is useful in many areas such as supply chain management, healthcare, environmental management, multi-objective engineering design, social sciences, etc. The IHFR-EDAS approach is typically static and may not work well in situations when preferences, criteria, or alternatives vary over time. The IHFR-EDAS approach might be overly complicated for small-scale decision-making situations when compared to more straightforward MADM methods like AHP and TOPSIS. Particularly, when working with complicated FSs, with a large number of alternatives, or criteria, the IHFR-EDAS approach can require a substantial amount of processing power. The IHFR-EDAS approach may not be suitable for simple problems or situations where clear data are adequate, even when it works effectively for complicated, unclear DM scenarios. The approach works independently and could be difficult to combine with other frameworks like machine learning, game theory, or simulation-based techniques. Future research will expand the suggested approach to include various aggregation operators, such as the Bonferroni mean, Einstein operations, Maclaurin symmetric mean operators, Hamacher operations, Dombi operations, the Choquet integral, and interaction aggregation operators.

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List of Symbols

FS	Fuzzy set
IFS	Intuitionistic fuzzy set
HF	Hesitant fuzzy set
IHFS	Intuitionistic hesitant fuzzy set
RS	Rough set
Non-MG	Grade of non-membership
$\boxplus$	Summation
$\boxtimes$	Product
$\vee$	Maximum
$\wedge$	Minimum
$\hat{R}(\xi)$	Collection of IHFRVs

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