

Advances in Difference Equations

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1. Introduction

This editorial concerns the Special Issue of *Axioms* entitled “Advances in Difference Equations”. It is well known that difference equations provide an extreme and yet widely recognized representation of complex dynamical systems. The kernel of non-integer order derivative operators holds significant relevance as an empirical explanation for these complex phenomena. In recent years, the theory of non-integer order derivative operators has been successfully applied to the study of anomalous behaviors in both social and physical sciences. This Special Issue thus highlights high-quality research papers featuring novel findings, with a focus on the theory and applications of differential and difference equations, particularly in the fields of science and engineering.

2. An Overview of the Published Papers

This Special Issue includes 10 papers that were accepted for publication following a thorough and rigorous review process.

In the first contribution, several new types of partial fractional derivatives in both continuous and discrete settings are introduced. Moreover, some classes of the abstract fractional differential equations and the abstract fractional difference equations depending on several variables are investigated.

The second contribution presents new nonlinear delayed integral inequalities which can be utilized to study the existence, stability, boundedness, uniqueness, and asymptotic behavior of solutions of nonlinear delayed integro-differential equations. These inequalities can be used in the symmetrical properties of functions and also generalize several well-known inequalities in the literature.

In the third contribution to this Special Issue, a class of nonlinear ordinary differential equations with impulses at variable times is considered. The existence and uniqueness of the solution are given. Simultaneously, the classical definitions of continuous dependence and Gâteaux differentiability are modified. The results provide a foundation to study optimal control problems of systems governed by differential equations with impulses at variable times.

The fourth contribution deals with the oscillatory behavior of solutions of a new class of second-order nonlinear differential equations. Some new criteria, that guarantee the oscillation of all solutions of the dynamical model without additional restrictions, are introduced. This new approach improves the standard integral averaging technique to obtain simpler oscillation theorems for new classes of nonlinear differential equations.

The Special Issue’s fifth contribution aims to describe the dynamics of a discrete fractional-order reaction–diffusion FitzHugh–Nagumo model. Acceptable requirements for the local asymptotic stability of the system’s unique equilibrium are established. Moreover, it is established that the constant equilibrium solution is globally asymptotically stable.

In the sixth contribution, the problem of synchronization-control in a fractional discrete nonlinear biological model is investigated using the Caputo h -difference operator and an L1 finite difference scheme. Furthermore, this research revealed that the L1 finite difference scheme and the second-order central difference scheme may successfully retain the properties of the related continuous system.



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In the seventh contribution to this Special Issue, topological degree and fixed point theorems are applied to investigate the existence, uniqueness, and multiplicity of solutions for a boundary value problem associated with a fractional-order difference equation. The results are validated by the provision of appropriate examples.

In eighth contribution, the qualitative properties of solutions to a general difference equation are investigated. Necessary and sufficient conditions for the existence of prime period-two and period-three solutions are provided. Furthermore, the boundedness and global stability of the solutions is investigated.

The ninth contribution forwards a neural network approach based on Lie series in Lie groups of differential equations to solve Burgers–Huxley nonlinear partial differential equations, where initial or boundary value terms in loss functions are investigated. The proposed technique yields closed analytic solutions that possess excellent generalization properties. Moreover, a thorough comparison with its exact solution is carried out to validate the practicality and effectiveness of the proposed method, using vivid graphics and detailed analysis.

Finally, the tenth contribution to this Issue offers novel adequate conditions for difference equations with forcing, positive, and negative terms to ensure non-oscillatory solutions. To help establish the main results, an analogous representation for the main equation, called a Volterra-type summation equation, is constructed. Two numerical examples are provided to demonstrate the validity of the theoretical findings.

Conflicts of Interest: The author declares no conflicts of interest.

List of Contributions:

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9. Wen, Y.; Chaolu, T. Study of Burgers–Huxley Equation Using Neural Network Method. *Axioms* **2023**, *12*, 429. <https://doi.org/10.3390/axioms12050429>
10. Alzabut, J.; Grace, S.R.; Jonnalagadda, J.M.; Santra, S.S.; Abdalla, B. Higher-Order Nabla Difference Equations of Arbitrary Order with Forcing, Positive and Negative Terms: Non-Oscillatory Solutions. *Axioms* **2023**, *12*, 325. <https://doi.org/10.3390/axioms12040325>

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