axioms

MDPI

Article

Robust State Feedback Control with D-Admissible Assurance for Uncertain Discrete Singular Systems

Chih-Peng Huang

Department of Computer Science, University of Taipei, Taipei 100, Taiwan; cphuang@utaipei.edu.tw or ponytony@seed.net.tw

Abstract: This study addresses the state feedback control associated with D-admissible assurance for discrete singular systems subjected to parameter uncertainties in both the difference term and system matrices. Firstly, a refined analysis criterion of D-admissible assurance is presented, where the distinct form embraces multiple slack matrices and has lessened linear matrix inequalities (LMIs) constraints, which may be beneficial for reducing the conservatism of admissibility analysis. In consequence, by hiring the state feedback control, controller design issues with pole locations, which directly dominate the system performance, are mainly treated. For all the presented criteria can be formulated by the strict LMIs, they are thus suitably solved via current LMI solvers to conduct a state feedback controller with specific poles' locations of system's performance requirements. Finally, two numerical examples illustrate that the presented results are efficient and practicable.

Keywords: uncertain systems; discrete singular systems; D-admissibility; linear matrix inequality (LMI)

MSC: 93C90; 93C05; 93D12; 93B52

check for updates

Citation: Huang, C.-P. Robust State Feedback Control with D-Admissible Assurance for Uncertain Discrete Singular Systems. *Axioms* **2024**, *13*, 634. https://doi.org/10.3390/axioms13090634

Academic Editors: Nohe R. Cazarez-Castro, Selene L. Cardenas-Maciel and Jorge A. Lopez-Renteria

Received: 28 July 2024
Revised: 14 September 2024
Accepted: 15 September 2024
Published: 17 September 2024

Copyright: © 2024 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

Singular systems have become an extended form of the traditional state-space systems, where, besides the dynamic behaviors, they can further integrate algebraic constraints into the systems. They are more ingeniously applicable in miscellaneous systems, e.g., economic model systems [1,2], circuit systems [3], chemical response processes [4], and power systems [5]. However, the stability analysis and controller design issues of singular systems are more intricate than traditional ones [6–8]. Since they usually embrace rank insufficient derivative term matrices (or difference term matrices, for discrete system cases) in the models, besides the stability verification, we must ensure the regularity and impulse immunity (or causality, for discrete system cases) simultaneously. Furthermore, the practical system models need to accommodate parameter uncertainties of circumstance variation, component aging, measuring inaccuracy, and so on. Thus, the robust admissible analysis and/or robust control for singular systems were deeply investigated in the past (see, e.g., [9–15] and the references therein). For practical implementation, digital signals are indispensable in real-time control or monitoring systems. Using signal transformation, discrete difference models with highly accurate forms could be attained [16]. Thus, many works have been inspired to cope with discrete singular systems subjected to various uncertainties (see, e.g., [17–19] and the references therein).

Moreover, when implementing a control system, stability is the minimum requirement of the system's behavior. The system's performance is directly dominated by the pole locations in the complex plane of a state-space system's model [20–24]. Subsequently, the D-admissible analysis and controller design issues for discrete singular systems have also been discussed recently ([25–29] and the references therein). By examining past results, they considered the difference term matrix of the discrete singular systems that needed to

be fixed. Then, they could attain some explicit admissible analysis and controller design criteria. However, for system modeling from a practical system, the parameter variations usually have individually varying durations and can be meaningfully embraced by both the derivative term (or difference term for discrete system cases) and system matrices with structural uncertainties [15,30–34]. For discrete singular systems, the robust D-admissibility issues with perturbation in both the difference term and the system's matrices have been previously investigated [33]. However, the proposed conditions involved a large number of LMI constraints, which not only cause computing exertion but also may introduce conservatism for the proposed criteria. In the work [35], the D-admissibility and controller design issues were discussed for uncertain discrete singular systems with state delay. However, the results are based on the augmented system's approach, which makes it challenging to tackle the uncertainties that exist in the difference term matrix. Concluding the aforementioned exploration, it seems that relatively few results deal with the state feedback control with D-admissible assurance for uncertain discrete singular systems subjected to parametric perturbation in both the difference term and system matrices, which inspired us to undertake this topic.

This study is devoted to the state feedback control with D-admissible assurance for discrete singular systems subjected to uncertainties in both the difference term and system matrices. Using matrix algebra and LMI techniques [36,37], we first present a distinct D-admissibility criterion of the considered systems. The new form not only has less LMI constraints but also involves some slack metrics, which may be beneficial for reducing the conservatism of analysis criteria. By hiring the state feedback control, the controller design issues associated with prescribed pole locations for the closed-loop singular system are further treated. All of the presented conditions can be expressed in terms of the strict LMI constraints, which can be easily evaluated by current LMI solvers [37] for the admissible analysis or by systematically conducting state feedback control. Ultimately, two examples illustrate the feasibility and applicability of the derived results. By comparing with the previous results, the contributions of our work are summarized in the following points:

1. To the best of our knowledge, few works focus on the state feedback control with D-admissible assurance for uncertain discrete singular systems subjected to parameter uncertainties in both the difference term and system matrices. This work is mainly devoted to the state feedback control with D-admissible issues for the considered systems.
2. For all of the design criteria that can be derived in terms of the strict LMIs, a state feedback controller with prescribed performance requirements can be readily conducted via current LMI tools.

The remaining information is outlined as follows. Section 2 introduces preliminary results and definitions for discrete singular systems. The D-admissibility and the state feedback control are mainly addressed in Section 3. Two numerical examples are involved to illustrate the feasibility and effectiveness of the proposed results in Section 4. Finally, we make some concluding remarks on this study in Section 5.

2. Preliminaries on Discrete Singular Systems

Consider an uncertain discrete singular model described by

$$\widetilde{E}x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)u(k),\ k > 0, \tag{1}$$

where $x(\cdot) \in R^n$ and $u(\cdot) \in R^r$ are the state vector and the control input, respectively. The uncertain difference term matrix $\widetilde{E}$ with $rank(\widetilde{E}) = m \leq n$ can be denoted by a polytopic form as

$$\Omega_E \equiv \left\{ \widetilde{E} : \widetilde{E} = \sum_{i=1}^{q_E} e_i E_i, e_i \geq 0, \sum_{i=1}^{q_E} e_i = 1 \right\}, \tag{2}$$

where q_E is the total number of the matrices' vertices, E_i. The nominal system matrices A and B have compatible dimensions and the uncertainties terms ΔA and ΔB are assumed to be norm-bounded with constant matrices M, N_A, N_B, and matrix Λ, $\Lambda^T\Lambda \leq I$, satisfying

$$\begin{bmatrix}\Delta A & \Delta B\end{bmatrix} = M\Lambda\begin{bmatrix}N_A & N_B\end{bmatrix}. \tag{3}$$

Remark 1. *The considered descriptor system in (1) involves both the uncertainties in (2) and (3) with distinct forms. They can be reasonably analyzed due to their inherited characteristics, including uncertain parameters that exist in the difference term and system matrices [34].*

Lemma 1 [38]. *Denote a matrix $\Omega = \Omega^T$, with compatible matrices M and N, such that*

$$\Omega + M\Lambda N + N^T\Lambda^T M^T < 0$$

where Λ satisfies $\Lambda\Lambda^T \leq I$, if and only if there exists a positive number α satisfying

$$\Omega + \alpha MM^T + \alpha^{-1}N^T N < 0.$$

To cope with the control issues of the discrete singular system (1), some definitions for the nominal form, $Ex(k+1) = Ax(k)$, are given as follows.

Definition 1 [8,33,35].

1. *The matrix pair $(E,\ A)$ is asserted to be regular if* $\det(zE - A) \neq 0$.
2. *The matrix pair $(E,\ A)$ is asserted to be causal if* $\det(zE - A) \neq 0$ *and deg[det(zE − A)] = rank(E).*
3. *Let the characteristic polynomial of the nominal system $Ex(k+1) = Ax(k)$ be $F(z) =$* $\det(zE - A)$. *The nominal system is asserted to be D-admissible, if it is regular, causal, and all of the finite solutions of $F(z) = 0$ satisfy $z \in D(a,\ r) = \{z : |z - a| < r\}$, where $|a| + r < 1$. And, the nominal system is asserted to be admissible, if it is regular, causal, and all of the finite solutions of $F(z) = 0$ satisfy $z \in D(0,\ 1)$.*

Some previous results are introduced below.

Lemma 2 [10]. *The nominal system, $Ex(k+1) = Ax(k)$, is admissible if and only if there exist two matrices, $P > 0$ and Q, satisfying*

$$A^T PA - E^T PE + QS^T A + A^T SQ^T < 0 \tag{4}$$

where $S \in R^{n\times(n-m)}$ with $E^T S = 0$ and rank(S) = $n - m$.

Lemma 3 [33]. *The nominal system, $Ex(k+1) = Ax(k)$, is D-admissible if and only if there exist two matrices, $P > 0$ and Q, satisfying*

$$\frac{(A - aE)}{r}^T P\frac{(A - aE)}{r} - E^T PE + QS^T A + A^T SQ^T < 0 \tag{5}$$

where $S \in R^{n\times(n-m)}$ with $E^T S = 0$ and rank(S) = $n - m$.

Deducing from the equivalent admissibility issues of the symmetric form of $(E,\ A)$ [39], we can replace $(E,\ A)$ in Equation (5) by the transpose pair $(E^T,\ A^T)$ and obtain a symmetric manner as follows.

Corollary 1. *The nominal system, $Ex(k+1) = Ax(k)$, is D-admissible if and only if there exist two matrices, $P > 0$ and Q, satisfying*

$$\frac{(A-aE)}{r}P\frac{(A-aE)^T}{r} - EPE^T + QS^TA^T + ASQ^T < 0 \tag{6}$$

where $S \in R^{n\times(n-m)}$ with $ES = 0$ and rank(S) = $n - m$.

For the controller design issue, we present a distinct form by replacing the matrix S by PS with the nonsingular matrix P.

Corollary 2. *The nominal system, $Ex(k+1) = Ax(k)$, is D-admissible if and only if there exist two matrices, $P > 0$ and Q, satisfying*

$$\frac{(A-aE)}{r}P\frac{(A-aE)^T}{r} - EPE^T + QS^TPA^T + APSQ^T < 0 \tag{7}$$

where $S \in R^{n\times(n-m)}$ with $EPS = 0$ and rank(S) = $n - m$.

3. D-Admissibility and State Feedback Control

A refined robust admissible analysis criterion for the uncertain discrete singular system (1) with (2) and (3) is derived as follows.

Theorem 1. *The system (1) subjected to the uncertainties (2) and (3) with free input is D-admissible, if there exist a set of matrices $P > 0$, Q_i, $i = 1, 2, \ldots, q_E$, and scalars $\alpha_{ij} > 0$, $\forall i \le j$, satisfying*

$$\begin{bmatrix} ASQ_i{}^T + Q_iS^TA^T - E_iPE_i^T + \alpha_{ii}MM^T & (A-aE_i)P & Q_iS^TN_A^T \\ P(A-aE_i)^T & -r^2P & PN_A^T \\ N_ASQ_i^T & N_AP & -\alpha_{ii}I \end{bmatrix} < 0,\ \forall i, \tag{8}$$

$$\begin{bmatrix} AS(Q_i+Q_j)^T + (Q_i+Q_j)S^TA^T - E_iPE_j^T - E_jPE_i^T + \alpha_{ij}MM^T & (2A-aE_i-aE_j)P & (Q_i+Q_j)S^TN_A^T \\ P(2A-aE_i-aE_j)^T & -2r^2P & 2PN_A^T \\ N_AS(Q_i+Q_j)^T & 2N_AP & -\alpha_{ij}I \end{bmatrix} < 0, \quad \forall i < j, \tag{9}$$

where $S \in R^{n\times(n-m)}$ with $E_iS = 0$, $\forall i$, and rank(S) = $n - m$.

Proof. Based on Corollary 1 associated with the Schur complement [36], Equation (6) can be equivalent to

$$\begin{bmatrix} ASQ^T + QS^TA^T - EPE^T & (A-aE)P \\ P(A-aE)^T & -r^2P \end{bmatrix} < 0.$$

Assume that matrices $P > 0$ and Q_i, $i = 1, 2, \ldots, q_E$, and scalars $\alpha_{ij} > 0$, $\forall i \le j$ satisfy the inequalities (8) and (9). Based on Corollary 1 with the above equation for the system (1) with uncertainties (2) and (3), and letting $\widetilde{Q} \equiv \sum_i e_iQ_i$, we can verify the D-admissibility by

$$
\begin{aligned}
&\begin{bmatrix} (A+\Delta A)S\widetilde{Q}^T+\widetilde{Q}S^T(A+\Delta A)^T-\widetilde{E}P\widetilde{E}^T & (A+\Delta A-a\widetilde{E})P \\ P(A+\Delta A-a\widetilde{E})^T & -r^2P \end{bmatrix} \\
&=\begin{bmatrix} (A+\Delta A)S\left(\sum_i e_iQ_i\right)^T+\left(\sum_i e_iQ_i\right)S^T(A+\Delta A)^T-\left(\sum_i e_iE_i\right)P\left(\sum_i e_iE_i\right)^T & \left(A+\Delta A-a\left(\sum_i e_iE_i\right)\right)P \\ P\left(A+\Delta A-a\left(\sum_i e_iE_i\right)\right)^T & -r^2P \end{bmatrix} \\
&=\sum_i e_i^2\begin{bmatrix} (A+\Delta A)SQ_i{}^T+Q_iS^T(A+\Delta A)^T-E_iPE_i^T & (A+\Delta A-\alpha E_i)P \\ P(A+\Delta A-\alpha E_i)^T & -r^2P \end{bmatrix} \\
&+\sum_{i<j} e_ie_j\begin{bmatrix} (A+\Delta A)S(Q_i+Q_j)^T+(Q_i+Q_j)S^T(A+\Delta A)^T-E_iPE_j^T-E_jPE_i^T & (2A+2\Delta A-aE_i-aE_j)P \\ P(2A+2\Delta A-aE_i-aE_j)^T & -2r^2P \end{bmatrix} \\
&<0.
\end{aligned}
$$

Substituting $\Delta A = M\Lambda N_A$ into the first terms of the above leads to

$$
\begin{aligned}
&\begin{bmatrix} (A+\Delta A)SQ_i{}^T+Q_iS^T(A+\Delta A)^T-E_iPE_i^T & (A+\Delta A-aE_i)P \\ P(A+\Delta A-aE_i)^T & -r^2P \end{bmatrix} \\
&=\begin{bmatrix} ASQ_i{}^T+Q_iS^TA^T-E_iPE_i^T & (A-aE_i)P \\ P(A-aE_i)^T & -r^2P \end{bmatrix}+\begin{bmatrix} \Delta ASQ_i{}^T+Q_iS^T\Delta A^T & \Delta AP \\ P\Delta A^T & 0 \end{bmatrix} \\
&=\begin{bmatrix} ASQ_i{}^T+Q_iS^TA^T-E_iPE_i^T & (A-aE_i)P \\ P(A-aE_i)^T & -r^2P \end{bmatrix}+\begin{bmatrix} M \\ 0 \end{bmatrix}\Lambda\begin{bmatrix} N_ASQ_i^T & N_AP \end{bmatrix}+\begin{bmatrix} Q_iS^TN_A^T \\ PN_A^T \end{bmatrix}\Lambda^T\begin{bmatrix} M^T & 0 \end{bmatrix}
\end{aligned}
\quad ,\forall i
$$

And according to Lemma 1, the above can be equivalent to

$$
\Rightarrow\begin{bmatrix} ASQ_i{}^T+Q_iS^TA^T-E_iPE_i^T & (A-aE_i)P \\ P(A-aE_i)^T & -r^2P \end{bmatrix}+\alpha_{ii}\begin{bmatrix} M \\ 0 \end{bmatrix}\begin{bmatrix} M^T & 0 \end{bmatrix}+\alpha_{ii}^{-1}\begin{bmatrix} Q_iS^TN_A^T \\ PN_A^T \end{bmatrix}\begin{bmatrix} N_ASQ_i^T & N_AP \end{bmatrix}
\quad ,\forall i
$$

Thus, by the Schur complement, they lead to

$$
\Rightarrow\begin{bmatrix} ASQ_i{}^T+Q_iS^TA^T-E_iPE_i^T+\alpha_{ii}MM^T & (A-aE_i)P & Q_iS^TN_A^T \\ P(A-aE_i)^T & -r^2P & PN_A^T \\ N_ASQ_i^T & N_AP & -\alpha_{ii}I \end{bmatrix},\ \forall i,
$$

which are identical to (8).

Similarly, the second terms can be deduced to be

$$
\Rightarrow\begin{bmatrix} AS(Q_i+Q_j)^T+(Q_i+Q_j)S^TA^T-E_iPE_j^T-E_jPE_i^T+\alpha_{ij}MM^T & (2A-aE_i-aE_j)P & (Q_i+Q_j)S^TN_A^T \\ P(2A-aE_i-aE_j)^T & -2r^2P & 2PN_A^T \\ N_AS(Q_i+Q_j)^T & 2N_AP & -\alpha_{ij}I \end{bmatrix}
\quad ,\ \forall i<j,
$$

which are identical to (9). Thus, if (8) and (9) are satisfied, we can conclude that the considered system is D-admissible according to Corollary 1. □

By Theorem 1, we can simply verify the system with admissibility by letting $a = 0$ and $r = 1$, i.e., the D-admissibility with the unit circle $D(0,\ 1)$, as follows.

Corollary 3. *The system (1) subjected to the uncertainties (2) and (3) with free input is admissible, if there exist matrices $P > 0$ and Q_i, $i = 1, 2, \ldots, q_E$, and scalars $\alpha_{ij} > 0$, $\forall i \leq j$, satisfying*

$$\begin{bmatrix} ASQ_i^T + Q_iS^TA^T - E_iPE_i^T + \alpha_{ii}MM^T & AP & Q_iS^TN_A^T \\ PA^T & -P & PN_A^T \\ N_ASQ_i^T & N_AP & -\alpha_{ii}I \end{bmatrix} < 0, \ \forall i, \tag{10}$$

$$\begin{bmatrix} AS(Q_i+Q_j)^T + (Q_i+Q_j)S^TA^T - E_iPE_j^T - E_jPE_i^T + \alpha_{ij}MM^T & 2AP & (Q_i+Q_j)S^TN_A^T \\ 2PA^T & -2P & 2PN_A^T \\ N_AS(Q_i+Q_j)^T & 2N_AP & -\alpha_{ij}I \end{bmatrix} < 0, \ \forall i < j, \tag{11}$$

where $S \in R^{n\times(n-m)}$ with $E_iS = 0$, $\forall i$, and rank(S) = $n - m$.

Remark 2. *Comparing with the previous result [33], the LMIs' constraint number is $q_A \times (q_E + C_{q_E}^2)$.* Nevertheless, in Theorem 1, the new approach can achieve a compact set of LMIs with $(q_E + C_{q_E}^2)$ and involves multiple slack matrices Q_i, $i = 1, 2, \ldots, q_E$. They both may be useful to lessen conservatism of admissible analysis.

Subsequently, by introducing the state feedback control, i.e., the considered system with $u(k) = Kx(k)$ in (1), the control design conditions are further presented.

Theorem 2. *The system (1) with $u(k) = Kx(k)$ subjected to the uncertainties (2) and (3) is D-admissible, if there exist matrices $P > 0$ and Q_i, $i = 1, 2, \ldots, q_E$, X, and scalars $\alpha_{ij} > 0$, $\forall i \leq j$, satisfying*

$$\begin{bmatrix} \Phi_1 & (A - aE_i)P + BX & Q_i(N_APS + N_BXS)^T \\ P(A - aE_i)^T + X^TB^T & -r^2P & PN_A^T + X^TN_B^T \\ (N_APS + N_BXS)Q_i^T & N_AP + N_BX & -\alpha_{ii}I \end{bmatrix} < 0, \ \forall i, \tag{12}$$

$$\begin{bmatrix} \Phi_2 & (2A - aE_i - aE_j)P + 2BX & (Q_i+Q_j)(N_APS + N_BXS)^T \\ P(2A - aE_i - aE_j)^T + 2X^TB^T & -2r^2P & 2PN_A^T + 2X^TN_B^T \\ (N_APS + N_BXS)(Q_i+Q_j)^T & 2N_AP + 2N_BX & -\alpha_{ij}I \end{bmatrix} < 0, \ \forall i < j, \tag{13}$$

where

$$\Phi_1 = APSQ_i^T + Q_iS^TPA^T + BXSQ_i^T + Q_iS^TX^TB^T - E_iPE_i^T + \alpha_{ii}MM^T$$

$$\Phi_2 = APS(Q_i+Q_j)^T + (Q_i+Q_j)S^TPA^T + BXS(Q_i+Q_j)^T + (Q_i+Q_j)S^TX^TB^T - E_iPE_j^T - E_jPE_i^T + \alpha_{ij}MM^T$$

where the matrix $S \in R^{n\times(n-m)}$ with $E_iPS = 0$, $\forall i$, and rank(S) = $n - m$. Then, a state feedback gain with a prescribed pole disk $D(a, r) \subset D(0, 1)$ with the center at a and the radius r can be determined by $K = XP^{-1}$.

Proof. Deducing from Corollary 2 associated with the Schur complement, Equation (7) can be equivalent to

$$\begin{bmatrix} APSQ^T + QS^TPA^T - EPE^T & (A - aE)P \\ P(A - aE)^T & -r^2P \end{bmatrix} < 0$$

Assume that matrices $P > 0$ and Q_i, $i = 1, 2, \ldots, q_E$, X, and scalars $\alpha_{ij} > 0$, $\forall i \leq j$ satisfy the inequalities (12) and (13). Based on Corollary 2 associated with the above for the resulting uncertain closed-loop singular system (1) with $u(t) = Kx(t)$, and letting $\widetilde{A}_C \equiv A + \Delta A + BK + \Delta BK$, $\widetilde{Q} \equiv \sum_i e_iQ_i$, we can verify the D-admissibility of (1) by

$$
\begin{aligned}
&\begin{bmatrix} \widetilde{A}_C PS\widetilde{Q}^T + \widetilde{Q}S^T P\widetilde{A}_C^T - \widetilde{E}P\widetilde{E}^T & (\widetilde{A}_C - a\widetilde{E})P \\ P(\widetilde{A}_C - a\widetilde{E})^T & -r^2P \end{bmatrix} \\
&= \begin{bmatrix} \widetilde{A}_C PS\left(\sum_i e_i Q_i\right)^T + \left(\sum_i e_i Q_i\right)S^T P\widetilde{A}_C^T - \left(\sum_i e_i E_i\right)P\left(\sum_i e_i E_i\right)^T & \left(\widetilde{A}_C - a\left(\sum_i e_i E_i\right)\right)P \\ P\left(\widetilde{A}_C - a\left(\sum_i e_i E_i\right)\right)^T & -r^2P \end{bmatrix} \\
&= \sum_i e_i^2 \begin{bmatrix} \widetilde{A}_C PSQ_i^T + Q_i S^T P\widetilde{A}_C^T - E_i PE_i^T & (\widetilde{A}_C - aE_i)P \\ P(\widetilde{A}_C - aE_i)^T & -r^2P \end{bmatrix} \\
&+ \sum_{i<j} e_i e_j \begin{bmatrix} \widetilde{A}_C PS(Q_i + Q_j)^T + (Q_i + Q_j)S^T P\widetilde{A}_C^T - E_i PE_j^T - E_j PE_i^T & (2\widetilde{A}_C - aE_i - aE_j)P \\ P(2\widetilde{A}_C - aE_i - aE_j)^T & -2r^2P \end{bmatrix} \\
&< 0.
\end{aligned}
$$

Denote $X = KP$. Substituting $\widetilde{A}_C = A + \Delta A + BK + \Delta BK$ and $\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = M\Lambda \begin{bmatrix} N_A & N_B \end{bmatrix}$ into the first terms of the above leads to

$$
\begin{aligned}
&\begin{bmatrix} \widetilde{A}_C PSQ_i^T + Q_i S^T P\widetilde{A}_C^T - E_i PE_i^T & (\widetilde{A}_C - aE_i)P \\ P(\widetilde{A}_C - aE_i)^T & -r^2P \end{bmatrix} \\
&= \begin{bmatrix} APSQ_i^T + Q_i S^T A^T + BKP + PK^T B^T - E_i PE_i^T & (A - aE_i)P + BKP \\ P(A - aE_i)^T + PK^T B^T & -r^2P \end{bmatrix} \\
&+ \begin{bmatrix} \Delta APSQ_i^T + Q_i S^T P\Delta A^T + \Delta BKPSQ_i^T + Q_i S^T PK^T \Delta B^T & \Delta AP + \Delta BKP \\ P\Delta A^T + PK^T \Delta B^T & 0 \end{bmatrix} \\
&= \begin{bmatrix} APSQ_i^T + Q_i S^T A^T + BX + X^T B^T - E_i PE_i^T & (A - aE_i)P + BX \\ P(A - aE_i)^T + X^T B^T & -r^2P \end{bmatrix} \\
&+ \begin{bmatrix} M \\ 0 \end{bmatrix} \Lambda \begin{bmatrix} N_A PSQ_i^T + N_B XSQ_i^T & N_A P + N_B X \end{bmatrix} + \begin{bmatrix} Q_i S^T PN_A^T + Q_i S^T X^T N_B^T \\ PN_A^T + X^T N_B^T \end{bmatrix} \Lambda^T \begin{bmatrix} M^T & 0 \end{bmatrix} \\
&\qquad\qquad\qquad

which are identical to (13). Thus, if (12) and (13) are satisfied, the considered system with $u(k) = Kx(k)$ is ensured to be D-admissible according to Corollary 2. □

Based on Theorem 2, we can simplify to conduct a state feedback control for the system (1) with admissible assurance by letting $a = 0$ and $r = 1$ in the following.

Corollary 4. *The system (1) with $u(k) = Kx(k)$ subjected to the uncertainties (2) and (3) is admissible, if there exist matrices $P > 0$ and Q_i, $i = 1, 2, \ldots, q_E$, X, and scalars $\alpha_{ij} > 0$, $\forall i \leq j$, satisfying*

$$\begin{bmatrix} \Phi_1 & AP + BX & Q_i(N_APS + N_BXS)^T \\ PA^T + X^TB^T & -P & PN_A^T + X^TN_B^T \\ (N_APS + N_BXS)Q_i^T & N_AP + N_BX & -\alpha_{ii}I \end{bmatrix} < 0, \ \forall i, \tag{14}$$

$$\begin{bmatrix} \Phi_2 & 2AP + 2BX & (Q_i + Q_j)(N_APS + N_BXS)^T \\ 2PA^T + 2X^TB^T & -2P & 2PN_A^T + 2X^TN_B^T \\ (N_APS + N_BXS)(Q_i + Q_j)^T & 2N_AP + 2N_BX & -\alpha_{ij}I \end{bmatrix} < 0, \ \forall i < j, \tag{15}$$

where

$$\Phi_1 = APSQ_i{}^T + Q_iS^TPA^T + BXSQ_i{}^T + Q_iS^TX^TB^T - E_iPE_i^T + \alpha_{ii}MM^T$$

$$\Phi_2 = APS(Q_i + Q_j)^T + (Q_i + Q_j)S^TPA^T + BXS(Q_i + Q_j)^T + (Q_i + Q_j)S^TX^TB^T - E_iPE_j^T - E_jPE_i^T + \alpha_{ij}MM^T$$

where the matrix $S \in R^{n\times(n-m)}$ with $E_iPS = 0$, $\forall i$, and rank(S) = $n - m$. Then, a state feedback gain with admissible assurance can be determined by $K = XP^{-1}$.

From the proposed design criteria in Theorem 2 and Corollary 4, we summarize the design steps as follows.

Design procedure:

Step 1: Based on the descriptor system (1) with (2) and (3), denote a set of *Ei* by (2), and M, N_A, and N_B by (3).
Step 2: Denote a matrix S which is of full-column rank and satisfies $E_iPS = 0$, $\forall i$.
Step 3: Initially denote Q_i with a compatible dimension.
Step 4: Construct, respectively, LMI constraint sets by (12) and (13) with D-admissible assurance or (14) and (15) with admissible assurance.
Step 5: Evaluate the constructed LMIs from the LMI tool [37] for existing solutions $P > 0$, X and scalars $\alpha_{ij} > 0$.
Step 6: If the LMIs are feasible, a satisfying control gain can be evaluated by $K = XP^{-1}$; otherwise, no satisfying control gain can be obtained. End the design procedure.

4. Illustrative Examples

Two numerical examples are introduced to verify the effectiveness and applicability as follows.

Example 1. *A discrete singular system subjected to parametric perturbation is represented as*

$$\left(\begin{bmatrix} 2 & 1+w_1 & 0 \\ 0 & 1+w_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) x(k+1) = \left(\begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.2 & -0.5 & 0 \\ 0.4 & -0.2 & 0.4 \end{bmatrix} + w_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.47 & 0 & 0.36 \end{bmatrix} \right) x(k),$$

where $|w_1| \leq 0.52$, $|w_2| \leq 0.19$, and $|w_3| \leq 1$.

Since the considered singular system involves the uncertain difference term matrix $\widetilde{E}$, some existing works [17,18,39] with a constant E are not applicable. Moreover, based on the previous work in [33], we first denote the matrices' vertices for the difference term and system's matrices as

$$E_1 = \begin{bmatrix} 2 & 1.52 & 0 \\ 0 & 1.19 & 0 \\ 0 & 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 2 & 0.48 & 0 \\ 0 & 1.19 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 2 & 1.52 & 0 \\ 0 & 0.81 & 0 \\ 0 & 0 & 0 \end{bmatrix}, E_4 = \begin{bmatrix} 2 & 0.48 & 0 \\ 0 & 0.81 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

with $q_E = 4$ and

$$A_1 = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.2 & -0.5 & 0 \\ 0.87 & -0.2 & 0.76 \end{bmatrix}, A_2 = \begin{bmatrix} 0.5 & 0.4 & 0.6 \\ 0.2 & -0.5 & 0 \\ -0.07 & -0.2 & 0.04 \end{bmatrix},$$

with $q_A = 2$. According to Theorem 3.1 in [33], we can form $q_A \times (q_E + C_{q_E}^2)$ = 20 LMIs for analyzing the admissibility of the considered system. However, when we evaluated them using the LMI solver [37], it showed infeasibility and we could not conclude the admissibility for this system. Furthermore, when evaluating the regarding system from the proposed approach in [19], the LMI solver also showed infeasibility.

However, in this work, the uncertain term of a system matrix can be alternatively described as $\Delta A = M\Lambda N_A$ with $M = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ and $N_a = \begin{bmatrix} 0.47 & 0 & 0.36 \end{bmatrix}$. Based on Corollary 3 with a denoted matrix $S = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$, $E_1 S = E_2 S = 0$, the admissible analyzing conditions for this system can be formed by the LMI constraints with the number $(q_E + C_{q_E}^2) = 10$ from (10) and (11). By evaluating the LMI, we can obtain satisfying feasible solutions such as

$$P = \begin{bmatrix} 154.8676 & -1.0631 & -125.8808 \\ -1.0631 & 20.7002 & -0.1751 \\ -125.8808 & -0.1751 & 145.8124 \end{bmatrix} \times 10^{-2} > 0, Q_1 = \begin{bmatrix} 111.1610 \\ -2.9668 \\ -60.6450 \end{bmatrix} \times 10^{-2},$$

$$Q_2 = \begin{bmatrix} 106.5411 \\ -8.5998 \\ -62.0371 \end{bmatrix} \times 10^{-2}, Q_3 = \begin{bmatrix} 83.2633 \\ -7.8827 \\ -45.4755 \end{bmatrix} \times 10^{-2}, Q_4 = \begin{bmatrix} 110.5696 \\ -9.5869 \\ -64.3897 \end{bmatrix} \times 10^{-2},$$

$$\min_{i \le j} \alpha_{ij} = \alpha_{33} = 0.2 > 0.$$

According to Corollary 3, we thus conclude that the regarded system with uncertainties is robustly admissible.

For verification, we denote the initial condition $x(0) = \begin{bmatrix} 20 & -10 & -25 \end{bmatrix}^T$ and the uncertainties terms as $\omega_1 = 0.52 \sin k\pi$, $\omega_2 = 0.19 \cos 2k\pi$, and $\omega_3 = \sin 4k\pi$. The regarded system, subjected to the given uncertainties, is then simulated. By observing Figure 1, it is evident that the state behaviors $x(k)$ have convergent trajectories with all the allowable uncertainties.

Example 2. *A third-order singular system with parametric perturbation is described by*

$$\begin{bmatrix} 1 & 0 & 0 \\ 1+w_1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} x(k+1) = \left(\begin{bmatrix} 1 & 0.8 & -1 \\ -1.2 & 1.5 & 2 \\ 0.6 & 0.4 & 1 \end{bmatrix} + w_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right) x(k) + \left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + w_3 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) u(k),$$

where the perturbed uncertainties are assumed to fulfill $|w_1| \le 1$, $0 \le w_2 \le 0.6$, *and* $0 \le w_3 \le 1$.

The vertices of matrices $\widetilde{\mathrm{E}}$ in (2) for the considered system can be denoted as

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The uncertain system matrices can be described by $\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = M\Lambda \begin{bmatrix} N_A & N_B \end{bmatrix}$ with $M = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$, $N_a = \begin{bmatrix} 0 & 0.6 & 0.6 \end{bmatrix}$, and $N_b = 1$. By primary evaluation, the nominal unforced system is originally unstable, i.e., the poles of the system with free input are not within $D(0,\ 1)$. When letting $x(0) = \begin{bmatrix} -3 & 7 & -1 \end{bmatrix}^T$, the system with free input is firstly simulated, and the state behaviors are drawn in Figure 2. By observation, the state responses of the system with free input are divergent. Thus, we need to conduct a proper control law for compensation. However, the previous work [19] cannot be applied to design a practicable controller with respect to D-admissible assurance.

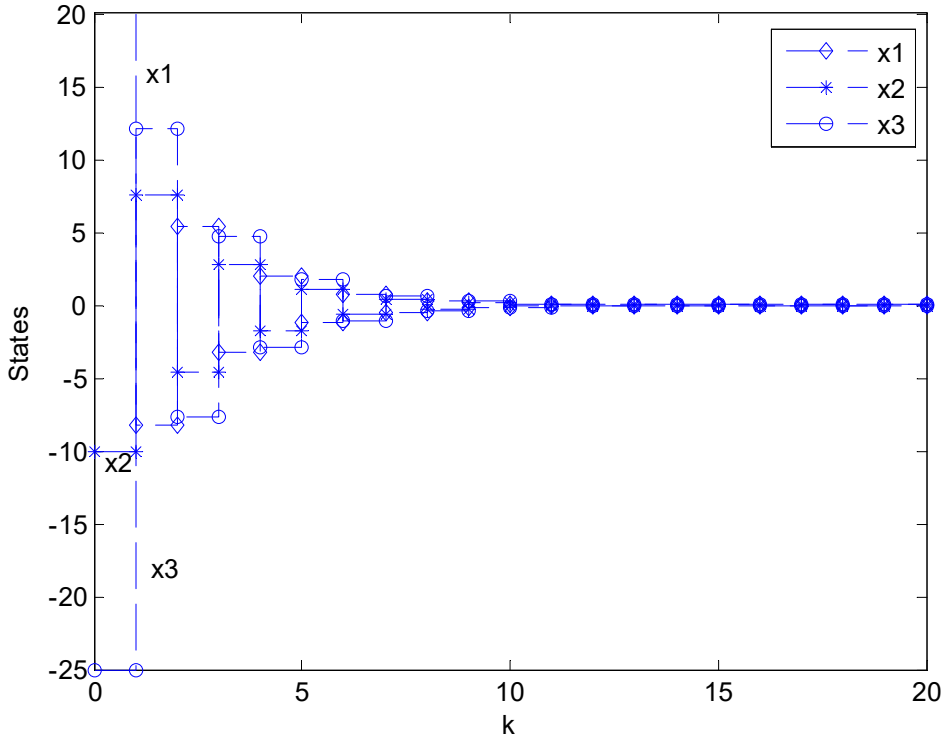

Figure 1. State responses of the considered system in Example 1.

By the design criteria of Theorem 2, we can conduct a state feedback controller with D-admissible assurance of a prescribed pole disk $D(0.2,\ 0.6)$. Thus, based on Equations (12) and (13), we correspondingly formulate three LMI constraints and denote

$$S = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, Q_1 = Q_2 = -10 \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ P = \begin{bmatrix} P_1 & 0 \\ 0 & P_3 \end{bmatrix},$$

with $P_1 \in R^{2\times 2}$, $P_3 \in R^{1\times 1}$ satisfying $E_i PS = 0, \forall i$. Using the LMI solver for evaluation, a set of feasible solutions can be attained by

$$
\begin{aligned}
P &= \begin{bmatrix} 3.3813 & -15.1082 & 0 \\ -15.1082 & 109.5238 & 0 \\ 0 & 0 & 2.0432 \end{bmatrix} \times 10^4 > 0, \\
X &= \begin{bmatrix} 6.7012 & -48.1456 & 3.4029 \end{bmatrix} \times 10^4, \\
\alpha_{11} &= 1.3965 \times 10^6, \\
\alpha_{12} &= 2.5093 \times 10^6, \\
\alpha_{22} &= 1.4135 \times 10^6.
\end{aligned}
$$

Thus, a satisfying state feedback gain with the pole region $D(0.2,\ 0.6)$ can be determined by

$$K = XP^{-1} = \begin{bmatrix} 4.6076 & -43.3235 & 166.5498 \end{bmatrix} \times 10^{-2}.$$

When given the same initial condition $x(0) = \begin{bmatrix} -3 & 7 & -1 \end{bmatrix}^T$, the compensated system with the state feedback controller is simulated once again. Furthermore, the state behaviors $x(k)$ and the control input $u(k)$ are depicted in Figures 3 and 4, respectively. From Figure 3, all the state responses have well-convergent trajectories. Thus, the considered system with the state feedback gain is ensured to be D-admissible, where the closed-loop system can meet the prescribed performance requirement with the pole region $D(0.2,\ 0.6)$.

Remark 3. *Using the proposed design scheme in Theorem 2 and Corollary 4, we can conduct a conventional state feedback controller with D-admissible assurance or admissible assurance for the considered discrete singular system subjected to the uncertainties, which has the advantages of easy implementation and low cost. The given numerical examples have illustrated the feasibility and superiority of the developed methods. For nonlinear systems, we can perform the linearization by the given operating point and then can approximate to a linear form with uncertainties around the operating point. However, in the case of multiple operating points, we can incorporate the proposed results with existing intelligent control methods, such as fuzzy control systems [40,41], for further study.*

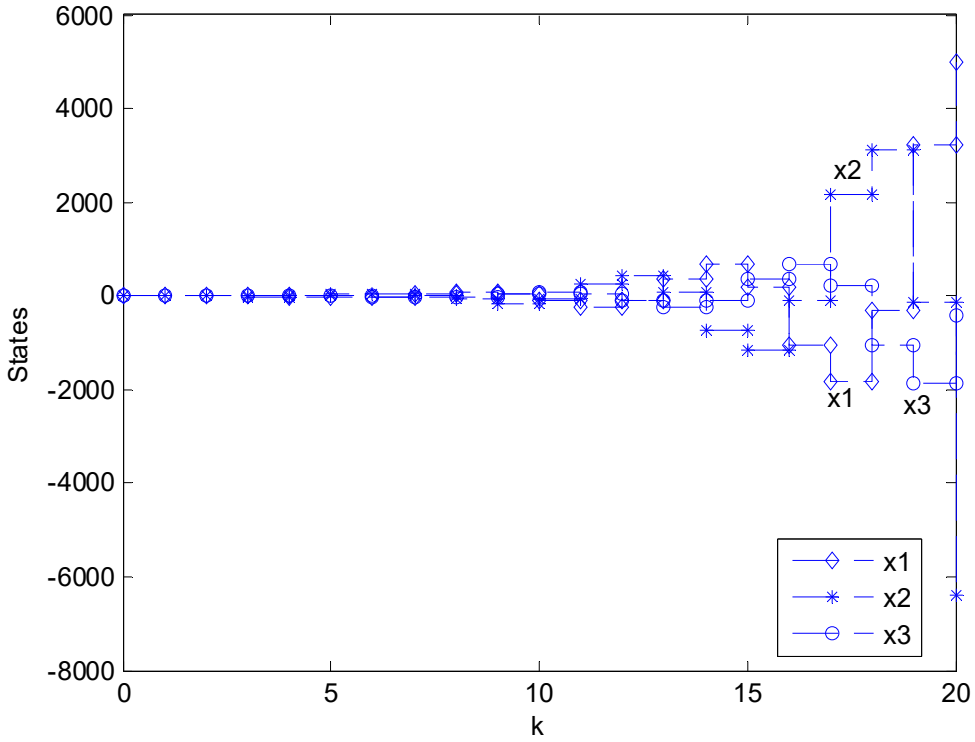

Figure 2. State responses of the open-loop system.

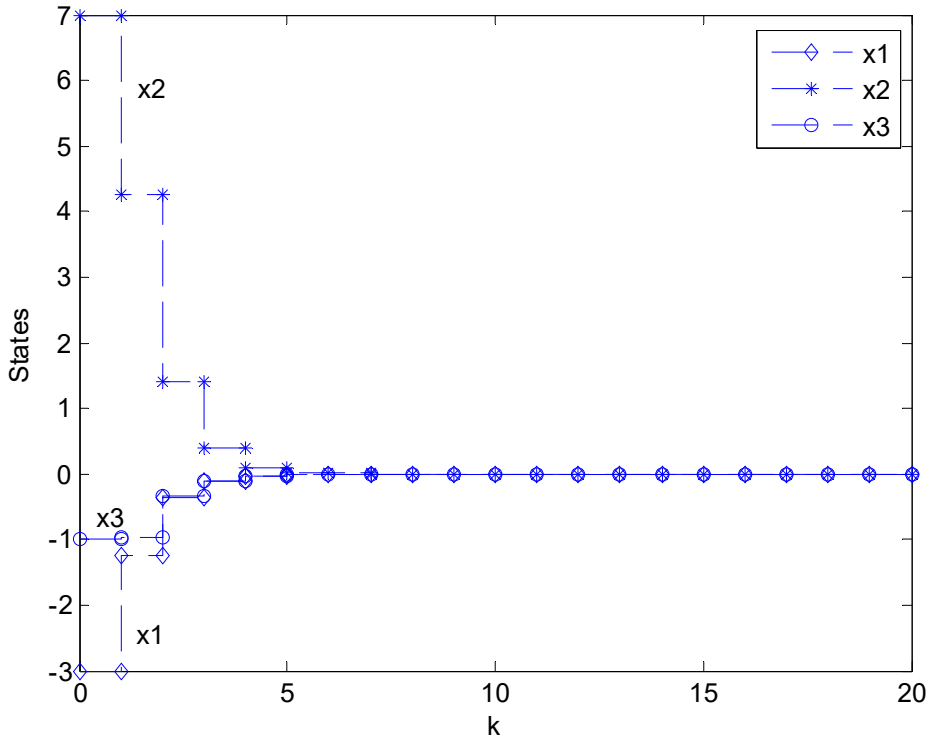

Figure 3. State responses with D-admissible assurance of the closed-loop system.

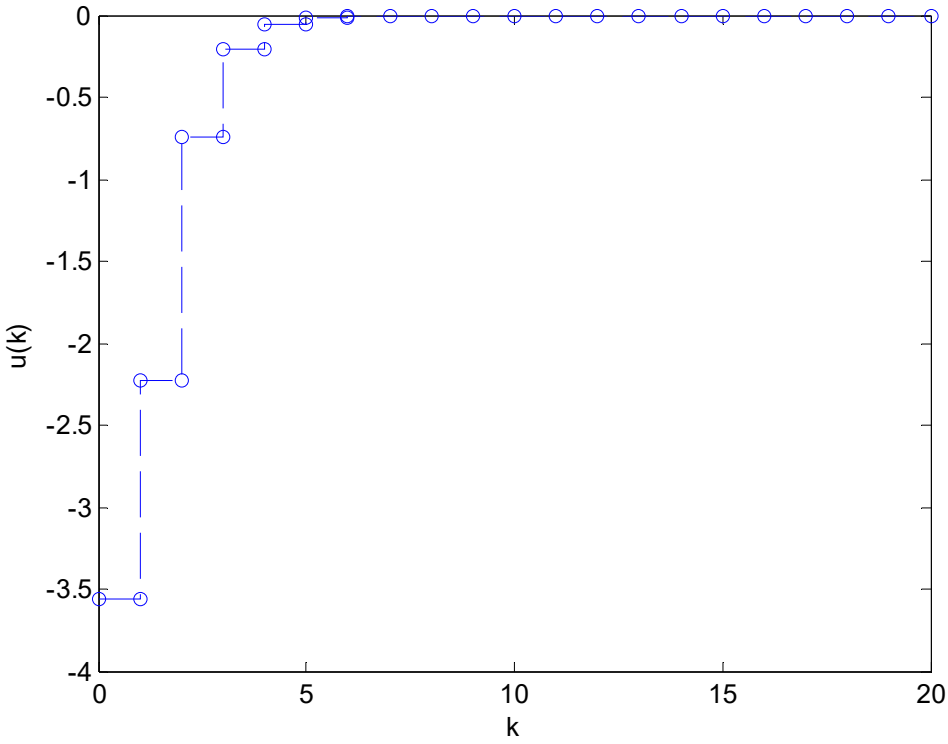

Figure 4. Control input trajectory.

5. Conclusions

In this work, we have dealt with state feedback control with D-admissible assurance for discrete singular systems subjected to the uncertain difference term and system matrices. Firstly, based on LMI techniques and matrix algebra manipulation, a refined D-admissible analysis criterion could be presented. The extended result has compact LMI constraints

and involves multiple slack matrices, which help to lessen the conservatism for admissible analysis. By introducing the state feedback control, we then focused on state feedback control with D-admissible assurance for the closed-loop system. Since the proposed design criteria can be formulated in terms of the strict LMIs, they are readily evaluated by current LMI tools to construct a state feedback controller with the prescribed system's performance. Finally, two illustrative examples demonstrate the feasibility and applicability of the derived results. In the future, we will incorporate the proposed design scheme with existing intelligent control to cope with real physical systems, nonlinear systems, and/or time-delay systems.

Funding: This research received no external funding.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Luenberger, D.G.; Arbel, A. Singular dynamical Leotief systems. *Econometrica* **1977**, *45*, 991–995. [CrossRef]
2. Zhang, Y.; Zhao, Y.; Li, N.; Wang, Y. Observer-based sliding mode controller design for singular bio-economic system with stochastic disturbance. *AIMS Math.* **2023**, *9*, 1472–1493. [CrossRef]
3. Newcomb, R.W. The semistate description of nonlinear time variable circuits. *IEEE Trans. Circuits Syst.* **1981**, *28*, 62–71. [CrossRef]
4. Kumar, A.; Daoutidis, P. Feedback control of nonlinear differential-algebraic-equation systems. *AIChE J.* **1995**, *41*, 619–636. [CrossRef]
5. Li, Y.; Geng, G.; Jiang, Q. A parallel contour integral method for eigenvalue analysis of power systems. *IEEE Trans. Power Syst.* **2017**, *32*, 624–632. [CrossRef]
6. Luenberger, D.G. Dynamic equations in descriptor form. *IEEE Trans. Autom. Control.* **1977**, *22*, 312–321. [CrossRef]
7. Lewis, F.L. A survey of linear singular systems. *Circuits Syst. Signal Process.* **1986**, *5*, 3–36. [CrossRef]
8. Dai, L. *Singular Control Systems. Lecture Notes in Control and Information Sciences*; Springer: New York, NY, USA, 1989; Volume 118.
9. Masubuchi, I.; Kamitane, Y.; Ohara, A.; Suda, N. H_∞ control for descriptor systems: A matrix inequalities approach. *Automatica* **1997**, *33*, 669–673. [CrossRef]
10. Xu, S.; Lam, J. Robust stability and stabilization of discrete singular systems: An equivalent characterization. *IEEE Trans. Autom. Control.* **2004**, *49*, 568–574. [CrossRef]
11. Yue, D.; Lam, J. Suboptimal robust mixed H_2/H_∞ controller design for uncertain descriptor systems with distributed delays. *Comput. Math. Appl.* **2004**, *47*, 1041–1055. [CrossRef]
12. Zhang, G.; Xia, Y.; Shi, P. New bounded real lemma for discrete-time singular systems. *Automatica* **2008**, *44*, 886–890. [CrossRef]
13. Chen, Y.; Xu, T.; Zeng, C.; Zhou, Z.; Zhang, Q. Simultaneous stabilization for uncertain descriptor systems with input saturation. *Int. J. Robust Nonlinear Control.* **2012**, *22*, 1938–1951. [CrossRef]
14. Zaghdoud, R.; Salhi, S.; Ksouri, M. State derivative feedback for singular systems. *IMA J. Math. Control. Inf.* **2018**, *35*, 611–626. [CrossRef]
15. Liu, R.R.; Xiong, J.; Zhang, Z. Reliable induced L_∞ control for polytopic uncertain continuous-time singular systems with dynamic quantization. *Appl. Math. Comput.* **2023**, *443*, 127765. [CrossRef]
16. Zhang, T.; Li, Y. Exponential Euler scheme of multi-delay Caputo–Fabrizio fractional-order differential equations. *Appl. Math. Lett.* **2022**, *124*, 107709. [CrossRef]
17. Belov, A.A.; Andrianova, O.G. Robust state feedback H_∞ control for discrete-time descriptor systems with norm-bounded parametric uncertainties. *Int. J. Syst. Sci.* **2019**, *50*, 1303–1312. [CrossRef]
18. Wang, J.; Wu, H.; Ji, X.; Liu, X. Robust finite-time stabilization for uncertain discrete-time linear singular systems. *IEEE Access* **2020**, *8*, 100645–100651. [CrossRef]
19. Huang, C.P. Admissibility analysis and controller design for discrete singular time-delay systems embracing uncertainties in the difference and systems' matrices. *Math. Probl. Eng.* **2021**, *2021*, 8845558. [CrossRef]
20. Chilali, M.; Gahinet, P. H_∞ design with pole placement constraints: An LMI approach. *IEEE Trans. Autom. Control.* **1996**, *41*, 358–367. [CrossRef]
21. Luo, J.S.; Bosch, P.P.J.; Weiland, S. New sufficient condition for robust root clustering of linear state space models with structured uncertainty. *IEE Proc.-Control. Theory Appl.* **1996**, *143*, 244–246. [CrossRef]
22. Chen, S.J.; Lin, J.L. Robust D-stability of discrete and continuous time interval systems. *J. Frankl. Inst.* **2004**, *341*, 505–517. [CrossRef]
23. Goncalves, E.N.; Palhares, R.M.; Takahashi, R.H.C.; Mesquita, R.C. New approach to robust D-stability analysis of linear time-invariant systems with polytope-bounded uncertainty. *IEEE Trans. Autom. Control.* **2006**, *51*, 1709–1714. [CrossRef]
24. Maamri, N.; Bachelier, O.; Mehdi, D. Pole placement in a union of regions with prespecified subregion allocation. *Math. Comput. Simul.* **2006**, *72*, 38–46. [CrossRef]

25. Xu, S.; Lam, J.; Zheng, L. Robust D-stability analysis for uncertain discrete singular systems with state delay. *IEEE Trans. Circuits Syst. I Fundam. Theory Appl.* **2002**, *49*, 551–555.
26. Chen, S.J. Robust D-stability analysis for linear discrete singular time-delay systems with structured and unstructured parameter uncertainties. *IMAJ. Math. Control. Inf.* **2003**, *20*, 153–165. [CrossRef]
27. Chen, S.H.; Chou, J.H. Robust D-stability analysis for linear uncertain discrete singular systems with state delay. *Appl. Math. Lett.* **2006**, *19*, 197–205. [CrossRef]
28. Chen, S.H.; Chou, J.H. Robust eigenvalue-clustering in a specified circular region for linear uncertain discrete singular systems with state delay. *Appl. Math. Comput.* **2007**, *186*, 1660–1670. [CrossRef]
29. Sari, B.; Bachelier, O.; Bosche, J.; Maamri, N.; Mehdi, D. Pole placement in non connected regions for descriptor models. *Math. Comput. Simul.* **2011**, *81*, 2617–2631. [CrossRef]
30. Lin, C.; Wang, J.L.; Yang, G.H.; Lam, J. Robust stabilization via state feedback for descriptor systems with uncertainties in the derivative matrix. *Int. J. Control.* **2000**, *73*, 407–415. [CrossRef]
31. Lin, C.; Lam, J.; Wang, J.; Yang, G.H. Analysis on robust stability for interval descriptor systems. *Syst. Control. Lett.* **2001**, *42*, 267–278. [CrossRef]
32. Lin, C.; Wang, Q.G.; Lee, T.H. Robust normalization and stabilization of uncertain descriptor systems with norm-bounded perturbations. *IEEE Trans. Autom. Control.* **2005**, *50*, 515–520.
33. Huang, C.P. Analysing robust D-admissibility for discrete singular systems subjected to parametric uncertainties in the difference matrix. *IMA J. Math. Control. Inf.* **2011**, *28*, 279–289. [CrossRef]
34. Huang, C.P. Specific D-admissibility and design issues for uncertain descriptor systems with parametric uncertainty in the derivative matrix. *Math. Probl. Eng.* **2016**, *2016*, 6142848. [CrossRef]
35. Mao, W.J. An LMI approach to D-stability and D-stabilization of linear discrete singular systems with state delay. *Appl. Math. Comput.* **2011**, *218*, 1694–1704. [CrossRef]
36. Boyd, S.; Ghaouim, L.E.; Feron, E.; Balakrishnan, V. *Linear Matrix Inequalities in Systems and Control Theory*; SIAM: Philadelphia, PA, USA, 1994.
37. Gahinet, P.; Nemirovski, A.; Jaub, A.J.; Chilali, M. *LMI Control Toolbox User's Guide*; The Mathworks Partner Series; The Mathworks, Inc.: Natick, MA, USA, 1995.
38. Petersen, I.R. A stabilization algorithm for a class of uncertain linear systems. *Syst. Control. Lett.* **1987**, *8*, 351–357. [CrossRef]
39. Sun, X.; Zhang, Q.; Yang, C.; Su, Z. Stability analysis and stabilization for discrete-time singular delay systems. *J. Syst. Eng. Electron.* **2011**, *22*, 482–487. [CrossRef]
40. Saravanakumar, T.; Lee, T.H. Hybrid-driven-based resilient control for networked T-S fuzzy systems with time-delay and cyber-attacks. *Int. J. Robust Nonlinear Control.* **2023**, *33*, 7869–7891. [CrossRef]
41. Tharanidharan, V.; Saravanakumar, T.; Anthoni, S.M. Robust and quantized repetitive tracking control for fractional-order fuzzy large-scale systems. *Int. J. Adapt. Control. Signal Process.* **2024**, *38*, 1496–1511. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.