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Sălăgean Differential Operator in Connection with Stirling Numbers

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Abstract: Sălăgean differential operator $\mathbb{D}^k$ plays an important role in the geometric function theory, where many studies are using this operator to introduce new subclasses of analytic functions defined in the open unit disk. Studies of Sălăgean differential operator $\mathbb{D}^k$ in connection with Stirling numbers are relatively new. In this paper, the differential operator $\mathbb{D}^k$ involving Stirling numbers is considered. A new sufficient condition involving Stirling numbers for the series $Y_\theta^s(z)$ written with the Pascal distribution are discussed for the subclass $\mathfrak{T}_k(\epsilon, b)$. Also, we provide a sufficient condition for the inclusion relation $\mathcal{I}_\theta^s(\mathcal{R}^\omega(\mathfrak{E}, \mathfrak{D})) \subset \mathfrak{T}_k(\epsilon, b)$. Further, we consider the properties of an integral operator related to Pascal distribution series. New special cases as a consequences of the main results are also obtained.

Keywords: analytic functions; Hadamard product; Stirling numbers; Pascal distribution series

MSC: 30C45



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1. Introduction

In recent years, several researchers used important distribution series, like the hypergeometric distribution series [1,2], Poisson distribution [3], Pascal distribution [4,5], Mittag–Leffler-type Poisson distribution [6,7], Miller–Ross-type Poisson distribution [8], binomial distribution [9,10], generalized distribution [11], confluent hypergeometric distribution [2], and hypergeometric-type probability distribution [12] to obtain some necessary and sufficient conditions for these distributions to belong to certain classes of analytic functions $\mathfrak{N}$, written as

$$h(z) = z + \sum_{\tau=2}^{\infty} a_{\tau} z^{\tau}, \quad (1)$$

and defined in the open unit disk $\wp = \{z \in \mathbb{C} : |z| < 1\}$.

For functions $h(z) \in \mathfrak{N}$ given by (1) and $g(z) \in \mathfrak{N}$ given by ([13,14])

$$g(z) = z + \sum_{\tau=2}^{\infty} c_{\tau} z^{\tau},$$

we define the Hadamard product (or convolution) of $h(z)$ and $g(z)$ by

$$(h * g)(z) = z + \sum_{k=1}^{\infty} a_{\tau} c_{\tau} z^{\tau} = (g * h)(z).$$

A variable X is said to be a Pascal distribution if it takes the values $0, 1, 2, 3, \dots$ with probabilities $(1 - \theta)^s, \frac{\theta s(1 - \theta)^s}{1!}, \frac{\theta^2 s(s + 1)(1 - \theta)^s}{2!}, \frac{\theta^3 s(s + 1)(s + 2)(1 - \theta)^s}{3!}, \dots$, respectively, where θ and s are called the parameters, and thus

$$P(X = v) = \binom{v + s - 1}{s - 1} \theta^v (1 - \theta)^s, \quad v = 0, 1, 2, 3, \dots$$

Let $\Lambda_\theta^s(\mathcal{K})$ be the following power series whose coefficients are probabilities of a Pascal distribution [4]:

$$\Lambda_\theta^s(\mathcal{K}) = \mathcal{K} + \sum_{\tau=2}^{\infty} \binom{\tau + s - 2}{s - 1} \theta^{\tau-1} (1 - \theta)^s \mathcal{K}^\tau, \quad \mathcal{K} \in \wp, \quad (2)$$

where $s \geq 1; 0 \leq \theta \leq 1$, and we note that, via the ratio test, the radius of convergence of the above series is infinity.

Now, we consider the linear operator (see, [4])

$$\mathcal{I}_\theta^s \mathcal{H}(\mathcal{K}) : \mathbb{N} \rightarrow \mathbb{N}$$

defined by the Hadamard product

$$\mathcal{I}_\theta^s \mathcal{H}(\mathcal{K}) = \Lambda_\theta^s(\mathcal{K}) * \mathcal{H}(\mathcal{K}) = \mathcal{K} + \sum_{\tau=2}^{\infty} \binom{\tau + s - 2}{s - 1} \theta^{\tau-1} (1 - \theta)^s a_\tau \mathcal{K}^\tau, \quad \mathcal{K} \in \wp. \quad (3)$$

Differential operators play an important role in the geometric function theory such as the Alexander integral operator [15], generalized Bernardi operator [16], Ruscheweyh derivative operator [17], Carlson–Shaffer operator [18], Sălăgean differential operator [19], Noor integral operator [20], Dziok–Srivastava operator [21], Al–Oboudi differential operator [22], and many others. Making use of these operators, many studies introduced new subclasses of $\mathbb{N}$ of analytic functions defined in the disk $\wp$ (see, for example, [23,24]).

For a function $\mathcal{H}(\mathcal{K})$ in $\mathbb{N}$, we define

$$\mathbb{D}^0 \mathcal{H}(\mathcal{K}) = \mathcal{H}(\mathcal{K}), \quad (4)$$

$$\mathbb{D}^1 \mathcal{H}(\mathcal{K}) = \mathcal{K} \mathcal{H}'(\mathcal{K}), \quad (5)$$

and, in general, we have

$$\mathbb{D}^\kappa \mathcal{H}(\mathcal{K}) = \mathcal{K} (\mathbb{D}^{\kappa-1} \mathcal{H}(\mathcal{K}))', \quad (\kappa \in \mathbb{N}). \quad (6)$$

The differential operator $\mathbb{D}^\kappa$ was introduced by Sălăgean [19].

The Stirling number of the second kind [25] is the number of ways to partition a set of κ objects into j non-empty subsets and is denoted by $S_{\kappa,j}$.

Studies of the Sălăgean operator $\mathbb{D}^\kappa$ in connection with Stirling numbers are relatively new, where in [26] (see [27]), Frasin noticed that operator $\mathbb{D}^\kappa$ can be written in terms of Stirling numbers, as follows:

$$\begin{aligned} \mathbb{D}^\kappa \mathcal{H}(\mathcal{K}) &= \mathcal{K} (\mathbb{D}^{\kappa-1} \mathcal{H}(\mathcal{K}))', \\ &= S_{\kappa,1} \mathcal{K} \mathcal{H}'(\mathcal{K}) + S_{\kappa,2} \mathcal{K}^2 \mathcal{H}''(\mathcal{K}) + S_{\kappa,3} \mathcal{K}^3 \mathcal{H}'''(\mathcal{K}) + \dots + S_{\kappa,\kappa} \mathcal{K}^\kappa \mathcal{H}^{(\kappa)}(\mathcal{K}), \\ &= \sum_{j=1}^{\kappa} S_{\kappa,j} \mathcal{K}^j \mathcal{H}^{(j)}(\mathcal{K}), \quad (\kappa \in \mathbb{N}), \end{aligned} \quad (7)$$

where

$$S_{\kappa,j} = j S_{\kappa-1,j} + S_{\kappa-1,j-1}, \text{ and } S_{\kappa,1} = S_{\kappa,\kappa} = 1. \quad (8)$$

For example,

(i) If $\kappa = 2$, we have

$$\begin{aligned}\mathbb{D}^2 h(\kappa) &= \kappa(\mathbb{D}^1 h(\kappa))', \\ &= \kappa h'(\kappa) + \kappa^2 h''(\kappa), \\ &= S_{2,1} \kappa h'(\kappa) + S_{2,2} \kappa^2 h''(\kappa),\end{aligned}$$

where

$$S_{2,1} = S_{2,2} = 1.$$

(ii) If $\kappa = 3$, we have

$$\begin{aligned}\mathbb{D}^3 h(\kappa) &= \kappa(\mathbb{D}^2 h(\kappa))', \\ &= \kappa h'(\kappa) + 3\kappa^2 h''(\kappa) + \kappa^3 h'''(\kappa), \\ &= S_{3,1} \kappa h'(\kappa) + S_{3,2} \kappa^2 h''(\kappa) + S_{3,3} \kappa^3 h'''(\kappa),\end{aligned}$$

where

$$\begin{aligned}S_{3,2} &= 2S_{2,2} + S_{2,1} = 3, \\ S_{3,1} &= S_{3,3} = 1.\end{aligned}$$

(iii) If $\kappa = 4$, we have

$$\begin{aligned}\mathbb{D}^4 h(\kappa) &= \kappa(\mathbb{D}^3 h(\kappa))', \\ &= \kappa h'(\kappa) + 7\kappa^2 h''(\kappa) + 6\kappa^3 h'''(\kappa) + \kappa^4 h^{(4)}(\kappa), \\ &= S_{4,1} \kappa h'(\kappa) + S_{4,2} \kappa^2 h''(\kappa) + S_{4,3} \kappa^3 h'''(\kappa) + S_{4,4} \kappa^4 h^{(4)}(\kappa),\end{aligned}$$

where

$$\begin{aligned}S_{4,2} &= 2S_{3,2} + S_{3,1} = 7, \\ S_{4,3} &= 3S_{3,3} + S_{3,2} = 6, \\ S_{4,1} &= S_{4,4} = 1.\end{aligned}$$

(iii) If $\kappa = 5$, we have

$$\begin{aligned}\mathbb{D}^5 h(\kappa) &= \kappa(\mathbb{D}^4 h(\kappa))', \\ &= \kappa h'(\kappa) + 15\kappa^2 h''(\kappa) + 25\kappa^3 h'''(\kappa) + 10\kappa^4 h^{(4)}(\kappa) + \kappa^5 h^{(5)}(\kappa), \\ &= S_{5,1} \kappa h'(\kappa) + S_{5,2} \kappa^2 h''(\kappa) + S_{5,3} \kappa^3 h'''(\kappa) + S_{5,4} \kappa^4 h^{(4)}(\kappa) + S_{5,5} \kappa^5 h^{(5)}(\kappa),\end{aligned}$$

where

$$\begin{aligned}S_{5,2} &= 2S_{4,2} + S_{4,1} = 15, \\ S_{5,3} &= 3S_{4,3} + S_{4,2} = 25, \\ S_{5,4} &= 4S_{4,4} + S_{4,3} = 10, \\ S_{5,1} &= S_{5,5} = 1.\end{aligned}$$

Table 1 below represents the coefficients $S_{\kappa,j}$ of $\kappa^j h^{(\kappa)}(\kappa)$; $1 \leq \kappa \leq 6$. For $\kappa = 2, 3, 4, 5$, we observe that

$$\begin{aligned}\mathbb{D}^2 h(\kappa) &= \kappa^2 h''(\kappa) + \kappa h'(\kappa), \\ \kappa + \sum_{\tau=2}^{\infty} \tau^2 a_{\tau} \kappa^{\tau} &= \kappa + \sum_{\tau=2}^{\infty} [\tau(\tau-1) + \tau] a_{\tau} \kappa^{\tau},\end{aligned}\tag{9}$$

$$\begin{aligned}\mathbb{D}^3 h(\kappa) &= \kappa^3 h'''(\kappa) + 3\kappa^2 h''(\kappa) + \kappa h'(\kappa), \\ \kappa + \sum_{\tau=2}^{\infty} \tau^3 a_{\tau} \kappa^{\tau} &= \kappa + \sum_{\tau=2}^{\infty} [\tau(\tau-1)(\tau-2) + 3\tau(\tau-1) + \tau] a_{\tau} \kappa^{\tau},\end{aligned}\quad (10)$$

$$\begin{aligned}\mathbb{D}^4 h(\kappa) &= \kappa^4 h^{(4)}(\kappa) + 6\kappa^3 h'''(\kappa) + 7\kappa^2 h''(\kappa) + \kappa h'(\kappa), \\ \kappa + \sum_{\tau=2}^{\infty} \tau^4 a_{\tau} \kappa^{\tau} &= \kappa + \sum_{\tau=2}^{\infty} [\tau(\tau-1)(\tau-2)(\tau-3) + 6\tau(\tau-1)(\tau-2) \\ &\quad + 7\tau(\tau-1) + \tau],\end{aligned}\quad (11)$$

and

$$\begin{aligned}\mathbb{D}^5 h(\kappa) &= \kappa^5 h^{(5)}(\kappa) + 15\kappa^4 h^{(4)}(\kappa) + 25\kappa^3 h'''(\kappa) + 10\kappa^2 h''(\kappa) + \kappa h'(\kappa) \\ \kappa + \sum_{\tau=2}^{\infty} \tau^5 a_{\tau} \kappa^{\tau} &= \kappa + \sum_{\tau=2}^{\infty} [\tau(\tau-1)(\tau-2)(\tau-3)(\tau-4) + 10\tau(\tau-1)(\tau-2)(\tau-3) \\ &\quad + 25\tau(\tau-1)(\tau-2) + 15\tau(\tau-1) + \tau] a_{\tau} \kappa^{\tau}.\end{aligned}\quad (12)$$

From (9)–(12), we conclude that

$$\begin{aligned}\tau &= (\tau-1) + 1 \\ &= S_{2,2}(\tau-1) + S_{2,1}, \\ \tau^2 &= (\tau-1)(\tau-2) + 3(\tau-1) + 1 \\ &= S_{3,3}(\tau-1)(\tau-2) + S_{3,2}(\tau-1) + S_{3,1}, \\ \tau^3 &= (\tau-1)(\tau-2)(\tau-3) + 6(\tau-1)(\tau-2) + 7(\tau-1) + 1 \\ &= S_{4,4}(\tau-1)(\tau-2)(\tau-3) + S_{4,3}(\tau-1)(\tau-2) + S_{4,2}(\tau-1) + S_{4,1},\end{aligned}$$

and

$$\begin{aligned}\tau^4 &= (\tau-1)(\tau-2)(\tau-3)(\tau-4) + 10(\tau-1)(\tau-2)(\tau-3) + 25(\tau-1)(\tau-2) \\ &\quad + 15(\tau-1) + 1 \\ &= S_{5,5}(\tau-1)(\tau-2)(\tau-3)(\tau-4) + S_{5,4}(\tau-1)(\tau-2)(\tau-3) + S_{5,3}(\tau-1)(\tau-2) \\ &\quad + S_{5,2}(\tau-1) + S_{5,1}.\end{aligned}$$

Table 1. First few possibilities for Stirling numbers of the second kind (see [28]).

	$\kappa h'(\kappa)$	$\kappa^2 h''(\kappa)$	$\kappa^3 h'''(\kappa)$	$\kappa^4 h^{(4)}(\kappa)$	$\kappa^5 h^{(5)}(\kappa)$	$\kappa^6 h^{(6)}(\kappa)$
$\mathbb{D}^1 h(\kappa)$	1	0	0	0	0	0
$\mathbb{D}^2 h(\kappa)$	1	1	0	0	0	0
$\mathbb{D}^3 h(\kappa)$	1	3	1	0	0	0
$\mathbb{D}^4 h(\kappa)$	1	7	6	1	0	0
$\mathbb{D}^5 h(\kappa)$	1	15	25	10	1	0
$\mathbb{D}^6 h(\kappa)$	1	31	90	65	15	1
$\vdots$	1	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{D}^{\kappa} h(\kappa)$	1	$S_{\kappa,2}$	$S_{\kappa,3}$	$S_{\kappa,4}$	$S_{\kappa,5}$	$S_{\kappa,6}$

In general, we have

$$\begin{aligned}\tau^\kappa &= S_{\kappa+1,1} + S_{\kappa+1,2}(\tau-1) + S_{\kappa+1,3}(\tau-1)(\tau-2) + \cdots \\ &\quad + S_{\kappa+1,\kappa+1}(\tau-1)(\tau-2)(\tau-3) \cdots (\tau-\kappa) \\ &= S_{\kappa+1,1} + \sum_{j=2}^{\kappa+1} S_{\kappa+1,j}(\tau-1)(\tau-2)(\tau-3) \cdots (\tau-(j-1)), \quad \kappa = 1, 2, 3, \dots\end{aligned}\quad (13)$$

Using Sălăgean differential operator $\mathbb{D}^\kappa$, we say that a function $h(\mathcal{z})$ belonging to $\mathbb{N}$ is said to be in the class $\mathfrak{T}_\kappa(\epsilon, b)$ if and only if

$$\Re\left\{1 + \frac{1}{b}\left[(1-\epsilon)\frac{\mathbb{D}^\kappa h(\mathcal{z})}{\mathcal{z}} + \epsilon(\mathbb{D}^\kappa h(\mathcal{z}))' - 1\right]\right\} > 0, \quad (\mathcal{z} \in \wp), \quad (14)$$

where $\epsilon \geq 0$; $\kappa \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $b \in \mathbb{C} \setminus \{0\}$.

Class $\mathfrak{T}_\kappa(\epsilon, b)$ was introduced and studied by Aouf [29], containing many well-known and new classes of analytic univalent functions of a complex order. For example,

1. $\mathfrak{T}_\kappa(0, b) = G_\kappa(b) = \left\{h \in \mathbb{N} : \Re\left\{1 + \frac{1}{b}\left[\frac{\mathbb{D}^\kappa h(\mathcal{z})}{\mathcal{z}} - 1\right]\right\} > 0\right\};$
2. $\mathfrak{T}_\kappa(1, b) = R_\kappa(b) = \left\{h \in \mathbb{N} : \Re\left\{(\mathbb{D}^\kappa h(\mathcal{z}))' - 1\right\} > 0\right\};$
3. $\mathfrak{T}_0(\epsilon, b) = G(\epsilon, b) = \left\{h \in \mathbb{N} : \Re\left\{1 + \frac{1}{b}\left[(1-\epsilon)\frac{h(\mathcal{z})}{\mathcal{z}} + \epsilon h'(\mathcal{z}) - 1\right]\right\} > 0\right\};$
4. $\mathfrak{T}_0(0, b) = G(b) = \left\{h \in \mathbb{N} : \Re\left\{1 + \frac{1}{b}\left[\frac{h(\mathcal{z})}{\mathcal{z}} - 1\right]\right\} > 0\right\};$
5. $\mathfrak{T}_0(1, b) = R(b) = \left\{h \in \mathbb{N} : \Re\left\{1 + \frac{1}{b}[h'(\mathcal{z}) - 1]\right\} > 0\right\}.$

For the above classes and their special cases, one can refer to [29].

Furthermore, we have the following two subclasses:

$$\mathfrak{T}_1(\epsilon, b) = M(\epsilon, b) = \Re\left\{1 + \frac{1}{b}[\mathcal{z}h'(\mathcal{z}) + \epsilon h''(\mathcal{z}) - 1]\right\} > 0;$$

and

$$\mathfrak{T}_2(\epsilon, b) = N(\epsilon, b) = \Re\left\{1 + \frac{1}{b}\left[(1-\epsilon)(h'(\mathcal{z}) + \mathcal{z}h''(\mathcal{z})) + \epsilon(\mathcal{z}h'(\mathcal{z}) + \mathcal{z}^2h''(\mathcal{z}))' - 1\right]\right\} > 0.$$

In [30], Dixit and Pal introduced the following subclass of analytic functions:

$$\mathbf{R}^\omega(\mathfrak{E}, \mathfrak{D}) = \left\{h \in \mathbb{N} : \left|\frac{h'(\mathcal{z}) - 1}{(\mathfrak{E} - \mathfrak{D})\omega - \mathfrak{D}[h'(\mathcal{z}) - 1]}\right| < 1, \quad \mathcal{z} \in \wp,\right.$$

where $\omega \in \mathbb{C} \setminus \{0\}$ and $-1 \leq \mathfrak{D} < \mathfrak{E} \leq 1$.

In Section 2, we recall some lemmas and useful relations, which will be useful to prove the main results. Section 3 is devoted to obtain a sufficient condition for $Y_\theta^s(\mathcal{z})$ to be in the class $\mathfrak{T}_\kappa(\epsilon, b)$. In Section 4, we provide a sufficient condition for $\mathcal{I}_\theta^s(\mathbf{R}^\omega(\mathfrak{E}, \mathfrak{D})) \subset \mathfrak{T}_\kappa(\epsilon, b)$. Finally, in Section 5, we give conditions for the integral operator $\mathcal{G}_\theta^s h(\mathcal{z}) = \int_0^{\mathcal{z}} \frac{\Lambda_\theta^s(t)}{t} dt$ to be in the class $\mathfrak{T}_\kappa(\epsilon, b)$.

2. Lemmas and Useful Relations

With simple calculations, we derive the following relations:

$$\sum_{\tau=2}^{\infty} \binom{\tau+s-2}{s-1} \theta^{\tau-1} = \frac{1}{(1-\theta)^s} - 1,$$

$$\begin{aligned}\sum_{\tau=5}^{\infty}(\tau-1)(\tau-2)(\tau-3)(\tau-4)\binom{\tau+s-2}{s-1}\theta^{\tau-1} &= \frac{24\theta^4\binom{s+3}{s-1}}{(1-\theta)^{s+4}}, \\ \sum_{\tau=4}^{\infty}(\tau-1)(\tau-2)(\tau-3)\binom{\tau+s-2}{s-1}\theta^{\tau-1} &= \frac{6\theta^3\binom{s+2}{s-1}}{(1-\theta)^{s+3}}, \\ \sum_{\tau=3}^{\infty}(\tau-1)(\tau-2)\binom{\tau+s-2}{s-1}\theta^{\tau-1} &= \frac{2\theta^2\binom{s+1}{s-1}}{(1-\theta)^{s+2}}, \\ \sum_{\tau=2}^{\infty}(\tau-1)\binom{\tau+s-2}{s-1}\theta^{\tau-1} &= \frac{\theta\binom{s}{s-1}}{(1-\theta)^{s+1}},\end{aligned}$$

and, in general, for $s = 1, 2, 3, \dots$, we have

$$\sum_{\tau=s+1}^{\infty}(\tau-1)(\tau-2)(\tau-3)\cdots(\tau-s)\binom{\tau+s-2}{s-1}\theta^{\tau-1} = s!\theta^s \frac{\binom{s+s-1}{s-1}}{(1-\theta)^{s+s}}. \quad (15)$$

To prove our main results, we need the following lemmas.

Lemma 1 ([29]). Let the function $\mathfrak{h}(\varkappa)$ be defined by (1). Then, $\mathfrak{h}(\varkappa) \in \mathfrak{T}_{\kappa}(\epsilon, \mathfrak{b})$ if and only if

$$\sum_{\tau=2}^{\infty} \tau^{\kappa} [1 + \epsilon(\kappa-1)] |a_{\tau}| \leq |\mathfrak{b}|, \quad (\varkappa \in \wp). \quad (16)$$

The result is sharp.

Lemma 2 ([30]). If $\mathfrak{h} \in \mathbf{R}^{\omega}(\mathfrak{E}, \mathfrak{D})$ is of the form (1), then

$$|a_{\tau}| \leq (\mathfrak{E} - \mathfrak{D}) \frac{|\omega|}{\tau}, \quad \omega \in \mathbb{N} - \{1\}. \quad (17)$$

The result is sharp.

Unless otherwise mentioned, we shall assume in this paper that $0 \leq \mathfrak{b} < 1$, $0 \leq \epsilon < 1$, $s \geq 1$ and $0 \leq \theta < 1$.

3. Sufficient Condition

First of all, with the help of Lemma 1, we obtain the following sufficient condition for $Y_{\theta}^s(\varkappa)$ to be in $\mathfrak{T}_{\kappa}(\epsilon, \mathfrak{b})$.

Theorem 1. The series $Y_{\theta}^s(\varkappa) \in \mathfrak{T}_{\kappa}(\epsilon, \mathfrak{b})$, $\kappa \in \mathbb{N}$, if

$$\begin{aligned}& \epsilon(\kappa+1)! \theta^{\kappa+1} \frac{\binom{s+\kappa}{s-1}}{(1-\theta)^{\kappa+1}} \\ & + \sum_{j=2}^{\kappa+1} (\epsilon S_{\kappa+2,j} + (1-\epsilon) S_{\kappa+1,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \\ & \leq |\mathfrak{b}|.\end{aligned} \quad (18)$$

Proof. In view of (16), we will show that $P_1 \leq |\mathfrak{b}|$, where

$$P_1 = \sum_{\tau=2}^{\infty} \tau^{\kappa} [1 + \epsilon(\kappa-1)] \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s.$$

Using (14) and (15), we have

$$\begin{aligned}
P_1 &= \sum_{\tau=2}^{\infty} [\epsilon \tau^{\kappa+1} + (1-\epsilon) \tau^{\kappa}] \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s \\
&= \sum_{\tau=2}^{\infty} [(\epsilon + \epsilon S_{\kappa+2, \kappa+2} (\tau-1)(\tau-2) \cdots (\tau-(\kappa+1))) \\
&\quad + \epsilon \sum_{j=2}^{\kappa+1} S_{\kappa+2, j} (\tau-1)(\tau-2) \cdots (\tau-(j-1)) \\
&\quad + (1-\epsilon) \left(1 + \sum_{j=2}^{\kappa+1} S_{\kappa+1, j} (\tau-1)(\tau-2) \cdots (\tau-(j-1)) \right)] \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s \\
&= (1-\theta)^s \left[\epsilon \sum_{\tau=\kappa+2}^{\infty} (\tau-1)(\tau-2)(\tau-3) \cdots (\tau-(\kappa+1)) \binom{\tau+s-2}{s-1} \theta^{\tau-1} \right. \\
&\quad + \sum_{j=2}^{\kappa+1} (\epsilon S_{\kappa+2, j} + (1-\epsilon) S_{\kappa+1, j}) \sum_{\tau=j}^{\infty} (\tau-1)(\tau-2) \cdots (\tau-(j-1)) \binom{\tau+s-2}{s-1} \theta^{\tau-1} \\
&\quad \left. + \sum_{\tau=2}^{\infty} \binom{\tau+s-2}{s-1} \theta^{\tau-1} \right] \\
&= \epsilon (\kappa+1)! \theta^{\kappa+1} \frac{\binom{s+\kappa}{s-1}}{(1-\theta)^{\kappa+1}} \\
&\quad + \sum_{j=2}^{\kappa+1} (\epsilon S_{\kappa+2, j} + (1-\epsilon) S_{\kappa+1, j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) \\
&\quad + (1 - (1-\theta)^s).
\end{aligned}$$

Therefore, we see that the last expression is bounded above by $|b|$ if (19) is satisfied. $\square$

By putting $\kappa = 1$ in Theorem 1, we obtain the following corollary.

Corollary 1. The series $Y_{\theta}^s(\kappa) \in M(\epsilon, b)$, if

$$\epsilon \theta^2 \frac{s(s+1)}{(1-\theta)^2} + (2\epsilon + 1) \left(\theta \frac{s}{(1-\theta)} \right) + (1 - (1-\theta)^s) \leq |b|. \quad (19)$$

By putting $\kappa = 2$ in Theorem 1, we obtain the following corollary.

Corollary 2. The series $Y_{\theta}^s(\kappa) \in N(\epsilon, b)$, if

$$\epsilon \frac{\theta^3 s(s+1)(s+2)}{(1-\theta)^3} + (5\epsilon + 1) \frac{\theta^2 s(s+1)}{(1-\theta)^2} + (4\epsilon + 3) \frac{\theta s}{(1-\theta)} + (1 - (1-\theta)^s) \leq |b|. \quad (20)$$

By putting $\epsilon = 0$ in Theorem 1, we obtain the following corollary.

Corollary 3. The series $Y_{\theta}^s(\kappa) \in G_{\kappa}(b)$, if

$$\sum_{j=2}^{\kappa+1} (S_{\kappa+1, j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \leq |b|. \quad (21)$$

By putting $\epsilon = 1$ in Theorem 1, we obtain the following corollary.

Corollary 4. The series $Y_{\theta}^s(\kappa) \in R_{\kappa}(b)$, if

$$\begin{aligned} & (\kappa + 1)! \theta^{\kappa+1} \frac{\binom{s+\kappa}{s-1}}{(1-\theta)^{\kappa+1}} \\ & + \sum_{j=2}^{\kappa+1} (S_{\kappa+2,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \\ & \leq |b|. \end{aligned} \quad (22)$$

4. Inclusion Properties

Making use of Lemma 2, we will study the inclusion relation $R^{\omega}(\mathfrak{E}, \mathfrak{D}) \subset G_{\kappa}(\epsilon, b)$.

Theorem 2. Let $\hbar \in R^{\omega}(\mathfrak{E}, \mathfrak{D})$. Then, $\mathcal{I}_{\theta}^s \hbar(\kappa) \in \mathfrak{T}_{\kappa}(\epsilon, b)$, $\kappa \geq 2$, if

$$\begin{aligned} & (\mathfrak{E} - \mathfrak{D})|\omega| \left[\epsilon \kappa! \theta^{\kappa} \frac{\binom{s+\kappa-1}{s-1}}{(1-\theta)^{\kappa}} \right. \\ & \left. + \sum_{j=2}^{\kappa} (\epsilon S_{\kappa+1,j} + (1-\epsilon) S_{\kappa,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \right] \leq |b|. \end{aligned} \quad (23)$$

Proof. From (16), it suffices to show that $P_2 \leq |b|$, where

$$P_2 = \sum_{\tau=2}^{\infty} \tau^{\kappa} [1 + \epsilon(\kappa-1)] \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s |a_{\tau}|. \quad (24)$$

Applying (17) of Lemma 2, we find from Equations (14) and (15) that

$$\begin{aligned} P_2 & \leq (\mathfrak{E} - \mathfrak{D})|\omega| \left[\sum_{\tau=2}^{\infty} \left(\epsilon \tau^{\kappa} + (1-\epsilon) \tau^{\kappa-1} \right) \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s \right] \\ & \leq (\mathfrak{E} - \mathfrak{D})|\omega| (1-\theta)^s \left[\sum_{\tau=2}^{\infty} [\epsilon + \epsilon S_{\kappa+1,\kappa+1} (\tau-1)(\tau-2) \cdots (\tau-\kappa) \right. \\ & \quad \left. + \epsilon \sum_{j=2}^{\kappa} S_{\kappa+1,j} (\tau-1)(\tau-2) \cdots (\tau-(j-1)) \right. \\ & \quad \left. + (1-\epsilon) \left(1 + \sum_{j=2}^{\kappa} S_{\kappa,j} (\tau-1)(\tau-2) \cdots (\tau-(j-1)) \right) \right] \binom{\tau+s-2}{s-1} \theta^{\tau-1} \\ & \leq (\mathfrak{E} - \mathfrak{D})|\omega| (1-\theta)^s \left[\epsilon \sum_{\tau=\kappa+1}^{\infty} (\tau-1)(\tau-2) \cdots (\tau-\kappa) \binom{\tau+s-2}{s-1} \theta^{\tau-1} \right. \\ & \quad \left. + \sum_{j=2}^{\kappa} (\epsilon S_{\kappa+1,j} + (1-\epsilon) S_{\kappa,j}) \sum_{\tau=j}^{\infty} (\tau-1)(\tau-2) \cdots (\tau-(j-1)) \binom{\tau+s-2}{s-1} \theta^{\tau-1} \right. \\ & \quad \left. + \sum_{\tau=2}^{\infty} \binom{\tau+s-2}{s-1} \theta^{\tau-1} \right] \\ & \leq (\mathfrak{E} - \mathfrak{D})|\omega| \left[\epsilon \kappa! \theta^{\kappa} \frac{\binom{s+\kappa-1}{s-1}}{(1-\theta)^{\kappa}} \right. \\ & \quad \left. + \sum_{j=2}^{\kappa} (\epsilon S_{\kappa+1,j} + (1-\epsilon) S_{\kappa,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) \right. \\ & \quad \left. + (1 - (1-\theta)^s) \right]. \end{aligned}$$

However, this last expression is bounded by $|b|$, if (23) holds. $\square$

By putting $\kappa = 2$ in Theorem 2, we obtain the following corollary.

Corollary 5. Let $h \in \mathbf{R}^\omega(\mathfrak{E}, \mathfrak{D})$. Then, $\mathcal{I}_\theta^s h(\varkappa) \in M(\epsilon, \flat)$ if

$$\begin{aligned} & (\mathfrak{E} - \mathfrak{D})|\omega| \left[\epsilon \theta^2 \frac{s(s+1)}{(1-\theta)^2} + (2\epsilon + 1)\theta \frac{s}{(1-\theta)} \right. \\ & \quad \left. + (1 - (1-\theta)^s) \right] \\ & \leq |\flat|. \end{aligned} \quad (25)$$

By putting $\epsilon = 0$ in Theorem 2, we obtain the following corollary.

Corollary 6. Let $h \in \mathbf{R}^\omega(\mathfrak{E}, \mathfrak{D})$. Then, $\mathcal{I}_\theta^s h(\varkappa) \in G_\kappa(\flat)$, $\kappa \geq 2$, if

$$(\mathfrak{E} - \mathfrak{D})|\omega| \left[\sum_{j=2}^{\kappa} (S_{\kappa,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \right] \leq |\flat|. \quad (26)$$

By putting $\epsilon = 1$ in Theorem 2, we obtain the following corollary.

Corollary 7. Let $h \in \mathbf{R}^\omega(\mathfrak{E}, \mathfrak{D})$. Then, $\mathcal{I}_\theta^s h(\varkappa) \in R_\kappa(\flat)$, $\kappa \geq 2$, if

$$\begin{aligned} & (\mathfrak{E} - \mathfrak{D})|\omega| \left[\kappa! \theta^\kappa \frac{\binom{s+\kappa-1}{s-1}}{(1-\theta)^\kappa} \right. \\ & \quad \left. + \sum_{j=2}^{\kappa} (S_{\kappa+1,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \right] \leq |\flat|. \end{aligned} \quad (27)$$

5. An Integral Operator

In this section, we consider the integral operator $\mathcal{G}_\theta^s$ defined by

$$\mathcal{G}_\theta^s h(\varkappa) = \int_0^\varkappa \frac{\Lambda_\theta^s(t)}{t} dt. \quad (28)$$

Theorem 3. The integral operator $\mathcal{G}_\theta^s h(\varkappa)$ defined by (28) is in the class $\mathfrak{T}_\kappa(\epsilon, \flat)$, $\kappa \geq 2$, if

$$\begin{aligned} & \epsilon \kappa! \theta^\kappa \frac{\binom{s+\kappa-1}{s-1}}{(1-\theta)^\kappa} + \sum_{j=2}^{\kappa} (\epsilon S_{\kappa+1,j} + (1-\epsilon) S_{\kappa,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) \\ & \quad + (1 - (1-\theta)^s) \\ & \leq |\flat|. \end{aligned} \quad (29)$$

Proof. From the definitions (2) and (28), we get

$$\mathcal{G}_\theta^s h(\varkappa) = \varkappa + \sum_{\tau=2}^{\infty} \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s \frac{\varkappa^\tau}{\tau}.$$

By virtue of Lemma 1, we need only to show that $P_3 \leq |\flat|$, where

$$P_3 = \sum_{\tau=2}^{\infty} \tau^\kappa [1 + \epsilon(\kappa-1)] \times \frac{1}{\tau} \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s,$$

or, equivalently,

$$P_3 = \sum_{\tau=2}^{\infty} (\epsilon \tau^\kappa + (1-\epsilon) \tau^{\kappa-1}) \binom{\tau+s-2}{s-1} \theta^{\tau-1} (1-\theta)^s.$$

The sufficient condition (29) can be proved on the same lines of Theorem 2. Hence, the theorem is proved. $\square$

By putting $\kappa = 2$ in Theorem 3, we obtain the following corollary.

Corollary 8. *The integral operator $\mathcal{G}_\theta^s \mathfrak{h}(\varkappa)$ defined by (28) is in the class $M(\epsilon, \flat)$, if*

$$\frac{\epsilon \theta^2 s(s+1)}{(1-\theta)^2} + (2\epsilon + 1) \frac{\theta s}{(1-\theta)} + (1 - (1-\theta)^s) \leq |\flat|. \quad (30)$$

By putting $\epsilon = 0$ in Theorem 2, we obtain the following corollary.

Corollary 9. *The integral operator $\mathcal{G}_\theta^s \mathfrak{h}(\varkappa)$ defined by (28) is in the class $G_\kappa(\flat)$, $\kappa \geq 2$, if*

$$\sum_{j=2}^{\kappa} (S_{\kappa,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \leq |\flat|. \quad (31)$$

By putting $\epsilon = 1$ in Theorem 2, we obtain the following corollary.

Corollary 10. *The integral operator $\mathcal{G}_\theta^s \mathfrak{h}(\varkappa)$ defined by (28) is in the class $R_\kappa(\flat)$, $\kappa \geq 2$, if*

$$\kappa! \theta^\kappa \frac{\binom{s+\kappa-1}{s-1}}{(1-\theta)^\kappa} + \sum_{j=2}^{\kappa} (S_{\kappa+1,j}) \left((j-1)! \theta^{j-1} \frac{\binom{s+j-2}{s-1}}{(1-\theta)^{j-1}} \right) + (1 - (1-\theta)^s) \leq |\flat|. \quad (32)$$

Remark 1. *Taking different choices of $\kappa \in \mathbb{N}$, one can obtain new sufficient conditions and inclusion relations for Pascal distribution series to be in the class $\mathfrak{T}_\kappa(\epsilon, \flat)$.*

6. Conclusions

Differential operators play an important role in the geometric function theory (GFT). In 1983, the Sălăgean operator introduced differential $\mathbb{D}^\kappa$, which bear its name [19]. This operator inspired many mathematicians to obtain new and interesting results by introducing many classes of analytic and univalent functions defined in the open unit disk (see, for example, [31,32]). Recently, several new operators were written, involving the Sălăgean operator [22,33]. Some mathematicians have used the Sălăgean differential in different topics of the GFT, such as differential subordination [34], fractional integral operators [35], and q-calculus [36]. Furthermore, this operator was also recently used to obtain results related to the Fekete–Szegő inequality [37]. Very recently, Frasin [26] found a connection between the Sălăgean differential operator and Stirling numbers. In the present paper, we obtained new conditions for Pascal distribution series to be in the class $\mathfrak{T}_\kappa(\epsilon, \flat)$ of analytic functions defined by the operator $\mathbb{D}^\kappa$, including Stirling numbers. In addition, we proved new conditions for the operator $\mathcal{G}_\theta^s$ to be in the class $\mathfrak{T}_\kappa(\epsilon, \flat)$. Some special cases of our main results were also obtained. This study could inspire researchers to find new conditions and inclusion relations for Pascal distribution series to be in other classes defined by the operator $\mathbb{D}^\kappa$.

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