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Fermatean Probabilistic Hesitant Fuzzy Power Bonferroni Aggregation Operators with Dual Probabilistic Information and Their Application in Green Supplier Selection

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Abstract: In the realm of management decision-making, the selection of green suppliers has long been a complex issue. Companies must take a holistic approach, evaluating potential suppliers based on their capabilities, economic viability, and environmental impact. The decision-making process, fraught with intricacies and uncertainties, urgently demands the development of a scientifically sound and efficient method for guidance. Since the concept of Fermatean fuzzy sets (FFSs) was proposed, it has been proved to be an effective tool for solving multi-attribute decision-making (MADM) problems in complicated realistic situations. And the Power Bonferroni mean (PBM) operator, combining the strengths of the power average (PA) and Bonferroni mean (BM), excels in considering attribute interactions for a thorough evaluation. To ensure a comprehensive and sufficient evaluation framework for supplier selection, this paper introduces innovative aggregation operators that extend the PBM and integrate probabilistic information into Fermatean hesitant fuzzy sets (FHFSs) and Fermatean probabilistic hesitant fuzzy sets (FPHFSs). It successively proposes the Fermatean hesitant fuzzy power Bonferroni mean (FHFPBM), Fermatean hesitant fuzzy weighted power Bonferroni mean (FHFWPBM), and Fermatean hesitant fuzzy probabilistic weighted power Bonferroni mean (FHFPWPBM) operators, examining their key properties like idempotency, boundedness, and permutation invariance. By further integrating PBM with probabilistic information into FPHFSs, three new Fermatean probabilistic hesitant fuzzy power Bonferroni aggregation operators are developed: the Fermatean probabilistic hesitant fuzzy power Bonferroni mean (FPHFPBM), Fermatean probabilistic hesitant fuzzy weighted power Bonferroni mean (FPHFWPBM), and Fermatean probabilistic hesitant fuzzy probabilistic weighted power Bonferroni mean (FPHFPWPBM). Subsequently, a MADM method based on these operators is constructed. Finally, a numerical example concerning the selection of green suppliers is presented to demonstrate the applicability and effectiveness of this method using the FPHFPWPBM operator.

Keywords: Fermatean probabilistic hesitant fuzzy set; power Bonferroni mean operators; probabilistic information; multi-attribute decision-making

MSC: 03B52; 03E72; 94D05

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1. Introduction

Multi-attribute decision-making (MADM) is frequently utilized to effectively rank or select one or more alternatives from a discrete set of candidate schemes in the presence of multiple, usually conflicting, criteria or attributes [1]. As human economies and societies rapidly evolve, the complexity of decision-making problems has escalated, compounded by the proliferation of data in everyday production and life. The presence of inaccurate and

incomplete information, coupled with the subjective ambiguity and cognitive constraints of decision-makers, frequently renders existing MADM methods inadequate for practical needs. Consequently, considerable efforts have been continuously made towards developing more effective approaches to mitigate the impact of subjectivity and uncertainty on the decision-making process [2]. In an idealized decision-making situation, the decision results are usually presented in the form of a precise numerical value or a set of specific orders. However, the reality is that the decision results often fall within a range of vague values, and there may be many potential outcomes of ranking. In response, Zadeh [3] raised the concept of fuzzy set (FS), allowing the evaluation results to be intermediate between several fuzzy numbers. Subsequent related studies have continuously enriched the FS theory and provided some effective methods for dealing with complex decision-making problems under uncertain circumstances.

Having recognized the inadequacy of FS containing only membership degree in evaluating the information, Atanassov [4] proposed intuitionistic fuzzy set (IFS) by adding non-membership degree to represent the negation degree of a certain attribute. Compared with the FS, the IFS can better describe the completeness and accuracy of decision information and reflect the degree of support, opposition, and neutrality of decision-makers. Numerous works have shed light on the capability of IFSs to navigate the challenges of uncertainty and fuzziness [5–10], including aggregation operators [5,6], distance measures [7], correlation coefficients [8], and the application of MADM methods in IFS [9,10]. With the increasing complexity of practical decision problems, the condition of IFSs on the sum of membership and non-membership degrees not greater than 1 gradually shows an inadequacy in data aggregation. Thus, Yager [11] proposed Pythagorean fuzzy set (PFS), which broadens the condition to allow the sum of membership and non-membership degrees greater than 1 with the sum of square less than or equal to 1, so it has a stronger ability to describe fuzzy phenomena than FS and IFS. PFS has been widely studied since it was proposed, mainly focusing on the application of information aggregation operator [12,13], information measure including distance, similarity and entropy measure [14–16], and combination with MADM methods [17,18]. Subsequently, to encompass a wider array of scenarios, Yager [19] extended the framework of PFSs, introducing a novel concept known as "q-rung orthopair fuzzy sets" (q-ROFSs), where the sum of qth power of the membership and non-membership is constrained such that $\eta^q(x) + \nu^q(x) \leq 1$ with $q \geq 1$. Obviously, q-ROFSs offer a broader spectrum of fuzzy information, making them a more versatile and suitable approach for addressing unpredictable situations. In recent years, the concept of q-ROFSs has garnered growing scholarly attention, thereby broadening its applications in the realms of information integration and decision-making [20–22]. Specifically, in order to address decision situations with more huge and complicated information, Senapati and Yager [23] adopted a similar approach and proposed Fermatean fuzzy set (FFS), further extending the condition that allow the sum of squares of membership and non-membership degrees greater than 1 with the sum of cubic less than or equal to 1. Obviously, compared to IFS and PFS, the FFS can help decision-makers describe complex information under uncertain environments more adequately. In recent years, the research on FFS has emerged and scholars have conducted extensive research on relevant aspects including information aggregation operators [24–26], information measures [27,28], and extensions of FFSs [29,30].

To accommodate the indecision between multiple evaluation values that often occurs in actual decision-making situation, Torra [31] suggested the concept of hesitant fuzzy set (HFS). Compared to the single value of the membership degrees in FS, the HFS allows membership degrees to be multiple values, which can better describe the hesitation of decision-makers. As a prominent tool for depicting hesitancy in decision-making processes, scholars have constantly enriched the extensions of HFSs, such as interval-valued hesitant fuzzy set (IVHFS) [32], intuitionistic hesitant fuzzy set (IHFS) [33], Pythagorean hesitant fuzzy set (PyHFS) [34], and Fermatean hesitant fuzzy set (FHFS) [35]. Combining the advantages of FFS and HFS, FHFS is acknowledged as an excellent tool for information integration in fuzzy environments, thus attracting a large number of scholars to conduct

research on it. Lai et al. [36] introduced the concept of hesitant Fermatean fuzzy sets (HFFSs) and advanced the CoCoSo method by integrating aggregation operators and information measures specific to HFFSs. Mishra et al. [37] brought forth the notion of interval-valued hesitant Fermatean fuzzy sets (IVHFFSs), enhancing the COPRAS method with IVHFF information. Liu and Luo [38] utilized the probabilistic hesitant Fermatean fuzzy set to illustrate the comprehensive evaluation information. Ruan et al. [39] explored a Fermatean hesitant fuzzy information aggregation method by combining the FHFSs with Heronian mean (HM) operators.

Although HFS can reflect the decision-maker's hesitancy, it ignores the decision-maker's preferences for different values within each membership degree. To address this, Xu et al. [40] introduced concept of probabilistic hesitant fuzzy set (PHFS) and its basic operational rules. By adding probabilities to each membership degree, the PHFS can effectively address the issues of individual expert preferences and the number of experts in the decision-making process. Therefore, since the definition of PHFS, it has achieved a series of research results in the field of MADM. Scholars have successively investigated probabilistic linguistic term set (PLTS) [41], probabilistic interval-valued hesitant fuzzy set (PIVHFS) [42], probabilistic interval-valued intuitionistic hesitant fuzzy set [43], and probabilistic interval-valued Fermatean hesitant fuzzy set (PIVFHFS) [44]. Combining the advantages of q-ROFSs and PHFSs, Ren et al. [45] proposed the concept of q-rung orthopair probabilistic hesitant fuzzy rough sets (q-ROPHFSs), which provides a more comprehensive framework for modeling complex and nuanced uncertainties. Chen et al. [46] improved the algorithms for q-ROPHFS by introducing the Schweizer-Sklar T-norm and developed a MADM model based on the newly proposed q-rung orthopair probabilistic hesitant fuzzy Schweizer-Sklar power weighted Hamy mean operator, enhancing the computational efficiency of the information aggregation operator within q-ROPHFS framework. Chen et al. [47] also explored the application of the q-rung orthopair probability hesitant fuzzy GRA-TOPSIS as a modeling tool for assessing airspace operation effectiveness. Additionally, Attaullah et al. [48] introduced the concept of q-rung orthopair probabilistic hesitant fuzzy rough set (q-ROPHFRS) by integrating rough set with q-ROPHFS and successfully applied it in a numerical case study regarding the selection of medical oxygen suppliers for hospitals. In recent years, as scholars have been enriching the research on q-ROPHFSs, the study of the specific fuzzy set paradigms associated with it has also been continuously evolving, i.e., Pythagorean probabilistic hesitant fuzzy set (PyPHFS) [49], and Fermatean probabilistic hesitant fuzzy set (FPHFS) [50]. Among them, the FPHFS, by integrating probabilistic information into Fermatean Hesitant Fuzzy Sets, is capable of more fully expressing the uncertainty of information and the hesitancy of decision-makers. For example, there is a Fermatean probabilistic hesitant fuzzy number $(\{0.4/0.3, 0.6/0.7\}, \{0.2/0.4, 0.3/0.6\})$ within a FPHFS used to denote the evaluation results for an element x regarding the attribute "good quality" by a group of experts. Obviously, we can consider that in the evaluation results given by the experts, for the membership degree, 30% show 0.4 and 70% show 0.6; for the non-membership degree, 40% show 0.2, and the remaining 60% show 0.3. Compared to the single numerical evaluation results in FFS, the multiple values contained in FPHFS can reflect the decision-makers' hesitancy; whereas in the FHFS, the possibility of evaluation values is assumed to be uniform by default, the FPHFS retain the probability of the evaluation values, thereby better reflecting the decision-makers' risk preferences.

In actual decision-making, various evaluation attributes may not be completely independent but may have varying degrees of correlation, such as complementary, redundant, and preference relationships, etc. Considering the interplay between evaluation attributes can directly affect the accuracy and rationality of decision outcomes, Yager [51] proposed the power average (PA) operator. This operator adjusts the weights of assessment values that deviate significantly from the overall information by calculating the support degree, effectively overcoming the impact of information interrelation on decision outcomes. The PA operator provides more information and versatility for the aggregation of information,

which has attracted widespread attention and in-depth research from subsequent scholars. On the basis of the PA operator, Xu and Yager [52] integrated the geometric mean (GM) operator into it, developing the power geometric (PG) operator, weighted power geometric (WPG) operator, and power ordered weighted geometric (POWG) operator. Zhang [53] extended the PA operator to the hesitant fuzzy environment and defined the hesitant fuzzy power average (HFPA) operator and the hesitant fuzzy power geometric (HFPG) operator. By generalizing the PA operator to the intuitive fuzzy environment, Xu [54,55] proposed a series of novel intuitionistic fuzzy power aggregation operators and interval-valued intuitionistic fuzzy power aggregation operators. He et al. [56] applied the PA operator to the Pythagorean fuzzy environment, defining several Pythagorean fuzzy power average aggregation operators and developed a decision-making method based on the proposed operators.

Another operator that considers the mutual influence between input variables for data aggregation is the Bonferroni mean (BM) operator. Yager [57] was the first to provide an interpretation of the BM operator, emphasizing that it can capture the interrelationship between criteria, which has laid a theoretical foundation for its decision-making applications. Beliakov et al. [58] extended Yager's results by proposing the generalized extensions of the BM operator. Shortly after, Xu [59] extended the BM operator to the decision-making environment with interval numbers, and proposed the intuitionistic fuzzy Bonferroni mean (IFBM) operator and the weighted intuitionistic fuzzy Bonferroni mean (WIFBM) operator along with Yager [60]. To integrate and capitalize on the strengths of the PA and BM operators, He et al. [61] combined the PA operator with the BM operator to propose the Power Bonferroni Mean (PBM) operator. Subsequently, the PBM operator was extended to various fuzzy environments, including intuitionistic fuzzy sets [62], interval-valued intuitionistic fuzzy sets [63], and linguistic intuitionistic fuzzy sets [64]. Luo et al. [65] extended the PBM operator to the Pythagorean fuzzy environment, proposing a series of Pythagorean fuzzy power Bonferroni aggregation operators. Furthermore, by integrating the PBM operator and the FFS information, Ruan et al. [66] introduced the Fermatean fuzzy power Bonferroni mean (FFPBM) and Fermatean fuzzy weighted power Bonferroni mean (FFWPBM) operators, thereby enriching the MADM methods. However, there is currently a scarcity of research on the extension and application of the PBM operator in FHFSs and FPHFSs, indicating that research in this field holds considerable potential for future exploration.

As environmental awareness increases and businesses pursue sustainable development, green supply chain management has become an important component of corporate competitiveness. At the same time, with the significant impact of the concept of sustainable development on business and marketing, the selection of green suppliers is increasingly critical for effective green supply chain management, garnering attention from both researchers and practitioners [67,68]. To select the appropriate green suppliers, companies need to consider various dimensions involving economic, environmental, and social aspects during the evaluation and decision-making process, making the green supplier selection a typical multi-attribute decision-making (MADM) problem [69]. In recent years, scholars have introduced MADM methods and attempted to build various decision models to explore how to effectively carry out the selection of green suppliers. Deng et al. [70] developed the D-AHP method, extending the classical analytic hierarchy process (AHP) method to incorporate so-called D numbers for uncertain information, and applied it to supplier selection. Boran ey al. [71] proposed a TOPSIS method combined with intuitionistic fuzzy set to select appropriate supplier in group decision making environment. Qu et al. [67] developed a framework combining fuzzy TOPSIS and ELECTRE approaches to screen appropriate green chain suppliers, validating its feasibility with a case study of a Chinese internet company. It is worth noting that the selection of green suppliers often faces inaccurate or incomplete information in uncertain environments, and the potential interplay between different evaluation criteria may also have a significant impact on the decision outcomes. However, existing methodologies for green supplier selection seldom account

for the interrelationships among attributes, and they inadequately address the fuzziness and randomness inherent in evaluation data. This limitation hampers their applicability to the supplier selection problem within dynamic and complex contexts. In real-world decision-making, to enhance the quality of decision outcomes, decision-makers must not only consider the potential interrelationships between attributes but also account for the fuzziness of evaluation data, the hesitancy of decision-makers, as well as the randomness arising in uncertain environments especially, which is typically expressed through probabilistic information. Consequently, this paper endeavors to extend the PBM operator to the extensions of FFS, introducing a novel MADM method that incorporates attribute interrelationships and complex probabilistic information, which is expected to serve as a robust framework to address the intricacies of green supplier selection.

This MADM method will be developed through a structured process. Initially, we consider integrating the PBM operator and probabilistic information into FHFSs, innovatively proposing relevant Fermatean hesitant fuzzy power Bonferroni aggregation operators, i.e., the Fermatean hesitant fuzzy power Bonferroni mean (FHFPBM), Fermatean hesitant fuzzy weighted power Bonferroni mean (FHFWPBM) and Fermatean hesitant fuzzy probabilistic weighted power Bonferroni mean (FHFPWPBM) operators. These operators can not only effectively combine evaluation information with attribute interrelations and random properties in complex fuzzy environments, but also reflect the hesitancy in the decision-makers' psychology. Subsequently, following a similar approach, we further extends the PBM operator and probabilistic information to FPHFSs, presenting relevant Fermatean probabilistic hesitant fuzzy power Bonferroni aggregation operators including the Fermatean probabilistic hesitant fuzzy power Bonferroni mean (FPHFPBM), Fermatean probabilistic hesitant fuzzy weighted power Bonferroni mean (FPHFWPBM) and Fermatean probabilistic hesitant fuzzy probabilistic weighted power Bonferroni mean (FPHFPWPBM) operators. It should be noted that the proposed FPHFPWPBM operator, due to the inclusion of external weighted probabilistic information and internal probabilistic information within the fuzzy sets, can fully capture the stochastic nature of the evaluation data and be better adapted to the generally complex situations in modern decision-making. Ultimately, a novel MADM method is constructed based on the FPHFPWPBM operator that uniquely encapsulates dual probabilistic characteristics of evaluation data, thereby be well-suited for application in the case study of green supplier selection.

Thus, the main contributions of this paper can be summarized as follows: (1) The PBM operator has been extended to the Fermatean hesitant fuzzy environment and the Fermatean probabilistic hesitant fuzzy environment, respectively, with corresponding aggregation operators being proposed and named as FHFPBM. FHFWPBM, FHFPWPBM and FPHFPBM. FPHFWPBM, FPHFPWPBM operators. (2) The probabilistic information is incorporated into these proposed aggregation operators, among which the FHFPWPBM operator takes into account the influence of positional weights and external probabilistic information (such as the degree of expert familiarity) during the attribute weighting process, while the FPHFPWPBM operator further considers the probabilistic information inherent in the evaluation data itself, making the decision-making more closely aligned with real-world conditions. (3) A novel Fermatean probabilistic hesitant fuzzy MADM method combining the PBM operator with probabilistic information has been developed, and its scientific validity and practicality have been substantiated through a numerical case study concerning the selection of green suppliers. This further enriches the field of Fermatean probabilistic hesitant fuzzy aggregation operators and expands the application domain of the PBM operator.

The remainder of this work is organized as follows. Section 2 provides a brief review of the basic concepts and operational rules concerning FFS, FHFS, FPHFS, and the PBM operator. Sections 3 and 4 develop the Fermatean hesitant fuzzy power Bonferroni aggregation operators and the Fermatean probabilistic hesitant fuzzy power Bonferroni aggregation operators, respectively. Section 5 introduces a multi-attribute decision-making

method based on the FPHFPWPBM operator and applies it to a numerical case study on green supplier selection. And this paper ends in Section 6 with some conclusions.

2. Preliminaries

2.1. Fermatean Fuzzy Set (FFS)

Definition 1 [23]. *Let X be a nonempty set. A Fermatean fuzzy set F on X is defined as:*

$$F = \{(x, \alpha_F(x), \beta_F(x)) : x \in X\} \tag{1}$$

where $\alpha_F(x) : x \to [0,1]$ is the membership degree and $\beta_F(x) : x \to [0,1]$ is the non-membership degree of each element $x \in X$, satisfying the condition $0 \leq \alpha_F^3(x) + \beta_F^3(x) \leq 1$. The indeterminacy degree of each element x in F is $\pi_F(x) = \sqrt[3]{1 - \alpha_F^3(x) - \beta_F^3(x)}$.

Definition 2 [23]. *Let $F = (\mu, \nu)$, $F_1 = (\mu_1, \nu_1)$, and $F_2 = (\mu_2, \nu_2)$ be three Fermatean fuzzy numbers (FFNs). Some basic operations are defined as follows:*

(1) $F_1 \cup F_2 = (\max\{\mu_1, \mu_2\}, \min\{\nu_1, \nu_2\})$;
(2) $F_1 \cap F_2 = (\min\{\mu_1, \mu_2\}, \max\{\nu_1, \nu_2\})$;
(3) $F^c = (\nu, \mu)$, $c > 0$;
(4) $F_1 \subseteq F_2$, *if and only if* $\mu_1 \leq \mu_2, \nu_1 \geq \nu_2$.

Definition 3 [23]. *Let $F = (\mu, \nu)$, $F_1 = (\mu_1, \nu_1)$, and $F_2 = (\mu_2, \nu_2)$ be three FFNs. Several basic operations between them are given as follows:*

(1) $F_1 \oplus F_2 \left(\sqrt[3]{{\mu_1}^3 + {\mu_2}^3 - {\mu_1}^3 {\mu_2}^3}, \nu_1 \nu_2 \right)$;
(2) $\mathrm{F}_1 \otimes \mathrm{F}_2 = \left(\mu_1 \mu_2, \sqrt[3]{{\nu_1}^3 + {\nu_2}^3 - {\nu_1}^3 {\nu_2}^3} \right)$;
(3) $cF = \left(\sqrt[3]{1 - (1 - \mu_3)^c}, \nu^3 \right)$, $c > 0$;
(4) $F^c = \left(\mu^3, \sqrt[3]{1 - (1 - \nu^3)^c} \right)$, $c > 0$.

2.2. Fermatean Hesitant Fuzzy Set (FHFS)

Torra [31] proposed the concept of hesitant fuzzy set (HFS), which allows the membership of each element belonging to a set to be represented as multiple possible values between 0 and 1.

Definition 4 [35]. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed set. A hesitant fuzzy set (HFS) E on X is a structure of the following form:*

$$E = \{\langle x, h_E(x) \rangle | x \in X\} \tag{2}$$

where $h_E(x)$ is the set of several values belonging to $[0,1]$, denoting some possible membership degrees of the element $x \in X$ to the set E.

Definition 5 [35]. *Let X be a universe of discourse. A Fermatean hesitant fuzzy set (FHFS) A on X can be defined as follows:*

$$A = (\langle x, U_A(x), V_A(x) | x \in X \rangle) \tag{3}$$

where $U_A(x)$ and $V_A(x)$ are nonempty finite subsets on $[0,1]$, and respectively represent the set of all the possible membership and non-membership degrees of each element. $\forall \mu_A(x) \in U_A(x)$, $\forall v_A(x) \in V_A(x)$, the condition $0 \leq \mu_A^3(x) + \nu_A^3(x) \leq 1$ holds for each element on X. For $x \in X$, $\langle \mu_A(x), \nu_A(x) \rangle$ is named as a Fermatean hesitant fuzzy number (FHFN). For convenience, we denote it as $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$ and Φ as all Fermatean hesitant fuzzy numbers.

Definition 6 [35]. *Let* $\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$, $\alpha_1 = \langle \mu_{\alpha_1}, \nu_{\alpha_1} \rangle$, *and* $\alpha_2 = \langle \mu_{\alpha_2}, \nu_{\alpha_2} \rangle$ *be three FHFNs, and* $\lambda > 0$, *then*

(1) $\alpha_1 \oplus \alpha_2 = \left\langle \bigcup\limits_{\mu_{\alpha_1} \in U_{\alpha_1}, \mu_{\alpha_2} \in U_{\alpha_2}} \left\{ \sqrt[3]{\mu_{\alpha_1}^3 + \mu_{\alpha_2}^3 - \mu_{\alpha_1}^3 \mu_{\alpha_2}^3} \right\}, \bigcup\limits_{\nu_{\alpha_1} \in V_{\alpha_1}, \nu_{\alpha_2} \in V_{\alpha_2}} \{ v_{\alpha_1} v_{\alpha_2} \} \right\rangle$;

(2) $\alpha_1 \otimes \alpha_2 = \left\langle \bigcup\limits_{\mu_{\alpha_1} \in U_{\alpha_1}, \mu_{\alpha_2} \in U_{\alpha_2}} \{ \mu_{\alpha_1} \mu_{\alpha_2} \}, \bigcup\limits_{\nu_{\alpha_1} \in V_{\alpha_1}, \nu_{\alpha_2} \in V_{\alpha_2}} \left\{ \sqrt[3]{v_{\alpha_1}^3 + v_{\alpha_2}^3 - v_{\alpha_1}^3 v_{\alpha_2}^3} \right\} \right\rangle$;

(3) $\lambda\alpha = \left\langle \bigcup\limits_{\mu_\alpha \in U_\alpha} \left\{ \sqrt[3]{1 - (1 - \mu_\alpha^3)^\lambda} \right\}, \bigcup\limits_{\nu_\alpha \in V_\alpha} \{ v_\alpha^\lambda \} \right\rangle$;

(4) $\alpha^\lambda = \left\langle \bigcup\limits_{\mu_\alpha \in U_\alpha} \{ \mu_\alpha^\lambda \}, \bigcup\limits_{\nu_\alpha \in V_\alpha} \left\{ \sqrt[3]{1 - (1 - v_\alpha^3)^\lambda} \right\} \right\rangle$.

2.3. Fermatean Probabilistic Hesitant Fuzzy Set (FPHFS)

Definition 7 [50]. *Let* X *be a fixed non-empty set. A Fermatean probabilistic hesitant fuzzy set (FPHFS)* ε *on* X *is presented as:*

$$\varepsilon = \{ \langle x, \mu_\varepsilon(x) / p_\varepsilon(x), \nu_\varepsilon(x) / q_\varepsilon(x) \rangle | x \in X \} \tag{4}$$

where $\mu_\varepsilon(x)$ *and* $\nu_\varepsilon(x)$ *are both non-empty finite sets on* $[0, 1]$, *representing all possible membership and non-membership degrees of the element* x *to the FPHFS* ε *respectively, and satisfying the condition* $(\max\{\mu_i\})^3 + (\min\{\nu_j\})^3 \leq 1$ *and* $(\min\{\mu_i\})^3 + (\max\{\nu_j\})^3 \leq 1$, $\forall \mu_i \in \mu_\varepsilon(x)$, $\nu_j(x) \in \nu_\varepsilon(x)$, $0 \leq \mu_i, \nu_j \leq 1$. $p_\varepsilon(x)$ *and* $q_\varepsilon(x)$ *respectively denotes all the set of probabilities corresponding to the possible membership and non-membership, under the condition that* $\forall p_i \in p_\varepsilon(x)$, $q_j \in q_\varepsilon(x)$ *with* $\sum_{i=1}^{l(p)} p_i \leq 1$ *and* $\sum_{j=1}^{l(q)} q_j \leq 1$, *where* $l(p)$ *and* $l(q)$ *respectively represents the cardinal number of the set* $p_\varepsilon(x)$ *and* $q_\varepsilon(x)$. *For ease, the Fermatean Probabilistic Hesitant Fuzzy Number (FPHFN) is symbolized by the pair* $(\mu_\varepsilon / p_\varepsilon, \nu_\varepsilon / q_\varepsilon)$.

In addition, it means that the probabilistic information is complete when $\sum_{i=1}^{l(p)} p_i = 1$ and $\sum_{j=1}^{l(q)} q_j = 1$, while there is some unknown probabilistic information when $\sum_{i=1}^{l(p)} p_i < 1$ or $\sum_{j=1}^{l(q)} q_j < 1$. In this case, the FPHFN is labeled as the weak Fermatean Probabilistic Hesitant Fuzzy Number, which can be corrected to the complete FPHFN by the following formula:

$$\hat{p}_i = p_i / \sum_{i=1}^{l(p)} p_i, \ \hat{q}_j = q_j / \sum_{j=1}^{l(q)} q_j \tag{5}$$

Definition 8 [50]. *Let* $\varepsilon_1 = (\mu_{\varepsilon_1} / p_{\varepsilon_1}, \nu_{\varepsilon_1} / q_{\varepsilon_1})$ *and* $\varepsilon_2 = (\mu_{\varepsilon_2} / p_{\varepsilon_2}, \nu_{\varepsilon_2} / q_{\varepsilon_2})$ *be two FPHFNs, the fundamental operational laws are defined as:*

(1) $\varepsilon_1 \cup \varepsilon_2 = \left\{ \bigcup\limits_{\substack{\mu_1 \in \mu_{\varepsilon_1(x)}, \mu_2 \in \mu_{\varepsilon_2(x)} \\ p_1 \in p_{\varepsilon_1}, p_2 \in p_{\varepsilon_2}}} \{ \max(\mu_1 / p_1, \mu_2 / p_2) \}, \bigcup\limits_{\substack{\nu_1 \in \nu_{\varepsilon_1(x)}, \nu_2 \in \nu_{\varepsilon_2(x)} \\ q_1 \in q_{\varepsilon_1}, q_2 \in q_{\varepsilon_2}}} \{ \min(\nu_1 / q_1, \nu_2 / q_2) \} \right\}$;

(2) $\varepsilon_1 \cap \varepsilon_2 = \left\{ \bigcup\limits_{\substack{\mu_1 \in \mu_{\varepsilon_1(x)}, \mu_2 \in \mu_{\varepsilon_2(x)} \\ p_1 \in p_{\varepsilon_1}, p_2 \in p_{\varepsilon_2}}} \{ \min(\mu_1 / p_1, \mu_2 / p_2) \}, \bigcup\limits_{\substack{\nu_1 \in \nu_{\varepsilon_1(x)}, \nu_2 \in \nu_{\varepsilon_2(x)} \\ q_1 \in q_{\varepsilon_1}, q_2 \in q_{\varepsilon_2}}} \{ \max(\nu_1 / q_1, \nu_2 / q_2) \} \right\}$;

(3) $\varepsilon_1^c = \left(\nu_{\varepsilon_1(x)} / q_{\varepsilon_1}, \mu_{\varepsilon_1(x)} / p_{\varepsilon_1} \right)$.

Definition 9 [50]. *Let* $\varepsilon_1 = (\mu_{\varepsilon_1} / p_{\varepsilon_1}, \nu_{\varepsilon_1} / q_{\varepsilon_1})$ *and* $\varepsilon_2 = (\mu_{\varepsilon_2} / p_{\varepsilon_2}, \nu_{\varepsilon_2} / q_{\varepsilon_2})$ *be two FPHFNs and* $\lambda > 0$, *then their operations are articulated as follows:*

(1) $\varepsilon_1 \oplus \varepsilon_2 = \left\langle \bigcup\limits_{\substack{\mu_1 \in \mu_{\varepsilon_{1(x)}}, \mu_2 \in \mu_{\varepsilon_{2(x)}} \\ p_1 \in p_{\varepsilon_1}, p_2 \in p_{\varepsilon_2}}} \left\{ \sqrt[3]{\mu_1^3 + \mu_2^3 - \mu_1^3 \mu_2^3} / p_1 p_2 \right\}, \bigcup\limits_{\substack{\nu_1 \in \nu_{\varepsilon_{1(x)}}, \nu_2 \in \nu_{\varepsilon_{2(x)}} \\ q_1 \in q_{\varepsilon_1}, q_1 \in q_{\varepsilon_2}}} \left\{ \sqrt[3]{\nu_1^3 + \nu_2^3 - \nu_1^3 \nu_2^3} / q_1 q_2 \right\} \right\rangle$;

(2) $\varepsilon_1 \otimes \varepsilon_2 = \left\langle \bigcup_{\substack{\mu_1 \in \mu_{\varepsilon_{1(x)}}, \mu_2 \in \mu_{\varepsilon_{2(x)}} \\ p_1 \in p_{\varepsilon_1}, p_2 \in p_{\varepsilon_2}}} \{\mu_1\mu_2/p_1p_2\}, \bigcup_{\substack{\nu_1 \in \nu_{\varepsilon_{1(x)}}, \nu_2 \in \nu_{\varepsilon_{2(x)}} \\ q_1 \in q_{\varepsilon_1}, q_2 \in q_{\varepsilon_2}}} \left\{\sqrt[3]{\nu_1^3 + \nu_2^3 - \nu_1^3\nu_2^3}/q_1q_2\right\} \right\rangle;$

(3) $\lambda\varepsilon_1 = \left\langle \bigcup_{\mu_1 \in \mu_{\varepsilon_{1(x)}}, p_1 \in p_{\varepsilon_1}} \left\{\sqrt[3]{1-(1-\mu_1^3)^\lambda}/p_1\right\}, \bigcup_{\nu_1 \in \nu_{\varepsilon_{1(x)}}, q_1 \in q_{\varepsilon_1}} \{\nu_1^\lambda/q_1\} \right\rangle;$

(4) $\varepsilon_1^\lambda = \left\langle \bigcup_{\mu_1 \in \mu_{\varepsilon_{1(x)}}, p_1 \in p_{\varepsilon_1}} \{\mu_1^\lambda/p_1\}, \bigcup_{\nu_1 \in \nu_{\varepsilon_{1(x)}}, q_1 \in q_{\varepsilon_1}} \left\{\sqrt[3]{1-(1-\nu_1^3)^\lambda}/q_1\right\} \right\rangle.$

Definition 10 [50]. *Let* $\varepsilon = \left\langle \mu_{h(x)}/p_x, \nu_{h(x)}/q_x \right\rangle$ *be a FPHFN,* $|\mu_\varepsilon|$ *and* $|\nu_\varepsilon|$ *denote the number of elements in the set* $\mu_{h(x)}$ *and* $\nu_{h(x)}$*, respectively. Then, the score function* S_ε *and accuracy function* P_ε *of* ε *are defined as follows:*

$$S_\varepsilon = \left(\frac{1}{|\mu_\varepsilon|}\sum_{\mu_i \in \mu_{h(x)}, p_i \in p_{h(x)}} (\mu_i p_i)\right)^3 - \left(\frac{1}{|\nu_\varepsilon|}\sum_{\nu_i \in \nu_{h(x)}, q_i \in q_{h(x)}} (\nu_i q_i)\right)^3 \tag{6}$$

$$p_\varepsilon = \left(\frac{1}{|\mu_\varepsilon|}\sum_{\mu_i \in \mu_{h(x)}, p_i \in p_{h(x)}} (\mu_i p_i)\right)^3 + \left(\frac{1}{|\nu_\varepsilon|}\sum_{\nu_i \in \nu_{h(x)}, q_i \in q_{h(x)}} (\nu_i q_i)\right)^3 \tag{7}$$

Theorem 1 [50]. *Let* $\varepsilon_1 = \left\langle \mu_{h_1(x)}/p_{1x}, \nu_{h_1(x)}/q_{1x} \right\rangle$ *and* $\varepsilon_2 = \left\langle \mu_{h_2(x)}/p_{2x}, \nu_{h_2(x)}/q_{2x} \right\rangle$ *be two FPHFNs, then*

(1) *If* $S_{\varepsilon_1} > S_{\varepsilon_2}$*, then* $\varepsilon_1 \succ \varepsilon_2$*;*
(2) *If* $S_{\varepsilon_1} < S_{\varepsilon_2}$*, then* $\varepsilon_1 \prec \varepsilon_2$*;*
(3) *If* $S_{\varepsilon_1} = S_{\varepsilon_2}$*, then*
  (i) *If* $P_{\varepsilon_1} > P_{\varepsilon_2}$*, then* $\varepsilon_1 \succ \varepsilon_2$*;*
  (ii) *If* $P_{\varepsilon_1} < P_{\varepsilon_2}$*, then* $\varepsilon_1 \prec \varepsilon_2$*.*

Given that the membership and non-membership are both information determined by the decision-maker and the accuracy function also represents the decision-maker's certain information, this paper proposes a function of indeterminacy degree based on the accuracy function of Definition 10 and the research results by Liang et al. [72] as follows.

Theorem 2 [72]. *Let* $\varepsilon = \left\langle \mu_{h(x)}/p_x, \nu_{h(x)}/q_x \right\rangle$ *be a FPHFN, the degree of indeterminacy of* ε *is*

$$\pi_\varepsilon = \sqrt[3]{1-P_\varepsilon} = \sqrt[3]{1 - \left(\left(\frac{1}{|\mu_\varepsilon|}\sum_{\mu_i \in \mu_{h(x)}, p_i \in p_{h(x)}} (\mu_i p_i)\right)^3 + \left(\frac{1}{|\nu_\varepsilon|}\sum_{\nu_i \in \nu_{h(x)}, q_i \in q_{h(x)}} (\nu_i q_i)\right)^3\right)} \tag{8}$$

When analyzing the comparison of FHFNs, we encounter that the number of their corresponding element may not be equal. To deal with this situation, we utilize the normalization method proposed by Zhu et al. [73] which is shown as follows.

Definition 11 [73]. *Let* $\varepsilon = \left\langle \mu_{h(x)}/p_x, \nu_{h(x)}/q_x \right\rangle$ *be a FPHFN.* $\overline{\mu} = \sigma\mu^+ + (1-\sigma)\mu^-$ *and* $\overline{\nu} = \sigma\nu^+ + (1-\sigma)\nu^-$ *are the added membership degree and the non-membership degree, respectively.* μ^+ *and* μ^- *are the maximum and the minimum elements of* μ*, respectively.* ν^+ *and* ν^- *are the maximum and the minimum elements of* ν*, respectively.* $\sigma \in [0,1]$ *is a parameter representing the decision maker's risk preference. Then we can obtain three special cases of the preference of the decision maker:*

(1) *When the value of* σ *is 1, the optimistic decision maker may add the maximum membership degree* μ^+ *and the maximum non-membership degree* ν^+*.*

(2) *When the value of σ is 0.5, the neutral decision maker may add the membership degree of $0.5(\mu^+ + \mu^-)$ and the non-membership degree of $0.5(\nu^+ + \nu^-)$.*
(3) *When the value of σ is 0, the pessimist decision maker may add the minimum membership degree μ^- and the minimum non-membership degree ν^-.*

For unification, the set elements are added by default from a pessimistic perspective, and represented by simplified symbols, and the elements in the set of membership, non-membership and indeterminacy are arranged in ascending order in this paper. Additionally, to preserve the original probabilistic distribution, the probability of the original elements is redistributed equally among themselves and the newly added elements, ensuring that the addition of new elements does not alter the existing probabilities.

Example 1. *Let $\varepsilon_1, \varepsilon_2$ be two FPHFN, which are set as*

$$\varepsilon_1 = \left\langle \mu_{h_1}/p_1, \nu_{h_1}/q_1 \right\rangle = \langle \{0.7/0.6, 0.8/0.4\}, \{0.4/0.7, 0.6/0.3\} \rangle$$

$$\varepsilon_2 = \left\langle \mu_{h_2}/p_2, \nu_{h_2}/q_2 \right\rangle = \langle \{0.3/0.2, 0.4/0.3, 0.7/0.5\}, \{0.8/1.\} \rangle$$

To reflect the fuzzy information completely, we add the indeterminacy degree based on Equation (8) respectively, namely

$$\pi_1 = \sqrt[3]{1 - P_1^3} = \sqrt[3]{1 - \left(\left(\frac{1}{2} \times (0.7 \times 0.6 + 0.8 \times 0.4) \right)^3 + \left(\frac{1}{2} \times (0.4 \times 0.7 + 0.6 \times 0.3) \right)^3 \right)} = 0.9786,$$

$$\pi_2 = \sqrt[3]{1 - P_2^3} = \sqrt[3]{1 - \left(\left(\frac{1}{3} \times (0.3 \times 0.2 + 0.4 \times 0.3 + 0.7 \times 0.5) \right)^3 + (1 \times (0.8 \times 1))^3 \right)} = 0.7843.$$

Then we can obtain

$$\varepsilon_1 = \langle \{0.7/0.6, 0.8/0.4\}, \{0.4/0.7, 0.6/0.3\}, \{0.9786\} \rangle,$$

$$\varepsilon_2 = \langle \{0.3/0.2, 0.4/0.3, 0.7/0.5\}, \{0.8/1\}, \{0.9980\} \rangle.$$

After adding the minimum elements from a pessimistic perspective respectively, the two FPHFNs are normalized as

$$\hat{\varepsilon}_1 = \langle \{0.7/0.3, 0.7/0.3, 0.8/0.4\}, \{0.4/0.7, 0.6/0.3\}, \{0.9786\} \rangle,$$

$$\hat{\varepsilon}_2 = \langle \{0.3/0.2, 0.4/0.3, 0.7/0.5\}, \{0.8/0.5, 0.8/0.5\}, \{0.9980\} \rangle.$$

Definition 12 [66]. *Let $\varepsilon_1 = \left\langle \mu_{h_1(x)}/p_{1x}, \nu_{h_1(x)}/q_{1x} \right\rangle$ and $\varepsilon_2 = \left\langle \mu_{h_2(x)}/p_{2x}, \nu_{h_2(x)}/q_{2x} \right\rangle$ be two FPHFNs, the support degree between them can be expressed as:*

$$Sup(\varepsilon_1, \varepsilon_2) = 1 - d(\varepsilon_1, \varepsilon_2) \tag{9}$$

where $d(\varepsilon_1, \varepsilon_2)$ represents the distance beween the two FHFNs ε_1 and ε_2, defined as follows:

$$d(\varepsilon_1, \varepsilon_2) = \frac{1}{2}\left(\frac{1}{t}\sum_{i=1}^{t}\left|(\mu_{1i}p_{1i})^3 - (\mu_{2i}p_{2i})^3\right| + \frac{1}{g}\sum_{k=1}^{g}\left|(\nu_{1k}q_{1k})^3 - (\nu_{2k}q_{2k})^3\right| + \left|\pi_1^3 - \pi_2^3\right|\right) \tag{10}$$

where $t = |\mu_{\varepsilon_1}| = |\mu_{\varepsilon_2}|$ and $g = |\nu_{\varepsilon_1}| = |\nu_{\varepsilon_2}|$, μ_{1i} and μ_{2i} are the $i-$th smallest values of the membership degrees of ε_1, ν_{1k} and ν_{2k} are the $k-$th smallest values of the non-membership degrees of ε_2, and π_i denotes the indeterminacy degree of ε_i.

2.4. Power Bonferroni Mean (PBM) Operator

Definition 13 [51]. *Let $x_i (i = 1, 2, \cdots, n)$ be a set of real numbers, power average (PA) operator can be defined as the following form:*

$$PA(x_1, x_2, \cdots, x_n) = \frac{\sum_{i=1}^{n} (1 + T(x_i)) x_i}{\sum_{i=1}^{n} (1 + T(x_i))} \tag{11}$$

where $T(x_i) = \sum_{i=1, i \neq j}^{n} Sup(x_i, x_j)$, $i = 1, 2, \ldots, n$, denoting the support degree between x_i and x_j.

Definition 14 [57]. *Let $x_i (i = 1, 2, \cdots, n)$ be a set of real numbers and $x_i \geq 0$. The parameters $p \geq 0$, $q \geq 0$ and they are not both zero simultaneously, then the Bonferroni mean (BM) operator can be shown as the following form:*

$$BM^{p,q}(x_1, x_2, \cdots, x_n) = \left(\frac{1}{n(n-1)} \sum_{i,j=1, i \neq j}^{n} {x_i}^p {x_j}^q \right)^{\frac{1}{p+q}} \tag{12}$$

Definition 15 [61]. *Let $x_i (i = 1, 2, \cdots, n)$ be a set of real numbers and $x_i \geq 0$, the parameters $p \geq 0$, $q \geq 0$ and they are not both zero simultaneously. A power Bonferroni mean (PBM) operator holds the following form:*

$$\begin{aligned} &PBM(x_1, x_2, \cdots, x_n) \\ &= \left(\frac{1}{n(n-1)} \sum_{i,j=1, i \neq j}^{n} \left(\left(\frac{n(1 + T(x_i))}{\sum_{k=1}^{n} (1 + T(x_k))} x_i \right)^p \times \left(\frac{n(1 + T(x_j))}{\sum_{k=1}^{n} (1 + T(x_k))} x_j \right)^q \right) \right)^{\frac{1}{p+q}} \end{aligned} \tag{13}$$

3. Fermatean Hesitant Fuzzy Power Bonferroni Aggregation Operators

In this section, we shall discuss several Fermatean hesitant fuzzy power Bonferroni aggregation operators and their classic properties.

Definition 16. *Suppose that $p \geq 0$, $q \geq 0$ and they are not both zero simultaneously, and $\kappa_i = (\mu_i, \nu_i)$ $(i = 1, 2, \cdots, n)$ is a set of FHFNs. The Fermatean hesitant fuzzy power Bonferroni mean (FHFPBM) operator is a mapping $FHFPBM: \kappa^n \to \kappa$ such that*

$$\begin{aligned} &FHFPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) \\ &= \left(\left(\frac{1}{n(n-1)} \bigoplus_{i,j=1, i \neq j}^{n} \left(\frac{n(1 + T(\kappa_i))}{\sum_{t=1}^{n} (1 + T(\kappa_t))} \kappa_i \right) \otimes \left(\frac{n(1 + T(\kappa_j))}{\sum_{t=1}^{n} (1 + T(\kappa_t))} \kappa_j \right)^q \right) \right)^{\frac{1}{p+q}} \end{aligned} \tag{14}$$

where $T(\kappa_i) = \sum_{j=1, j \neq i}^{n} Sup(\kappa_i, \kappa_j)$, $i = 1, 2, \ldots, n$, and $Sup(\kappa_i, \kappa_j)$ indicates the support degree between Fermatean hesitant fuzzy sets κ_i and κ_j.

Theorem 3. *Let $\kappa_i = (\mu_i, \nu_i) (i = 1, 2, \cdots, n)$ be a collection of FHFNs, the parameters $p \geq 0$, $q \geq 0$ and they cannot be both zero simultaneously. The result obtained after the aggregation of FHFPBM operator is still FHFNs, and*

$$FHFPBM^{p,q}(\kappa_1,\kappa_2,\cdots,\kappa_n)=$$

$$\left\langle \left(\left(\sqrt[3]{1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\left(1-\mu_{i\lambda}^{3}\right)^{\frac{n(1+T(\kappa_i))}{\sum_{t=1}^{n}(1+T(\kappa_t))}} \right)^{p} \left(1-\left(1-\mu_{j\lambda}^{3}\right)^{\frac{n(1+T(\kappa_j))}{\sum_{t=1}^{n}(1+T(\kappa_t))}} \right)^{q} \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right), \right.$$

$$\left. \sqrt[3]{1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\nu_{i\theta}^{\frac{3n(1+T(\kappa_i))}{\sum_{t=1}^{n}(1+T(\kappa_t))}} \right)^{p} \left(1-\nu_{j\theta}^{\frac{3n(1+T(\kappa_j))}{\sum_{t=1}^{n}(1+T(\kappa_t))}} \right)^{q} \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}} \right\rangle \tag{15}$$

where $\mu_{i\lambda}$ and $\mu_{j\lambda}$ are the $\lambda-$ th smallest values of the membership degrees of the FHFNs κ_i and κ_j, respectively. And $\nu_{i\theta}$ and $\nu_{j\theta}$ are the $\theta-$ th smallest values of non-membership degrees of κ_i and κ_j, respectively. $\lambda=1,2,\ldots,\ |\mu_\kappa|$ and $\theta=1,2,\ldots,|\nu_\kappa|$, where $|\mu_\varepsilon|$ and $|\nu_\varepsilon|$ denote the number of elements contained in the membership and non-membership degrees of each FHFN, respectively.

Proof. Let $w_i{}' = \frac{n(1+T(\kappa_i))}{\sum_{t=1}^{n}(1+T(\kappa_t))}$, $w_j{}' = \frac{n(1+T(\kappa_j))}{\sum_{t=1}^{n}(1+T(\kappa_t))}$, then Equations (14) and (15) can be simplified as the following forms, respectively:

$$FHFPBM^{p,q}(\kappa_1,\kappa_2,\cdots,\kappa_n)=\left(\frac{1}{n(n-1)}\bigoplus_{i,j=1,i\neq j}^{n}(nw_i{}'\kappa_i)^p\otimes(nw_j{}'\kappa_j)^q\right)^{\frac{1}{p+q}} \tag{16}$$

$$FHFPBM^{p,q}(\kappa_1,\kappa_2,\cdots,\kappa_n)=$$

$$\left\langle \left(\left(\sqrt[3]{1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\left(1-\mu_{i\lambda}^{3}\right)^{nw_i{}'} \right)^{p} \left(1-\left(1-\mu_{j\lambda}^{3}\right)^{nw_j{}'} \right)^{q} \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right), \right.$$

$$\left. \sqrt[3]{1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\nu_{i\theta}^{3nw'_i} \right)^{p} \left(1-\nu_{j\theta}^{3nw'_j} \right)^{q} \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}} \right\rangle \tag{17}$$

According to the operational rules of FHFNs in Definition 6, it is easy to obtain

$$nw_i{}'\kappa_i=\left\langle \sqrt[3]{1-(1-\mu_i^3)^{nw_i{}'}},\nu_i^{nw_i{}'}\right\rangle,$$

$$nw_j{}'\kappa_j=\left\langle \sqrt[3]{1-(1-\mu_j^3)^{nw_j{}'}},\nu_j^{nw_j{}'}\right\rangle.$$

and

$$(nw_i{}'\kappa_i)^p=\left\langle \left(\sqrt[3]{1-(1-\mu_i^3)^{nw_i{}'}}\right)^p,\sqrt[3]{1-(1-\nu_i^{3nw_i{}'})^p}\right\rangle,$$

$$(nw_j'\kappa_j)^q = \left\langle \left(\sqrt[3]{1-(1-\mu_j^3)^{nw_j'}} \right)^q, \sqrt[3]{1-(1-\nu_j^{3nw_j'})^q} \right\rangle.$$

Then

$$(nw_i'\kappa_i)^p \otimes (nw_j'\kappa_j)^q = \left\langle \left(\sqrt[3]{1-(1-\mu_i^3)^{nw_j'}} \right)^p \left(\sqrt[3]{1-(1-\mu_j^3)^{nw_j'}} \right)^q, \sqrt[3]{1-(1-\nu_i^{3nw_i'})^p (1-\nu_j^{3nw_j'})^q} \right\rangle.$$

Hence

$$FHFPBM^{p,q}(\kappa_1,\kappa_2,\cdots,\kappa_n) = \left(\frac{1}{n(n-1)} \overset{n}{\underset{i,j=1,i\neq j}{\oplus}} (nw_i'\kappa_i)^p \otimes (nw_j'\kappa_j)^q \right)^{\frac{1}{p+q}}$$

$$= \left\langle \left(\left(\sqrt[3]{1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\left(1-\mu_{i\lambda}^3\right)^{nw_i'}\right)^p \left(1-\left(1-\mu_{j\lambda}^3\right)^{nw_j'}\right)^q \right) \right)^{\frac{1}{n(n-1)}}} \right)^{\frac{1}{p+q}} \right),$$

$$\sqrt[3]{1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\nu_{i\theta}^{3\,nw_i'}\right)^p \left(1-\nu_{j\theta}^{3\,nw_j'}\right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}} \Bigg\rangle$$

Since $0 \le \mu_i \le 1, 0 \le \nu_i \le 1$ and $0 \le {\mu_i}^3 + {\nu_i}^3 \le 1$, it can be derived as

$$0 \le \left(\sqrt[3]{1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\left(1-\mu_{i\lambda}^3\right)^{nw_i'}\right)^p \left(1-\left(1-\mu_{j\lambda}^3\right)^{nw_j'}\right)^q \right) \right)^{\frac{1}{n(n-1)}}} \right)^{\frac{1}{p+q}} \le 1,$$

$$0 \le \sqrt[3]{1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\nu_{i\theta}^{3nw_i'}\right)^p \left(1-\nu_{j\theta}^{3nw_j'}\right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}} \le 1.$$

and

$$0 \le \left(1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\left(1-\mu_{i\lambda}^3\right)^{nw_i'}\right)^p \left(1-\left(1-\mu_{j\lambda}^3\right)^{nw_j'}\right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}$$

$$+ \left(1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n} \left(1-\left(1-\nu_{i\theta}^{3nw_i'}\right)^p \left(1-\nu_{j\theta}^{3nw_j'}\right)^q \right) \right)^{\frac{1}{n(n-1)}} \right) \right) \le 1.$$

Therefore, the Theorem 3 has been proved according to the above. □

In addition, the FHFPBM operator owns several properties such as idempotency, boundedness and permutation invariance, which will be discussed in detail as follows.

Property 1 (Idempotency). *Let* $\kappa_i = (\mu_i, \nu_i)(i = 1, 2, \cdots, n)$ *be a set of FHFNs. If* $\kappa_1 = \kappa_2 = \cdots = \kappa_n = \kappa$, *then*

$$FHFPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) = \kappa \tag{18}$$

Property 2 (Boundedness). *Let* $\kappa_i = (\mu_i, \nu_i)(i = 1, 2, \cdots, n)$ *be a Fermatean hesitant fuzzy set, then*

$$\min\{\kappa_i\} \leq FHFPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) \leq \max\{\kappa_i\} \tag{19}$$

Property 3 (Permutation invariance). *Let* $\kappa_i = (\mu_i, \nu_i)(i = 1, 2, \cdots, n)$ *be a set of FHFNs. If* $\kappa_i' = (\kappa_1', \kappa_2', \cdots, \kappa_n')$ *is any permutation of* $\kappa_i = (\kappa_1, \kappa_2, \cdots, \kappa_n)$, *then*

$$FHFPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) = FHFPBM^{p,q}(\kappa_1', \kappa_2', \cdots, \kappa_n') \tag{20}$$

Given that the indicators of decision-making are usually assigned unequal weights in real life, this section further propose Fermatean hesitant fuzzy weighted power Bonferroni mean (FHFWPBM) operator, which takes the weights of indicators into account and is shown as follows.

Definition 17. *Let* $\kappa_i = (\mu_i, \nu_i)(i = 1, 2, \cdots, n)$ *be a set of FHFNs, and* $p, q \geq 0$, *p and q can not both be zero simultaneously. The Fermatean hesitant fuzzy weighted power Bonferroni mean (FHFWPBM) operator is a mapping* $FHFWPBM: \kappa^n \to \kappa$ *such that*

$$\begin{aligned} &FHFWPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) \\ &= \left(\frac{1}{n(n-1)} \underset{i,j=1, i\neq j}{\overset{n}{\oplus}} \left(\left(\frac{n w_i (1 + T(\kappa_i))}{\sum_{t=1}^{n} (1 + T(\kappa_t))} \kappa_i \right)^p \otimes \left(\frac{n w_j (1 + T(\kappa_j))}{\sum_{t=1}^{n} (1 + T(\kappa_t))} \kappa_j \right)^q \right) \right)^{\frac{1}{p+q}} \end{aligned} \tag{21}$$

where $w = (w_1, w_2, \ldots, w_n)$ *is the weighting vector of* κ_i, $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, *and* $T(\kappa_i) = \sum_{j=1, j\neq i}^{n} w_j Sup(\kappa_i, \kappa_j)$, $i = 1, 2, \cdots, n$. $Sup(\kappa_i, \kappa_j)$ *represents the support degree between* κ_i *and* κ_j.

Theorem 4. *Let* $\kappa_i = (\mu_i, \nu_i)(i = 1, 2, \cdots, n)$ *be a collection of FHFNs, the parameters* $p \geq 0$, $q \geq 0$ *and they cannot be both zero simultaneously. The result obtained after the aggregation of FHFWPBM operator is still FHFNs, and*

$$\begin{aligned} &FHFWPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) = \\ &\left\langle \left(\left(\sqrt[3]{1 - \left(\prod_{i,j=1, i\neq j}^{n} \left(1 - \left(1 - \left(1 - \mu_{i\lambda}^3 \right)^{\frac{n w_i (1 + T(\kappa_i))}{\sum_{t=1}^{n} w_t (1 + T(\kappa_t))}} \right)^p \left(1 - \left(1 - \mu_{j\lambda}^3 \right)^{\frac{n w_j (1 + T(\kappa_j))}{\sum_{t=1}^{n} w_t (1 + T(\kappa_t))}} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \right), \right. \\ &\left. \sqrt[3]{1 - \left(1 - \left(\prod_{i,j=1, i\neq j}^{n} \left(1 - \left(1 - \nu_{i\theta}^{\frac{3 n w_i (1 + T(\kappa_i))}{\sum_{t=1}^{n} w_t (1 + T(\kappa_t))}} \right)^p \left(1 - \nu_{j\theta}^{\frac{3 n w_j (1 + T(\kappa_j))}{\sum_{t=1}^{n} w_t (1 + T(\kappa_t))}} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}} \right\rangle \end{aligned} \tag{22}$$

where $\mu_{i\lambda}$ *and* $\mu_{j\lambda}$ *are the* $\lambda -$ th *smallest values of the membership degrees of the FHFNs* κ_i *and* κ_j, *respectively. And* $\nu_{i\theta}$ *and* $\nu_{j\theta}$ *are the* $\theta -$ th *smallest values of non-membership degrees of* κ_i *and* κ_j,

respectively. $\lambda = 1, 2, \ldots, \ |\mu_\kappa|$ *and* $\theta = 1, 2, \ldots, |\nu_\kappa|$, *where* $|\mu_\varepsilon|$ *and* $|\nu_\varepsilon|$ *denote the number of elements contained in the membership and non-membership degrees of each FHFN, respectively.*

The proving process of Theorem 4 is similar to the proof of Theorem 3 which is shown before. And the FHFWPBM operator has excellent properties such as idempotency, boundedness and permutation invariance, which are omitted here.

Definition 18. *Let* $p \geq 0$, $q \geq 0$ (p *and* q *cannot both be zero simultaneously), and* $\kappa_i = (\mu_i, \nu_i)$ $(i = 1, 2, \cdots, n)$ *be a Fermatean hesitant fuzzy set. A Fermatean hesitant fuzzy probabilistic weighted power Bonferroni mean (FHFPWPBM) operator for* κ_i *is a mapping* $FHFPWPBM : \kappa^n \to \kappa$ *such that*

$$FHFPWPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n)$$
$$= \left(\frac{1}{n(n-1)} \bigoplus_{i,j=1, i \neq j}^{n} \left(\left(\frac{n\widetilde{p}_i(1 + T(\kappa_i))}{\sum_{t=1}^{n}(1 + T(\kappa_t))} \kappa_i \right)^p \otimes \left(\frac{n\widetilde{p}_j(1 + T(\kappa_j))}{\sum_{t=1}^{n}(1 + T(\kappa_t))} \kappa_j \right)^q \right) \right)^{\frac{1}{p+q}} \quad (23)$$

where $\widetilde{p}_j$ *represents the comprehensive weighted probability,* $\widetilde{p}_j = \delta w_j + (1 - \delta)\overline{p}_j$ *with* $\delta \in [0, 1]$. $w = (w_1, w_2, \ldots, w_n)$ *is the weighting vector of* κ_i, $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, *and* $p = (p_1, p_2, \ldots, p_n)$ *is the probabilistic vector with* $p_i \in [0, 1]$ *and* $\sum_{i=1}^{n} p_i = 1$. *Suppose* τ_j *is the j-th largest element of* κ_i, *then* $\overline{p}_j$ *denotes the probability corresponding to* τ_j. $T(\kappa_i) = \sum_{j=1, j \neq i}^{n} \widetilde{p}_j Sup(\kappa_i, \kappa_j)$, $i = 1, 2, \cdots, n$, *where* $Sup(\kappa_i, \kappa_j)$ *representing the support degree between* κ_i *and* κ_j.

Particularly, when $\delta = 1$, the FHFPWPBM operator degenerates to the FHFWPBM operator. When $\delta = 0$, the FHFPWPBM operator degenerates to the Fermatean hesitant fuzzy probabilistic power Bonferroni mean (FHFPPBM) operator. And a numerical example is provided below to specify the process of obtaining the comprehensive weighted probability $\widetilde{p}_j$.

Example 2. *Given three FHFNs:* $\kappa_1 =< \{0.6, 0.7\}, \{0.3, 0.5\} >$, $\kappa_2 =< \{0.3, 0.5\}, \{0.4\} >$, *and* $\kappa_3 =< \{0.4, 0.6\}, \{0.3\} >$. *The weighting vector is* $w = (0.2, 0.3, 0.5)$ *and the probabilistic vector is* $p = (0.3, 0.3, 0.4)$, $\delta = 0.6$.

First, let's calculate the score of each FHFN and sort them according to their score values. Since

$$S_{\kappa_1} = (0.6^3 + 0.7^3)/2 - (0.3^3 + 0.5^3)/2 = 0.2035,$$
$$S_{\kappa_2} = (0.3^3 + 0.5^3)/2 - 0.2^3 = 0.0680,$$
$$S_{\kappa_3} = (0.4^3 + 0.6^3)/2 - 0.3^3 = 0.1130,$$

we have

$$\kappa_1 \succ \kappa_3 \succ \kappa_2,$$

Then the ordered probability are

$$\overline{p}_1 = p_1 = 0.3, \ \overline{p}_2 = p_3 = 0.4, \ \overline{p}_3 = p_2 = 0.3.$$

Consequently, the comprehensive weighted probability of each one can be calculated as

$$\widetilde{p}_1 = 0.6 \times 0.2 + 0.4 \times 0.3 = 0.24,$$

$$\widetilde{p}_2 = 0.6 \times 0.5 + 0.4 \times 0.3 = 0.42,$$

$$\widetilde{p}_3 = 0.6 \times 0.3 + 0.4 \times 0.4 = 0.34.$$

Theorem 5. *Let $\kappa_i = (\mu_i, \nu_i)(i = 1, 2, \cdots, n)$ be a collection of FHFNs with a corresponding probabilistic vector $p = (p_1, p_2, \ldots, p_n)$ and its weight vector is $w = (w_1, w_2, \ldots, w_n)$. The parameters $p \geq 0$, $q \geq 0$ and they cannot be both zero simultaneously. The result obtained after the aggregation of FHFPWPBM operator is still FHFNs.*

$$FHFPWPBM^{p,q}(\kappa_1, \kappa_2, \cdots, \kappa_n) = \left\langle \left(\sqrt[3]{1 - \left(\prod_{i,j=1, i \neq j}^{n} \left(1 - \left(1 - \left(1 - \mu_{i\lambda}^3 \right)^{\frac{n\widetilde{p}_i(1+T(\kappa_i))}{\sum_{t=1}^{n} \widetilde{p}_t(1+T(\kappa_t))}} \right)^p \left(1 - \left(1 - \mu_{j\lambda}^3 \right)^{\frac{n\widetilde{p}_j(1+T(\kappa_j))}{\sum_{t=1}^{n} \widetilde{p}_t(1+T(\kappa_t))}} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}, \right.$$
$$\left. \sqrt[3]{1 - \left(1 - \left(\prod_{i,j=1, i \neq j}^{n} \left(1 - \left(1 - \nu_{i\theta}^{\frac{3n\widetilde{p}_i(1+T(\kappa_i))}{\sum_{t=1}^{n} \widetilde{p}_t(1+T(\kappa_t))}} \right)^p \left(1 - \nu_{j\theta}^{\frac{3n\widetilde{p}_j(1+T(\kappa_j))}{\sum_{t=1}^{n} \widetilde{p}_t(1+T(\kappa_t))}} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}}} \right\rangle \tag{24}$$

where $\widetilde{p}_j$ represents the comprehensive weighted probability combining the weight vector $w = (w_1, w_2, \ldots, w_n)$ and probabilistic vector $p = (p_1, p_2, \ldots, p_n)$ by $\widetilde{p}_j = \delta w_j + (1 - \delta)\overline{p}_j$ with $\delta \in [0, 1]$, $\overline{p}_j$ is the probability corresponding to τ_j that denotes the j-th largest element of κ_i. And $\mu_{i\lambda}$ and $\mu_{j\lambda}$ are the $\lambda -$ th smallest values of the membership degrees of the FHFNs κ_i and κ_j, respectively. Similarly, $\nu_{i\theta}$ and $\nu_{j\theta}$ are the $\theta -$ th smallest values of non-membership degrees of κ_i and κ_j, respectively. $\lambda = 1, 2, \ldots, |\mu_\kappa|$ and $\theta = 1, 2, \ldots, |\nu_\kappa|$, where $|\mu_\varepsilon|$ and $|\nu_\varepsilon|$ represent the number of components contained in the membership and non-membership degrees of each FHFN, respectively.

4. Fermatean Probabilistic Hesitant Fuzzy Power Bonferroni Aggregation Operators

With the increasing uncertainty and complexity of the evaluation information in real decision-making problems, the FPHFSs, which can better describe the probabilistic information than FHFSs during decision-making process, deserve to be introduced for handling MADM problems. Therefore, in order to better characterize the randomness of information evaluation results and maximize the completeness and accuracy of the information integration, a series of Fermatean probabilistic hesitant fuzzy power Bonferroni aggregation operators will be presented next, including the Fermatean probabilistic hesitant fuzzy power Bonferroni mean (FPHFPBM) operator, the Fermatean probabilistic hesitant fuzzy weighted power Bonferroni mean (FPHFPWPBM) operator and the Fermatean probabilistic hesitant fuzzy probabilistic weighted power Bonferroni mean (FPHFPWPBM) operator. And the excellent properties of each are investigated as well.

Definition 19. *Suppose that $p \geq 0, q \geq 0$ and they are not both zero simultaneously, and $\varepsilon_i = \left\langle \mu_{h_i(x)} / p_{ix}, \nu_{h_i(x)} / q_{ix} \right\rangle (i = 1, 2, \ldots, n)$ is a set of FPHFNs. The Fermatean probabilistic hesitant fuzzy power Bonferroni mean (FPHFPBM) operator is a mapping $FPHFPBM : \varepsilon^n \to \varepsilon$ such that*

$$FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \left(\left(\frac{1}{n(n-1)} \bigoplus_{i,j=1, i \neq j}^{n} \left(\frac{n(1+T(\varepsilon_i))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))} \varepsilon_i \right) \otimes \left(\frac{n(1+T(\varepsilon_j))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))} \varepsilon_j \right)^q \right) \right)^{\frac{1}{p+q}} \tag{25}$$

where $T(\varepsilon_i) = \sum_{j=1, j\neq i}^{n} Sup(\varepsilon_i, \varepsilon_j)$, $i = 1, 2, \ldots, n$, *and* $Sup(\varepsilon_i, \varepsilon_j)$ *indicates the support degree between the FPHFNs* ε_i *and* ε_j.

Theorem 6. *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)} / p_{ix}, \nu_{h_i(x)} / q_{ix} \right\rangle (i = 1, 2, \ldots, n)$ *be a collection of FPHFNs, the parameters* $p \geq 0$, $q \geq 0$ *and they cannot be both zero simultaneously. The result obtained after the aggregation of FPHFPBM operator is still FPHFNs, and*

$$FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) =$$

$$\left\langle \bigcup_{\substack{\mu_i \in \mu_{h_{i(x)}}, \mu_j \in \mu_{h_{j(x)}} \\ p_i \in p_{ix}, p_j \in p_{jx}}} \left\{ \left(\sqrt[3]{1 - \left(\prod_{i,j=1, i\neq j}^{n} \left(1 - \left(1 - (1 - \mu_{i\lambda}^3)^{\frac{n(1+T(\varepsilon_i))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}} \right)^p \left(1 - \left(1 - \mu_{j\lambda}^3 \right)^{\frac{n(1+T(\varepsilon_j))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}} \right)^q \right) \right)^{\frac{1}{n(n-1)}} } \right)^{\frac{1}{p+q}} \Big/ \prod_{i,j=1, i\neq j}^{n} p_i p_j \right\}, \right.$$

$$\left. \bigcup_{\substack{\nu_i \in \nu_{h_{i(x)}}, \nu_j \in \nu_{h_{j(x)}} \\ q_i \in q_{ix}, q_j \in q_{jx}}} \left\{ \sqrt[3]{1 - \left(1 - \left(\prod_{i,j=1, i\neq j}^{n} \left(1 - \left(1 - \nu_{i\theta}^{\frac{3n(1+T(\varepsilon_j))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}} \right)^p \left(1 - \nu_{j\theta}^{\frac{3n(1+T(\varepsilon_j))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} } \Big/ \prod_{i,j=1, i\neq j}^{n} q_i q_j \right\} \right\rangle \tag{26}$$

where $\mu_{i\lambda}$ *and* $\mu_{j\lambda}$ *are the* $\lambda -$ th *smallest values of the membership degrees of* ε_i *and* ε_j*, respectively. And* $\nu_{i\theta}$ *and* $\nu_{j\theta}$ *are the* $\theta -$ th *smallest values of non-membership degrees of* ε_i *and* ε_j*, respectively.* $\lambda = 1, 2, \ldots,$ $|\mu_\varepsilon|$ *and* $\theta = 1, 2, \ldots, |\nu_\varepsilon|$*, where* $|\mu_\varepsilon|$ *and* $|\nu_\varepsilon|$ *denote the number of elements contained in the membership and non-membership degrees of each FPHFN, respectively.*

Proof. Let $\omega_i' = \frac{n(1+T(\varepsilon_i))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}$, $\omega_j' = \frac{n\left(1+T(\varepsilon_j)\right)}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}$, then Equations (25) and (26) can be simplified as the following forms, respectively:

$$FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \left(\frac{1}{n(n-1)} \bigoplus_{i,j=1, i\neq j}^{n} (nw_i'\varepsilon_i)^p \otimes (nw_j'\varepsilon_j)^q \right)^{\frac{1}{p+q}} \tag{27}$$

$$FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) =$$

$$\left\langle \bigcup_{\substack{\mu_i \in \mu_{h_{i(x)}}, \mu_j \in \mu_{h_{j(x)}} \\ p_i \in p_{ix}, p_j \in p_{jx}}} \left\{ \left(\sqrt[3]{1 - \left(\prod_{i,j=1, i\neq j}^{n} \left(1 - \left(1 - (1 - \mu_{i\lambda}^3)^{nw_i'} \right)^p \left(1 - \left(1 - \mu_{j\lambda}^3 \right)^{nw_j'} \right)^q \right) \right)^{\frac{1}{n(n-1)}} } \right)^{\frac{1}{p+q}} \Big/ \prod_{i,j=1, i\neq j}^{n} p_i p_j \right\}, \right.$$

$$\left. \bigcup_{\substack{\nu_i \in \nu_{h_{i(x)}}, \nu_j \in \nu_{h_{j(x)}} \\ q_i \in q_{ix}, q_j \in q_{jx}}} \left\{ \sqrt[3]{1 - \left(1 - \left(\prod_{i,j=1, i\neq j}^{n} \left(1 - \left(1 - \nu_{i\theta}^{3\,nw_i'} \right)^p \left(1 - \nu_{j\theta}^{3\,nw_j'} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} } \Big/ \prod_{i,j=1, i\neq j}^{n} q_i q_j \right\} \right\rangle \tag{28}$$

According to the operational rules of FPHFNs in Definition 9, it can be derived that

$$nw_i'\varepsilon_i = \left\langle \bigcup_{\mu_i \in \mu_{h_{i(x)}}, p_i \in p_{ix}} \left\{ \sqrt[3]{1 - (1 - \mu_i^3)^{nw_i'}} \Big/ p_i \right\}, \bigcup_{\nu_i \in \nu_{h_i(x)}, q_i \in q_{ix}} \left\{ \nu_i^{nw_i'} \Big/ q_i \right\} \right\rangle$$

$$nw_j'\varepsilon_j = \left\langle \bigcup_{\mu_j \in \mu_{h_{j(x)}}, p_j \in p_{jx}} \left\{ \sqrt[3]{1 - (1 - \mu_j^3)^{nw_j'}} \Big/ p_j \right\}, \bigcup_{\nu_i \in \nu_{h_j(x)}, q_i \in q_{jx}} \left\{ \nu_j^{nw_j'} \Big/ q_j \right\} \right\rangle,$$

and

$$\left(nw_i'\varepsilon_i\right)^p = \left\langle \bigcup_{\mu_i\in\mu_{h_{i(x)}},p_i\in p_{ix}} \left\{ \left(\sqrt[3]{1-\left(1-\mu_i^3\right)^{nw_i'}}\right)^p \middle/ p_i \right\}, \bigcup_{\nu_i\in\nu_{h_{i(x)}},q_i\in q_{ix}} \left\{ \sqrt[3]{1-\left(1-\nu_i^{3nw_i'}\right)^p} \middle/ q_i \right\} \right\rangle$$

$$\left(nw_j'\varepsilon_j\right)^q = \left\langle \bigcup_{\mu_j\in\mu_{h_{j(x)}},p_j\in p_{jx}} \left\{ \left(\sqrt[3]{1-\left(1-\mu_j^3\right)^{nw_j'}}\right)^p \middle/ p_j \right\}, \bigcup_{\nu_j\in\nu_{h_{j(x)}},q_j\in q_{jx}} \left\{ \sqrt[3]{1-\left(1-\nu_j^{3nw_j'}\right)^p} \middle/ q_j \right\} \right\rangle.$$

Then

$$\left(nw_i'\kappa_i\right)^p \otimes \left(nw_j'\kappa_j\right)^q = \left\langle \bigcup_{\substack{\mu_i\in\mu_{h_{i(x)}},\mu_j\in\mu_{h_{j(x)}} \\ p_i\in p_{ix},p_j\in p_{jx}}} \left\{ \left(\sqrt[3]{1-\left(1-\mu_i^3\right)^{nw_j'}}\right)^p \left(\sqrt[3]{1-\left(1-\mu_j^3\right)^{nw_j'}}\right)^q \middle/ p_ip_j \right\}, \right.$$
$$\left. \bigcup_{\substack{\nu_i\in\nu_{h_{i(x)}},\nu_j\in\nu_{h_{j(x)}} \\ q_i\in q_{ix},q_j\in q_{jx}}} \left\{ \sqrt[3]{1-\left(1-\nu_i^{3nw_i'}\right)^p\left(1-\nu_j^{3nw_j'}\right)^q} \middle/ q_iq_j \right\} \right\rangle.$$

Hence

$$FPHFPBM^{p,q}(\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n) = \left(\frac{1}{n(n-1)} \overset{n}{\underset{i,j=1,i\neq j}{\oplus}} \left(nw_i'\varepsilon_i\right)^p \otimes \left(nw_j'\varepsilon_j\right)^q\right)^{\frac{1}{p+q}}$$

$$= \left\langle \bigcup_{\substack{\mu_i\in\mu_{h_{i(x)}},\mu_j\in\mu_{h_{j(x)}} \\ p_i\in p_{ix},p_j\in p_{jx}}} \left\{ \left(\sqrt[3]{1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\left(1-\left(1-\mu_{i\lambda}^3\right)^{nw_i'}\right)^p\left(1-\left(1-\mu_{j\lambda}^3\right)^{nw_j'}\right)^q\right)\right)^{\frac{1}{n(n-1)}}}\right)^{\frac{1}{p+q}} \middle/ \prod_{i,j=1,i\neq j}^{n} p_ip_j \right\}, \right.$$
$$\left. \bigcup_{\substack{\nu_i\in\nu_{h_{i(x)}},\nu_j\in\nu_{h_{j(x)}} \\ q_i\in q_{ix},q_j\in q_{jx}}} \left\{ \sqrt[3]{1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\left(1-\nu_{i\theta}^{3\,nw_i'}\right)^p\left(1-\nu_{j\theta}^{3\,nw_j'}\right)^q\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}} \middle/ \prod_{i,j=1,i\neq j}^{n} q_iq_j \right\} \right\rangle.$$

Since $0 \le \mu_i \le 1$, $0 \le \nu_i \le 1$ and $0 \le {\mu_i}^3 + {\nu_i}^3 \le 1$, it is easy to obtain that

$$0 \le \left(\sqrt[3]{1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\left(1-\left(1-\mu_{i\lambda}^3\right)^{nw_i'}\right)^p\left(1-\left(1-\mu_{j\lambda}^3\right)^{nw_j'}\right)^q\right)\right)^{\frac{1}{n(n-1)}}}\right)^{\frac{1}{p+q}} \le 1,$$

$$0 \le \sqrt[3]{1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\left(1-\nu_{i\theta}^{3nw_i'}\right)^p\left(1-\nu_{j\theta}^{3nw_j'}\right)^q\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}} \le 1,$$

and

$$0 \leq \left(1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\left(1-\left(1-\mu_{i\lambda}^{3}\right)^{nw_i'}\right)^{p}\left(1-\left(1-\mu_{j\lambda}^{3}\right)^{nw_j'}\right)^{q}\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{p+q}}$$
$$+\left(1-\left(1-\left(\prod_{i,j=1,i\neq j}^{n}\left(1-\left(1-\nu_{i\theta}^{3nw_i'}\right)^{p}\left(1-\nu_{j\theta}^{3nw_j'}\right)^{q}\right)\right)^{\frac{1}{n(n-1)}}\right)\right)\leq 1.$$

Also, given that $p_i, q_i \in [0,1]$, $p_j, q_j \in [0,1]$, it is evident that

$$0 \leq \prod_{i,j=1,i\neq j}^{n} p_i p_j \leq 1,\ 0 \leq \prod_{i,j=1,i\neq j}^{n} q_i q_j \leq 1.$$

Therefore, the Theorem 6 has been proved according to the above. □

In addition, the FPHFPBM operator owns several properties such as idempotency, boundedness and permutation invariance, which will be discussed in detail as follows.

Property 4 (Idempotency). *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)}/p_{ix}, \nu_{h_i(x)}/q_{ix} \right\rangle (i = 1,2,\ldots,n)$ *be a set of FPHFNs. If* $\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_n = \varepsilon$, *then*

$$FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \varepsilon \tag{29}$$

Property 5 (Boundedness). *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)}/p_{ix}, \nu_{h_i(x)}/q_{ix} \right\rangle (i = 1,2,\ldots,n)$ *be a Fermatean probabilistic hesitant fuzzy set, then*

$$\min\{\varepsilon_i\} \leq FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) \leq \max\{\varepsilon_i\} \tag{30}$$

Property 6 (Permutation invariance). *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)}/p_{ix}, \nu_{h_i(x)}/q_{ix} \right\rangle (i = 1,2,\ldots,n)$ *be a set of FPHFNs. If* $\varepsilon_i' = (\varepsilon_1', \varepsilon_2', \cdots, \varepsilon_n')$ *is any permutation of* $\varepsilon_i = (\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n)$, *then*

$$FPHFPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = FPHFPBM^{p,q}(\varepsilon'_1, \varepsilon'_2, \cdots, \varepsilon'_n) \tag{31}$$

Definition 20. *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)}/p_{ix}, \nu_{h_i(x)}/q_{ix} \right\rangle (i = 1,2,\ldots,n)$ *be a set of FPHFNs, and* $p, q \geq 0$, *p and q can not both be zero simultaneously. The Fermatean probabilistic hesitant fuzzy weighted power Bonferroni mean (FPHFWPBM) operator is a mapping* $FPHFWPBM: \varepsilon^n \to \varepsilon$ *such that*

$$FPHFWPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n)$$
$$= \left(\frac{1}{n(n-1)} \overset{n}{\underset{i,j=1,i\neq j}{\oplus}} \left(\left(\frac{nw_i(1+T(\varepsilon_i))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}\varepsilon_i\right)^{p} \otimes \left(\frac{nw_j(1+T(\varepsilon_j))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))}\varepsilon_j\right)^{q}\right)\right)^{\frac{1}{p+q}} \tag{32}$$

where $w = (w_1, w_2, \ldots, w_n)$ *is the weighting vector of* ε_i, $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, *and* $T(\kappa_i) = \sum_{j=1,j\neq i}^{n} w_j Sup(\varepsilon_i, \varepsilon_j)$, $i = 1,2,\cdots,n$. $Sup(\varepsilon_i, \varepsilon_j)$ *represents the support degree between* ε_i *and* ε_j.

Theorem 7. *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)} / p_{ix}, \nu_{h_i(x)} / q_{ix} \right\rangle (i = 1, 2, \ldots, n)$ *be a set of FPHFNs, the parameters* $p \geq 0, q \geq 0$ *and they cannot be both zero simultaneously. The result obtained after the aggregation of FPHFWPBM operator is still FPHFNs, and*

$$FPHFWPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \left\langle \bigcup_{\substack{\mu_i \in \mu_{h_{i(x)}}, \mu_j \in \mu_{h_{j(x)}} \\ p_i \in p_{ix}, p_j \in p_{jx}}} \left\{ \left(\sqrt[3]{1 - \left(\prod_{i,j=1, i \neq j}^{n} \left(1 - \left(1 - \left(1 - \mu_{i\lambda}^3 \right) \frac{nw_i(1+T(\varepsilon_i))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^p \left(1 - \left(1 - \mu_{j\lambda}^3 \right) \frac{nw_j(1+T(\varepsilon_j))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \Big/ \prod_{i,j=1, i \neq j}^{n} p_i p_j \right\}, \right.$$

$$\left. \bigcup_{\substack{\nu_i \in \nu_{h_{i(x)}}, \nu_j \in \nu_{h_{j(x)}} \\ q_i \in q_{ix}, q_j \in q_{jx}}} \left\{ \sqrt[3]{1 - \left(1 - \left(\prod_{i,j=1, i \neq j}^{n} \left(1 - \left(1 - \nu_{i\theta}^3 \right) \frac{3nw_i(1+T(\varepsilon_i))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^p \left(1 - \nu_{j\theta}^3 \frac{3nw_j(1+T(\varepsilon_j))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \Big/ \prod_{i,j=1, i \neq j}^{n} q_i q_j \right\} \right\rangle \tag{33}$$

The proving process of Theorem 7 is similar to the proof of Theorem 3 which is shown before. And the FPHFWPBM operator has excellent properties such as idempotency, boundedness and permutation invariance, which are omitted here.

Definition 21. *Let* $p \geq 0$, $q \geq 0$ *(*p *and* q *cannot both be zero simultaneously), and* $\varepsilon_i = \left\langle \mu_{h_i(x)} / p_{ix}, \nu_{h_i(x)} / q_{ix} \right\rangle (i = 1, 2, \ldots, n)$ *be a Fermatean probabilistic hesitant fuzzy set. A Fermatean probabilistic hesitant fuzzy probabilistic weighted power Bonferroni mean (FPHFPWPBM) operator for* ε_i *is a mapping* $FPHFPWPBM : \varepsilon^n \to \varepsilon$ *such that*

$$FPHFPWPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \left(\frac{1}{n(n-1)} \bigoplus_{i,j=1, i \neq j}^{n} \left(\left(\frac{n\widetilde{p}_i(1+T(\varepsilon_i))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))} \varepsilon_i \right)^p \otimes \left(\frac{n\widetilde{p}_j(1+T(\varepsilon_j))}{\sum_{t=1}^{n}(1+T(\varepsilon_t))} \varepsilon_j \right)^q \right) \right)^{\frac{1}{p+q}} \tag{34}$$

where $T(\kappa_i) = \sum_{j=1, j \neq i}^{n} \widetilde{p}_j Sup(\varepsilon_i, \varepsilon_j)$, $i = 1, 2, \cdots, n$. $Sup(\varepsilon_i, \varepsilon_j)$ *represents the support degree between* ε_i *and* ε_j. $\widetilde{p}_j = \delta w_j + (1-\delta)\overline{p}_j$ *with* $\delta \in [0, 1]$, *and* $\overline{p}_j$ *is the ordered probability related to* τ_j, *which is the* j*-th largest element of* ε_i.

Definition 22. *Let* $\varepsilon_i = \left\langle \mu_{h_i(x)} / p_{ix}, \nu_{h_i(x)} / q_{ix} \right\rangle (i = 1, 2, \ldots, n)$ *be a set of FPHFNs, and* $p \geq 0$, $q \geq 0$ *(*p *and* q *cannot both be zero simultaneously), then*

$$FPHFPWPBM^{p,q}(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) = \left\langle \bigcup_{\substack{\mu_i \in \mu_{h_{i(x)}}, \mu_j \in \mu_{h_{j(x)}} \\ p_i \in p_{ix}, p_j \in p_{jx}}} \left\{ \left(\sqrt[3]{1 - \left(\prod_{i,j=1, i \neq j}^{n} \left(1 - \left(1 - \left(1 - \mu_{i\lambda}^3 \right) \frac{n\widetilde{p}_i(1+T(\varepsilon_i))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^p \left(1 - \left(1 - \mu_{j\lambda}^3 \right) \frac{n\widetilde{p}_j(1+T(\varepsilon_j))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \Big/ \prod_{i,j=1, i \neq j}^{n} p_i p_j \right\}, \right.$$

$$\left. \bigcup_{\substack{\nu_i \in \nu_{h_{i(x)}}, \nu_j \in \nu_{h_{j(x)}} \\ q_i \in q_{ix}, q_j \in q_{jx}}} \left\{ \sqrt[3]{1 - \left(1 - \left(\prod_{i,j=1, i \neq j}^{n} \left(1 - \left(1 - \nu_{i\theta}^3 \right) \frac{3n\widetilde{p}_i(1+T(\varepsilon_i))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^p \left(1 - \nu_{j\theta}^3 \frac{3n\widetilde{p}_j(1+T(\varepsilon_j))}{\sum_{t=1}^{n} \widetilde{p}_t (1+T(\varepsilon_t))} \right)^q \right) \right)^{\frac{1}{n(n-1)}} \right)^{\frac{1}{p+q}} \Big/ \prod_{i,j=1, i \neq j}^{n} q_i q_j \right\} \right\rangle \tag{35}$$

where $\widetilde{p}_j = \delta w_j + (1-\delta)\overline{p}_j$ *with* $\delta \in [0,1]$, $w = (w_1, w_2, \ldots, w_n)$ *is the weighting vector of* ε_i *with* $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$, *and* $p = (p_1, p_2, \ldots, p_n)$ *is the probabilistic vector with* $p_i \in [0,1]$, $\sum_{i=1}^{n} p_i = 1$. $\overline{p}_j$ *is the ordered probability associated with* τ_j *which denotes the j-th largest element of* κ_i. $\mu_{i\lambda}$ *and* $\mu_{j\lambda}$ *are the* $\lambda -$ th *smallest values of the membership degrees of* ε_i *and* ε_j, *respectively. And* $v_{i\theta}$ *and* $v_{j\theta}$ *are the* $\theta -$ th *smallest values of non-membership degrees of* ε_i *and* ε_j, *respectively.* $\lambda = 1, 2, \ldots,\ |\mu_\varepsilon|$ *and* $\theta = 1, 2, \ldots, |\nu_\varepsilon|$, *where* $|\mu_\varepsilon|$ *and* $|\nu_\varepsilon|$ *denote the number of elements contained in the membership and non-membership degrees of each FPHFN, respectively.*

The proposed FPHFPWPBM operator innovatively combines the PBM operator and probabilistic information for incorporating the weight allocation under the Fermatean probabilistic hesitant fuzzy environment. It extends the traditional Fermatean hesitant fuzzy aggregation operators to the Fermatean hesitant fuzzy power Bonferroni aggregation operators with dual-probabilistic information, which can certainly provide a novel technique and method to deal with more complex MADM problems with uncertain information.

5. Application of the FPHFPWPBM Operator in Green Supplier Selection

5.1. Decision-Making Process

Suppose that the expert group is currently planning to hold a specific discussion on a multi-attribute decision-making (MADM) issue. Let $X_i = \{X_1, X_2, \ldots, X_m\}$ be a set of alternatives and $A_j = \{A_1, A_2, \ldots, A_n\}$ be a collection of attributes. $\omega_i = \{\omega_1, \omega_2, \cdots, \omega_n\}^T$ is the weighting vector corresponding to each attribute such that $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$. $\alpha_{ij} = \langle \mu_{ij}/p_{ij}, \nu_{ij}/q_{ij} \rangle$ $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ is a set of Fermatean probabilistic hesitant fuzzy numbers, where α_{ij} denotes the evaluation value of alternative X_i with respect to attribute A_j, p_{ij} and q_{ij} represent the probabilities attached to each element in the membership and non-membership degrees, respectively. Then, the Fermatean probabilistic hesitant fuzzy decision matrix $D = (\alpha_{ij})_{m\times n}$ is obtained. In light of this issue this paper proposes a Fermatean probabilistic hesitant fuzzy MADM method based on FPHFPWPBM operator, and the specific steps are as follows:

Step 1. Relevant experts are invited to give evaluation information for each alternative and a Fermatean probabilistic hesitant fuzzy decision matrix $D = (\alpha_{ij})_{m\times n}$ is established. Then judge whether the matrix is normalized or not. If it is not normalized, normalize it according to the following Formula.

$$\overline{D}_{ij} = \begin{cases} \alpha_{ij} = (\mu_{ij}/p_{ij}, \nu_{ij}/q_{ij}), A_j \text{ is benefit-type attribute} \\ \overline{\alpha}_{ij} = (\nu_{ij}/q_{ij}, \mu_{ij}/p_{ij}), A_j \text{ is cost-type attribute} \end{cases} \tag{36}$$

Step 2. Add constant elements to make the cardinal numbers of the FPHFSs under each alternative equal and the processed Fermatean probabilistic hesitant fuzzy decision matrix is obtained. From a pessimistic perspective, the smallest elements are added to the set to equalize the cardinal numbers. And the probability values attached to the newly added elements are assigned based on the criteria outlined in Definition 11.

Step 3. Calculate the support degree $Sup(\overline{\alpha}_{lk}, \overline{\alpha}_{lj})$ between varibles.

$$\begin{aligned} &Sup(\overline{\alpha}_{lk}, \overline{\alpha}_{lj}) = 1 - d(\overline{\alpha}_{lk}, \overline{\alpha}_{lj}), \\ &k, j = 1, 2, \ldots, m;\ l = 1, 2, \ldots, n. \end{aligned} \tag{37}$$

Step 4. Calculate the corresponding support degree $T(\overline{\alpha}_{lk})$ connected to FPHFN $\overline{\alpha}_{ij}$ based on the comprehensive weighted probability $\widetilde{p}_j$, and further obtain the support degree index ξ_{lk} of the variables.

$$\widetilde{p}_j = \delta\omega_j + (1-\delta)\overline{p}_j, \delta \in [0,1] \tag{38}$$

$$\xi_{lk} = \frac{n\widetilde{p}_k(1+T(\overline{\alpha}_{lk}))}{\sum_{t=1}^{n}\widetilde{p}_t(1+T(\overline{\alpha}_{lt}))}, k = 1,2,\ldots,m,\ l = 1,2,\ldots,n. \tag{39}$$

Step 5. Use the definition of FPHFPWPBM operator to aggregate the attribute value $\alpha_{ij}(j = 1,2,\ldots,n)$ corresponding to each alternative $X_i = (X_1, X_2, \ldots, X_m)$ so as to obtain the comprehensive attribute value $V_i(i = 1,2,\ldots,m)$ of scheme X_i.

Step 6. The score function (Formula (4)) and accuracy function (Formula (5)) are used to sort the comprehensive attribute values, and then the best scheme is selected based on the sorting results.

Given that the probability values contained in $V_i(i = 1,2,\ldots,m)$ might be so tiny, it is advisable to cope with the probability values by the following Formula (40), which can help to facilitate the numerical calculation without affecting the score results. Then the comprehensive attribute value $\hat{V}_i = (\mu_i/\hat{p}_i, \nu_i/\hat{q}_i)(i = 1,2,\ldots,m)$ used to calculate the final scores are obtained. The detailed flowchart of this method is shown in Figure 1.

$$\hat{p}_i = \frac{p_i}{\sum_{k=1}^{m} p_k},\ \hat{q}_i = \frac{q_i}{\sum_{k=1}^{m} q_k}\ (i = 1,2,\ldots,m) \tag{40}$$

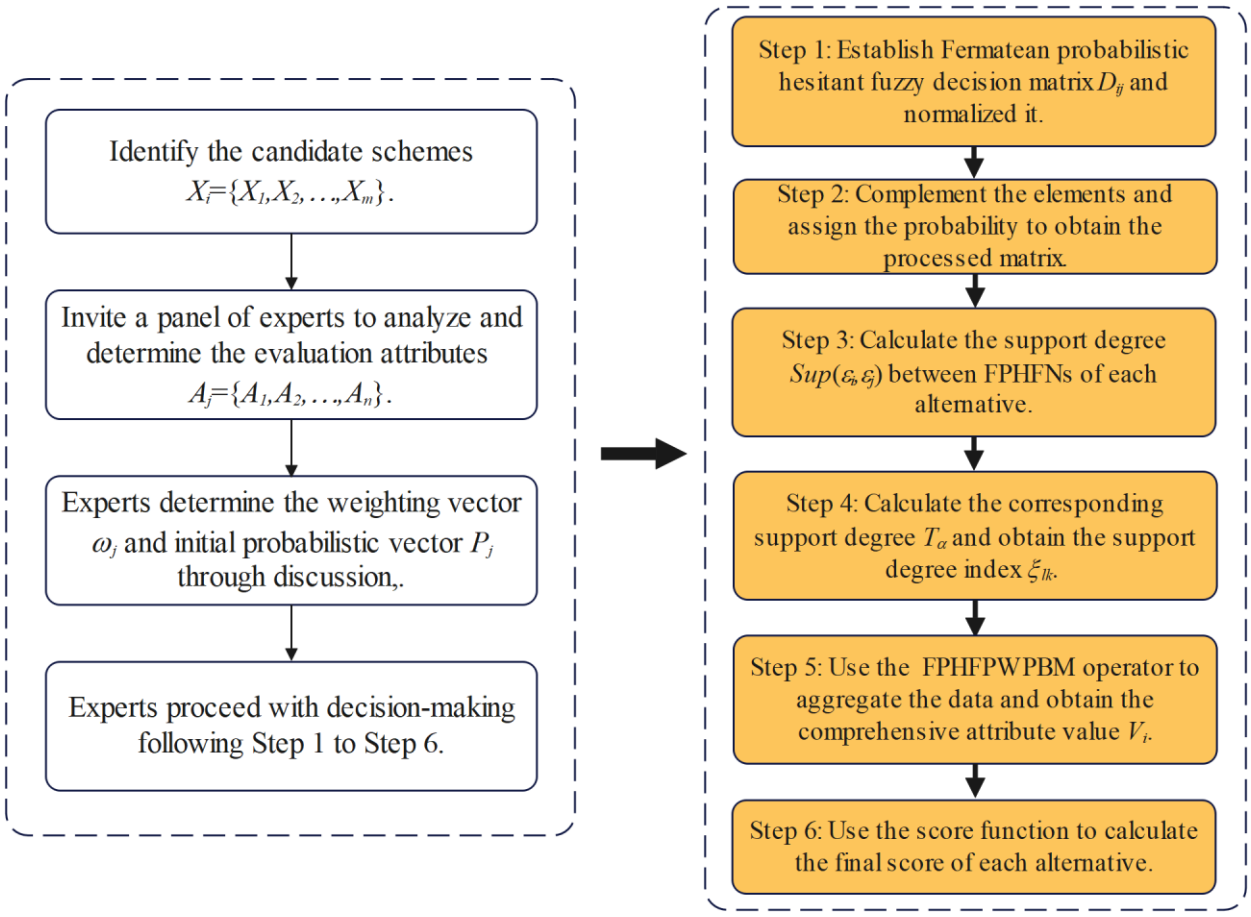


Figure 1. The flow chart of the proposed method.

5.2. Numerical Example

With the continuous development of the economy and society, and the increasingly severe global environmental issues, more and more countries and regions have begun to advocate for green and low-carbon production and lifestyles. The demand for sustainable products among consumers has gradually increased. In order to better win consumers' favor for green products, more and more enterprises are beginning to transition towards green

and low-carbon practices. Green supply chain management has now become an important way for enterprises to enhance competitiveness and achieve sustainable development [74].

First proposed in 1996, the concept of green supply chain management refers to a supply chain system that meets environmental protection requirements throughout the entire life cycle of a product from initial design to recycling and disposal, so as to realize the dual unity of economic benefits and environmental benefits. Now widely applied in various fields such as manufacturing, automotive industry, construction industry and service industry, the key link of Green supply chain management is the selection of green suppliers [75]. By choosing green suppliers that are compatible with their development strategies and production programme, the core enterprises can guide the green behaviors of upstream and downstream enterprises, help them to achieve environmental goals, improve economic performance, and thus improve the green and sustainability of the entire supply chain network. Therefore, it is particularly important for green enterprises to establish and implement scientific, reasonable, and effective evaluation and selection methods for green suppliers, which has become a common challenge faced in the process of enterprise development. In recent years, under the background of proposal of the "dual carbon goals" in China, many enterprises have begun to get involved in the evaluation and selection of green suppliers [76]. In July 2024, the State Council of China proposed to accelerate the comprehensive green transformation of economic and social development, and to promote the green and low-carbon transformation and upgrading of traditional industries, thereby clarifying the direction for green supply chain management. In pursuit of sustainable development, supply chain enterprises must fully consider the green development capabilities of their supply chain when assessing potential suppliers. The assessment content often involves a rigorous comparative analysis of key attributes, such as the supplier's product quality, price performance, and service level.

Suppose that a new energy vehicle manufacturer is looking for the best auto parts suppliers that can meet the development needs of the enterprise in order to produce better cars and meet the requirements of green production. After the investigation and screening of the parts suppliers on the market, the manufacturer initially identify four candidate suppliers $X_i = \{X_1, X_2, X_3, X_4\}$. To further select the suitable suppliers, the manufacturer decides to organize an expert panel to carry out relevant evaluation. After clarifying the evaluation criteria and objectives for the selection of green suppliers, the manufacturing company invited a panel of 11 experts with relevant professional backgrounds and experience. These experts, including renowned scholars and practitioners, have extensive experience in technological innovation and supply chain management. By collecting pertinent information, they provided a detailed explanation of the development status and evaluation objectives of the candidate companies. Through in-depth discussions and opinion exchanges, the panel have reached a consensus on the four foundational evaluation indicators as follows, informed by their collective and integrated assessment.

G_1: Product quality.

Product factors serve as tangible indicators of a supplier's green innovation capabilities and are of significant importance for the market operations of upstream and downstream enterprises as well as for the stable development of a green supply chain. Product quality is a critical indicator, reflecting not only the supplier's quality assurance system but also their capability to manufacture specific low-carbon, eco-friendly products that align with green demand. Existing research has found that the product quality and delivery compliance of suppliers are particularly significant for the manufacturing industry in emerging economies [77], which indicates a direction for systematic improvement in supply chain systems.

G_2: Price performance.

When selecting suppliers, companies must consider their product offerings, carbon emissions, transportation, recycling, and other aspects of price performance. Stable, good, and competitive pricing is an important factor in ensuring corporate profits and supply chain performance. Shah et al. [78] has demonstrated, through the construction of a

mathematical model, that the green efforts incorporated in products have a significant impact on corporate pricing decisions, which in turn can regulate the sales performance of the products. Therefore, the pricing performance of suitable green suppliers should be congruent with the corporate objectives of green development.

G_3: Service capability.

In the context of fierce market competition, suppliers with a high level of service are increasingly favored. Typically, the degree of management involvement, service attitude, timeliness of delivery, and market responsiveness are critical indicators for assessing the excellence of a supplier's comprehensive service capabilities. Wang et al. [79] consider the service level to be a key element affecting logistics quality in their construction of evaluation indicators for green suppliers. Consequently, enterprises must meticulously assess the service capabilities of potential suppliers in their selection process.

G_4: Social responsibility.

To select green suppliers for sustainable development, companies should consider their social responsibility performance, including but not limited to environmental responsibilities, employee rights protection, and social ethical responsibilities.These factors not only affect the company's environmental performance and green innovation capabilities but can also have a significant impact on the company's economic benefits and social reputation. Moreover, governance mechanisms of social responsibility based on supplier selection and supplier development have a positive impact on the supply chain core enterprise's social and environmental performance [80].

After determining the evaluation indicators, the panel of experts, through extensive discussion, unanimously decide that the associated weighting vector is $\omega = (0.35, 0.3, 0.15, 0.2)$. Now the experts are invited to conduct an anonymous evaluation of each alternative in the form of Fermatean hesitant fuzzy numbers, where the repeated membership and non-membership degrees are considered only once in order to ensure the scientificity and rationality of the decision results. And the familiarity of these experts with each evaluation attribute is regarded as the initial probabilistic vector $p = (0.1, 0.2, 0.4, 0.3)$. Then a Fermatean probabilistic hesitant fuzzy decision matrix $D = (\alpha_{ij})_{4\times 4}$ is established and shown as Table 1, where $\alpha_{ij} = (\mu_{ij}/p_{ij}, \nu_{ij}/q_{ij})$ denotes a Fermatean probabilistic hesitant fuzzy set. Each of the four candidate suppliers is evaluated and the optimal one is selected according to the evaluation information and specific decision-making steps given as follows.

Table 1. The Fermatean probabilistic hesitant fuzzy decision matrix.

	G_1	G_2	G_3	G_4
X_1	<{0.6/0.4, 0.7/0.6}, {0.1/0.7, 0.3/0.3}>	<{0.5/0.8, 0.7/0.2}, {0.3/0.3, 0.4/0.7}>	<{0.5/0.5, 0.7/0.3, 0.8/0.2}, {0.4/1}>	<{0.7/0.5, 0.8/0.5}, {0.3/0.4, 0.5/0.6}>
X_2	<{0.5/0.6, 0.8/0.4}, {0.2/0.5, 0.3/0.5}>	<{0.4/0.3, 0.6/0.4, 0.7/0.3}, {0.2/1}>	<{0.3/0.6, 0.6/0.4}, {0.2/0.7, 0.4/0.3}>	<{0.5/0.4, 0.8/0.6}, {0.1/0.6, 0.2/0.4}>
X_3	<{0.5/0.6, 0.7/0.3, 0.9/0.1}, {0.2/1}>	<{0.7/0.5, 0.8/0.5}, {0.2/0.6, 0.3/0.4}>	<{0.6/0.3, 0.8/0.7}, {0.2/0.4, 0.3/0.6}>	<{0.7/0.6, 0.9/0.4}, {0.3/0.7, 0.4/0.3}>
X_4	<{0.6/0.8, 0.8/0.2}, {0.3/0.6, 0.4/0.4}>	<{0.5/0.3, 0.6/0.7}, {0.1/0.4, 0.4/0.6}>	<{0.5/0.5, 0.7/0.5}, {0.1/0.6, 0.3/0.4}>	<{0.8/1}, {0.2/0.7, 0.5/0.2, 0.6/0.1}>

Step 1. Judge whether the established Fermatean probabilistic hesitant fuzzy decision matrix needs to be normalized or not. Since the attribute G_4: Cost price belongs to the cost-type attribute while other three indicators are all benefit-type attributes, it is only necessary to normalize G_4 based on Formula (36) and the normalized matrix is shown as Table 2.

Table 2. The normalized Fermatean probabilistic hesitant fuzzy decision matrix.

	G_1	G_2	G_3	G_4
X_1	<{0.6/0.4, 0.7/0.6}, {0.1/0.7, 0.3/0.3}>	<{0.5/0.8, 0.7/0.2}, {0.3/0.3, 0.4/0.7}>	<{0.5/0.5, 0.7/0.3, 0.8/0.2}, {0.4/1}>	<{0.3/0.4, 0.5/0.6}, {0.7/0.5, 0.8/0.5}>
X_2	<{0.5/0.6, 0.8/0.4}, {0.2/0.5, 0.3/0.5}>	<{0.4/0.3, 0.6/0.4, 0.7/0.3}, {0.2/1}>	<{0.3/0.6, 0.6/0.4}, {0.2/0.7, 0.4/0.3}>	<{0.1/0.6, 0.2/0.4}, {0.5/0.4, 0.8/0.6}>
X_3	<{0.5/0.6, 0.7/0.3, 0.9/0.1}, {0.2/1}>	<{0.7/0.5, 0.8/0.5}, {0.2/0.6, 0.3/0.4}>	<{0.6/0.3, 0.8/0.7}, {0.2/0.4, 0.3/0.6}>	<{0.3/0.7, 0.4/0.3}, {0.7/0.6, 0.9/0.4}>
X_4	<{0.6/0.8, 0.8/0.2}, {0.3/0.6, 0.4/0.4}>	<{0.5/0.3, 0.6/0.7}, {0.1/0.4, 0.4/0.6}>	<{0.5/0.5, 0.7/0.5}, {0.1/0.6, 0.3/0.4}>	<{0.2/0.7, 0.5/0.2, 0.6/0.1}, {0.8/1}>

Step 2. Supplement elements of the FPHFNs with smaller from a pessimistic perspective, namely adding minimal elements to the certain sets. Then the processed Fermatean probabilistic hesitant fuzzy decision matrix which is shown as Table 3 can be obtained.

Table 3. The processed Fermatean hesitant fuzzy decision matrix.

	G_1	G_2	G_3	G_4
X_1	<{0.6/0.2, 0.6/0.2, 0.7/0.6}, {0.1/0.7, 0.3/0.3}>	<{0.5/0.4, 0.5/0.4, 0.7/0.2}, {0.3/0.3, 0.4/0.7}>	<{0.5/0.5, 0.7/0.3, 0.8/0.2}, {0.4/0.5, 0.4/0.5}>	<{0.3/0.2, 0.3/0.2, 0.5/0.6}, {0.7/0.5, 0.8/0.5}>
X_2	<{0.5/0.3, 0.5/0.3, 0.8/0.4}, {0.2/0.5, 0.3/0.5}>	<{0.4/0.3, 0.6/0.4, 0.7/0.3}, {0.2/0.5, 0.2/0.5}>	<{0.3/0.3, 0.3/0.3, 0.6/0.4}, {0.2/0.7, 0.4/0.3}>	<{0.1/0.3, 0.1/0.3, 0.2/0.4}, {0.5/0.6, 0.8/0.4}>
X_3	<{0.5/0.6, 0.7/0.3, 0.9/0.1}, {0.2/0.5, 0.2/0.5}>	<{0.7/0.25, 0.7/0.25, 0.8/0.5}, {0.2/0.6, 0.3/0.4}>	<{0.6/0.15, 0.6/0.15, 0.8/0.7}, {0.2/0.4, 0.3/0.6}>	<{0.3/0.35, 0.3/0.35, 0.4/0.3}, {0.7/0.6, 0.9/0.4}>
X_4	<{0.6/0.4, 0.6/0.4, 0.8/0.2}, {0.3/0.6, 0.4/0.4}>	<{0.5/0.15, 0.5/0.15, 0.6/0.7}, {0.1/0.4, 0.4/0.6}>	<{0.5/0.25, 0.5/0.25, 0.7/0.5}, {0.1/0.6, 0.3/0.4}>	<{0.2/0.7, 0.5/0.2, 0.6/0.1}, {0.8/0.5, 0.8/0.5}>

Step 3. Calculate the support degree among the four attributes for each alternative to derive the corresponding support matrix as follows:

$$Sup_1 = \begin{pmatrix} 1 & 0.98 & 0.98 & 0.94 \\ 0.98 & 1 & 0.99 & 0.95 \\ 0.98 & 0.99 & 1 & 0.95 \\ 0.94 & 0.95 & 0.95 & 1 \end{pmatrix}$$

$$Sup_2 = \begin{pmatrix} 1 & 0.99 & 0.99 & 0.95 \\ 0.99 & 1 & 0.99 & 0.95 \\ 0.99 & 0.99 & 1 & 0.95 \\ 0.95 & 0.95 & 0.95 & 1 \end{pmatrix}$$

$$Sup_3 = \begin{pmatrix} 1 & 0.98 & 0.96 & 0.94 \\ 0.98 & 1 & 0.98 & 0.94 \\ 0.96 & 0.98 & 1 & 0.92 \\ 0.94 & 0.94 & 0.92 & 1 \end{pmatrix}$$

$$Sup_4 = \begin{pmatrix} 1 & 0.98 & 0.98 & 0.94 \\ 0.98 & 1 & 0.99 & 0.93 \\ 0.98 & 0.99 & 1 & 0.93 \\ 0.94 & 0.93 & 0.93 & 1 \end{pmatrix}$$

Step 4. Calculate the support degree $T(\overline{\alpha}_{lk})$ between overall variables based on the comprehensive weighted probability $\widetilde{p}_j$ and further figure out the support degree index ζ_{lk} of the variables, where $\widetilde{p}_j$ can be calculated for the following results according to the given weighting vector $\omega = (0.35, 0.3, 0.15, 0.2)$ and initial probabilistic vector $p = (0.1, 0.2, 0.4, 0.3)$. And the value of δ is assumed to be 0.6.

For example, the score values of $\alpha_{1j}(j = 1, 2, 3, 4)$ in Table 3 are calculated and sorted as follows: $\alpha_{11} \succ \alpha_{13} \succ \alpha_{12} \succ \alpha_{14}$. Then, calculate the value of the probability $\widetilde{p}_{1j}$ according to the ordering of α_{1j}:$\widetilde{p}_{11} = 0.6 \times 0.35 + 0.4 \times 0.1 = 0.25$, $\widetilde{p}_{13} = 0.6 \times 0.3 + 0.4 \times 0.4 = 0.34$, $\widetilde{p}_{12} = 0.6 \times 0.15 + 0.4 \times 0.2 = 0.17$, $\widetilde{p}_{14} = 0.6 \times 0.2 + 0.4 \times 0.3 = 0.24$. The rest of the $\widetilde{p}_{ij}$ can be calculated by the same principle.

$$(\widetilde{p}_{ij})_{4\times4} = \begin{pmatrix} 0.25 & 0.17 & 0.34 & 0.24 \\ 0.25 & 0.26 & 0.25 & 0.24 \\ 0.13 & 0.29 & 0.34 & 0.24 \\ 0.22 & 0.17 & 0.37 & 0.24 \end{pmatrix}$$

$$T = \begin{pmatrix} 0.725 & 0.497 & 0.992 & 0.683 \\ 0.734 & 0.764 & 0.735 & 0.685 \\ 0.374 & 0.840 & 0.972 & 0.671 \\ 0.639 & 0.493 & 1.076 & 0.673 \end{pmatrix}$$

$$\xi_{4\times4} = \begin{pmatrix} 0.9764 & 0.5760 & 1.5335 & 0.9141 \\ 1.0021 & 1.0603 & 1.0027 & 0.9349 \\ 0.4006 & 1.1965 & 1.5035 & 0.8994 \\ 0.8083 & 0.5690 & 1.7223 & 0.9003 \end{pmatrix}$$

Step 5. Use the proposed FPHFPWPBM operator to aggregate the corresponding attribute values of each alternative. Then the comprehensive attribute values $V_i(i = 1, 2, 3, 4)$ regarding alternatives $X_i(i = 1, 2, 3, 4)$ are calculated as the following results. In particular, for a convenient calculation, we let the parameters $p = 1$ and $q = 1$, thereby the results are obtained as follows:

$$V_1 = \langle\{0.4876/0.0285, 0.5541/0.0013, 0.6806/0.9702\}, \{0.4391/0.5000, 0.5288/0.5000\}\rangle$$

$$V_2 = \langle\{0.3636/0.0054, 0.4314/0.0306, 0.6359/0.9640\}, \{0.2864/0.9341, 0.4548/0.0659\}\rangle$$

$$V_3 = \langle\{0.5582/0.1507, 0.5912/0.0024, 0.7480/0.8469\}, \{0.4102/0.9193, 0.5029/0.0807\}\rangle$$

$$V_4 = \langle\{0.4746/0.9188, 0.5187/0.0005, 0.6770/0.0807\}, \{0.4314/0.9193, 0.5699/0.0807\}\rangle$$

Step 6. Use the score function to calculate the final scores $S_i(i = 1, 2, 3, 4)$ of corresponding comprehensive attribute values $V_i(i = 1, 2, 3, 4)$ respectively, which will serve as the basis for ranking the alternatives.

$$S_1 = -0.0028,\ S_2 = 0.0059,\ S_3 = 0.0047,\ S_4 = -0.0065$$

Based on the score values above, it is clear that $S_2 > S_3 > S_1 > S_4$. Therefore, the sorting of the four candidate suppliers should be $X_2 \succ X_3 \succ X_1 \succ X_4$, which indicates that X_2 is the optimal choice.

5.3. Sensitivity Analysis

The above results are obtained under the assumption that the parameter $p = q = 1$. To observe the variation of the optimal alternative under different parameter values, without loss of generality, we assign different values of 0.3, 0.7, 1.5, 2.4, 3, 5, 10 to both p and q. And the specific sorting results calculated with FHFPWPBM operator are shown in Table 4 below.

Figure 2 below clearly illustrates the score values and their trend of change for each alternative as the equal values of parameters p and q vary.

Table 4. Sorting results calculated with FPHFPWPBM operator when p, q are equal.

Parameter Value	S_1	S_2	S_3	S_4	Ranking
p = q = 0.3	0.0208	0.0213	0.0234	0.0145	$X_3 \succ X_2 \succ X_1 \succ X_4$
p = q = 0.7	0.0055	0.0107	0.0112	−0.0002	$X_3 \succ X_2 \succ X_1 \succ X_4$
p = q = 1.5	−0.0131	0.0006	−0.0034	−0.0133	$X_2 \succ X_3 \succ X_1 \succ X_4$
p = q = 2.4	−0.0254	−0.0052	−0.0132	−0.0209	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = q = 3	−0.0313	−0.0078	−0.0180	−0.0245	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = q = 5	−0.0445	−0.0139	−0.0284	−0.0334	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = q = 10	−0.0616	−0.0241	−0.0407	−0.0475	$X_2 \succ X_3 \succ X_4 \succ X_1$

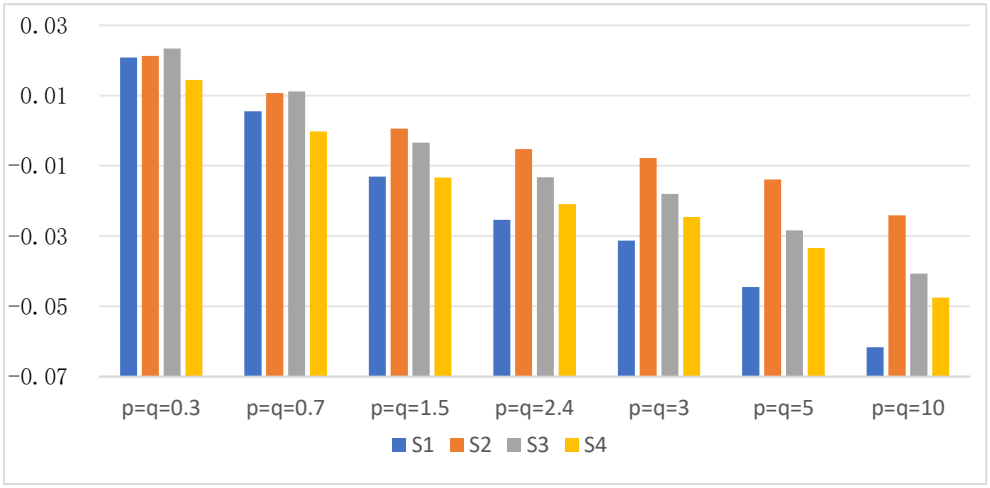


Figure 2. The score for each alternative when p, q are equal.

According to the results shown in Table 4 and Figure 2, it can be observed that the ranking of the schemes changes slightly with the variation in parameter values when $p = q$. Specifically, when the parameters p and q are assigned less than 1.5, the sorting results remain as $X_3 \succ X_2 \succ X_1 \succ X_4$. However, when both p and q are set equal to 1.5, the ranking changes with X_2 and X_3 exchanging priority. Moreover, starting from $p = q = 2.4$, the ordering results consistently show $X_2 \succ X_3 \succ X_4 \succ X_1$ as the parameter values increase, demonstrating the stability and reliability of the MADM method based on the FPHFPWPBM operator. Further analysis finds that the comprehensive score of each alternative will decrease as the parameters p and q increase, which may be a result of the change in risk preference of the decision-makers in different contexts. Therefore, decision-makers can choose the appropriate parameters p and q according to their risk preference during the decision-making process.

The following Figure 3 further clarifies the sensitivity of the score values of each scheme to the variation of parameter values when p and q are equal.

From Figure 3, it can be observed that when the parameter values are increased synchronously from 1 to 7, S_1 exhibits the greatest change, followed in order by S_3, S_4 and S_2. This indicates that alternative X_1 is the most sensitive to the parameter changes in this range, while X_2 is the least sensitive among the options presented. This indicates to a certain extent that, compared to other candidate schemes, X_1 is more significantly affected by parameter changes, and therefore decision-makers may need to optimize their decisions based on their specific preferences.

In addition, since the above results are calculated when parameters p and q are assigned the same value, we are completely interested in taking results with different values when p and q are not equal. Table 5 below shows the comprehensive score and sorting results in that case.

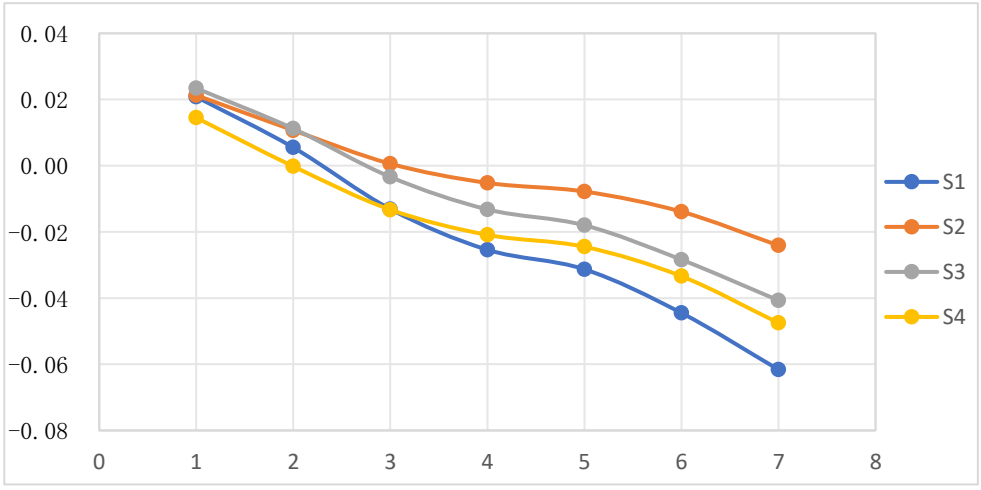


Figure 3. The parameter sensitivity of the score values for each alternative when p, q are equal.

Table 5. Sorting results calculated with FPHFPWPBM operator when p, q are not equal.

Parameter Value	S_1	S_2	S_3	S_4	Ranking
p = 0.1, q = 0.5	0.0222	0.0224	0.0246	0.0158	$X_3 \succ X_2 \succ X_1 \succ X_4$
p = 0.5, q = 1	0.0047	0.0100	0.0105	−0.0009	$X_3 \succ X_2 \succ X_1 \succ X_4$
p = 1.5, q = 2.6	−0.0204	−0.0032	−0.0094	−0.0178	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = 3.5, q = 4.8	−0.0394	−0.0115	−0.0245	−0.0298	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = 5.4, q = 6.8	−0.0493	−0.0165	−0.0320	−0.0370	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = 8, q = 10	−0.0589	−0.0223	−0.0388	−0.0450	$X_2 \succ X_3 \succ X_4 \succ X_1$
p = 10, q = 20	−0.0697	−0.0312	−0.0460	−0.0557	$X_2 \succ X_3 \succ X_4 \succ X_1$

Similarly, the following Figure 4 depicts the value and change trend of the score for each alternative when the parameters p and q are not equal and their value changes.

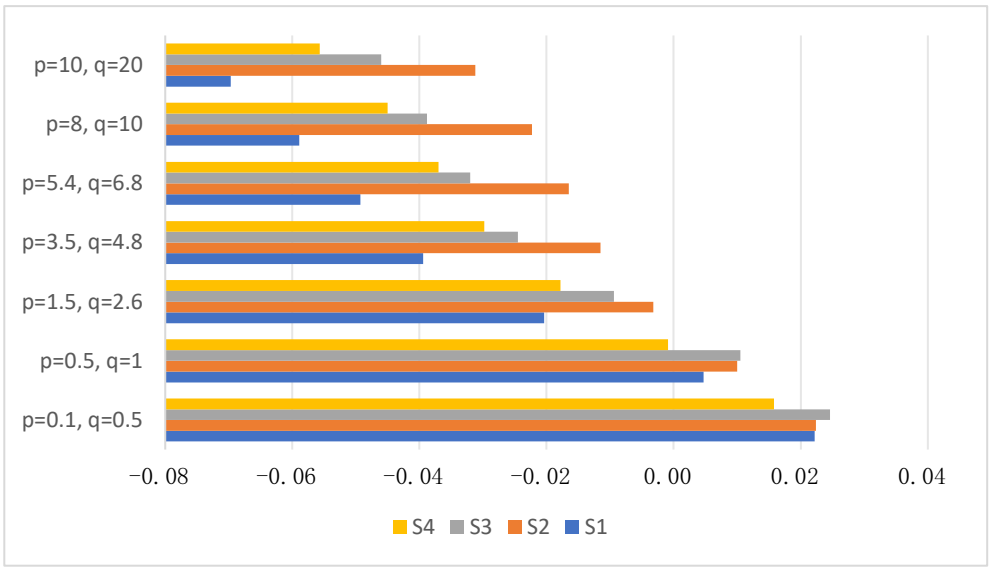


Figure 4. The score for each alternative when p, q are not equal.

As it can be seen from the Table 5 and Figure 4, the values of parameters p and q will affect the scoring results and ranking of each alternative in the case where they are not equal. To be precise, the score value of each scheme will decrease as p and q increase. And when the parameters p and q are set at a relatively low level, for example, $p = 0.5$ and $q = 1$, X_3 remains the optimal alternative according to the ranking results. Nevertheless,

when p and q are taken at a relatively high level, for example, $p = 1.5$ and $q = 2.6$, the optimal alternative changes to X_2 and the ranking remains stable as $X_2 \succ X_3 \succ X_4 \succ X_1$ even as p and q increase. In a word, according to the ranking results above, the optimal scheme is X_2 when p and q take values at a lower level, while X_3 becomes the best option when the parameters take values at a higher level. Therefore, in the practical application of this MADM method, the decision-makers are advised to assign the parameters p and q based on the degree of their risk aversion, and the values are advised to be assigned at not less than 3 to ensure the reliability and practicality of the decision-making results.

Next, to further clarify the sensitivity of each scheme's score to parameter value changes when p and q are not equal, a radar chart is used for description, and the results are shown in Figure 5 below.

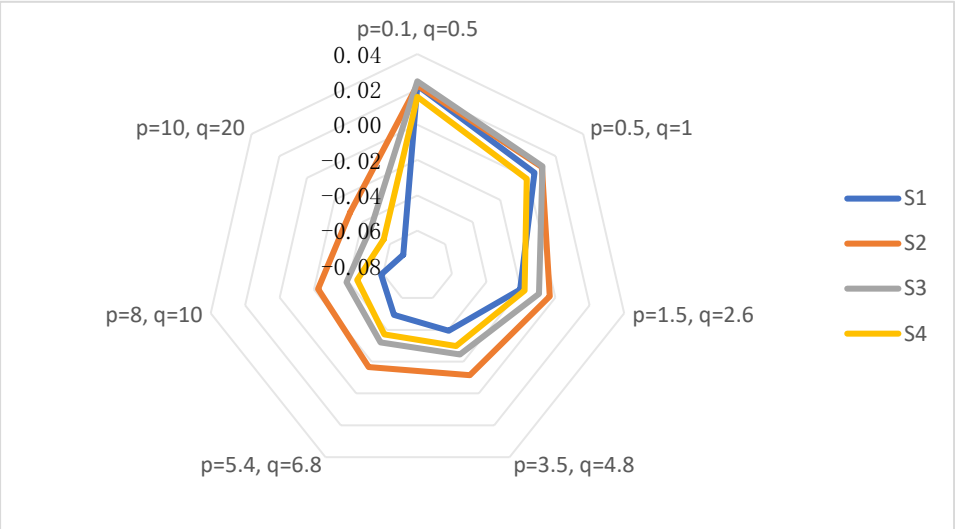


Figure 5. The parameter sensitivity of the score values for each alternative when p, q are not equal.

Furthermore, Figure 5 illustrates the impact of the simultaneous change in parameters p and q on the score values of each alternative. It can be observed that, as the parameter values increase from small to large, the score values of all schemes correspondingly decrease, depicted graphically by the inward movement of the scores. Among the alternatives, S_1 exhibits the largest absolute decrease in score, while S_2 shows the smallest. S_4 and S_3 demonstrate intermediate levels of change, positioned between the extremes of the above two. This suggests that the variations in parameter values exert the most pronounced impact on S_1, with subsequent notable effects on S_4 and S_3, and the least influence on S_2. Consequently, we can infer that the alternative X_1 exhibits the highest sensitivity to parameter value fluctuations, whereas X_2 appears to be a more robust option across various scenarios with varying parameter values, which is similar to the situation when parameters p and q are equal.

In summary, decision makers can adjust the parameter value of p and q according to risk preferences in the decision-making process of using this MADM method with FPHFPWPBM operator, so as to enhance the fairness, impartiality and objectivity of decision results.

5.4. Comparative Analysis

In order to further verify the scientificity and effectiveness of the MADM method with FHFPOWPBM operator proposed in this paper, a series of related aggregation operators will be used for the comparative analysis. First of all, the FHFWPBM, FHFPWPBM and FPHFWPBM operators, as presented above, will be applied for aggregating the evaluation data from the aforementioned numerical example to explore the impact of different and varying degrees of probabilistic information, taking into account the correlation of attributes.

Then several Fermatean hesitant fuzzy aggregation operators involving ordered weighted average (OWA) method presented by Ruan et al. [78], including the FHFPA, FHFOWA, FHFPOWA and the GFHFPOWA operators, are used to perform a comparative analysis. In addition, two types of Fermatean probabilistic hesitant fuzzy aggregation operators mentioned by Qahtan et al. [46] are also included in the consideration.

Table 6 below compares the differences in the properties of these operators to more clearly describe their respective characteristics. In this context, the external probabilistic information refers to the probability involved in the weight allocation process (such as the probability indicating the level of expert familiarity), while the internal probabilistic information refers to the probability carried by the evaluation data itself (such as the frequency with which a value appears in the evaluation data). It can be observed that the four operators proposed in this paper for comparative analysis all take into account attribute correlation. However, the FPHFPWPBM operator, additionally, considers dual probabilistic information (that is, internal and external probabilistic information). In contrast, the other operators used for comparison do not take into account attribute correlation. Specifically, the operators proposed by Ruan et al. [81] do not involve internal probabilistic information, and those introduced by Qahtan et al. [50] do not involve external probabilistic information.

Table 6. Comparison with existing operators, in which "+" indicates compliance with the characteristic, and "−" indicates non-compliance with the characteristic.

Operator	Considering Correlation of Attributes	Considering External Probabilistic Information	Considering Internal Probabilistic Information
FHFWPBM	+	−	−
FHFPWPBM	+	+	−
FPHFWPBM	+	−	+
FPHFPWPBM	+	+	+
FHFPA [81]	−	+	−
FHFOWA [81]	−	−	−
FHFPOWA [81]	−	+	−
GFHFPOWA [81]	−	+	−
FPHFAM [50]	−	−	+
FPHFWM [50]	−	−	+

Subsequently, these operators are individually adopted to aggregate the evaluation data from the numerical example. To facilitate computation, the parameters of these comparison operators are uniformly set to 1. The results of the comparative analysis are shown in Table 7 below.

Table 7. Comparison of the ranking results for different operators.

Operator	S_1	S_2	S_3	S_4	Ranking
FHFWPBM	0.0582	0.0607	0.1895	0.0883	$X_3 \succ X_4 \succ X_2 \succ X_1$
FHFPWPBM	0.0881	0.0710	0.1777	0.0577	$X_3 \succ X_1 \succ X_2 \succ X_4$
FPHFWPBM	−0.0063	0.0035	0.0058	−0.0027	$X_3 \succ X_2 \succ X_1 \succ X_4$
FPHFPWPBM	−0.0028	0.0059	0.0047	−0.0065	$X_2 \succ X_3 \succ X_1 \succ X_4$
FHFPA [81]	0.1602	0.0944	0.2716	0.1666	$X_3 \succ X_4 \succ X_1 \succ X_2$
FHFOWA [81]	0.2015	0.1867	0.3394	0.2094	$X_3 \succ X_4 \succ X_1 \succ X_2$
FHFPOWA [81]	0.1970	0.1526	0.3059	0.1982	$X_3 \succ X_4 \succ X_1 \succ X_2$
GFHFPOWA [81]	−0.1203	−0.1226	0.0211	−0.0749	$X_3 \succ X_4 \succ X_1 \succ X_2$
FPHFAM [50]	−0.0728	−0.0427	−0.0612	−0.0636	$X_2 \succ X_3 \succ X_4 \succ X_1$
FPHFWM [50]	−0.0518	−0.0335	−0.0483	−0.0634	$X_2 \succ X_3 \succ X_1 \succ X_4$

From the results above, it can be easily find that the optimal choice of sorting results is either X_2 or X_3, which is consistent with the results of the parameter analysis, indicating that the MADM method proposed in this paper has a certain degree of stability

and reliability. Specifically, when considering the correlation of attributes, the optimal scheme obtained by the FPHFPWPBM operator is X_2 while other operators including the FHFWPBM, FHFPWPBM and FPHFWPBM operators yield X_3. This is primarily due to the different probabilistic information and the extent to which it is considered. For instance, the FHFWPBM operator only takes into account the hesitancy of the evaluation data and the interaction between attributes. The FHFPWPBM operator incorporates external probabilistic information for ordered weighting on top of the FHFWPBM. Meanwhile, the FPHFWPBM operator considers the internal probabilistic information of the evaluation data on the basis of the FHFWPBM. Different from the others, the FPHFPWPBM operator incorporates dual probabilistic information on the basis of the FHFWPBM operator, which means it not only includes the external probabilities in the weighting process but also takes into account the probabilistic information inherent in the evaluation data itself. As a result, the sorting result of the FPHFPWPBM operator is likely to be more objective, accurate and comprehensive compared to the other three operators.

Then, let's pay attention to several other Fermatean hesitant fuzzy aggregation operators and Fermatean probabilistic hesitant fuzzy aggregation operators that involve the OWA method. The OWA operators primarily integrates information through the method of arithmetic averaging, which can effectively reflect the concentration of a set of data and are widely applied to traditional weighted decision-making problems. The new series of operators proposed in this paper also involve weighted averaging, hence it is necessary to conduct a comparative analysis among them. From Table 6, we can observe that the ranking results of the FHFPA, FHFOWA, FHFPOWA and the GFHFPOWA operators are consistently $X_3 \succ X_4 \succ X_1 \succ X_2$, but the FPHFAM and FPHFWM operators yield the optimal option as X_2, although there is a slight difference in the ranking of the schemes between the two. Obviously, the sorting result of the FPHFPWPBM operator is more closely aligned with that of the FPHFAM and FPHFWM operators, which indicates the consistency of the Fermatean probabilistic hesitant fuzzy aggregation operators when considering the internal probabilistic information of the data. In contrast, the Fermatean hesitant fuzzy aggregation operators only consider external weighted probabilities at most, thus it is reasonable that there are differences in the ranking results. Therefore, the information integration result of the FPHFPWPBM operator may be more convincing and representative when dealing with more complex MADM problems in real life.

6. Conclusions

This paper develops a novel MADM method which combines the PBM operator theory and probabilistic information under Fermatean hesitant fuzzy environment and Fermatean probabilistic hesitant fuzzy environment, along with newly proposed Fermatean hesitant fuzzy power Bonferroni aggregation operators and Fermatean probabilistic hesitant fuzzy power Bonferroni aggregation operators. Firstly, given that the PBM operator is recognized for its ability to mitigate the effects of inappropriate aggregation values and for considering the interaction between evaluation attributes during decision-making process, we propose several Fermatean hesitant fuzzy power Bonferroni aggregation operators to further expand the application of the PBM operator in FHFS, including the FHFPBM, FHFWPBM and FHFPWPBM operators. Among them, the FHFPWPBM operator not only develops a combination of PBM operator and FHFS, but takes into account the weighted probability of combining degrees of expert familiarity and attribute weights, assisting in managing complex decision-making problems characterized by inaccuracy and uncertainty.

In real decision-making, as the probability value is an effective way to describe information uncertainty, it is essential to pay attention to the probabilistic information in the study of MADM methods. Accordingly, we introduce the PBM operator into FPHFS by presenting several Fermatean probabilistic hesitant fuzzy power Bonferroni aggregation operators, including the FPHFPBM, FPHFWPBM and FPHFPWPBM operators. In particular, the FPHFPWPBM operator, further developed from the FHFPWPBM operator, incorporates dual probabilistic information. One is the aforementioned external weighted probability,

and the other is the internal probability value attached to the evaluation data itself. It undoubtedly indicates that the information integration result of the FPHFPWPBM operator not only considers the correlation between evaluation attributes, but also better reflects the stochastic nature in the data evaluation process, effectively quantifying the importance of attributes. Then a new MADM method is proposed based on these operators. Finally, this method is applied to a numerical example about a manufacturer enterprise selecting the most suitable green supplier. Furthermore, the scientificity and practicability of this method are demonstrated and verified through the parameter analysis and comparative analysis of the information integration results.

In conclusion, this paper creatively proposes a series of aggregation operators by introducing PBM operator and probabilistic information into FHFSs and FPHFSs, and constructs a new MADM method based on these operators, which provides a new technique and path for solving complex and uncertain decision-making problems in real life. Future research is expected to expand the PBM operator to more complex fuzzy environment such as interval-valued Fermatean hesitant fuzzy environment and probabilistic interval-valued Fermatean hesitant fuzzy environment, so as to further enrich the information aggregation operator under the theoretical system of Fermatean fuzzy information. Moreover, it is valuable to consider the application of the proposed method in different practical domains or scenarios, such as venture capital proposal evaluation, product scheme selection, quality assessment in talent recruitment, among others.

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Abbreviations

List of Main Abbreviations:

Sr.NO	Complete Word	Abbreviation
1	Multi-attribute decision-making	MADM
2	Fuzzy set	FS
3	Intuitionistic fuzzy set	IFS
4	Pythagorean fuzzy set	PFS
5	Fermatean fuzzy set	FFS
6	q-Rung orthopair fuzzy set	q-ROFS
7	Hesitant fuzzy set	HFS
8	Interval-valued hesitant fuzzy set	IVHFS
9	Intuitionistic hesitant fuzzy set	IHFS
10	Pythagorean hesitant fuzzy set	PyHFS
11	Fermatean hesitant fuzzy set	FHFS
12	Interval-valued hesitant Fermatean fuzzy set	IVHFFS
13	Probabilistic hesitant fuzzy set	PHFS
14	Probabilistic linguistic term set	PLTS
15	Probabilistic interval-valued hesitant fuzzy set	PIVHFS
16	Probabilistic interval-valued Fermatean hesitant fuzzy set	PIVFHFS
17	q-Rung orthopair probabilistic hesitant fuzzy rough set	q-ROPHFS
18	q-Rung orthopair probabilistic hesitant fuzzy rough set	q-ROPHFRS
19	Pythagorean probabilistic hesitant fuzzy set	PyPHFS
20	Fermatean probabilistic hesitant fuzzy set	FPHFS
21	Power average	PA

22	Bonferroni mean	BM
23	Power Bonferroni Mean	PBM
24	Fermatean fuzzy power Bonferroni mean	FFPBM
25	Fermatean fuzzy weighted power Bonferroni mean	FFWPBM
26	Fermatean hesitant fuzzy power Bonferroni mean	FHFPBM
27	Fermatean hesitant fuzzy weighted power Bonferroni mean	FHFWPBM
28	Fermatean hesitant fuzzy probabilistic weighted power Bonferroni mean	FHFPWPBM
29	Fermatean probabilistic hesitant fuzzy power Bonferroni mean	FPHFPBM
30	Fermatean probabilistic hesitant fuzzy weightedpower Bonferroni mean	FPHFWPBM
31	Fermatean probabilistic hesitant fuzzy probabilistic weighted power Bonferroni mean	FPHFPWPBM
32	Ordered weighted average	OWA
33	Fermatean probabilistic hesitant fuzzy average mean	FPHFAM
34	Fermatean probabilistic hesitant fuzzy weighted average	FPHFWA

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