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Uncertainty Degradation Model for Initiating Explosive Devices Based on Uncertain Differential Equations

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Abstract: The performance degradation of initiating explosive devices is influenced by various internal and external factors, leading to uncertainties in their reliability and lifetime predictions. This paper proposes an uncertain degradation model based on uncertain differential equations, utilizing the Liu process to characterize the volatility in degradation rates. The ignition delay time is selected as the primary performance parameter, and the uncertain distributions, expected values and confidence intervals are derived for the model. Moment estimation techniques are employed to estimate the unknown parameters within the model. A real data analysis of ignition delay times under accelerated storage conditions demonstrates the practical applicability of the proposed method.

Keywords: uncertain differential equation; initiating explosive devices; performance degradation; reliability assessment and lifetime evaluation

MSC: 34N05

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1. Introduction

Initiating explosive devices are a class of products with high reliability, which are widely used in various long-storage-life equipment [1]. The failure life data of such products are difficult to obtain, and the traditional reliability assessment method based on failure life data cannot effectively predict their life. As the first component of a critical system, the performance of pyrotechnic products has a direct impact on the safe and reliable operation of the whole system [2].

Some of the performance indicators of this type of product will show a degradation trend over time. For example, the critical resistance of a certain type of electrical connector will degrade in the course of use, with the resistance value gradually becoming larger, and failure occurs when the performance degradation reaches the failure threshold [3]. In the traditional approach, the degradation trend of a pyrotechnic product can be expressed based on an ordinary differential equation and it is assumed that the degradation is entirely driven by deterministic mechanisms. However, the degradation of pyrotechnics is affected by various external storage environments, such as temperature and humidity changes. Therefore, these deterministic models are not suitable for pyrotechnics. Some scholars have modified their degradation trends by adding stochastic processes to model stochastic pyrotechnic performance degradation using stochastic differential equations in a probabilistic framework. For example, Whitmore [4] used the Wiener model to analyze the performance degradation process under the consideration of measurement error.

However, due to the specific nature of long-storage-life products, sample sizes are generally small. These circumstances motivate us to search for more rational methods to characterize the performance degradation of pyrotechnic products. In addition to probability theory, uncertainty theory based on normality, parity, subadditivity and product axioms is another axiomatic mathematical system for dealing with uncertainty [5]. In the framework of uncertainty theory, the Liu process is designed as the counterpart of the Wiener process. It is a smooth independent incremental process whose increments are uncertain variables rather than random variables. Many researchers have made significant contributions to the development of uncertain differential equations. For example, Chen [6] gave an existence uniqueness theorem for uncertain differential equations. Liu [7] derived analytic solutions of some special forms of uncertain differential equations. In general, numerical methods were pioneered by Yao and Chen and have been followed by many researchers. Yao and Liu [8] proposed moment estimation for unknown parameters in uncertain differential equations, which was subsequently generalized to minimum coverage estimation. Uncertain differential equations play an important role in many fields such as engineering practice [9], drug concentration prediction [10] and so on.

This article introduces uncertain differential equations into the performance degradation process of pyrotechnic products in order to better describe their degradation trends. The remaining parts of this article are arranged as follows. In Section 2, we will introduce some basic definitions and theorems of uncertainty theory. We are going to propose an uncertain differential equation for the key performance parameters (ignition time) of pyrotechnic devices in order to better describe their degradation characteristics in Section 3. In Section 4, we will provide an evaluation method for the reliability and reliable lifespan of pyrotechnic products. Section 5 will provide parameter estimates for unknown parameters in the uncertain degradation model. Section 6 will apply our method to a set of real data. Finally, some conclusions will be given in Section 7.

2. Preliminaries

This section mainly introduces some basic definitions and theorems of uncertainty theory and provides theoretical support for the degradation process of pyrotechnic devices.

The uncertain measure is a class of aggregate functions that satisfy the axioms of uncertainty theory. It is used to express the degree of belief that an uncertain event may occur [11]. The uncertain measure is $\mathcal{M}$ on the σ-algebra $\mathcal{L}$. $\mathcal{M}\{\Lambda\}$ is assigned to the event Λ to indicate the belief degree with which we believe Λ will happen.

Definition 1 (Uncertain variable, Liu [5])**.** *An uncertain variable is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that $\{\xi \in B\}$; it is an event for any Borel set B of real numbers.*

Definition 2 (Uncertainty distribution, Liu [5])**.** *The uncertainty distribution ξ of an uncertain variable is defined by*

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}, \tag{1}$$

and is applicable for any real number x.

Definition 3 (Normal uncertainty distribution, Liu [5])**.** *An uncertain variable ξ is called normal if it has a normal uncertainty distribution*

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \ x \in \Re \tag{2}$$

denoted by $N(e,\sigma)$, where e and σ are real numbers when $\sigma > 0$.

Definition 4 (Inverse uncertainty distribution, Liu [11])**.** *Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$. Then, the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.*

Theorem 1 (Liu [11])**.** *A function Φ^{-1} is an inverse uncertainty distribution of an uncertain variable ξ, but only if*

$$\mathcal{M}\left\{\xi < \Phi^{-1}(\alpha)\right\} = \alpha \tag{3}$$

for all $\alpha \in [0,1]$.

Theorem 2 (Sufficient and necessary condition, Liu [12]). *A function* $\Phi^{-1}(\alpha) : (0,1) \to \Re$ *is an inverse uncertainty distribution if and only if it is a continuous and strictly increasing function with respect to* α.

Theorem 3 (Liu [11]). *Let* $\xi_1, \xi_2, \cdots, \xi_n$ *be independent uncertain variables with regular uncertainty distributions* $\Phi_1, \Phi_2, \cdots, \Phi_n$, *respectively. If the function* $f(x_1, x_2, \cdots, x_n)$ *is strictly increasing with respect to* $x_1, x_2, \cdots, x_m$ *and strictly decreasing with respect to* $x_{m+1}, x_{m+2}, \cdots, x_n$*, then the uncertain variable*

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n) \tag{4}$$

has an inverse uncertainty distribution.

$$\Psi^{-1}(\alpha) = f\Big(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots \Phi_n^{-1}(1-\alpha)\Big). \tag{5}$$

Definition 5 (Liu [5]). *Let be an uncertainty space and let* T *be a totally ordered set (e.g., time). An uncertain process is a function* $X_t(\gamma)$ *from* $T \times (\Gamma, \mathcal{L}, \mathcal{M})$ *to the set of real numbers such that* $\{X_t \in B\}$ *is an event for any Borel set* B *of real numbers at each time* t.

Definition 6 (Liu [13]). *Suppose* f *and* g *are continuous functions and* C_t *is a Liu process. Then,*

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t \tag{6}$$

is called an uncertain differential equation. A solution is an uncertain process X_t *that satisfies (6) identically in* t.

Definition 7 (Chen [14]). *Let* C_t *be a canonical Liu process and* μ_s *and* σ_s *be two uncertain processes. Then, the uncertain process*

$$X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dC_s \tag{7}$$

is called a Liu process with drift μ_t *and diffusion* σ_t.

3. Uncertain Degradation Models

The sensitivity parameters of initiating explosive devices are generally selected as performance degradation characteristic parameters. Typically, the sensitivity parameter is singular; at most, it is two when difficult to distinguish. When the sensitivity parameters exceed the range specified in the design task book, the product experiences degradation failure. Therefore, performance degradation failure characteristic parameters must meet two conditions: first, they must be accurately defined and monitorable; second, they must exhibit a clear trend change with the extension of the product's working or testing time, objectively reflecting the working state of the initiating explosive device. For example, the sensitivity parameters of electric detonators are the average firing sensitivity and activation time: for igniters, it is the ignition pressure, and for explosive bolts, it is the separation impulse.

Therefore, this paper selects the ignition delay time as the performance parameter for evaluating the lifetime and reliability of initiating explosive devices. The delay in ignition time is primarily due to changes in the performance of the propellant. For LTNR (lead thiocyanate/lead nitride), with the extension of storage time, the number of activated molecules in the initiating explosive device propellant increases, and the amplitude and frequency of activated molecular vibrations increase. This raises the probability of the weakest bonds breaking in the initiating explosive device propellant molecules, causing the activation energy to increase over storage time. As a result, it becomes more difficult for the propellant to react under the same excitation conditions, thus delaying the ignition time. For lead thiocyanate/potassium chlorate ignition powder, the propellant components undergo slow reactions in the storage environment, reducing the oxidizers and combustibles, thereby delaying the ignition time.

These chemical reaction processes are subject to various internal and external noises, which vary over time in the rate of increase in ignition time. To model these noises, it is assumed that the fluctuation rate of ignition time increase is $\mu + \sigma\dot{C}_t$, where C_t is the Liu process [15], $\dot{C}_t = dC_t/dt$ is a disturbance and k and σ are positive constants. Therefore, the change in ignition time in the period [0, t] is

$$\int_0^t (\mu + \sigma C_s)X_s ds. \tag{8}$$

We obtain

$$X_t - X_0 = \int_0^t (\mu + \sigma C_s)X_s ds \tag{9}$$

which means that ignition time X_t follows the uncertain differential equation

$$dX_t = \mu X_t dt + \sigma X_t dC_t,\ X_0 = x_0 \tag{10}$$

where X_t is the ignition time at time t, x_0 is the initial ignition time, μ and σ are positive constants, σ represents the noise level in the chemical reaction process and C_t is a Liu process.

Theorem 4 (Liu [5])**.** *Let u_t and v_t be two continuous functions of t. Then, the uncertain differential equation*

$$dX_t = u_t X_t dt + v_t X_t dC_t \tag{11}$$

has a solution

$$X_t = X_0 \exp\left(\int_0^t u_s ds + \int_0^t v_s dC_s\right). \tag{12}$$

Through Theorem 4, it can be determined that the uncertain degradation mode

$$dX_t = \mu X_t dt + \sigma X_t dC_t,\ X_0 = x_0 \tag{13}$$

has a solution

$$X_t = x_0 \exp(\mu t + \sigma C_t). \tag{14}$$

Suppose the uncertain distribution of X_t is $\Phi_t(x)$. Then, we can determine that

$$\Phi_t(x) = \mathcal{M}\{X_t \leq x\} = \mathcal{M}\left\{C_t \leq \frac{1}{\sigma}\left(-\mu t + \ln\frac{x}{x_0}\right)\right\}. \tag{15}$$

Since C_t is a normal uncertain variable with the uncertainty distribution

$$\mathcal{M}\{C_t \leq x\} = \left(1 + \exp\left(-\frac{\pi x}{\sqrt{3}t}\right)\right)^{-1}, \tag{16}$$

we obtain

$$\Phi_t(x) = \left(1 + \exp\left(\frac{\pi}{\sqrt{3}\sigma t}\left(\ln\frac{x_0}{x} + \mu t\right)\right)\right)^{-1}. \tag{17}$$

4. Reliability and Lifetime Assessment

Reliability is defined as the probability that a product, system or service will perform its intended function adequately for a specified period of time or will operate in a defined environment without failure. Assuming that the critical performance parameter p and its corresponding performance threshold p_{th} are both uncertain variables, where the uncertain distribution of p is $\Phi_p(x)$ and the uncertain distribution of p_{th} is $\Phi_{p_{th}}(x)$ (or $\Phi_{p_{th,L}}(x)$ and $\Phi_{p_{th,U}}(x)$), then the confidence reliability based on the performance margin is

$$R_B = \begin{cases} \sup\limits_{x\in\mathbb{R}}((1-\Phi_p(x)) \wedge \Phi_{p_{th}}(x)), \\ \quad \text{if p is a performance parameter to be maximized} \\ \sup\limits_{x\in\mathbb{R}}(\Phi_p(x) \wedge (1-\Phi_{p_{th}}(x))), \\ \quad \text{if p is a performance parameter to be minimized} \\ \sup\limits_{x\in\mathbb{R}}(\Phi_p(x) \wedge (1-\Phi_{p_{th},U}(x))) - \sup\limits_{x\in\mathbb{R}}(\Phi_p(x) \wedge (1-\Phi_{p_{th},L}(x))), \\ \quad \text{if p is a nominal-the-best performance parameter} \end{cases} \tag{18}$$

The ignition time of initiating explosive devices is a performance parameter to be minimized, and its threshold value is a constant. According to Equation (18), we can obtain the reliability of the initiating explosive devices:

$$R(t) = \mathcal{M}\{X_t \le p_{th}\} = \Phi_p(p_{th}). \tag{19}$$

Then, we can obtain the reliable lifetime of the initiating explosive devices:

$$T = \sup_t\{R(t) \ge \alpha\}. \tag{20}$$

Definition 8 (Yao-Chen [16])**.** *Let α be a number between 0 and 1. An uncertain differential equation*

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t, \tag{21}$$

is said to have an α-path $X\alpha t$ if it solves the corresponding ordinary differential equation

$$dX_t^\alpha = f(t, X_t^\alpha)dt + \left|g(t, X_t^\alpha)\right|\Phi^{-1}(\alpha)dt, \tag{22}$$

where $\Phi^{-1}(\alpha)$ is the inverse standard normal uncertainty distribution, i.e.,

$$\Phi^{-1}(\alpha) = \frac{\sqrt{3}}{\pi}\ln\left(\frac{\alpha}{1-\alpha}\right). \tag{23}$$

According to Definition 8, the ignition time X_t has an α-path

$$X_t^\alpha = X_0\exp\left(\mu t + \frac{\sqrt{3}\sigma t}{\pi}\ln\left(\frac{\alpha}{1-\alpha}\right)\right). \tag{24}$$

Based on Formula (24), the reliable lifetime of initiating explosive devices at the α-path level can be derived as

$$T_\alpha = \sup_t\{X_t^\alpha \le p_{th}\}. \tag{25}$$

5. Parameter Estimation

As we can see, there are two unknown parameters μ and σ in the uncertain degradation model. In this section, we employ the method of moments [8] to estimate these unknown parameters.

Give n positive observations $X_{t_1}, X_{t_2}, \ldots, X_{t_n}$ of the solution of X_t at times $t_1, t_2, \ldots, t_n$ with $t_1 < t_2 < \cdots < t_n$, respectively, before the complete failure of the initiating explosive devices. Note that Equation(13) has a difference form

$$X_{t_i} - X_{t_{i-1}} = \mu X_{t_{i-1}}(t_i - t_{i-1}) + \sigma X_{t_{i-1}}(C_{t_i} - C_{t_{i-1}}), \tag{26}$$

which can be rewritten as

$$\frac{X_{t_i} - X_{t_{i-1}} - \mu X_{t_{i-1}}(t_i - t_{i-1})}{\sigma X_{t_{i-1}}(t_i - t_{i-1})} = \frac{C_{t_i} - C_{t_{i-1}}}{t_i - t_{i-1}}. \tag{27}$$

It follows from the Liu process's properties [15] that the right term is a standard normal uncertain variable, whose uncertainty distribution and kth moments are

$$\Phi(x) = \left(1 + \exp\left(-\frac{\pi x}{\sqrt{3}}\right)\right)^{-1}, \tag{28}$$

and

$$\left(\frac{\sqrt{3}}{\pi}\right)^k \int_0^1 \left(\ln \frac{\alpha}{1-\alpha}\right)^k d\alpha,\ k = 1, 2, \ldots n, \tag{29}$$

respectively.

Consider the uncertain degradation model (13) with two unknown parameters μ and σ to be estimated. Give n observations $x_{t_1}, x_{t_2}, \ldots, x_{t_n}$ of the solution of x_t of the ignition time at times $t_1, t_2, \ldots, t_n$ with $t_1 < t_2 < \cdots < t_n$, respectively.

Based on the idea of moment estimation, the kth sample moments provide good estimates of the kth population moments. μ^* and σ^* are solutions of the following system of equations:

$$\begin{cases} \dfrac{1}{n-1}\sum\limits_{i=2}^{n} \dfrac{X_{t_i} - X_{t_{i-1}} + \mu^* X_{t_{i-1}}(t_i - t_{i-1})}{\sigma X_{t_{i-1}}(t_i - t_{i-1})} = \dfrac{\sqrt{3}}{\pi}\int_0^1 \ln \dfrac{\alpha}{1-\alpha} d\alpha = 0 \\ \dfrac{1}{n-1}\sum\limits_{i=2}^{n} \left(\dfrac{X_{t_i} - X_{t_{i-1}} + \mu^* X_{t_{i-1}}(t_i - t_{i-1})}{\sigma X_{t_{i-1}}(t_i - t_{i-1})}\right)^2 = \left(\dfrac{\sqrt{3}}{\pi}\right)^2 \int_0^1 \left(\ln \dfrac{\alpha}{1-\alpha}\right)^2 d\alpha = 1. \end{cases} \tag{30}$$

Therefore,

$$\begin{cases} \mu^* = \dfrac{1}{n-1}\sum\limits_{i=2}^{n} \dfrac{X_{t_i} - X_{t_{i-1}}}{X_{t_{i-1}}(t_i - t_{i-1})} \\ \sigma^* = \sqrt{\dfrac{1}{n-1}\sum\limits_{i=2}^{n} \left(\dfrac{X_{t_i} - X_{t_{i-1}} - \mu^* X_{t_{i-1}}(t_i - t_{i-1})}{\sigma X_{t_{i-1}}(t_i - t_{i-1})}\right)^2}. \end{cases} \tag{31}$$

Definition 9 (Ye-Liu [17])**.** *Let ξ be an uncertain variable with uncertainty distribution Φ_θ, where θ is an unknown vector of parameters. A rejection region $W \subset \Re^n$ is said to be a test for the hypotheses*

$$H_0 : \theta = \theta_0 \ \ versus \ \ H_1 : \theta \neq \theta_0 \tag{32}$$

at significance level α (e.g., 0.05). (i) For any value of $(z_1, z_2, \cdots, z_n) \in W$, there are more than α values of index i with $1 \leq i \leq n$ such that

$$\mathcal{M}_{\theta_0}\{\xi \geq z_i\} \wedge \mathcal{M}_{\theta_0}\{\xi \leq z_i\} < \frac{\alpha}{2}; \tag{33}$$

and (ii) for some instances where $\theta_1 \neq \theta_0$ and some where $(z_1, z_2, \cdots, z_n) \in W$, there are at least $1-\alpha$ values of index i with $1 \leq i \leq n$ and more than α values of index j with $1 \leq j \leq n$ such that

$$\mathcal{M}_{\theta_1}\{\xi \geq z_i\} \wedge \mathcal{M}_{\theta_1}\{\xi \leq z_i\} > \mathcal{M}_{\theta_0}\{\xi \geq z_j\} \wedge \mathcal{M}_{\theta_0}\{\xi \leq z_j\}. \tag{34}$$

Theorem 5 (Ye-Liu [17])**.** *Let ξ be an uncertain variable that follows a normal uncertainty distribution $\mathrm{N}(e, \sigma)$ with unknown expected value e and unknown variance σ^2. Then, the test for the hypotheses*

$$H_0 : e = e_0 \text{ and } \sigma = \sigma_0 \ \ versus \ \ H_1 : e \neq e_0 \text{ or } \sigma \neq \sigma_0 \tag{35}$$

at significance level α *is*

$$W = \Big\{(z_1, z_2, \cdots, z_n): \textit{ there are more than } e \textit{ values of indexes } i's \textit{ with } \\ 1 \le i \le n \textit{ such that } z_i < \Phi_0^{-1}\big(\tfrac{\alpha}{2}\big) \textit{ or } z_i > \Phi_0^{-1}\big(1-\tfrac{\alpha}{2}\big)\Big\} \tag{36}$$

where Φ_0^{-1} *is the inverse uncertainty distribution of* $\mathrm{N}(e_0, \sigma_0)$ *, i.e.,*

$$\Phi_0^{-1}(\alpha) = e_0 + \frac{\sigma_0\sqrt{3}}{\pi} \ln \frac{\alpha}{1-\alpha}. \tag{37}$$

6. Real Data Analysis

In this section, we will provide a detailed demonstration of our method for the ignition delay time of a pyrotechnic agent. This set of data shows the ignition delay time of initiating explosive devices under accelerated storage conditions, with a temperature of 71 °C and humidity of 80%. According to the generalized Eileen model, one year under accelerated storage conditions is equivalent to 1.25 years under normal temperature and humidity conditions (21 °C, 70%). The specific ignition delay time data of initiating explosive devices are shown in Table 1.

Table 1. Data of the ignition delay time for initiating explosive devices.

t	X_t	t	X_t
0	34.74	21	50.49
3	34.96	24	50.17
6	35.53	27	49.76
9	38.39	30	50.51
12	45.95	33	51
15	47.74	36	51.21
18	50.18		

According to Formula (31), it can be obtained that

$$dX_t = 0.011408 X_t dt + 0.018289 X_t dC_t,\ X_0 = x_0 = 34.74. \tag{38}$$

Note that the solution for model (16) is

$$X_t = 34.74 \exp(0.011408t + 0.018289C_t). \tag{39}$$

It follows from (17) that at any given time t, the uncertainty distribution of the ignition delay time X_t is

$$\Phi_t(x) = \left(1 + exp\left(\frac{\pi}{\sqrt{3} \times 0.018289t}\left(ln\frac{34.74}{x} + 0.011408t\right)\right)\right)^{-1}. \tag{40}$$

The uncertainty distribution $\Phi_{20}(x)$ of the ignition delay time X_t at $t = 20$ day is illustrated in Figure 1. And Figure 2 gives the uncertain measure $\Phi_t(40)$, i.e., $\mathcal{M}\{X_t \le 40\}$.

According to Definition 8, X_t has an α-path

$$X_t^\alpha = 34.74 \exp\left(0.011408t + \frac{\sqrt{3} \times 0.018289t}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right) \tag{41}$$

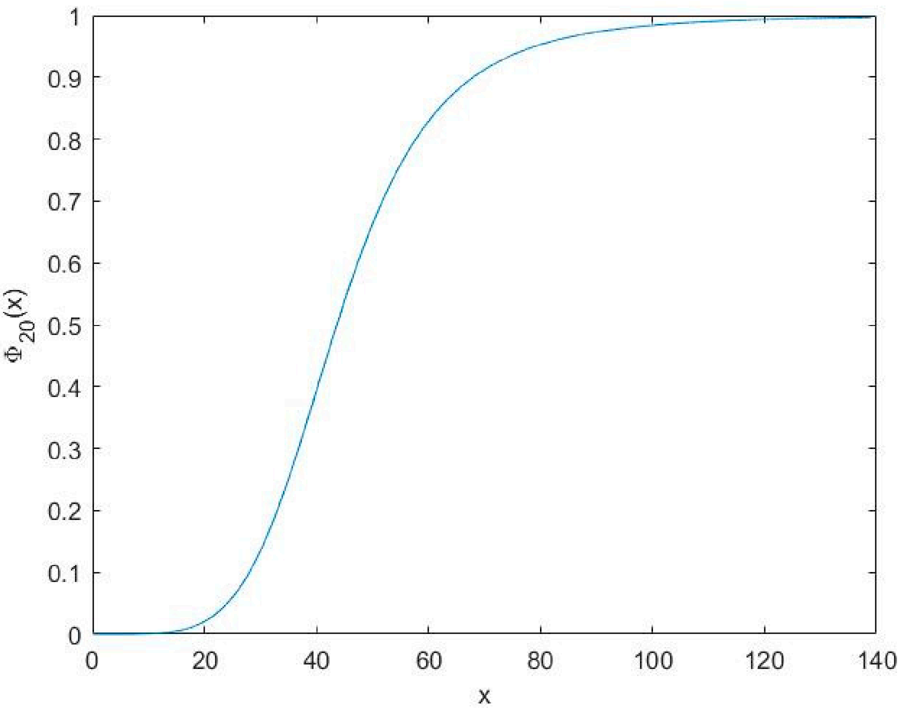

Figure 1. The uncertainty distribution $\Phi_{20}(x)$ for the ignition delay time X_{20}.

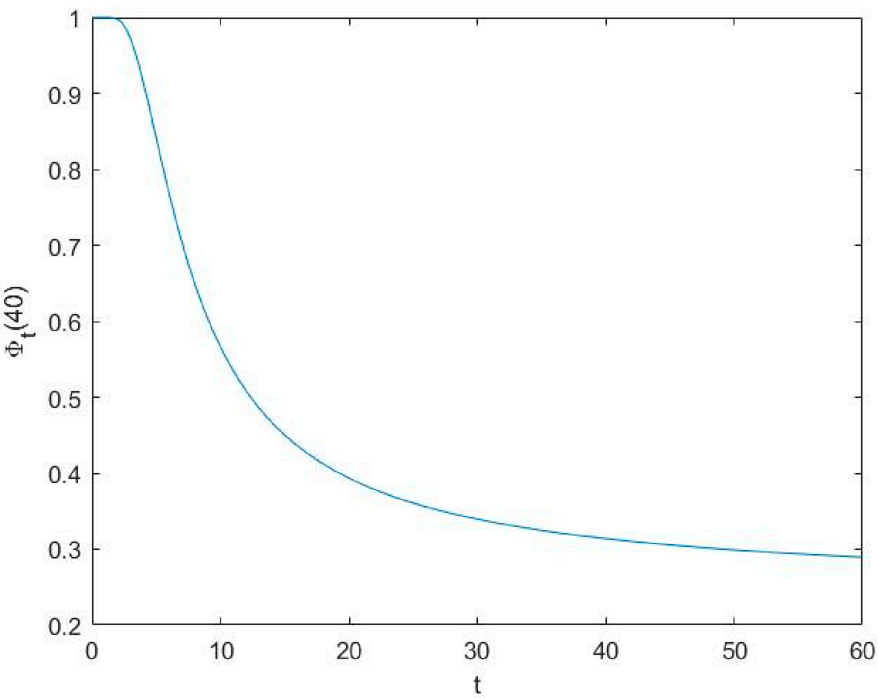

Figure 2. The uncertain measure $\Phi_t(40)$, i.e., $\mathcal{M}\{X_t \leq 40\}$ for the ignition delay time X_t.

The α-path for the ignition delay time is shown in Figure 3.

Then, we will solve the hypothesis test for the uncertain distribution. Note that the inverse uncertainty distribution of $\mathrm{N}(0.011408,\ 0.018289)$ is

$$\Phi_t^{-1}(\alpha) = 34.74 \exp\left(0.011408t + \frac{\sqrt{3} \times 0.018289t}{\pi} \ln\left(\frac{\alpha}{1-\alpha}\right)\right) \tag{42}$$

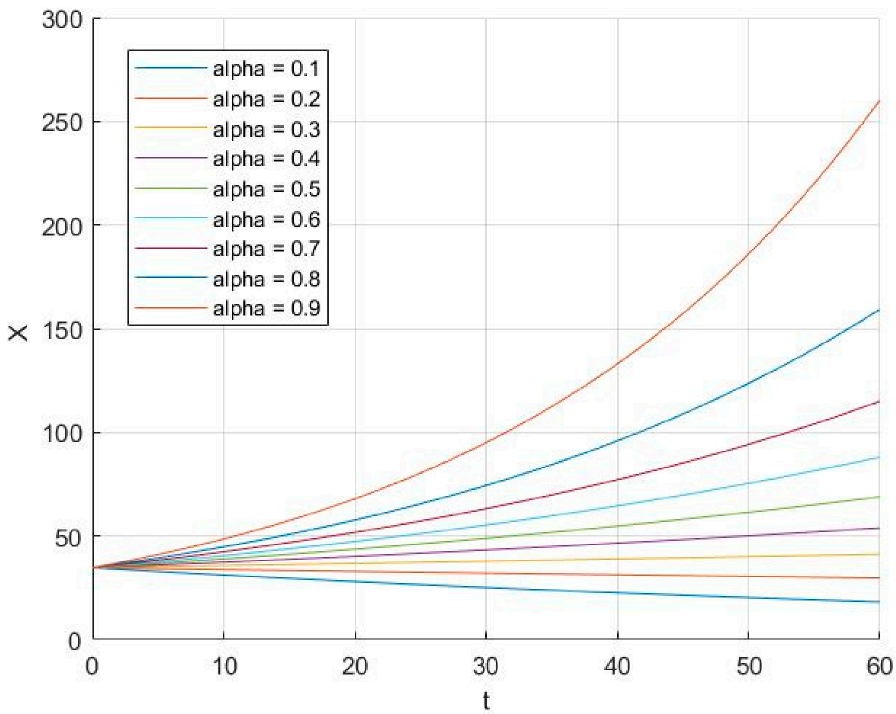

Figure 3. The α-path for the ignition delay time X_t.

Given a significance level $\alpha = 0.1$, we obtain the hypothesis test values as shown in Table 2.

Table 2. Hypothesis test values for ignition delay time of pyrotechnic devices.

t	$\Phi_t^{-1}(\frac{\alpha}{2})$	$\Phi_t^{-1}(1-\frac{\alpha}{2})$	t	$\Phi_t^{-1}(\frac{\alpha}{2})$	$\Phi_t^{-1}(1-\frac{\alpha}{2})$
0	34.74	34.74	21	23.6646	82.3468
3	32.886	39.2984	24	22.4017	93.152
6	31.131	44.455	27	21.2061	105.375
9	29.4696	50.2882	30	20.0744	119.202
12	27.8968	56.8867	33	19.0031	134.843
15	26.408	64.3511	36	17.9889	152.536
18	24.9987	72.795			

It follows from $\alpha \times 13 = 1.3$ and Theorem 5 that the test is

$$\begin{aligned} W = \{(z_1, z_2, \cdots, z_{13}) : \ & \textit{there are at least } 2 \textit{ of indexes } i's \textit{ with} \\ & 1 \le i \le 13 \textit{ such that } z_i < \Phi_t^{-1}\left(\tfrac{\alpha}{2}\right) \textit{ or } z_i > \Phi_t^{-1}\left(1 - \tfrac{\alpha}{2}\right)\Big\} \end{aligned} \tag{43}$$

Since there is no z_i that is not within the range of $\left[\Phi_t^{-1}\left(\frac{\alpha}{2}\right), \Phi_t^{-1}\left(1 - \frac{\alpha}{2}\right)\right]$, we obtain $(z_1, z_2, \cdots, z_{13}) \notin W$. Thus the normal uncertainty distribution N(0.011408, 0.018289) is a good fit to the ignition delay time. As shown in the Figure 4, all real data points are included in the envelope of $\alpha = 0.1 \sim 0.9$.

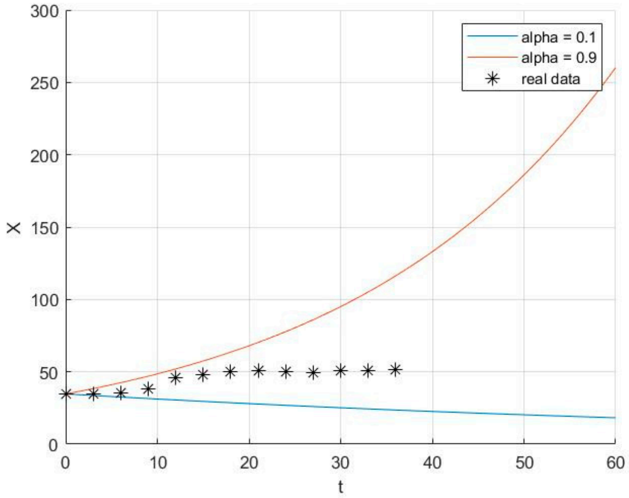

Figure 4. Envelope lines with a range of $\alpha = 0.1 \sim 0.9$ and real data points.

7. Conclusions

This study developed an uncertain degradation model for initiating explosive devices using uncertain differential equations, addressing the limitations of traditional models. Focusing on ignition delay time, the model incorporates internal and external noise affecting degradation. The Liu process within the uncertainty theory framework provided accurate lifetime predictions, demonstrated through real data. Moment-based parameter estimation ensured the model's adaptability. Future work can involve more complex environmental factors being integrated and the model being extended to other high-reliability products. This research provides a robust method for predicting the lifetime of initiating explosive devices, offering a practical tool for improving reliability assessment in key product applications.

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