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Editorial

Mathematical Models and Simulations

Giovanni Nastasi

Department of Mathematics and Computer Science, University of Catania, Viale Andrea Doria 6, 95125 Catania, Italy; giovanni.nastasi@unict.it

1. Introduction

In this editorial, we present the Special Issue of the scientific journal *Axioms* entitled "Mathematical Models and Simulations". Mathematical models constitute a fundamental tool for understanding physical phenomena, biological systems, and finance and engineering. In addition to theoretical aspects, simulations play a primary role in applications, because they allow for the prediction of the behavior of quantities of interest. We collected papers in the field of mathematical physics, where different categories of mathematical models are presented, both deterministic, i.e., based on ordinary or partial differential equations, and stochastic, i.e., defined by stochastic processes or based on stochastic differential equations. The study of mathematical aspects of the presented models has been tackled. To provide realistic applications, numerical simulations play an important role. Several numerical methods suited to the specific problem have been adopted. Moreover, in some cases, simulations have been performed by adopting real data for the parameters, and optimization procedures have been carried out.

2. Overview of the Published Papers

This Special Issue contains 13 papers that were accepted for publication after a rigorous review process.

In contribution 1, the authors E. El-Zahar and A. Ebaid study the pantograph delay differential equation. They determine the analytic solution of such an equation in a closed series form regarding exponential functions. The convergence of such a series is analyzed.

In contribution 2, B. Telli, M. Souid, and I Stamova present a paper devoted to boundary-value problems for Riemann–Liouville-type fractional differential equations of variable order involving finite delays. The existence of solutions is first studied using Darbo's fixed-point theorem and the Kuratowski measure of noncompactness. Second, the Ulam–Hyers stability criteria are examined.

In contribution 3, S. Bagchi proposes a generalized finite-dimensional algebraic analysis of the solution spaces of second-order ODEs equipped with periodic Dirac delta forcing. The proposed algebraic analysis establishes the conditions for the convergence of responses within the solution spaces without requiring the relative smoothness of the forcing functions. The analysis shows that smooth and locally finite responses can be admitted in an exponentially stable solution space.

In contribution 4, A. Elaiw, R. Alsulami, and A. Hobiny present a COVID-19 and Influenza Co-Infection Model with Time Delays and Humoral Immunity. The model considers the interactions among uninfected epithelial cells (ECs), SARS-CoV-2-infected ECs, IAV-infected ECs, SARS-CoV-2 particles, IAV particles, SARS-CoV-2 antibodies, and IAV antibodies. The model is constructed using a system of delayed ordinary differential equations (DODEs), which includes four time delays. They establish the non-negativity and boundedness of the solutions, examine the existence and stability of all equilibria, and perform numerical simulations to support the theoretical results.

In contribution 5, O. Muscato focuses on electron transport and heat generation in a Resonant Tunneling Diode semiconductor device. A new electrothermal Monte Carlo

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method is introduced. The method couples a Monte Carlo solver of the Boltzmann–Wigner transport equation with a steady-state solution of the heat diffusion equation. This methodology provides an accurate microscopic description of the spatial distribution of self-heating and its effect on the detailed nonequilibrium carrier dynamics.

In contribution 6, B. K. Singh, H. M. Baskonus, N. Singh, M. Gupta, and D. G. Prakasha analyze the dynamical behavior of two space-dimensional nonlinear time-fractional models governing the unsteady flow of polytropic gas that occurs in cosmology and astronomy. They adopt two efficient hybrid methods, the so-called optimal homotopy analysis $\mathbb{J}$-transform method and the $\mathbb{J}$-variational iteration transform method. The convergence of these methods is proven, and the numerical results demonstrate that both of the developed techniques perform better for the considered time-fractional model governing the unsteady flow of polytropic gas.

In contribution 7, F. M. Al-Askar, C. Cesarano, and W. W. Mohammed consider the stochastic Kadomtsev–Petviashvili equation with fractional beta-derivative. They find exact solutions employing the Riccati equation method and the Jacobi elliptic function method. The obtained solutions can also be used in practical applications, such as designing improved tsunami warning systems or optimizing wave energy converters. They investigate the effect of beta-derivatives and noise on the analytical solutions of the equation using graphs.

In contribution 8, A. Shehata, G. S. Khammash, and C. Cattani derive some classical and fractional properties of the *rRs* matrix function using the Hilfer fractional operator. The theory of special matrix functions is the theory of those matrices that correspond to special matrix functions such as the gamma, beta, and Gauss hypergeometric matrix functions. They also show the relationship with other generalized special matrix functions in the context of the Konhauser and Laguerre matrix polynomials.

In contribution 9, V. Sobchuk, O. Barabash, A. Musienko, I. Tsyganivska, and O. Kurylko propose a mathematical model of the process of cyber risk management in an enterprise, which is based on the distribution of piecewise continuous analytical approximating functions of cyberattacks in the Fourier series. This model makes it possible to move the system of the regulatory control of cyber threats of the enterprise from a discrete to a continuous automated process of regulatory control.

In contribution 10, L. Sánchez, G. Ibacache-Pulgar, C. Marchant, and M. Riquelme develop varying-coefficients quantile regression models based on the family of log-symmetric distributions. Moreover, they estimate the parameters of the model using the maximum penalized likelihood technique and a back-fitting algorithm. They incorporate the nonparametric structure through natural cubic smoothing splines and calculate local influence techniques for model diagnostics by assessing the normal curvatures under different perturbation scenarios. Further, they implement the obtained outcomes computationally within the R programming environment and apply these results to real data related to atmospheric pollutants in Padre Las Casas (Chile), recognized as one of the most contaminated cities in Latin America and the Caribbean.

In contribution 11, Y. Chong, A. J. Kashyap, S. Chen, and F. Chen study a class of discrete-time commensalism systems with additive Allee effects on the host species. First, the single species with additive Allee effects is analyzed for existence and stability; then, the existence of fixed points of discrete systems is given, and the local stability of fixed points is given by characteristic root analysis. Second, the bifurcation of a codimension of one of the systems at non-hyperbolic fixed points is examined. Furthermore, this work uses the hybrid chaos method to control the chaos that occurs in the flip bifurcation of the system. Finally, the analysis conclusions were verified by numerical simulations.

In contribution 12, J. F. Sánchez-Pérez, J. Solano-Ramírez, E. Castro, M. Conesa, F. Marín-García, and G. García-Ros apply the non-dimensionalization methodology to the Burgers–Huxley equation to obtain a universal solution to the problem posed. In this case, the symmetry condition is applied to one of the boundary conditions, and a constant value of the variable is applied to the other boundary condition (Dirichlet condition). Another

objective is to study the weight of the variables in the problem. For the construction of the universal curves, the Network Simulation Method was used, which has demonstrated its effectiveness in solving this problem, as well as other engineering problems.

In contribution 13, C. Feng considers the oscillatory behavior of the solutions for a Parkinson's disease model with discrete and distributed delays. The distributed delay terms can be changed to new functions such that the original model is equivalent to a system in which it only has discrete delays. The stability analysis is performed employing the linearization technique. By analyzing the linearized system at the smallest delay, some sufficient conditions to guarantee the existence of oscillatory solutions for a delayed Parkinson's disease system can be obtained. It is found that under suitable conditions of the parameters, a time delay affects the stability of the system. Some numerical simulations are provided to illustrate the theoretical result.

Conflicts of Interest: The authors declare no conflict of interest.

List of Contributions:

1. El-Zahar, E.; Ebaid, A. Analytical and Numerical Simulations of a Delay Model: The Pantograph Delay Equation. *Axioms* **2022**, *11*, 741. https://doi.org/10.3390/axioms11120741.
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3. Bagchi, S. Analysis of Finite Solution Spaces of Second-Order ODE with Dirac Delta Periodic Forcing. *Axioms* **2023**, *12*, 85. https://doi.org/10.3390/axioms12010085.
4. Elaiw, A.; Alsulami, R.; Hobiny, A. Dynamic Behaviors of a COVID-19 and Influenza Co-Infection Model with Time Delays and Humoral Immunity. *Axioms* **2023**, *12*, 151. https://doi.org/10.3390/axioms12020151.
5. Muscato, O. Electrothermal Monte Carlo Simulation of a GaAs Resonant Tunneling Diode. *Axioms* **2023**, *12*, 216. https://doi.org/10.3390/axioms12020216.
6. Singh, B.; Baskonus, H.; Singh, N.; Gupta, M.; Prakasha, D. Study of Time-Fractional Nonlinear Model Governing Unsteady Flow of Polytropic Gas. *Axioms* **2023**, *12*, 285. https://doi.org/10.3390/axioms12030285.
7. Al-Askar, F.; Cesarano, C.; Mohammed, W. Effects of the Wiener Process and Beta Derivative on the Exact Solutions of the Kadomtsev–Petviashvili Equation. *Axioms* **2023**, *12*, 748. https://doi.org/10.3390/axioms12080748.
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13. Feng, C. Oscillatory Behavior of the Solutions for a Parkinson's Disease Model with Discrete and Distributed Delays. *Axioms* **2024**, *13*, 75. https://doi.org/10.3390/axioms13020075.

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