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Editorial

Differential Geometry and Its Application

Mića S. Stanković

Faculty of Science and Mathematics, University of Niš, 18000 Niš, Serbia; mica.stankovic@pmf.edu.rs

1. Introduction

We have launched a Special Issue of *Axioms* which focuses on the generalization of Riemannian spaces and their mappings. This Special Issue provides a platform to showcase the latest achievements in many branches of theoretical and practical studies of mathematics, which relate to the theory of Riemannian and generalized Riemannian spaces and their mappings. The scope of this Special Issue includes Riemannian Spaces and generalizations, Kenmotsu manifolds, Kaehler manifolds, manifolds with non-symmetric linear connection, cosymplectic manifolds, contact manifolds, statistical manifolds, Minkowski spaces, geodesic mappings, almost geodesic mappings, holomorphically projective mappings, warped product of manifolds, complex space forms, quaternionic space forms, golden manifolds, inequalities, invariants, immersions, etc. Potential authors are encouraged to submit papers that present new ideas in the field of differential geometry in addition to the above topics. Given the broad scope and widespread interest in this topic, more works should be published in this area. It is expected that a follow-up Special Issue, "Differential Geometry and Its Application II", will be published in due course.

2. Overview of the Published Papers

This Special Issue contains 16 papers which were accepted for publication after a rigorous reviewing process.

In their study (Contribution 1), Cornelia-Livia Bejan, Galia Nakova and Adara M. Blaga use a Kähler B-manifold (i.e., a Kähler manifold with a Norden metric) to obtain some corresponding results from the Kählerian and para-Kählerian context concerning the Bochner curvature. They prove that such a manifold is of constant totally real sectional curvatures if and only if it is a holomorphic Einstein, Bochner flat manifold. Moreover, they provide the necessary and sufficient conditions for a gradient Ricci soliton or a holomorphic η-Einstein Kähler manifold with a Norden metric to be Bochner flat. Finally, they show that a Kähler B-manifold is of quasi-constant totally real sectional curvatures if and only if it is a holomorphic η-Einstein, Bochner flat manifold.

In another study (Contribution 2), Jamal Oudetallah, Rehab Alharbi and Iqbal M. Batiha define the so-called pairwise r-compactness in topological and bitopological spaces. In particular, several inferred properties of the r-compact spaces and their connections with other topological and bitopological spaces are studied theoretically. As a result, several novel theorems of the r-compact space are generalized in the pairwise r-compact space. The results established in this research paper are new in the field of topology.

The authors of Contribution 3, Halil İbrahim Yoldaş, Abdul Haseeb and Fatemah Mofarreh, investigate a Kenmotsu manifold satisfying certain curvature conditions endowed with $\bigstar - \eta$-Ricci solitons. First, they find some necessary conditions for such a manifold to be ϕ-Einstein. Then, they study the notion of $\bigstar - \eta$-Ricci soliton on this manifold and prove some significant results related to this notion. Finally, they construct a nontrivial example of three-dimensional Kenmotsu manifolds to verify some of their results.

The authors of Contribution 4, Jae Min Lee and Byungdo Park, construct a model of differential K-theory using superbundles with a $Z/2Z$-graded connection and a differential

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form on the base manifold and prove that their model is isomorphic to the Freed–Lott–Klonoff model of differential K-theory.

In Contribution 5, Samer Al Ghour introduces soft complete continuity as a strong form of soft continuity and he introduces soft strong continuity as a strong form of soft complete continuity. Several characterizations, compositions, and restriction theorems are obtained. Moreover, several preservation theorems regarding soft compactness, soft Lindelofness, soft connectedness, soft regularity, soft normality, soft almost regularity, soft mild normality, soft almost compactness, soft almost Lindelofness, soft near compactness, soft near Lindelofness, soft paracompactness, soft near paracompactness, soft almost paracompactness, and soft metacompactness are obtained. This study also explores the correlation between their new concepts in soft topology and their corresponding concepts in general topology; as a result, this authors demonstrates that soft complete continuity (resp. soft strong continuity) in soft topology is an extension of complete continuity (resp. strong continuity) in soft topology.

The authors of Contribution 6, Marilena Jianu, Sever Achimescu, Leonard Dăuş, Ion Mierluş-Mazilu, Adela Mihai and Daniel Tudor, define a surface that interpolates the ballot numbers in the Catalan triangle corresponding to every pair of nonnegative integers (except for the origin). They study the geometric properties of this surface and prove that it contains exactly five half-lines. The mean curvature and the Gauss curvature of the surface are also calculated.

In the research presented in Contribution 7, Mohd Bilal, Sushil Kumar, Rajendra Prasad, Abdul Haseeb and Sumeet Kumar introduce and study h-quasi-hemi-slant (in short, h-qhs) Riemannian maps and almost h-qhs Riemannian maps from almost quaternionic Hermitian manifolds to Riemannian manifolds. They investigate some fundamental results, mainly on h-qhs Riemannian maps: the integrability of distributions, the geometry of foliations, the condition for such maps to be totally geodesic, etc. At the end of this article, they give two non-trivial examples of this notion.

The author of Contribution 8, Samer Al Ghour, introduces soft rgw-closed sets as a new class of soft sets that strictly contain the classes of soft rg-closed sets and soft gw-closed sets. This study also explores the behavior of soft rgw-closed sets with respect to soft unions, soft intersections, and soft subspaces, as well as induced soft topologies. Moreover, soft $w - T_{1/2}$ spaces, which are a weaker form of soft $T_{1/2}$ spaces, are defined and investigated. In addition to these, the characterizations of soft $rg - T_{1/2}$ spaces and soft $rgw - T_{1/2}$ spaces are discussed. The work also examines the relationship between their novel notions in soft topological spaces and their analogs in topological spaces.

In the research presented in Contribution 9, Yanlin Li, Rajendra Prasad, Abdul Haseeb, Sushil Kumar and Sumeet Kumar characterize Clairaut semi-invariant Riemannian maps from cosymplectic manifolds to Riemannian manifolds. Moreover, they provide a nontrivial example of such a Riemannian map.

The authors of Contribution 10, Hassan Al-Zoubi, Alev Kelleci Akbay, Tareq Hamadneh and Mutaz Al-Sabbagh, define surfaces of revolution without parabolic points in three-dimensional Lorentz–Minkowski space. Then, they classify this class of surfaces under the condition $\Delta^{III}x = Ax$, where Δ^{III} is the Laplace operator regarding the third fundamental form, and A is a real square matrix of order 3. They prove that such surfaces are either catenoids or surfaces of Enneper, or pseudo spheres or hyperbolic spaces centered at the origin.

In the research presented in Contribution 11, Yanlin Li, Mohan Khatri, Jay Prakash Singh and Sudhakar K. Chaubey derive Chen's inequalities involving Chen's δ-invariant δ_M, Riemannian invariant $\delta(m_1, \ldots, m_k)$, Ricci curvature, and Riemannian invariant θk $(2 \leq k \leq m)$, the scalar curvature and the squared of the mean curvature for submanifolds of generalized Sasakian-space-forms endowed with a quarter-symmetric connection. As an application of the obtain inequality, they begin by deriving the Chen inequality for the bi-slant submanifold of generalized Sasakian-space-forms.

The author of Contribution 12, Nenad O. Vesić, Vladislava M. Milenković and Mića S. Stanković obtain two invariants for mappings of affine connection spaces with a special form of deformation tensors. At the end of this paper, they use these forms to obtain two invariants for third-type almost-geodesic mappings of symmetric affine connection.

In the research presented in Contribution 13, Samer Al Ghour defines a new class of soft open sets, namely soft R_w-open sets, and investigates their main features. With the help of examples, he shows that the class of soft R_w-open sets lies strictly between the classes of soft regular open sets and soft open sets. He shows that soft R_w-open subsets of a soft locally countable soft topological space coincide with the soft open sets. Moreover, he shows that soft R_w-open subsets of a soft anti-locally countable coincide with the soft regular open sets. Also, he shows that the class of soft R_w-open sets is closed under finite soft intersection, and concludes that this class forms a soft base for some soft topology. In addition, he defines the soft δ_w-closure operator as a new operator in soft topological spaces. Moreover, via the soft δ_w-closure operator, he introduces soft δ_w-open sets as a new class of soft open sets which form a soft topology. Moreover, he studies the correspondence between soft δ_w-open in soft topological spaces and δ_w-open in topological spaces.

The authors of Contribution 14, Muhittin Evren Aydın, Adela Mihai and Cihan Özgür, introduce the notion of a Pythagorean hypersurface immersed into an $(n+1)$-dimensional pseudo-Riemannian space form of constant sectional curvature $c \in \{-1, 0, 1\}$. By using this definition, they prove in a Riemannian setting that if an isoparametric hypersurface is Pythagorean, then it is totally umbilical with sectional curvature $\phi + c$, where ϕ is the Golden Ratio. They also extend this result to Lorentzian ambient space, observing the existence of a non-totally umbilical model.

In Contribution 15, Zachary McGuirk and Byungdo Park consider the next property: in the homotopy theory of spaces, the image of a continuous map is contractible to a point in its cofiber. This property does not apply when these authors discretize spaces and continuous maps to directed graphs and their morphisms. In this paper, the authors present a construction of a cofiber of a directed graph map whose image is contractible in the cofiber. Their work reveals that a theoretically correct construction in a continuous setup is no longer correct when it is discretized, highlighting canonical constructions in category theory in a different perspective.

The authors of Contribution 16, Vladimir Rovenski, Sergey Stepanov and Irina Tsyganok, are devoted to geometrical aspects of conformal mappings of complete Riemannian and Kählerian manifolds. Their work uses the Bochner technique, one of the oldest and most important techniques in modern differential geometry. One feature of this article is that the results presented here are easily obtained using a generalized version of the Bochner technique due to theorems on the connection between the geometry of a complete Riemannian manifold and the global behavior of its subharmonic, superharmonic, and convex functions.

3. Conclusions

A total of 16 papers were published in this Special Issue "Differential Geometry and Its Application". In these works, researchers interested in various aspects of Riemannian space theory and related topics will find interesting insights and inspiring results, leading to increased citation of these contributions; thus, this Special Issue will be exposed to a wider audience.

List of Contributions:

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3. Yoldaş, H.İ.; Haseeb, A.; Mofarreh, F. Certain Curvature Conditions on Kenmotsu Manifolds and $*$-η-Ricci Solitons. Axioms 2023, 12, 140. https://doi.org/10.3390/axioms12020140.

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11. Li, Y.; Khatri, M.; Singh, J.P.; Chaubey, S.K. Improved Chen's Inequalities for Submanifolds of Generalized Sasakian-Space-Forms. Axioms 2022, 11, 324. https://doi.org/10.3390/axioms11070324.
12. Vesić, N.O.; Milenković, V.M.; Stanković, M.S. Two Invariants for Geometric Mappings. Axioms 2022, 11, 239. https://doi.org/10.3390/axioms11050239.
13. Al Ghour, S. Soft $R\omega$-Open Sets and the Soft Topology of Soft δ_ω-Open Sets. Axioms 2022, 11, 177. https://doi.org/10.3390/axioms11040177.
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15. McGuirk, Z.; Park, B. A Model of Directed Graph Cofiber. Axioms 2022, 11, 32. https://doi.org/10.3390/axioms11010032.
16. Rovenski, V.; Stepanov, S.; Tsyganok, I. A Generalized Bochner Technique and Its Application to the Study of Conformal Mappings. Axioms 2021, 10, 333. https://doi.org/10.3390/axioms10040333.

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