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An Interval Type-3 Fuzzy–Fractal Approach for Plant Monitoring

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Abstract: In this article, a plant monitoring approach based on a hybrid mixture of type-3 fuzzy logic (T3FL) and the fractal dimension (FD) is presented. The main reason for combining type-3 and the fractal dimension is to take advantage of both their capabilities in solving the problem of monitoring a plant. Basically, T3FL helps in handling the uncertainty in monitoring the variables of a nonlinear system, while the FD helps to capture the signal complexity by finding key or hidden patterns in the data. The FD is utilized to estimate data complexity of the process variables being monitored. We utilize the box counting algorithm to approximate the values of the FD. A set of T3FL rules is utilized to model monitoring knowledge. The proposed approach was tested with a plant studied in previous works, which was solved with type-1 and type-2 fuzzy logic, and now type-3 is able to surpass the performance of previous approaches for this problem. The main contribution is the T3FL and FD hybrid proposal for plant monitoring, which has not been presented before in the literature. Simulation results illustrate the potential advantage of utilizing the T3FL and FD combination in this area.

Keywords: type-3 fuzzy sets; fractal theory; monitoring

MSC: 03B52; 03E72; 62P30

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1. Introduction

The use of intelligent techniques in plant monitoring has been receiving increasing attention. Recently, we have seen the utilization of neural networks, evolutionary computing, and fuzzy systems in this area. In the particular case of fuzzy logic, we can find that most of the works in the literature for monitoring are based on the simplest form of fuzzy logic [1–3], which is called type-1, like the works that can be reviewed in [4–12]. More recently, type-2 has also been considered in this area, as a way to model uncertainty in a better fashion and achieve better results, as can be verified in [13,14]. In addition, there are also works where fractal theory has been solely applied for achieving efficient monitoring of complex systems, like the works presented in [15–23]. In this case, fractal theory constructs are utilized to analyze the complexity of monitoring data to find hidden structure in the data. In addition, hybrid approaches for monitoring have also been proposed, like: genetic fuzzy, neuro-fuzzy, fuzzy–fractal, and others, as in [12–14]. In these hybrids, the idea is taking advantage of learning and optimization, provided by other techniques (genetic and neural algorithms), to improve the performance of fuzzy models. However, in this article we propose for the first time the utilization of T3FL in the monitoring area, as well as its hybrid combination with fractal theory, expecting that this combination will produce better results. The hybrid of T3FL with FD is a novel proposal for this area and we envision that will also help in solving other problems in different areas in the near future.

Recently, the area of T3FL has also been receiving increasing attention, as a new way of handling higher levels of uncertainty in decision making situations. In particular, we can find applications of type-3 in different areas, like control [24–30], quality control [31], time series prediction [32], and other significant applications that are being developed at the moment. However, currently there are no papers reporting the utilization of T3FL for the monitoring area, and we can consider that this is a research gap in the literature, which (of course) motivated our current research work presented in this paper. In this regard, this work on type-3 can be viewed as an innovative contribution in this area.

In this work, a type-3 fuzzy–fractal monitoring method is put forward. The construct of the FD is utilized to estimate time series complexity of process variables. The type-3 rules are utilized to encapsulate process monitoring knowledge. In these rules, the FD is utilized as a variable to aid in finding particular data patterns that are the key in detecting possible problems in the process. The contribution is the type-3 fuzzy–fractal monitoring approach, which has not been previously proposed in the existing literature. We also believe that related problems of diagnosis could be solved in a better fashion by utilizing type-3 fuzzy theory. Monitoring has the goal of detecting a problem in a system, while diagnosis consists in identifying what the actual reason for a problem is, which could be more complex (requiring additional expert knowledge of the problem) and we plan to deal with this diagnosis area in the future with a similar approach.

The article is arranged as: Section 2 reviews concepts on monitoring and diagnostics. In Section 3, some basic definitions of type-3 fuzzy logic are presented. In Section 4, the plant monitoring approach with interval type-3 is outlined. Section 5 summarizes the results and comparison with alternative approaches. Lastly, Section 6 outlines the conclusions of this work.

2. Monitoring and Diagnostics Concepts

The relevance of performing monitoring for plant processes is now mostly accepted due to the fact that it contributes to enhanced productivity, increased quality, and cost minimization [4,5]. The commonly utilized monitoring methods include pattern recognition methods [6], fuzzy systems [7], knowledge methods [8], neural networks [12], and metaheuristic methods [7]. It is noteworthy that despite the fact that these methods are quite different in their theory, they share a common design, when being applied, as depicted in Figure 1. Basically, the goal of monitoring is estimating the condition of a system by using the signal measurements and this is reflected, in a succinct way, in the block diagram of Figure 1. Diagnosis could be thought of as finding the reason for a problem that has been found by the monitoring process, but diagnosis is not considered in this work and can be viewed as worthy of interesting future research work.

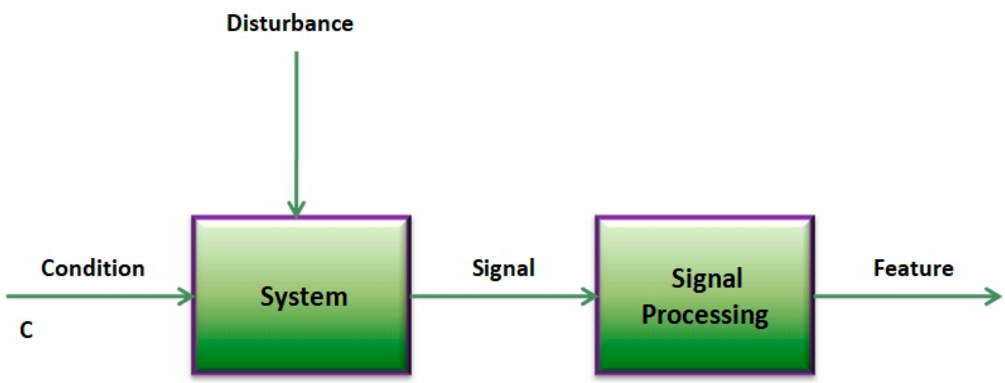

Figure 1. Monitoring structure.

The "health" or status of a machine or a process (which is called condition and it is indicated by C in Figure 1) is represented by $C \in \{c_1, c_2, \ldots, c_m\}$, the disturbance is viewed as "noise", and signals are system "outputs". Usually, signals are computed by a system, then they are mapped onto feature signals, symbolized by $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$. The possible conditions of the system are usually established a priori, such as stable, low, critical, high, etc., so that we can know in which state the system is at the moment. In the particular case of a fuzzy system, the fuzzy rules are established in such a way that they are capable of mapping the signals (x) to the condition (C). If a neural network is used, then the network is trained to learn this mapping from x to C. Of course, metaheuristics (such as particle swarm optimization, grey wolf optimizer, firefly algorithm, and other nature-inspired methods) could be utilized to optimize the fuzzy system or the neural network constructed for achieving efficient monitoring of a system. In this paper, the model is built with a mixture of T3FL and FD theory to achieve an efficient monitoring for nonlinear processes, which is a better way to model the uncertainty of the problem. In addition, the combination of T3FL with FD has not been put forward before for this type of problem and can be highlighted as the prominent contribution of the article. Finally, we can state that we believe that this combination could be a viable option for other decision-making problems, like diagnosis and prediction, and we intend to work on this area and its applications in the near future.

3. Basic Concepts of Type-3 Fuzzy Sets and Fractal Dimension

We outline in this section some basic concepts of both theoretical sides of this work, meaning the type-3 fuzzy and fractal theories.

3.1. Type-3 Fuzzy Definitions

We start by postulating type-3 concepts.

Definition 1. *A type-3 fuzzy set (T3 FS) [33–35], which can be written as $A^{(3)}$, is distinguished by a membership function (MF) of $A^{(3)}$, in Cartesian product $X \times [0,\ 1] \times [0,\ 1]$ in $[0,1]$, where X is the primary variable universe of $A^{(3)}$, x. The MF of $\mu_{A^{(3)}}$ is postulated by $\mu_{A^{(3)}}(x,u,v)$ (or $\mu_{A^{(3)}}$) and is written as a type-3 MF (T3 MF):*

$$\begin{gathered}\mu_{A^{(3)}} : X \times [0,\ 1] \times [0,\ 1] \to [0,\ 1] \\ A^{(3)} = \left\{ \left(x, u(x), v(x,u), \mu_{A^{(3)}}(x,u,v)\right) \mid x \in X,\ u \in U \subseteq [0,\ 1], v \in V \subseteq [0,\ 1] \right\}\end{gathered} \tag{1}$$

where U and V are the universes for secondary and tertiary variables u and v, respectively. A T3 FS, $A^{(3)}$, is postulated as:

$$A^{(3)} = \int_{x \in X} \int_{u \in [0,1]} \int_{v \in [0,1]} \mu_{A^{(3)}}(x,u,v) / (x,u,v) \tag{2}$$

$$A^{(3)} = \int_{x \in X} \left[\int_{u \in [0,1]} \left[\int_{v \in [0,1]} \mu_{A^{(3)}}(x,u,v) / v \right] / u \right] / x \tag{3}$$

where $\iiint$ denotes union of all x, u, v values

If $\mu_{A^{(3)}}(x,u,v) = 1$, the T3 FS $A^{(3)}$ is simplified to an interval type-3 fuzzy set (IT3 FS) denoted $\mathbb{A}$, postulated by Expression (4).

$$\mathbb{A} = \int_{x \in X} \left[\int_{u \in [0,1]} \left[\int_{v \in [\underline{\mu}_{\mathbb{A}}(x,u),\ \ \mu_{\mathbb{A}}(x,u)} 1/v \right] / u \right] / x \tag{4}$$

where

$$\mu_{\mathbb{A}(x,u)}(v) = \int_{v \in [\underline{\mu}_{\mathbb{A}}(x,u),\ \ \mu_{\mathbb{A}}(x,u)} 1/v$$

$$\mu_{\mathbb{A}(x)}(u,v) = \int_{u \in [0,1]} \left[\int_{v \in [\underline{\mu}_{\mathbb{A}}(x,u),\ \ \mu_{\mathbb{A}}(x,u)} 1/v \right] / u$$

$$\mathbb{A} = \int_{x \in X} \mu_{\mathbb{A}_{(x)}}(u,v) / x$$

Based on previous definitions of type-3 fuzzy sets, we can also mathematically define fuzzy relations, fuzzy logic, fuzzy inference, type reduction, fuzzy systems, and in general all the concepts and operations for type-3 [33,35]. The definitions are similar to the ones for type-2, but in general we can say that the structure of the fuzzy systems remains the same and the main change when going into type-3 is in the form of the MFs and the corresponding changes in the operations to calculate the global output. The main idea of extending the fuzzy sets to the type-3 fuzzy form is having a more powerful way to handle the underlying uncertainty in modeling nonlinear processes. In other words, we are elevating the fuzzy models fom type-1 to type-3 with the main goal of achieving a better approximation to uncertainty in real problems. For more details on the theory and corresponding definitions of type-3 that have been developed to date, the reader can check the key works found in [33–35]. In addition, some successful real applications, as an illustration of the potential of this new area, such as in prediction and control, can be found in [24–32]. In addition, it is expected that type-n could be theoretically constructed in the near future (with a kind of induction approach), as it would be a direct extension of the ideas mentioned here, and we believe that it could present even more powerful tools to solve real problems in a wide range of application areas.

3.2. Fractal Dimension

Recently, significant progress has been realized in comprehending the complexity of an object through the utilization of fractal constructs [21]. For example, time series in finance and engineering exhibit properties suggesting a fractal structure [22,23]. In addition, applications in medicine (like in COVID-19), robotics, control, and others can be found in the recent literature [24]. The fractal dimension is postulated as:

$$d = \lim\, [\ln N(r)]/[\ln(1/r)] \tag{5}$$

where N(r) stands for number of boxes achieving coverage and r is the box size. d expressed in (5) is approximated utilizing logarithmic regression for calculating an estimate of the d value. The approximation can be obtained with:

$$\ln N(r) = \ln\beta - d \ln r \tag{6}$$

where d is the dimension. This is what is called the box counting approximation of the fractal dimension. In the particular case of this paper, we characterize the complexity of the process by utilizing the fractal concept, as will be outlined later in the paper.

4. Type-3 Fuzzy Fractal Monitoring Approach

First, we exhibit in Figure 2 the type-3 fuzzy system structure, where we can find the inputs (temperature, pressure, and FD) and output (condition). The system is a chemical reactor, which was described in more detail in [4]. In Table 1, we can find the complete fuzzy rule base consisting of 27 interval type-3 fuzzy rules. The 27 rules were postulated with expert knowledge about the problem. The linguistic values of temperature, pressure and FD are: low, normal, and high. The MFs for the output represent the 9 possible conditions in which the plant can be at any time. In Table 2, we exhibit the parameterization for the Gaussian MFs utilized for all the values mentioned above. Figures 3–5 illustrate the input interval type-3 (IT3) MFs. On the other hand, Figure 6 illustrates the MFs of the output (condition). The Gaussian MFs for all variables can be appreciated in Figure 2 in the global view of the system.

The rules shown in Table 1 were extracted in part from the knowledge of experts in the monitoring of processes. Also, the use of the fractal dimension is based on previous works on monitoring [4,13].

The parameters of the Gaussian membership functions shown in Table 2 were obtained by a trial and error process, but in the future, we expect to optimize the parameter values with a metaheuristic algorithm to improve the results.

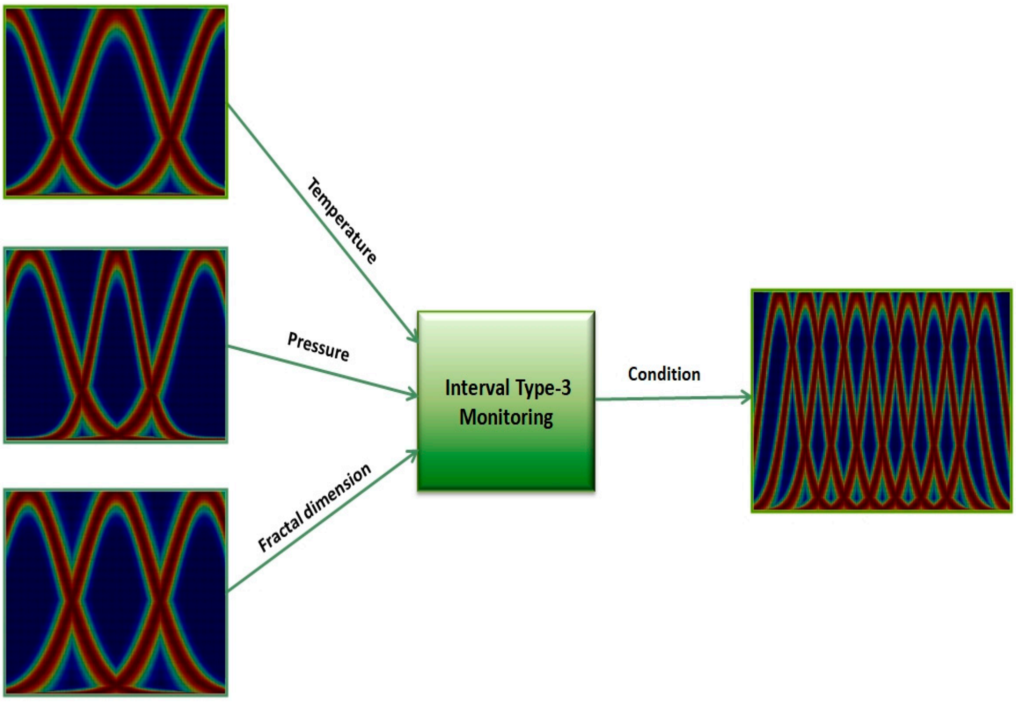

Figure 2. Type-3 fuzzy monitoring system.

Table 1. Rules for monitoring.

	IF	AND	AND	THEN
Number	**Temperature**	**Pressure**	**Fractal Dimension**	**Condition**
1	Low	Low	Low	Low_L
2	Low	Low	Normal	Low_N
3	Low	Low	High	Low_H
4	Low	Normal	Low	Low_N
5	Low	Normal	Normal	$Normal_L$
6	Low	Normal	High	Low_H
7	Low	High	Low	Low_H
8	Low	High	Normal	Low_H
9	Low	High	High	$High_N$
10	Normal	Low	Low	Low_N
11	Normal	Low	Normal	$Normal_L$
12	Normal	Low	High	$Normal_L$
13	Normal	Normal	Low	$Normal_L$
14	Normal	Normal	Normal	$Normal_N$
15	Normal	Normal	High	$Normal_H$
16	Normal	High	Low	$Normal_H$
17	Normal	High	Normal	$Normal_H$
18	Normal	High	High	$High_N$
19	High	Low	Low	Low_H
20	High	Low	Normal	$High_L$
21	High	Low	High	$High_L$
22	High	Normal	Low	$High_N$
23	High	Normal	Normal	$Normal_H$

Table 1. *Cont.*

	IF	AND	AND	THEN
Number	**Temperature**	**Pressure**	**Fractal Dimension**	**Condition**
24	High	Normal	High	$High_N$
25	High	High	Low	$High_L$
26	High	High	Normal	$High_N$
27	High	High	High	$High_H$

Table 2. Parameterization of the fuzzy system.

Variable	MFs	σ	m
Input 1	Low	8.30	80.0
Input 1	Normal	8.10	100.0
Input 1	High	8.30	120.0
Input 2	Low	18.30	90.0
Input 2	Normal	12.10	130.0
Input 2	High	18.30	170.0
Input 3	Low	0.20	1.10
Input 3	Medium	0.20	1.50
Input 3	High	0.20	1.90
Output	LowL	0.081	0.10
Output	LowN	0.081	0.20
Output	LowH	0.081	0.30
Output	NormalL	0.081	0.40
Output	NormalN	0.081	0.50
Output	NormalH	0.081	0.60
Output	HighL	0.081	0.70
Output	HighN	0.081	0.80
Output	HighH	0.081	0.90

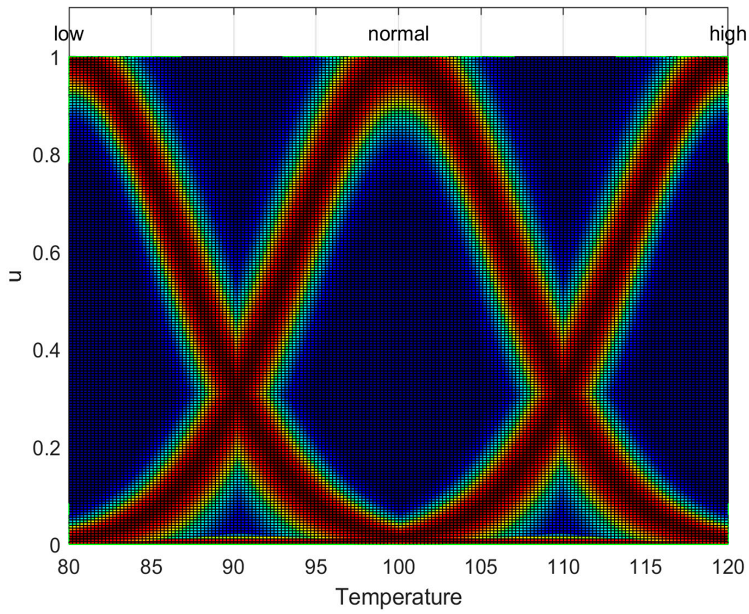

Figure 3. IT3 MFs for temperature.

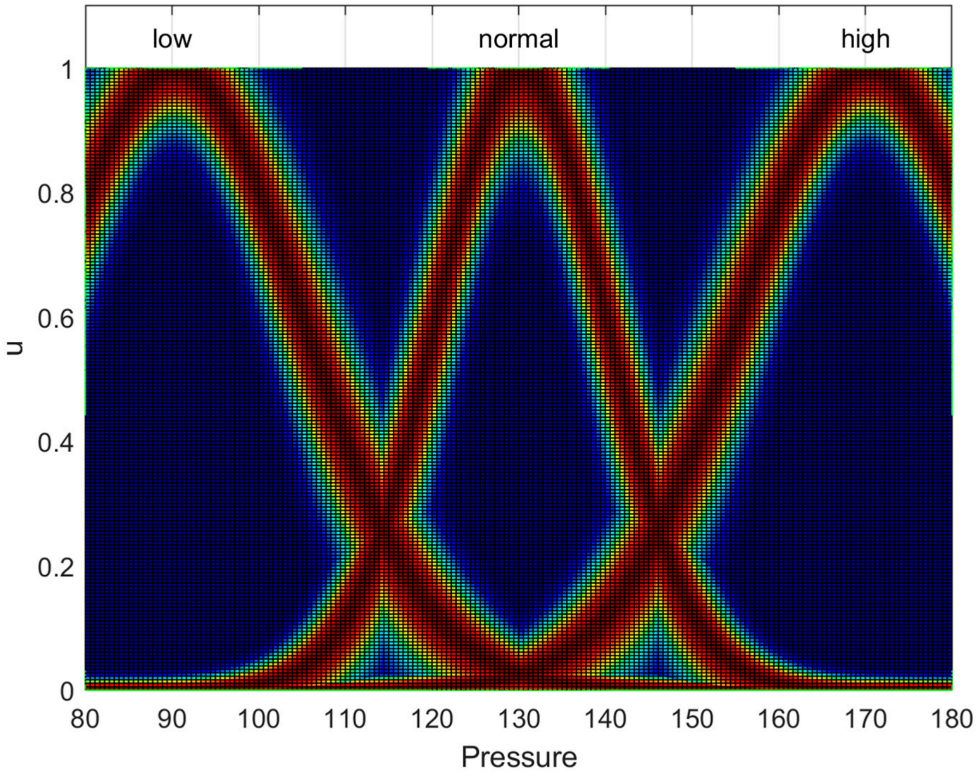

Figure 4. IT3 MFs for pressure.

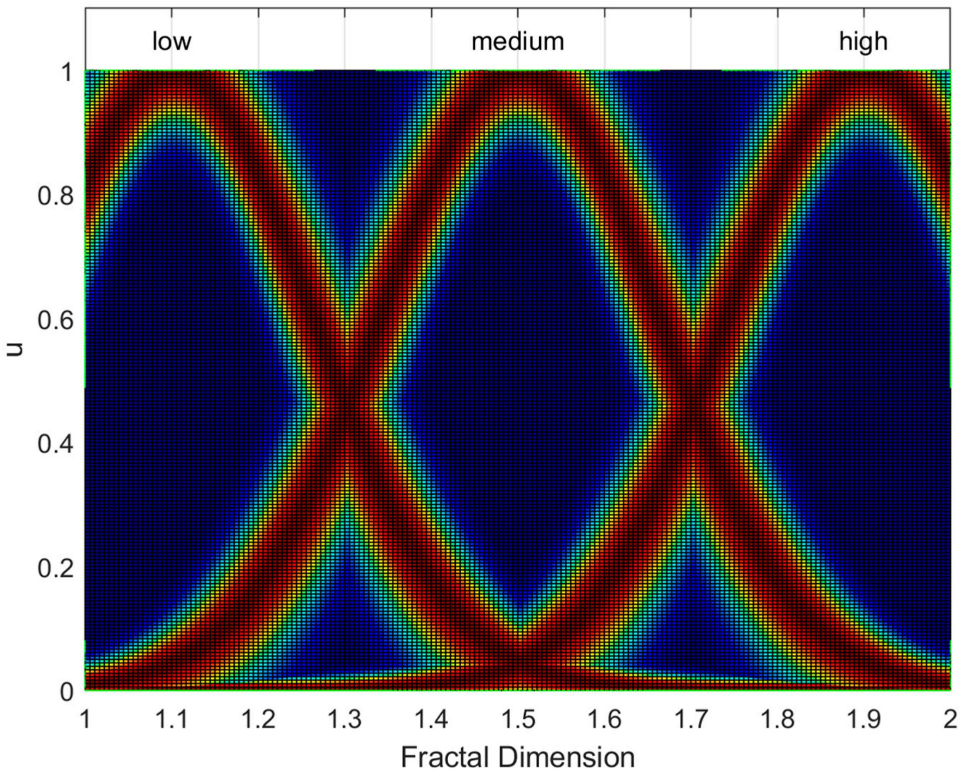

Figure 5. IT3 MFs for fractal dimension.

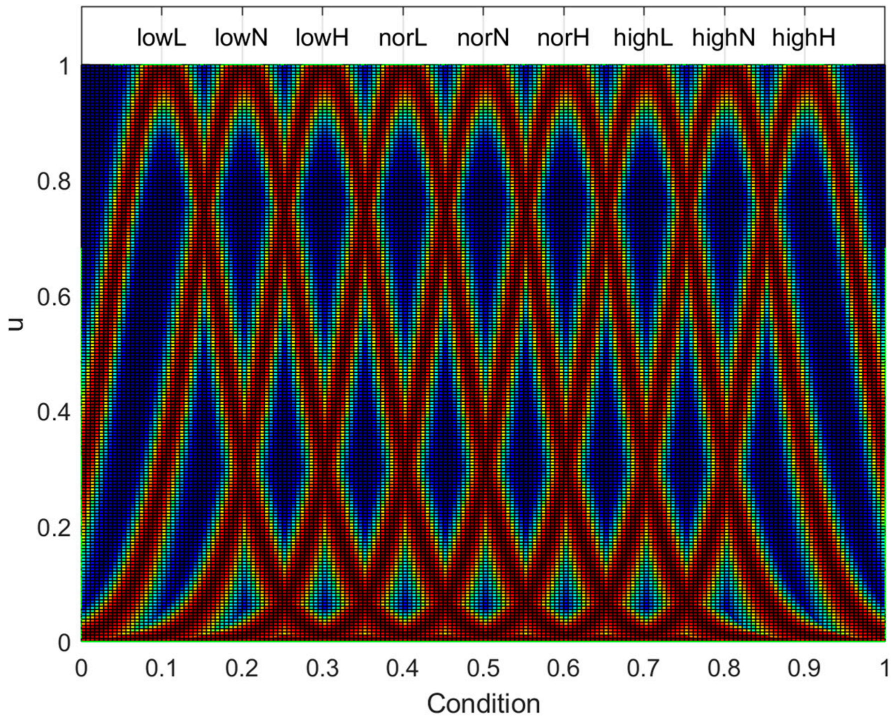

Figure 6. IT3 MFs for the output variable.

5. Simulation Results

The results of utilizing the fuzzy monitoring system are presented in this section. In Figures 7 and 8, we illustrate two different perspectives of the nonlinear surface for the fuzzy monitoring model. From these two last figures, we can notice the actual complexity and nonlinearity of the problem, as the type-3 fuzzy model is capturing this complexity to be able to solve the problem.

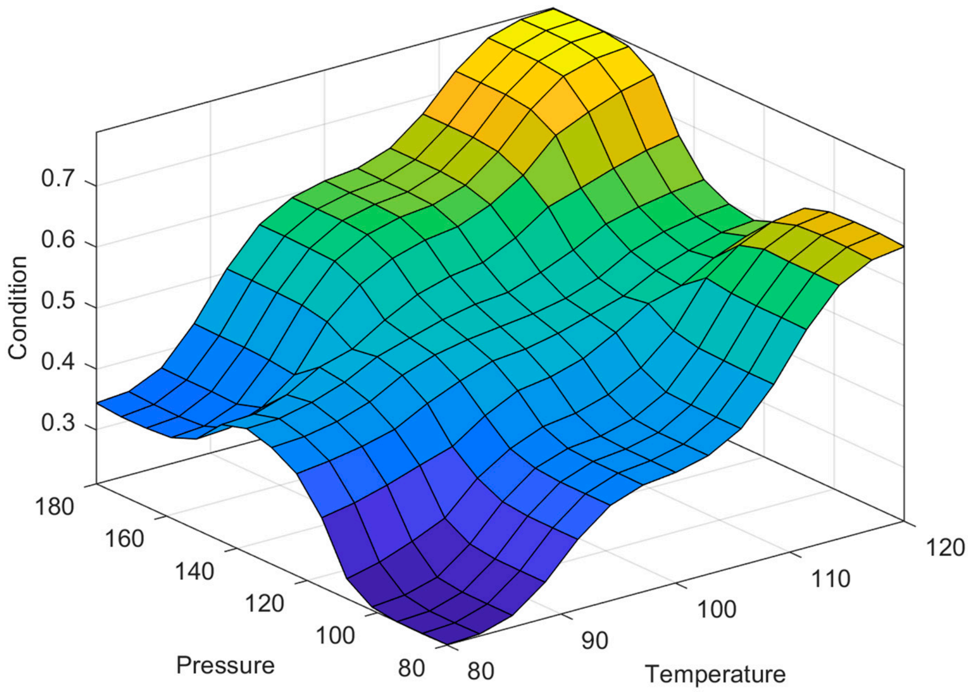

Figure 7. Nonlinear Surface of Condition with respect to pressure and temperature.

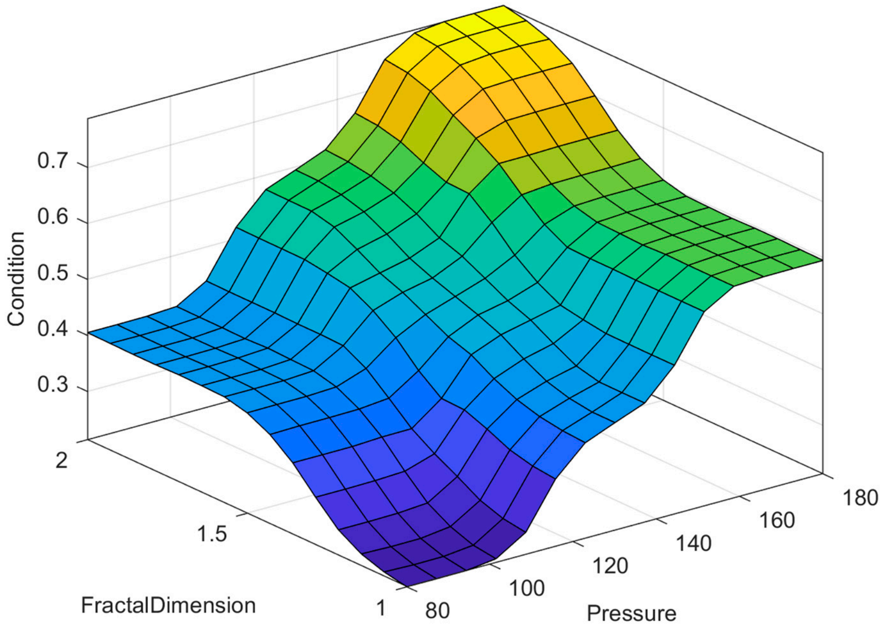

Figure 8. Nonlinear Surface of Condition with respect to fractal dimension and pressure.

In Table 3, the results with type-3 for 10 cases are presented. Also, the results obtained previously with type-2 and type-1 are shown for comparison. We show results of 2 designs for type-3, 1 for 10 rules (selected from the 27 as a subset), and another with all 27 rules. The system with 27 rules comprises all possible rules that can be outlined in this case, and the 10 rules are a selected subset of the total set of rules. The main idea of experimenting with a smaller number of rules was to find out if results would be almost the same with a simpler system. Of course, the selection of the 10 rules was carried out manually by the authors of the paper with the idea of maintaining what we believe are the most important rules. This process could be carried out in the future in a more systematic (computational) way by using something like a genetic algorithm that will explore all possible designs to verify if there could be an optimal design with fewer rules. We have to mention that the results shown in Table 3 for type-1 and type-2 are also for 27 rules.

Table 3. Results with type-3 and a comparison with lower fuzzy types.

Input			Type-1	Type-2	Type-3	Type-3
Temperature	**Pressure**	**FD**	**[4]**	**[13]**	**10 Rules**	**27 Rules**
105	130	1.6	0.4498	0.5030	0.5426	0.5430
100	120	1.5	0.2688	0.2775	0.4130	0.4756
95	115	1.4	0.2263	0.2539	0.3741	0.3789
90	110	1.3	0.2460	0.2783	0.2947	0.2875
102	122	1.7	0.3604	0.4210	0.5101	0.5431
85	90	1.2	0.2690	0.2750	0.2107	0.1605
75	100	1.8	0.2652	0.3039	0.5130	0.2868
55	105	1.3	0.2700	0.2701	0.3600	0.3729
130	90	1.1	0.5710	0.5855	0.6079	0.3621
112	115	1.6	0.4136	0.4138	0.6754	0.5979

A group of three experts on monitoring the chemical process were used to validate the results. Regarding the validation process, we can mention that the experts were not familiar with the fuzzy methods that we considered in the study and, in fact, in the validation process we only gave the experts the conditions and they made their estimation of the outputs, and after that we made the comparisons with respect to the different fuzzy systems. The validation was carried out in this fashion to ensure as much as possible that there was no bias in the results. We can analyze the results of some of the cases in Table 3 as follows. Regarding the case of (75, 100, 1.8), the value for type-3 with 10 rules looks to be bigger because of the fact that we did not use all 27 rules, while type-1, type-2, and type-3 with 27 rules have similar values in the table. Regarding the case of (100, 120, 1.5), we appreciate that a higher type of the fuzzy system gives higher results, which in this case is better (closer to experts) and we conclude that type-3 modeled better in this case. Regarding the case of (85, 90, 1.2), we appreciate that a higher type of fuzzy system gives lower results, which in this case is better (closer to experts) and we can say that type-3 modeled better in this case. Regarding the case of (130, 90, 1.1), we can say that lower values are better (according to experts), so type-3 with 27 rules gave the best results, but type-3 with 10 rules did not provide good results due to the lower number of rules and it was not able to model the problem adequately.

Based on the group of experts, we determined that in all cases the type-3 fuzzy fractal approach with 27 rules was the closest to their average estimates of the state (health) of the process. In particular, the design with 27 rules was slightly better according to the opinion of the experts. As a consequence, we can say that the presented approach (T3FL and FD) is shown to be the best for this application, but we will consider other monitoring applications in the near future to find other situations where this can also be true. Also, we intend to apply optimization techniques, such as metaheuristics, to find the best parameter values in the fuzzy systems to further improve the results. We recognize that one limitation of this study is that we relied on only three experts for the validation of the results and it would be desirable, statistically speaking, to have more experts available for the validation; this is also possible work to undertake in the future. In addition, other similar problems (like diagnosis and time series) could be considered with the T3FL and FD approach, which may be worthy of future work and it is envisioned that we will also consider undertaking this interesting research.

6. Conclusions

In this article, an approach for plant monitoring constructed with a hybrid combination of T3FL and the FD has been outlined and illustrated with an application. The main reason for mixing type-3 and the fractal dimension was to take advantage of both their capabilities in solving the problem of monitoring a plant. The FD has been previously applied in monitoring, as was already mentioned, but this was carried out as a standalone method. Also, fuzzy approaches were applied to the same area, but only with type-1 and type-2 forms. In this way, the innovative proposal of this article was putting forward the idea of mixing T3FL with FD to combine in a prudent fashion both theories in achieving an efficient solution to monitoring. Basically, T3FL helps in handling the uncertainty in monitoring the variables of nonlinear systems, while the FD helps to capture the complexity of the signals by finding hidden structures or patterns in the data. A set of interval type-3 rules was utilized to represent the monitoring knowledge. In the T3FL rules, the FD is utilized to aid in finding key patterns in processing data. The proposal was tested with a plant utilized in previous works, which was solved originally with type-1 and type-2 with acceptable results. Now, in this work, the results with the proposal based on T3FL are compared against type-2 and type-1, highlighting that type-3 is able to surpass the performance of its counterparts in this application. As future work, we plan to use the type-3 fuzzy fractal approach in human patient monitoring and other related health care applications, as we consider these applications very relevant for society. On the theoretical side, we also plan to consider a hybridation of type-3 with intuitionistic fuzzy theory for this kind of problem, as it is possible that we could handle an even higher degree of the existing uncertainty in monitoring. Another worthwhile idea could be establishing a mixture of type-3 with mediative fuzzy logic to potentially capture other kinds of uncertainty sources existing in real problems in diverse application areas.

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