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General New Results on $(\phi, \mathcal{F})$–Contractions in b–Metric-like-Spaces

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Abstract: Thispaper recognizes a general approach related to recent fixed point results about the classes of interpolative and hybrid contractions in metric space and general metric spaces. Considering auxiliary functions, so called Wardowski functions, and a rich set of implicit relations, we introduce types of $(\alpha_{v^q}, \phi, \mathcal{F})$–contractions and r–order hybrid $(\alpha_{v^q}, \phi, \mathcal{F})$–contractions in the setting of b–metric-like spaces. They generate and simplify many forms of contractions widely used in the literature. The resulting theorems significantly extend, generalize, and unify an excellent work on fixed point theory.

Keywords: $(\alpha_{v^q}, \phi, \mathcal{F})$–contraction; r–order hybrid $(\alpha_{v^q}, \phi, \mathcal{F})$–contraction; b–metric-like space; fixed point

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1. Introduction

The theory of fixed point has been studiedfor a long time and the fundamental concept linked to this theory is the concept of Banach's contraction [1]. It is well known for its simple nature and for being an applicable model forstudying the solutions of integral equations, differential equations, BVP problems, and many other problems in nonlinear analysis. Since then, many researchers have scientifically developed important extensions and generalized notions of metric space and the contractive map. Interesting scientific research is related to different abstract general metric settings and finding appropriate contractive conditions.

We emphasize some of the maingeneralizations that provide great developments to the fixed point theory, such as the concepts of $b-$metric [2,3] and $b-$metric-like [4]; many scientists have contributed to this theory with papers and essential results, and furthermore we can list references [5–14]. In 2012, Samet defined $\alpha-$admissible mapping [15], and further triangular $\alpha-$admissible mapping [16]. In the same year, Wardowski [17] established the notion of $\mathcal{F}-$contraction by using an auxiliary function under some imposed conditions, and later in 2018 introduced the notion of $(\phi, \mathcal{F})-$contraction [18]. The classes of $\mathcal{F}-$contraction and $(\phi, \mathcal{F})-$contraction, revisited simultaneously with $\alpha-$admissible mapping, are still a main focus and have been considered in the literature widely, and many fixed point theorems have beenpresented in metric space, $b-$metric and $b-$metric-like space (for short $b-m.l.s$), and other spaces. For a valuable work anddetails on these notions, see [19–28]. Later, Karapinar [29] came up with the notion of interpolative contraction, ongoing together with $r-$hybrid contractions, as defined by M. Sh. Shagari [30]. In this regard, and to become familiar with more accurate information, interested readers can browse references [31–36].

In the presented paper, the general types of $\left(\alpha_{v^q},\phi,\mathcal{F}\right)-$contractions and $r-$order hybrid $\left(\alpha_{v^q},\phi,\mathcal{F}\right)-$contractions are introduced, as a variant of Wardowski contractions and $\alpha-$contractions in the setting of $b-m.l.s$. By using these classes of contractions, we set up general new results which expand, generalize, and unify the repertoire for fixedpoint results beyond the types of interpolative and hybrid contractions discussed so far.

2. Preliminaries

First, let us obtain an understanding of the preliminary base concepts and notations.

Definition 1 ([4]). *Let Y be a nonempty set and a parameter $v\geq 1$. A mapping $b:Y\times Y\rightarrow[0,+\infty)$ is called a $b-$metric-like if for all $\kappa,s,z\in Y$, the following conditions are satisfied:*

$$b(\kappa,z)=0 \text{ implies } \kappa=z;$$
$$b(\kappa,z)=b(z,\kappa);$$
$$b(\kappa,z)\leq v\left[b(\kappa,s)+b(s,z)\right].$$

The pair (Y,b) is called a $b-$metric-like space. ($b-m.l.s$ for short).

Definition 2 ([4]). *Let (Y,b) be a $b-m.l.s$ with parameter v, and let $\{\kappa_n\}$ be any sequence in Y and $\kappa\in Y$. Then, the following applies:*

(a) *$\{\kappa_n\}$ converges to κ, iff $\lim_{n\rightarrow+\infty}b(\kappa_n,\kappa)=b(\kappa,\kappa)$;*

(b) *$\{\kappa_n\}$ is Cauchy sequence in (Y,b), iff $\lim_{n,m\rightarrow+\infty}b(\kappa_n,\kappa_m)$ exists and is finite;*

(c) *(Y,b) is complete $b-m.l.s$, iff for every Cauchy sequence $\{\kappa_n\}$ in Y, there exists $\kappa\in Y$ such that $\lim_{n,m\rightarrow+\infty}b(\kappa_n,\kappa_m)=\lim_{n\rightarrow+\infty}b(\kappa_n,\kappa)=b(\kappa,\kappa)$.*

Definition 3 ([4]). *Let (Y,b) be a $b-m.l.s$ with parameter v, and a function $f:Y\rightarrow Y$. We say that the function f is continuous if for each sequence $\{\kappa_n\}\subset Y$ the sequence $f\kappa_n\rightarrow f\kappa$ whenever $\kappa_n\rightarrow\kappa$ as $n\rightarrow+\infty$, that is, if $\lim_{n\rightarrow+\infty}b(\kappa_n,\kappa)=b(\kappa,\kappa)$ yields $\lim_{n\rightarrow+\infty}b(f\kappa_n,f\kappa)=b(f\kappa,f\kappa)$.*

In a $b-$m.l.s, it is remarked that if $\lim_{n,m\rightarrow+\infty}b(\kappa_n,\kappa_m)=0$, then the limit of the sequence $\{\kappa_n\}$ is unique if it exists.

Definition 4 ([15]). *Let Y be a non-empty set. Let $f:Y\rightarrow Y$ and $\alpha:Y\times Y\rightarrow R^+$ be given functions. We say that f is an $\alpha-$ admissible mapping if $\alpha(\kappa,z)\geq 1$ implies that $\alpha(f\kappa,fz)\geq 1$ for all $\kappa,z\in Y$.*

Definition 5 ([10]). *Let* (Y,b) *be a* $b-m.l.s$, $f:Y\to Y$ *and* $\alpha:Y\times Y\to R^{+}$ *be given mappings, and let* $q\ge 1$ *be an arbitrary constant. We can say that* f *is an* $\alpha_{v^q}-$ *admissible mapping if* $\alpha(\kappa,z)\ge v^{q}$, *which implies* $\min\{\alpha(f\kappa,z),\alpha(z,f\kappa)\}\ge v^{q}$ *for all* $\kappa,z\in Y$.

For illustrative examples belonging to this category of $\alpha_{v^q}-$admissible functions, one can search in references [7,10].

Definition 6. *Let* (Y,b) *be a* $b-m.l.s$ *with parameter* $v\ge 1$, $f:Y\to Y$, $\alpha:Y\times Y\to R^{+}$. *Then, the function* f *satisfies* $\alpha_{v^q}-$ *admissible property; if a sequence* $\{\kappa_n\}\subset Y$ *with* $\kappa_n\to\kappa\in Y$ *and* $\alpha(\kappa_n,\kappa_{n+1})\ge v^{q}$ *and* $\alpha(\kappa_{n+1},\kappa_n)\ge v^{q}$, *then there exists a subsequence* $\{\kappa_{n_k}\}$ *of* $\{\kappa_n\}$ *with* $\alpha(\kappa_{n_k},\kappa)\ge v^{q}$ *and* $\alpha(\kappa,\kappa_{n_k})\ge v^{q}$ *for all* $k\ge 0$ *and* $q\ge 1$.

Definition 7. *Let* (Y,b) *be a* $b-m.l.s$ *with parameter* $v\ge 1$, $f:Y\to Y$, $\alpha:Y\times Y\to R^{+}$. *Then, the function* f *fulfills the unique* $\alpha_{v^q}-$*property, if for all* κ,z $(\kappa\ne z)$ *fixed points of f ,we have* $\alpha(\kappa,z)\ge v^{q}$ *where* $q\ge 1$.

Lemma 1 ([4]). *Let* (Y,b) *be a* $b-m.l.s$ *with parameter* $v\ge 1$, *and suppose that* $\{\kappa_n\}$ *is convergent to* κ, *and* $b(\kappa,\kappa)=0$. *Then, for each* $z\in Y$, *we have*

$$v^{-1}b(\kappa,z)\le\liminf_{n\to+\infty}b(\kappa_n,z)\le\limsup_{n\to+\infty}b(\kappa_n,z)\le vb(\kappa,z).$$

Lemma 2 ([7]). *Let* (Y,b) *be a* $b-m.l.s$ *with parameter* $v\ge 1$. *Then, the following applies:*

(a) $b(\kappa,z)=0,$ *implies* $b(\kappa,\kappa)=b(z,z)=0;$

(b) *If for* $\{\kappa_n\}\subset Y$, $\lim_{n\to+\infty}b(\kappa_n,\kappa_{n+1})=0$, *then we have* $\lim_{n\to+\infty}b(\kappa_n,\kappa_n)=\lim_{n\to+\infty}b(\kappa_{n+1},\kappa_{n+1})=0$;

(c) $\kappa\ne z$ *implies* $b(\kappa,z)>0.$

Lemma 3 ([9]). *Let* $\{\kappa_n\}$ *be a sequence in a complete* $b-m.l.s$ (Y,b) *with parameter* $v\ge 1$, *such that*

$$\lim_{n\to+\infty}b(\kappa_n,\kappa_{n+1})=0.$$

If $\lim_{n,m\to+\infty}b(\kappa_n,\kappa_m)\ne 0$, *then there exists* $\varepsilon>0$ *and sequences* $\{m(k)\}_{k=1}^{\infty}$ *and* $\{n(k)\}_{k=1}^{\infty}$ *of natural numbers with* $n_k>m_k>k$, *(positive integers) such that*

$$b(\kappa_{m_k},\kappa_{n_k})\ge\varepsilon,\qquad b(\kappa_{m_k},\kappa_{n_k-1})<\varepsilon,\qquad \varepsilon/v^{2}\le\limsup_{k\to+\infty}b(\kappa_{m_k-1},\kappa_{n_k-1})\le\varepsilon v,$$

$$\varepsilon/v\le\limsup_{k\to+\infty}b(\kappa_{n_k-1},\kappa_{m_k})\le\varepsilon v^{2}\quad\text{and}\quad\varepsilon/v\le\limsup_{k\to+\infty}b(\kappa_{m_k-1},\kappa_{n_k})\le\varepsilon v^{2}.$$

3. Results

In this section, we introduce the new notion of general types of $(\alpha_{v^q},\phi,\mathcal{F})-$contractions in the setting of $b-m.l.s$ that can be seen as a new form of linear and nonlinear contractions involving classical contractions, Wardowski type $\mathcal{F}-$contractions, and interpolative and hybrid contractions.

Before proposing our definitions and theorems, we will use these formal notations:

$\mathbb{F} = \{\mathcal{F} : (0,+\infty) \to R / \mathcal{F} \text{ is continuous and strictly increasing}\}$,

$\Theta = \left\{\phi : (0,+\infty) \to (0,+\infty), \textit{satisfying condition} : \liminf_{m\to\partial^+} \phi(m) > 0 \textit{ for all } \partial > 0\right\}$,

Ω_n is the set of all continuous functions $\Gamma : [0,\infty)^n \to [0,\infty)$ where $n \ge 2$, satisfying the conditions:

- Γ is non-decreasing withrespect to each variable;
- $\Gamma(u,u,u,\ldots,u) \le u$ for $u \in [0,+\infty)$.

Definition 8. *Let (Y,b) be a $b-$ m.l.s with parameter $v \ge 1$ and $f : Y \to Y$, $\alpha : Y \times Y \to [0,+\infty)$. The self-mapping f on Y, is named a generalized $(\alpha_{v^q}, \phi, \mathcal{F})-$contraction, if there exist $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ such that*

$$\phi(b(\kappa,z)) + \mathcal{F}(\alpha(\kappa,z)b(f\kappa, fz)) \le \mathcal{F}(L(\kappa,z)) \tag{1}$$

for all $\kappa, z \in Y$ with $f\kappa \ne fz$ and $\alpha(\kappa,z) \ge v^q$;

where $L(\kappa,z) = \Gamma\left(b(\kappa,z), b(\kappa, f\kappa), b(z, fz), \dfrac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right)$ for some $\Gamma \in \Omega_4$.

Remark 1. *Some specific evidence of this definition ispointed out: aking the function $\phi \in \Theta$ as constant function, we derived a generalized $(\alpha_{v^q}, \mathcal{F})-$contraction. his definition extends and generalizes some definitions in [6,7,18,19,25,35]. By taking the function $\phi : (0,+\infty) \to (0,+\infty)$ as a constant function or $\alpha(\kappa,z) = v^q$, we can obtain other new definitions in the same metric structure. Also, this definition respectively holds validity in metric settings where $v = 1$, obtaining: Generalized $(\alpha,\phi,\mathcal{F})-$, $(\alpha,\mathcal{F})-$ contractions.*

Theorem 1. *Let (Y,b) be a complete $b-$ m.l.s with parameter $v \ge 1$ and a self-mapping $f : Y \to Y$ satisfying the conditions:*

C1. there exists $\kappa_0 \in Y$ with $\alpha(\kappa_0, f\kappa_0) \ge v^q$;

C2. f is $\alpha_{v^q}-$admissible mapping and satisfies $\alpha_{v^q}-$admissible property;

C3. f is a generalized $(\alpha_{v^q}, \phi, \mathcal{F})-$contraction.

Then, f has a fixed point in Y. Moreover, it is unique if f satisfies the unique $\alpha_{v^q}-$property.

Proof. Since C1 is satisfied, $\kappa_0 \in Y$ existssuch that $\alpha(\kappa_0, f\kappa_0) \ge v^q$. We construct the sequence $\{\kappa_n\}$ via aniterative equation $\kappa_{n+1} = f(\kappa_n)$ for all $n \in N \cup \{0\}$. Then, $\alpha(\kappa_0,\kappa_1) = \alpha(\kappa_0, f\kappa_0) \ge v^q$ and since f is $\alpha_{v^q}-$ admissible it follows $\alpha(\kappa_1,\kappa_2) = \alpha(\kappa_1, f\kappa_1) \ge v^q$, so inductively it can be concluded that $\alpha(\kappa_n,\kappa_{n+1}) = \alpha(\kappa_n, f\kappa_n) \ge v^q$. Now, if we suppose that $n_0 \in N$ exists with $f\kappa_{n_0-1} = f\kappa_{n_0}$, then the proof is completed. Therefore, we assume $f\kappa_n \ne f\kappa_{n-1}$ for all $n \in N \cup \{0\}$. (it means $b(f\kappa_n, f\kappa_{n-1}) > 0$). Applying inequality (1) and property of $\mathcal{F}$,we have

$$\begin{aligned}\phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)+\mathcal{F}\left(\alpha\left(\kappa_{n-1},\kappa_n\right)b\left(\kappa_n,\kappa_{n+1}\right)\right)&=\phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)+\mathcal{F}\left(\alpha\left(\kappa_{n-1},\kappa_n\right)b\left(f\lambda_{n-1},f\lambda_n\right)\right)\\&\leq\mathcal{F}\left(L\left(\kappa_{n-1},\kappa_n\right)\right)\\ \Rightarrow\qquad \phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)+\mathcal{F}\left(v^q b\left(\kappa_n,\kappa_{n+1}\right)\right)&\leq\mathcal{F}\left(L\left(\kappa_{n-1},\kappa_n\right)\right),\end{aligned} \tag{2}$$

where

$$\begin{aligned}L\left(\kappa_{n-1},\kappa_n\right)&=\Gamma\left(b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_{n-1},f\kappa_{n-1}\right),b\left(\kappa_n,f\kappa_n\right),\frac{b\left(\kappa_{n-1},f\kappa_n\right)+b\left(\kappa_n,f\kappa_{n-1}\right)}{4v}\right)\\&=\Gamma\left(b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_n,\kappa_{n+1}\right),\frac{b\left(\kappa_{n-1},\kappa_{n+1}\right)+b\left(\kappa_n,\kappa_n\right)}{4v}\right)\\&\leq\Gamma\left(b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_n,\kappa_{n+1}\right),\frac{vb\left(\kappa_{n-1},\kappa_n\right)+vb\left(\kappa_n,\kappa_{n+1}\right)+2vb\left(\kappa_{n-1},\kappa_n\right)}{4v}\right)\\&=\Gamma\left(b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_n,\kappa_{n+1}\right),\frac{vb\left(\kappa_n,\kappa_{n+1}\right)+3vb\left(\kappa_{n-1},\kappa_n\right)}{4v}\right).\end{aligned}$$

If we assume that $b\left(\kappa_{n-1},\kappa_n\right)\leq b\left(\kappa_n,\kappa_{n+1}\right)$, then

$$\begin{aligned}L\left(\kappa_{n-1},\kappa_n\right)&=\Gamma\left(b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_{n-1},\kappa_n\right),b\left(\kappa_n,\kappa_{n+1}\right),\frac{vb\left(\kappa_n,\kappa_{n+1}\right)+3vb\left(\kappa_{n-1},\kappa_n\right)}{4v}\right)\\&\leq\Gamma\left(b\left(\kappa_n,\kappa_{n+1}\right),b\left(\kappa_n,\kappa_{n+1}\right),b\left(\kappa_n,\kappa_{n+1}\right),b\left(\kappa_n,\kappa_{n+1}\right)\right)\\&\leq b\left(\kappa_n,\kappa_{n+1}\right).\end{aligned}$$

And, from inequality (2), it follows that

$$\phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)+\mathcal{F}\left(v^q b\left(\kappa_n,\kappa_{n+1}\right)\right)\leq\mathcal{F}\left(L\left(\kappa_{n-1},\kappa_n\right)\right)\leq\mathcal{F}\left(b\left(\kappa_n,\kappa_{n+1}\right)\right). \tag{3}$$

The inequality (3) generates

$$\begin{aligned}\mathcal{F}\left(v^q b\left(\kappa_n,\kappa_{n+1}\right)\right)&\leq\mathcal{F}\left(b\left(\kappa_n,\kappa_{n+1}\right)\right)-\phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)\\&<\mathcal{F}\left(b\left(\kappa_n,\kappa_{n+1}\right)\right),\end{aligned} \tag{4}$$

which is a contradiction. Therefore, we have for all $n\in N$:

$$b\left(\kappa_n,\kappa_{n+1}\right)<b\left(\kappa_{n-1},\kappa_n\right). \tag{5}$$

Hence, $\left\{b\left(\kappa_n,\kappa_{n+1}\right)\right\}$ is a decreasing sequence of nonnegative numbers. So, there exists $\delta\geq 0$ such that $b\left(\kappa_n,\kappa_{n+1}\right)\to\delta$ as $n\to+\infty$. If we suppose that $\delta>0$, then using (5) and the property of $\mathcal{F}$, Equation (3) can be written as

$$\phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)+\mathcal{F}\left(v^q b\left(\kappa_n,\kappa_{n+1}\right)\right)\leq\mathcal{F}\left(b\left(\kappa_{n-1},\kappa_n\right)\right). \tag{6}$$

If we take the limit along (6) as $n\to+\infty$, then

$$\liminf_{n\to+\infty}\phi\left(b\left(\kappa_{n-1},\kappa_n\right)\right)+\mathcal{F}\left(v^q\delta\right)\leq\mathcal{F}(\delta),$$

which is a contradiction. Thus, we state that $\delta=0$.

Hence,

$$0=\lim_{n\to+\infty} b\left(\kappa_n,\kappa_{n+1}\right). \tag{7}$$

Next, we show that $\lim\limits_{n,m\to+\infty} b\left(\kappa_n,\kappa_m\right)=0$. Then, we suppose the contrary, that is, $\lim\limits_{n,m\to+\infty} b\left(\kappa_n,\kappa_m\right)>0$. Then, according to Lemma 3, $\varepsilon>0$ exists, as well as sequences $\{m_k\}$ and $\{n_k\}$ of positive integers with $n_k>m_k>k$, such that

$$\begin{aligned}
&b(\kappa_{m_k},\kappa_{n_k})\geq\varepsilon, \qquad b(\kappa_{m_k},\kappa_{n_k-1})<\varepsilon, \qquad \text{and}\\
&\varepsilon/v^2\leq\limsup_{k\to+\infty} b(\kappa_{m_k-1},\kappa_{n_k-1})\leq\varepsilon v,\\
&\varepsilon/v\leq\limsup_{k\to+\infty} b(\kappa_{n_k-1},\kappa_{m_k})\leq\varepsilon v,\\
&\varepsilon/v\leq\limsup_{k\to+\infty} b(\kappa_{m_k-1},\kappa_{n_k})\leq\varepsilon v^2.
\end{aligned} \tag{8}$$

Since $0<\varepsilon\leq b(\kappa_{m_k},\kappa_{n_k})=b\left(f\kappa_{m_k-1},f\kappa_{n_k-1}\right)$, we apply condition (1) as

$$\begin{aligned}
\phi\left(b(\kappa_{m_k-1},\kappa_{n_k-1})\right)+\mathcal{F}\left(v^q b(\kappa_{m_k},\kappa_{n_k})\right)&=\phi\left(b(\kappa_{m_k-1},\kappa_{n_k-1})\right)+\mathcal{F}\left(v^q b(f\kappa_{m_k-1},f\kappa_{n_k-1})\right)\\
&\leq\phi\left(b(\kappa_{m_k-1},\kappa_{n_k-1})\right)+\mathcal{F}\left(\alpha(\kappa_{m_k-1},\kappa_{n_k-1})b\left(f\kappa_{m_k-1},f\kappa_{n_k-1}\right)\right)\\
&\leq\mathcal{F}\left(L(\kappa_{m_k-1},\kappa_{n_k-1})\right)
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
L(\kappa_{m_k-1},\kappa_{n_k-1})&=\Gamma\left(b(\kappa_{m_k-1},\kappa_{n_k-1}),b(\kappa_{m_k-1},f\kappa_{m_k-1}),b(\kappa_{n_k-1},f\kappa_{n_k-1}),\frac{b(\kappa_{m_k-1},f\kappa_{n_k-1})+b(\kappa_{n_k-1},f\kappa_{m_k-1})}{4v}\right)\\
&=\Gamma\left(b(\kappa_{m_k-1},\kappa_{n_k-1}),b(\kappa_{m_k-1},\kappa_{m_k}),b(\kappa_{n_k-1},\kappa_{n_k}),\frac{b(\kappa_{m_k-1},\kappa_{n_k})+b(\kappa_{n_k-1},\kappa_{m_k})}{4v}\right).
\end{aligned} \tag{10}$$

By taking the limit superior in Equation (10) and using Lemma 3, and result (7), we obtain

$$\begin{aligned}
&\limsup_{n\to+\infty} L\left((\kappa_{m_k-1},\kappa_{n_k-1})\right)=\\
&=\limsup_{n\to+\infty}\Gamma\left(b(\kappa_{m_k-1},\kappa_{n_k-1}),b(\kappa_{m_k-1},\kappa_{m_k}),b(\kappa_{n_k-1},\kappa_{n_k}),\frac{b(\kappa_{m_k-1},\kappa_{n_k})+b(\kappa_{n_k-1},\kappa_{m_k})}{4v}\right)\\
&=\Gamma\left(\begin{array}{l}\limsup\limits_{n\to+\infty} b(\kappa_{m_k-1},\kappa_{n_k-1}),\limsup\limits_{n\to+\infty} b(\kappa_{m_k-1},\kappa_{m_k}),\limsup\limits_{n\to+\infty} b(\kappa_{n_k-1},\kappa_{n_k}),\\ \dfrac{\limsup\limits_{n\to+\infty} b(\kappa_{m_k-1},\kappa_{n_k})+\limsup\limits_{n\to+\infty} b(\kappa_{n_k-1},\kappa_{m_k})}{4v}\end{array}\right)\\
&\leq\Gamma\left(\varepsilon v,0,0,\frac{\varepsilon+\varepsilon v^2}{4v}\right)\leq\Gamma\left(\varepsilon v,0,0,\frac{\varepsilon v}{2}\right)\leq\varepsilon v.
\end{aligned} \tag{11}$$

Taking the upper limit as $k\to+\infty$ along (9) and using (11), we have

$$\begin{aligned}
\liminf_{n\to+\infty}\phi\left(b(\kappa_{m_k},\kappa_{n_k})\right)+\mathcal{F}\left(v^q\varepsilon\right)&\leq\liminf_{n\to+\infty}\phi\left(b(\kappa_{m_k},\kappa_{n_k})\right)+\mathcal{F}\left(\limsup_{n\to+\infty} v^q b(\kappa_{m_k},\kappa_{n_k})\right)\\
&\leq\mathcal{F}\left(\limsup_{n\to+\infty} L\left((\kappa_{m_k-1},\kappa_{n_k-1})\right)\right)\\
&\leq\mathcal{F}\left(\varepsilon v\right).
\end{aligned} \tag{12}$$

Hence, the acquired inequality

$$\liminf_{n\to+\infty} \phi\left(b(\kappa_{m_k},\kappa_{n_k})\right)+\mathcal{F}\left(\varepsilon v^q\right)<\mathcal{F}\left(\varepsilon v\right),$$

is a contradiction since $\varepsilon>0$ and $\liminf_{n\to+\infty} \phi\left(b(\kappa_{m_k},\kappa_{n_k})\right)>0.$

Thus,

$$\lim_{n,m\to+\infty} b\left(\kappa_n,\kappa_m\right)=0. \tag{13}$$

Hence, $\{\kappa_n\}$ is a Cauchy sequence in (Y,b). Since it is complete, there is some $\kappa\in Y$, such that the sequence $\{\kappa_n\}$ is convergent to κ. Thus, according to def 2/(c) and (13), we have

$$\lim_{n\to+\infty} b\left(\kappa_n,\kappa\right)=\lim_{n,m\to+\infty} b\left(\kappa_n,\kappa_m\right)=b\left(\kappa,\kappa\right)=0. \tag{14}$$

Since f satisfies the $\alpha_{v^q}-$admissible property, there exists a subsequence $\{\kappa_{n_k}\}$ of $\{\kappa_n\}$ such that $\alpha\left(\kappa_{n_k},\kappa\right)\geq v^q$ for all $k\geq 0.$ Again, by using (1) and the property of $\mathcal{F}$, we have

$$\begin{aligned}\phi\left(b\left(\kappa_{n_k},\kappa\right)\right)+\mathcal{F}\left(v^q b\left(\kappa_{n_k+1},f\kappa\right)\right)&=\phi\left(b\left(\kappa_{n_k},\kappa\right)\right)+\mathcal{F}\left(v^q b\left(f\kappa_{n_k},f\kappa\right)\right)\\&\leq\phi\left(b\left(\kappa_{n_k},\kappa\right)\right)+\mathcal{F}\left(\alpha\left(\kappa_{n_k},\kappa\right)b\left(f\kappa_{n_k},f\kappa\right)\right)\\&\leq\mathcal{F}\left(L\left(\kappa_{n_k},\kappa\right)\right),\end{aligned} \tag{15}$$

where

$$\begin{aligned}L\left(\kappa_{n_k},\kappa\right)&=\Gamma\left(b\left(\kappa_{n_k},\kappa\right),b\left(\kappa_{n_k},f\kappa_{n_k}\right),b\left(\kappa,f\kappa\right),\frac{b\left(\kappa_{n_k},f\kappa\right)+b\left(\kappa,f\kappa_{n_k}\right)}{4v}\right)\\&=\Gamma\left(b\left(\kappa_{n_k},\kappa\right),b\left(\kappa_{n_k},\kappa_{n_k+1}\right),b\left(\kappa,f\kappa\right),\frac{b\left(\kappa_{n_k},f\kappa\right)+b\left(\kappa,\kappa_{n_k+1}\right)}{4v}\right).\end{aligned} \tag{16}$$

Taking the limit superior as $k\to+\infty$, in $L\left(\kappa_{n_k},\kappa\right)$ and in view of Lemma 1, we obtain

$$\begin{aligned}&\limsup_{k\to+\infty} L\left(\kappa_{n_k},\kappa\right)=\\&=\limsup_{k\to+\infty}\Gamma\left(b\left(\kappa_{n_k},\kappa\right),b\left(\kappa_{n_k},f\kappa_{n_k}\right),b\left(\kappa,f\kappa\right),\frac{b\left(\kappa_{n_k},f\kappa\right)+b\left(\kappa,f\kappa_{n_k}\right)}{4v}\right)\\&=\Gamma\left(\limsup_{k\to+\infty} b\left(\kappa_{n_k},\kappa\right),\limsup_{k\to+\infty} b\left(\kappa_{n_k},\kappa_{n_k+1}\right),\limsup_{k\to+\infty} b\left(\kappa,f\kappa\right),\frac{\limsup_{k\to+\infty} b\left(\kappa_{n_k},f\kappa\right)+\limsup_{k\to+\infty} b\left(\kappa,f\kappa_{n_k}\right)}{4v}\right)\\&\leq\Gamma\left(0,0,b\left(\kappa,f\kappa\right),\frac{vb\left(\kappa,f\kappa\right)+0}{4v}\right)\\&\leq b\left(\kappa,f\kappa\right).\end{aligned} \tag{17}$$

Again, taking the upper limit as $k\to+\infty$ in (15), and according to Lemma 1, the result (17) and property of $\mathcal{F}$, it follows that

$$\liminf_{k\to+\infty}\phi\left(b\left(\kappa_{n_k},\kappa\right)\right)+\mathcal{F}\left(v^{q-1}b\left(\kappa,f\kappa\right)\right)\le\mathcal{F}\left(b\left(\kappa,f\kappa\right)\right). \tag{18}$$

Hence, since $q\ge 1$, the inequality (18) implies that $b\left(\kappa,f\kappa\right)=0$ and so $f\kappa=\kappa$. Therefore, κ is a fixed point for which

$$b\left(\kappa,f\kappa\right)=0=b\left(\kappa,\kappa\right). \tag{19}$$

If we suppose that κ and s are two fixed points of f where $f\kappa=\kappa$ and $fs=s$ with $\kappa\ne s$, that is $f\kappa\ne fs.$, then using (19) and the property of $\Gamma\in\Omega_4$, for $L\left(\kappa,s\right)$, we have

$$\begin{aligned}
L\left(\kappa,s\right)&=\Gamma\left(b\left(\kappa,s\right),b\left(\kappa,f\kappa\right),b\left(s,fs\right),\frac{b\left(\kappa,fs\right)+b\left(s,f\kappa\right)}{4v}\right)\\
&\le\Gamma\left(b\left(\kappa,s\right),b\left(\kappa,\kappa\right),b\left(s,s\right),\frac{b\left(\kappa,s\right)+b\left(s,\kappa\right)}{4v}\right)\\
&=\Gamma\left(b\left(\kappa,s\right),b\left(\kappa,\kappa\right),b\left(s,s\right),\frac{b\left(\kappa,s\right)}{2v}\right)\\
&=\Gamma\left(b\left(\kappa,s\right),0,0,\frac{b\left(\kappa,s\right)}{2v}\right)\\
&\le\Gamma\left(b\left(\kappa,s\right),b\left(\kappa,s\right),b\left(\kappa,s\right),b\left(\kappa,s\right)\right)\\
&\le b\left(\kappa,s\right).
\end{aligned}$$

Since, f satisfies the $\alpha_{v^q}-$unique property, then $\alpha\left(\kappa,s\right)\ge v^q$, and from condition (1) and the above inequality, we have

$$\phi\left(\kappa,s\right)+\mathcal{F}\left(\alpha\left(\kappa,s\right)b\left(f\kappa,fs\right)\right)\le\mathcal{F}\left(L\left(\kappa,s\right)\right)\le\mathcal{F}\left(b\left(\kappa,s\right)\right). \tag{20}$$

$$\Rightarrow$$

$$\phi\left(\kappa,s\right)+\mathcal{F}\left(v^q b\left(\kappa,s\right)\right)\le\mathcal{F}\left(b\left(\kappa,s\right)\right) \tag{21}$$

and (21) leads to a contradiction that implies $b\left(\kappa,s\right)=0$. Therefore, $\kappa=s$. □

Corollary 1. *Let $\left(Y,b\right)$ be a $b-$m.l.s with parameter $v\ge 1$ and the mappings $f:Y\to Y$, $\alpha:Y\times Y\to\left[0,+\infty\right)$. If there exist $\mathcal{F}\in\mathbb{F}$, $\tau>0$ such that fulfill the condition C_1,C_2 and*

$$\tau+\mathcal{F}\left(\alpha\left(\kappa,z\right)b\left(f\kappa,fz\right)\right)\le\mathcal{F}\left(L(\kappa,z)\right) \tag{22}$$

for all $\kappa,z\in Y$ with $f\kappa\ne fz$ and $\alpha\left(\kappa,z\right)\ge v^q$; where
$L(\kappa,z)=\Gamma\left(b(\kappa,z),b(\kappa,f\kappa),b(z,fz),\frac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right)$ *for* $\Gamma\in\Omega_4$

Then, f has a fixed point in Y. Moreover, it is unique if f satisfies the unique $\alpha_{v^q}-$property.

Proof. Take the function $\phi:\left(0,+\infty\right)\to\left(0,+\infty\right)$ as constant $\phi\left(t\right)=\tau>0.$ □

Example 1. *Let* (Y,b) *be a complete* $b-$ *m.l.s with a coefficient* $v=2$, *where* $Y=[0,+\infty)$, $b:Y\times Y\to[0,+\infty)$ *with* $b(\kappa,z)=(\kappa+z)^2$ *for all* $\kappa,z\in Y$. *Consider the maps:* $f:Y\to Y$, $\alpha:Y\times Y\to[0,+\infty)$, *respectively, given by*

$$fy=\begin{cases}\dfrac{\kappa}{8} & \text{if } \kappa\in[0,1)\\ \dfrac{1}{10} & \text{if } \kappa\in[1,+\infty)\end{cases}\quad ;\alpha(\kappa,z)=\begin{cases}v^2 & \text{if } \kappa,z\in[0,1]\\ 0 & \text{otherwise.}\end{cases}$$

For $\kappa,z\in Y$ with $\alpha(\kappa,z)\geq 4=v^2$, $f\kappa,fz\in\left[0,\frac{1}{8}\right]$ and $\alpha(f\kappa,fz)\geq v^2$, that is, f is $\alpha_{v^q}-$admissible mapping. Consider $\mathcal{F}\in\mathbb{F}$ as $\mathcal{F}(t)=t$, $\phi\in\Theta$ as $\phi(t)=\frac{1}{16}t$, and $\Gamma\in\Omega_4$ as $\Gamma(u_1,u_2,u_3,u_4)=\max\{u_1,u_2,u_3,u_4\}$, and then
In the case of $\kappa,z\in[0,1)$, we have

$$\begin{aligned}&\phi(b(\kappa,z))+F(\alpha(\kappa,z)b(f\kappa,fz))=\\&=\phi(b(\kappa,z))+F(v^2b(f\kappa,fz))\\&=\phi(b(\kappa,z))+F\left(v^2b\left(\frac{\kappa}{8},\frac{z}{8}\right)\right)=\phi(b(\kappa,z))+F\left(4\left(\frac{\kappa+z}{8}\right)^2\right)\\&=\phi(b(\kappa,z))+F\left(\frac{1}{16}b(\kappa,z)\right)=\frac{1}{16}(b(\kappa,z))+\frac{1}{16}(b(\kappa,z))\\&=\frac{1}{8}(b(\kappa,z))\leq F(b(\kappa,z))\leq F(L(\kappa,z)).\end{aligned}$$

In the case of $\kappa\in[0,1)$ and $z=1$, we have

$$\begin{aligned}&\phi(b(\kappa,z))+F(\alpha(\kappa,z)b(f\kappa,fz))=\\&=\phi(b(\kappa,z))+F(v^2b(f\kappa,fz))\\&=\phi(b(\kappa,1))+F\left(v^2b\left(\frac{\kappa}{8},\frac{1}{10}\right)\right)=\phi(b(\kappa,1))+F\left(4\left(\frac{\kappa}{8}+\frac{1}{10}\right)^2\right)\\&=\phi(b(\kappa,1))+F\left(4\left(\frac{\kappa+1}{8}\right)^2\right)=\phi(b(\kappa,1))+F\left(\frac{1}{16}b(\kappa,1)\right)\\&=\frac{1}{16}(b(\kappa,1))+\frac{1}{16}(b(\kappa,1))=\frac{1}{8}(b(\kappa,1))\\&\leq F(b(\kappa,1))\leq F(L(\kappa,1))\leq F(L(\kappa,z)).\end{aligned}$$

Obviously, the conditions of Theorem 1 are confirmed and f has $y=0$ as a unique fixed point. Already, this theorem is not applicable in the frame of metric space and $b-$metric space, as we can see from the additional dates.

For $z=1$, $\kappa=99/100$ using the usual metric $b(\kappa,z)=|\kappa-z|$ (and $v=1$,), and taking $\Gamma(u_1,u_2,u_3,u_4)=u_1$, any increasing function $\mathcal{F}\in\mathbb{F}$, and any function $\phi\in\Theta$, we have

$$\begin{aligned}
&\phi\big(b(\kappa,z)\big)+F\big(\alpha(\kappa,z)b(f\kappa,fz)\big)=\\
&=\phi\big(b(\kappa,z)\big)+F\big(b(f\kappa,fz)\big)\\
&=\phi\left(b\left(\frac{99}{100},1\right)\right)+F\left(b\left(\frac{99}{800},\frac{1}{10}\right)\right)=\phi\left(\frac{1}{100}\right)+F\left(\frac{19}{800}\right)\\
&>F\left(\frac{1}{100}\right)=F\left(b\left(\frac{99}{100},1\right)\right)=F\big(b(\kappa,z)\big)\\
&=F\big(L(\kappa,z)\big).
\end{aligned}$$

Hence, we can say that the $(\alpha,\phi,\mathcal{F})$ and $(\alpha_{v^q},\phi,\mathcal{F})-$contractive condition is not satisfied. Also, we can remark the same in a b-metric space with b-metric $b(\kappa,z)=|\kappa-z|^2$ where

$$\begin{aligned}
&\phi\big(b(\kappa,z)\big)+F\big(\alpha(\kappa,z)b(f\kappa,fz)\big)=\\
&=\phi\big(b(\kappa,z)\big)+F\big(2^2b(f\kappa,fz)\big)\\
&=\phi\big(b(\kappa,z)\big)+F\left(4b\left(\frac{99}{800},\frac{1}{10}\right)\right)=\phi\big(b(\kappa,z)\big)+F\left(4\left(\frac{19}{800}\right)^2\right)\\
&>F\left(\left(\frac{1}{10000}\right)^2\right)=F\left(b\left(\frac{99}{100},1\right)\right)=F\big(b(\kappa,z)\big)\\
&=F\big(L(\kappa,z)\big).
\end{aligned}$$

Corollary 2. *Let (Y,b) be a $b-m.l.s$ with parameter $v\geq 1$ and a self-mapping $f:Y\to Y$. If there exist $\mathcal{F}\in\mathbb{F}$, $\phi\in\Theta$ such that*

$$\phi\big(b(\kappa,z)\big)+\mathcal{F}\big(v^q b(f\kappa,fz)\big)\leq\mathcal{F}\big(L(\kappa,z)\big). \tag{23}$$

for all $\kappa,z\in Y$ with $f\kappa\neq fz$, and some $q\geq 1$, where $L(\kappa,z)=\Gamma\left(b(\kappa,z),b(\kappa,f\kappa),b(z,fz),\frac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right)$ for some $\Gamma\in\Omega_4$.

Then, f has a unique fixed point in Y.

Proof. Take the function $\alpha(\kappa,z)=v^q$. □

In the following theorem, we will use another function to help cover rational expressions in the set $L(\kappa,z)$.

Theorem 2. *Let (Y,b) be a $b-m.l.s$ with parameter $v\geq 1$ and $f:Y\to Y$, $\alpha:Y\times Y\to[0,+\infty)$. If there exist $\mathcal{F}\in\mathbb{F}$, $\phi\in\Theta$ such that the following conditions are met:*

C1. There exists $\kappa_0\in Y$ with $\alpha(\kappa_0,f\kappa_0)\geq v^q$;

C2. f is $\alpha_{v^q}-$admissible mapping and satisfies $\alpha_{v^q}-$admissible property;

C3.

$$\phi\big(b(\kappa,z)\big)+\mathcal{F}\big(\alpha(\kappa,z)b(f\kappa,fz)\big)\leq\mathcal{F}\big(L_\psi(\kappa,z)\big) \tag{24}$$

for all $\kappa,z\in Y$ with $f\kappa\neq fz$ and $\alpha(\kappa,z)\geq v^q$, where $L_\psi(\kappa,z)=\Gamma\left(b(\kappa,z),b(\kappa,f\kappa),b(z,fz)\psi\big(b(\kappa,f\kappa),b(\kappa,z),\frac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right)$ for some $\Gamma\in\Omega_4$,

and $\psi:[0,+\infty)^2\to[0,+\infty)$ *is a continuous function with* $\psi(m,m)\le 1,$ *for all* $m>0.$
Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} *– property.*

Proof. Repeating the same process as in the previous theorem, for $\kappa_0\in Y$ with $\alpha(\kappa_0, f\kappa_0)\ge v^q$; build the iterative sequence $\{\kappa_n\}$ by $\kappa_{n+1}=f(\kappa_n)$ such that $\alpha(\kappa_n,\kappa_{n+1})\ge v^q$ for all $n\in N\cup\{0\}$. The proof is clear in the case that $n_0\in N$ exists, with $\kappa_{n_0+1}=\kappa_{n_0}$ .Assuming that $\kappa_{n+1}\ne\kappa_n\Leftrightarrow f\kappa_n\ne f\kappa_{n-1}$ for all $n\in N\cup\{0\}$, taking into account (1) for $\kappa=\kappa_{n-1}, z=\kappa_n$,we have

$$\begin{aligned}\phi(b(\kappa_{n-1},\kappa_n))+\mathcal{F}(v^q b(\kappa_n,\kappa_{n+1}))&\le\phi(b(\kappa_{n-1},\kappa_n))+\mathcal{F}(\alpha(\kappa_{n-1},\kappa_n)b(f\kappa_{n-1},f\kappa_n))\\&\le\mathcal{F}(L_\psi(\kappa_{n-1},\kappa_n)),\end{aligned}\tag{25}$$

where

$$\begin{aligned}L_\psi(\kappa_{n-1},\kappa_n)&=\Gamma\left(\begin{array}{l}b(\kappa_{n-1},\kappa_n),b(\kappa_{n-1},f\kappa_{n-1}),b(\kappa_n,f\kappa_n)\psi(b(\kappa_{n-1},\kappa_n),b(\kappa_{n-1},f\kappa_{n-1})),\\ \dfrac{b(\kappa_{n-1},f\kappa_n)+b(\kappa_n,f\kappa_{n-1})}{4v}\end{array}\right)\\&=\Gamma\left(\begin{array}{l}b(\kappa_{n-1},\kappa_n),b(\kappa_{n-1},\kappa_n),b(\kappa_n,\kappa_{n+1})\psi(b(\kappa_{n-1},\kappa_n),b(\kappa_{n-1},\kappa_n)),\\ \dfrac{b(\kappa_{n-1},\kappa_{n+1})+b(\kappa_n,\kappa_n)}{4v}\end{array}\right)\\&\le\Gamma\left(b(\kappa_{n-1},\kappa_n),b(\kappa_{n-1},\kappa_n),b(\kappa_n,\kappa_{n+1}),\frac{b(\kappa_{n-1},\kappa_{n+1})+b(\kappa_n,\kappa_n)}{4v}\right)\\&=L(\kappa_{n-1},\kappa_n).\end{aligned}$$

The result is the same as in Theorem 1, and the proof goes along the same lines.□

Corollary 3. *Let* (Y,b) *be a* $b-m.l.s$ *with parameter* $v\ge 1$ *and* $f:Y\to Y,\ \alpha:Y\times Y\to[0,+\infty)$. *If there exist* $\mathcal{F}\in\mathbb{F},\ \phi\in\Theta,\Gamma\in\Omega_4$ *such that satisfy theconditions* C_1,C_2 *and*

$$\begin{aligned}&\phi(b(\kappa,z))+\mathcal{F}(\alpha(\kappa,z)b(f\kappa,fz))\le\\&\mathcal{F}\left(\Gamma\left\{b(\kappa,z),b(\kappa,f\kappa),b(z,fz)\frac{b(z,fz)[1+b(\kappa,f\kappa)]}{1+b(\kappa,z)},\frac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right\}\right)\end{aligned}\tag{26}$$

for all $\kappa,z\in Y$ *with* $f\kappa\ne fz$ *and* $\alpha(\kappa,z)\ge v^q$.

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} *– property.*

Proof. This comes from Theorem 2 by taking $\psi(m,n)=\dfrac{1+n}{1+m}$. □

Corollary 4. *Let* (Y,b) *be a* $b-m.l.s$ *with parameter* $v\ge 1$ *and* $f:Y\to Y,\ \alpha:Y\times Y\to[0,+\infty)$. *If there exist* $\mathcal{F}\in\mathbb{F},\ \phi\in\Theta$ *such that satisfy the conditions* C_1,C_2 *and*

$$\phi(b(\kappa,z))+\mathcal{F}(\alpha(\kappa,z)b(f\kappa,fz))\le\mathcal{F}\left(\max\left(b(\kappa,z),b(\kappa,f\kappa),b(z,fz),\frac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right)\right)\tag{27}$$

for all $\kappa, z \in Y$ *with* $f\kappa \neq fz$ *and* $\alpha(\kappa, z) \geq v^q$.

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} – *property.*

Proof. The proof is consideredcompletedwhenthe substitution in Corollary 3 with $\Gamma \in \Omega_4$ as $\Gamma(u_1, u_2, u_3, u_4) = \max\{u_1, u_2, u_3, u_4\}$ is made.□

Corollary 5. *Let* (Y, b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f : Y \to Y$, $\alpha : Y \times Y \to [0, +\infty)$. *If there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *such that satisfy the conditions* C_1, C_2 *and*

$$\phi(b(\kappa, z)) + \mathcal{F}(\alpha(\kappa, z) b(f\kappa, fz)) \leq \mathcal{F}\left(a_1 b(\kappa, z) + a_2 b(\kappa, f\kappa) + a_3 b(z, fz) + a_4 \frac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right) \tag{28}$$

for all $\kappa, z \in Y$ *with* $f\kappa \neq fz$ *and* $\alpha(\kappa, z) \geq v^q$.

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} – *property.*

Proof. This comes from Theorem 1 by taking $\Gamma \in \Omega_4$ as $\Gamma(u_1, u_2, u_3, u_4) = c_1 u_1 + c_2 u_2 + c_3 u_3 + c_4 u_4$ with $0 < c_1 + c_2 + c_3 + c_4 < 1$. □

Corollary 6. *Let* (Y, b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f : Y \to Y$, $\alpha : Y \times Y \to [0, +\infty)$. *If there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *such that satisfy the conditions* C_1, C_2 *and*

$$\phi(b(\kappa, z)) + \mathcal{F}(\alpha(\kappa, z) b(f\kappa, fz)) \leq \mathcal{F}\left(\max\left\{b(\kappa, z), \frac{b(z, fz)[1 + b(\kappa, fk)]}{1 + b(\kappa, z)}\right\}\right) \tag{29}$$

for all $\kappa, z \in Y$ *with* $f\kappa \neq fz$ *and* $\alpha(\kappa, z) \geq v^q$.

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} – *property.*

Proof. The proof is completed by taking $\Gamma \in \Omega_4$ as $\Gamma(u_1, u_2, u_3, u_4) = \max\{u_1, u_3\}$ and the function $\psi(m, n) = \frac{1 + n}{1 + m}$ in Theorem 2.□

Corollary 7. *Let* (Y, b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f : Y \to Y$, $\alpha : Y \times Y \to [0, +\infty)$. *If there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *such that satisfy the condition* C_1, C_2 *and*

$$\phi(b(\kappa, z)) + \mathcal{F}(\alpha(\kappa, z) b(f\kappa, fz)) \leq \mathcal{F}\left(a_1 b(\kappa, z) + a_2 \frac{b(z, fz)[1 + b(\kappa, f\kappa)]}{1 + b(\kappa, z)}\right) \tag{30}$$

for all $\kappa, z \in Y$ *with* $f\kappa \neq fz$ *and* $\alpha(\kappa, z) \geq v^q$.

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} – *property.*

Proof. It comes from Theorem 2 by taking $\Gamma \in \Omega_4$ as $\Gamma(u_1, u_2, u_3, u_4) = a_1 u_1 + a_2 u_3$ with $0 < a_1 + a_2 < 1$, and the function $\psi(m, n) = \frac{1 + n}{1 + m}$. □

Remark 2. *The above corollaries 6; 7 correspond to the* $\left(\alpha_{v^q}, \phi, \mathcal{F}\right)-$ *Dass-Gupta and Jaggi contractions. They are the generalization and extension of the theorems in [6,10,35–37].*

In the following part, we generalize some previous definitions that have to do with interpolation and hybrid contractions in metric and generalized metric spaces. The theorems established for these classes of contractions, in the sequel and in the published literature, are a common important focus of Theorems 1 and 2.

Definition 9. *Let* (Y,b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f: Y \to Y$, $\alpha: Y \times Y \to [0,+\infty)$. *Theself-mapping* f *on* Y, *is named aHardy–Rogers type interpolative* $\left(\alpha_{v^q}, \phi, \mathcal{F}\right)-$ *contraction, if there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *and* $a_1, a_2, a_3, a_4 \in (0,1)$ *with* $0 < a_1 + a_2 + a_3 + a_4 = 1$, *such that*

$$\phi\left(b(\kappa,z)\right) + \mathcal{F}\left(\alpha(\kappa,z)b(f\kappa, fz)\right) \leq \mathcal{F}\left(L_a(\kappa,z)\right) \tag{31}$$

for all $\kappa, z \in Y \setminus Fix(f)$ *with* $f\kappa \neq fz$ *and* $\alpha(\kappa,z) \geq v^q$,
where

$$L_a(\kappa,z) = \left(b(\kappa,z)\right)^{a_1} \cdot \left(b(\kappa, f\kappa)\right)^{a_2} \cdot \left(b(z, fz)\right)^{a_3} \cdot \left(\frac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right)^{a_4}$$

Definition 10. *Let* (Y,b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f: Y \to Y$, $\alpha: Y \times Y \to [0,+\infty)$. *Theself-mapping* f *on* Y, *is named a Hardy–Rogers* $r-$ *order hybrid* $\left(\alpha_{v^q}, \phi, \mathcal{F}\right)-$*contraction if there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *such that*

$$\phi\left(b(\kappa,z)\right) + \mathcal{F}\left(\alpha(\kappa,z)b(f\kappa, fz)\right) \leq \mathcal{F}\left(L_a^r(\kappa,z)\right) \tag{32}$$

for all $\kappa, z \in Y \setminus Fix(f)$ *with* $f\kappa \neq fz$ $\alpha(\kappa,z) \geq v^q$, $r \geq 0$ *and* $a_i \geq 0, i = 1,2,3,4$ *such that* $0 < a_1 + a_2 + a_3 + a_4 = 1$,
where

$$L_a^r(\kappa,z) = \begin{cases} \left[a_1\left(b(\kappa,z)\right)^r + a_2\left(b(\kappa, f\kappa)\right)^r + a_3\left(b(z, fz)\right)^r + a_4\left(\dfrac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right)^r\right]^{\frac{1}{r}} & \text{for } r > 0, \kappa \neq z \\ \left(b(\kappa,z)\right)^{a_1}\left(b(\kappa, f\kappa)\right)^{a_2}\left(b(z, fz)\right)^{a_3}\left(\dfrac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right)^{a_4} & \text{for } r = 0; \kappa, z \in Y \setminus Fix(f). \end{cases}$$

Remark 3. *Taking* $v = 1$, *consequently, we obtain the corresponding definitions in metric spaces. Taking* $\alpha(\kappa,z) \geq v^q$, *we derive the* $r-$ *order hybrid* $(v,q,\phi,\mathcal{F})-$ *contraction. Taking* $\phi(t) = \tau > 0$, *we naturally obtain the* $r-$*order hybrid* $(\alpha,\mathcal{F})-$*contraction. Definition 10 generates an* $r-$*order interpolative* $(\phi,\mathcal{F})-$*contraction and* $r-$*order* $(\alpha,\mathcal{F})-$ *contraction. The general Definition 8 generates the above Definitions 9 and 10, and all classical contractions for certain types of* $\Gamma \in \Omega_4$ *and all classical contractions. It integrates many new forms of contractions that have been recently defined.*

Theorem 3. *Let* (Y,b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f: Y \to Y$, $\alpha: Y \times Y \to [0,+\infty)$. *If* f *is an interpolative Hardy–Rogers type*$\left(\alpha_{v^q}, \phi, \mathcal{F}\right)-$*contraction, and* *conditions* C_1, C_2 *hold,then* f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} *– property.*

Proof. It is derived from Theorem 1 by taking $\Gamma \in \Omega_4$ as $\Gamma(u_1, u_2, u_3, u_4) = u_1^{a_1} \cdot u_2^{a_2} \cdot u_3^{a_3} \cdot u_4^{1-a_1-a_2-a_3}$, where $a_1, a_2, a_3 \in (0,1)$ and $a_1 + a_2 + a_3 < 1$. □

Theorem 4. *Let* (Y, b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$, *and* $f : Y \to Y$, $\alpha : Y \times Y \to [0, +\infty)$. *If there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *such that satisfy the conditions* C_1, C_2 *and*

$$\phi(b(\kappa, z)) + \mathcal{F}(\alpha(\kappa, z) b(f\kappa, fz)) \leq$$
$$\mathcal{F}\left(\left[a_1(b(\kappa, z))^r + a_2(b(\kappa, f\kappa))^r + a_3(b(z, fz))^r + a_4\left(\frac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right)^r\right]^{\frac{1}{r}}\right)$$

for all $\kappa, z \in Y$ *with* $f\kappa \neq fz$, $\alpha(\kappa, z) \geq v^q$, *and* $r > 0$.

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} *– property.*

Proof. Itcomes from Theorem 1 by taking $\Gamma \in \Omega_4$ as $\Gamma(u_1, u_2, u_3, u_4) = \left[a_1 u_1^r + a_2 u_2^r + a_3 u_3^r + a_4 u_4^r\right]^{\frac{1}{r}}, r > 0$, where $0 < a_1 + a_2 + a_3 + a_4 < 1$. □

Theorem 5. *Let* (Y, b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$ *and* $f : Y \to Y$, $\alpha : Y \times Y \to [0, +\infty)$. *If there exist* $\mathcal{F} \in \mathbb{F}$, $\phi \in \Theta$ *such that hold conditions* C_1, C_2 *and*

$$\phi(b(\kappa, z)) + \mathcal{F}(\alpha(\kappa, z) b(f\kappa, fz)) \leq$$
$$\mathcal{F}\left(\left[\max\left\{(b(\kappa, z))^r, (b(\kappa, f\kappa))^r, (b(z, fz))^r, \left(\frac{b(\kappa, fz) + b(z, f\kappa)}{4v}\right)^r\right\}\right]^{\frac{1}{r}}\right)$$

for all $\kappa, z \in Y$ *with* $f\kappa \neq fz$, $\alpha(\kappa, z) \geq v^q$, *and* $r > 0$.
Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{v^q} *– property.*

Proof. It is a special case of Theorem 1 when we take $\Gamma \in \Omega_4$ as

$$\Gamma(u_1, u_2, u_3, u_4) = \left[\max\left\{u_1^r, u_2^r, u_3^r, u_4^r\right\}\right]^{\frac{1}{r}}, r > 0$$

. □

Theorem 6. *Let* (Y, b) *be a* $b-$ *m.l.s with parameter* $v \geq 1$, *and mappings* $f : Y \to Y$, $\alpha : Y \times Y \to [0, +\infty)$, *if the following conditions are satisfied:*

C1. There exists $\kappa_0 \in Y$ *with* $\alpha(\kappa_0, f\kappa_0) \geq v^q$;

C2. f *is* α_{v^q} *– admissible mapping and satisfies* α_{v^q} *– admissible property;*

C3. f *is a generalized* r *– order hybrid* $(\alpha_{v^q}, \phi, \mathcal{F})$ *– contraction.*

Then, f has a fixed point in Y. Moreover, it is unique if f satisfies the unique α_{v^q} – property.

Proof. In Theorem 1, we define

$$L_a^r(\kappa,z)=\begin{cases}\Gamma_1(u_1,u_2,u_3,u_4) & for\ r>0,\\ \Gamma_2(u_1,u_2,u_3,u_4) & for\ r=0.\end{cases}$$

And, we take functions $\Gamma_1,\Gamma_2\in\Omega_4$ as $\Gamma_1(u_1,u_2,u_3,u_4)=\left(a_1u_1{}^r+a_2u_2{}^r+a_3u_3{}^r+a_4u_4{}^r\right)^{\frac{1}{r}}$; $\Gamma_2(u_1,u_2,u_3,u_4)=u_1{}^{a_1}\cdot u_2{}^{a_2}\cdot u_3{}^{a_3}\cdot u_4{}^{a_4}$ where $a_1,a_2,a_3,a_4\geq 0$ with $0<a_1+a_2+a_3+a_4=1$, $r\geq 0$.

Then,

$$L_a^r(\kappa,z)=\begin{cases}\Gamma_1(u_1,u_2,u_3,u_4)=\left[a_1(b(\kappa,z))^r+a_2(b(\kappa,f\kappa))^r+a_3(b(z,fz))^r+a_4\left(\dfrac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right)^r\right]^{\frac{1}{r}} & for\ r>0\\ \Gamma_2(u_1,u_2,u_3,u_4)=(b(\kappa,z))^{a_1}(b(\kappa,f\kappa))^{a_2}(b(z,fz))^{a_3}\left(\dfrac{b(\kappa,fz)+b(z,f\kappa)}{4v}\right)^{a_4} & for\ \ r=0.\end{cases}$$

. □

Theorem 7. *Let (Y,b) be a $b-$ m.l.s with parameter $v\geq 1$, and $f:Y\to Y$, $\alpha:Y\times Y\to[0,+\infty)$. If there exist $\mathcal{F}\in\mathbb{F}$, $\phi\in\Theta$ such that satisfy the conditions C_1,C_2 and*

$$\phi(b(\kappa,z))+\mathcal{F}(\alpha(\kappa,z)b(f\kappa,fz))\leq\mathcal{F}\left(L_a{}^r(\kappa,z)\right)\tag{33}$$

for all $\kappa,z\in Y\setminus Fix(f)$ with $f\kappa\neq fz$ $\alpha(\kappa,z)\geq v^q$, $r\geq 0$ and $a_i\geq 0, i=1,2,3$ such that $0<a_1+a_2+a_3=1$, where

$$L_a^r(\kappa,z)=\begin{cases}\left[a_1(b(\kappa,z))^r+a_2(b(\kappa,f\kappa))^r+a_3(b(z,fz))^r\right]^{\frac{1}{r}} & for\ \ r>0,\kappa\neq z\\ (b(\kappa,z))^{a_1}(b(\kappa,f\kappa))^{a_2}\cdot(b(z,fz))^{a_3} & for\ \ r=0\ \ \kappa,z\in Y-Fix(f)\end{cases}$$

Then, f has a fixed point in Y. Moreover, it is unique if f satisfies the unique α_{v^q} – property.

Proof. The proof is considered completed using Theorem 1, if we take $\Gamma_1,\Gamma_2\in\Omega_4$ and define

$$L_a^r(\kappa,z)=\begin{cases}\Gamma_1(u_1,u_2,u_3,u_4) & for\ r>0\\ \Gamma_2(u_1,u_2,u_3,u_4) & for\ r=0,\end{cases}$$

where

$$\Gamma_1(u_1,u_2,u_3,u_4)=\left(a_1u_1{}^r+a_2u_2{}^r+a_3u_3{}^r\right)^{\frac{1}{r}}$$
$$\Gamma_2(u_1,u_2,u_3,u_4)=(u_1)^{a_1}\cdot(u_2)^{a_2}\cdot(u_3)^{a_3}.$$

. □

Theorem 8. *Let (Y,b) be a $b-$ m.l.s with parameter $v\geq 1$, and $f:Y\to Y$, $\alpha:Y\times Y\to[0,+\infty)$. If there exist $\mathcal{F}\in\mathbb{F}$, $\phi\in\Theta$ such that satisfy the condition C_1,C_2 and*

$$\phi\big(b(\kappa,z)\big)+\mathcal{F}\big(\alpha(\kappa,z)b(f\kappa, fz)\big)\le\mathcal{F}\big(L_a^{\,r}(\kappa,z)\big)$$

for all $\kappa,z\in Y\setminus Fix(f)$ *with* $f\kappa\ne fz$ *and* $\alpha(\kappa,z)\ge v^q$, $r\ge 0$ *and* $a_i\ge 0, i=1,2$ *such that* $0<a_1+a_2=1$, *where*

$$L_a^r(\kappa,z)=\begin{cases}\left[a_1\big(b(\kappa,z)\big)^r+a_2\left(\dfrac{\big(1+b(\kappa, f\kappa)\big)\cdot b(z, fz)}{1+b(\kappa,z)}\right)^r\right]^{\frac{1}{r}} & for\ r>0,\kappa\ne z\\ \big(b(\kappa, f\kappa)\big)^{a_1}\cdot\big(b(z, fz)\big)^{a_2} & for\ r=0\ ;\kappa,z\in Y\setminus Fix(f)\end{cases}$$

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{s^q} *– property.*

Proof. We take Theorem 2 $\psi(m,n)=\dfrac{1+n}{1+m}$, $\Gamma_1,\Gamma_2\in\Omega_4$ and define

$$\Gamma_a^r(\kappa,z)=\begin{cases}\Gamma_1(u_1,u_2,u_3,u_4)=a_1u_1^{\,r}+a_2u_3^{\,r} & for\ \ r>0,\\ \Gamma_2(u_1,u_2,u_3,u_4)=(u_2)^{a_1}\cdot(u_3)^{a_2} & for\ \ r=0.\end{cases}$$

. □

Remark 4. *Theorem 8 can be considered as new result for Jaggi* $r-$*order hybrid* $\big(\alpha_{v^q},\phi,\mathcal{F}\big)-$*contractions and Dass and Gupta type* $r-$*order hybrid* $\big(\alpha_{v^q},\phi,\mathcal{F}\big)-$*contractions extended in* $b-$*m.l.s; they are also a special case of generalized* $\big(\alpha_{v^q},\phi,\mathcal{F}\big)-$*contractions.*

Theorem 9. *Let* (Y,b) *be a* $b-$ *m.l.s with parameter* $v\ge 1$, *and* $f:Y\to Y$, $\alpha:Y\times Y\to[0,+\infty)$. *If there exist* $\mathcal{F}\in\mathbb{F}$, $\phi\in\Theta$ *such that satisfy the condition* C_1,C_2 *and*

$$\phi\big(b(\kappa,z)\big)+\mathcal{F}\big(\alpha(\kappa,z)b(f\kappa, fz)\big)\le\mathcal{F}\big(L_a^r(\kappa,z)\big)$$

for all $\kappa,z\in Y\setminus Fix(f)$ *with* $f\kappa\ne fz$ *and* $\alpha(\kappa,z)\ge v^q$, $r\ge 0$ *and* $a_i\ge 0, i=1,2,3,4$ *such that* $0<a_1+a_2+a_3+a_4=1$, *where*

$$L_a^r(\kappa,z)=\begin{cases}\left[\max\left\{\big(b(\kappa,z)\big)^r,\big(b(\kappa, f\kappa)\big)^r,\big(b(z, fz)\big)^r,\left(\dfrac{b(\kappa, fz)+b(z, f\kappa)}{4v}\right)^r\right\}\right]^{\frac{1}{r}} & for\ r>0,\kappa\ne z\\ \big(b(\kappa,z)\big)^{a_1}\cdot\big(b(\kappa, f\kappa)\big)^{a_2}\cdot\big(b(z, fz)\big)^{a_3}\cdot\left(\dfrac{b(\kappa, fz)+b(z, f\kappa)}{4v}\right)^{a_4} & for\ r=0;\kappa,z\in Y\setminus Fix(f)\end{cases}$$

Then, f *has a fixed point in* Y. *Moreover, it is unique if* f *satisfies the unique* α_{s^q} – *property.*

Proof. We take Theorem 1 with the function $\Gamma_1,\Gamma_2\in\Omega_4$ and define

$$L_a^r(\kappa,z)=\begin{cases}\Gamma_1(u_1,u_2,u_3,u_4)=\left[\max\left\{u_1^{\,r},u_2^{\,r},u_3^{\,r},u_4^{\,r}\right\}\right]^{\frac{1}{r}} & for\ \ r>0,\\ \Gamma_2(u_1,u_2,u_3,u_4)=u_1^{\,a_1}\cdot u_2^{\,a_2}\cdot u_3^{\,a_3}\cdot u_4^{\,a_4} & for\ \ r=0.\end{cases}$$

. □

Remark 5. *In Definition 9 and also Theorem 3*

- *By taking* $\Gamma(u_1,u_2,u_3,u_4)=c\cdot u_1^{a_1}\cdot u_2^{a_2}\cdot u_3^{a_3}\cdot u_4^{a_4}$ *where* $0<c<1$, $a_1,a_2,a_3,a_4\ge 0$ *with* $0<a_1+a_2+a_3+a_4=1$, *and* $v=1$ *we obtain Definition 2 and, respectively, Theorem 4in [31] in the* $\left(\alpha_{v^q},\phi,\mathcal{F}\right)$ *version; so, our result is an extension, generalization, and new result in the framework of metric and b-metric-like-spaces.*
- *By taking* $\Gamma(u_1,u_2,u_3,u_4)=\lambda\cdot u_1^{a_1}\cdot u_2^{a_2}\cdot u_3^{a_3}$ *where* $0<\lambda<1$, $a_1,a_2,a_3\ge 0$ *with* $0<a_1+a_2+a_3=1$, *and* $v=1$,*we obtain Definition 3 and, respectively, Theorem 4 in [34] in the* $\left(\alpha_{v^q},\phi,\mathcal{F}\right)$*version.*
- *By taking* $\alpha(y,z)=v^q$, *we obtain some theorems in [8].*
- *By taking* $\Gamma(u_1,u_2,u_3,u_4)=\varphi\left(u_1^{a_1}\cdot u_2^{a_2}\cdot u_3^{a_3}\right)$ *where* φ *is non decreasing function on* $[0,\infty)$ *such that* $\sum\limits_{n=1}^{\infty}\varphi^n(t)<\infty$ *for* $t>0$ *and* $\varphi(t)<t$ *for each* $t>0$, *also* $a_1,a_2,a_3\ge 0$ *with* $0<a_1+a_2+a_3=1$, $v=1$, *we obtain Definition 3 and, respectively, Theorem 1 in [32], in the* $\left(\alpha_{v^q},\phi,\mathcal{F}\right)$ *version.*
- *In view of the implicit relation set* Ω_n , *our definitions and, respectively, the stated Theorems have a general character and unifying power.*
- *The same consequences are present for Definition 10 and related theorems.*

4. Conclusions

In this work, we established some new valuable and significant fixed point results in a general metric space like a $b-m.l.s$. Moreover, we used these results to obtain several interesting results related to linear and nonlinear contractions recently elaborated on under the name of interpolative contraction and $r-$hybrid $\mathcal{F}-$contraction in such spaces. Our results extend, generalize, and significantly unify a great work on fixed point theory.

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