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Modeling of COVID-19 in View of Rough Topology

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Abstract: Rough-based topology has become an important technique for decision making in numerous real-life problems. The main purpose of this research paper is to use some topological notions in the approximation space of a rough set. Firstly, some concepts of topological near open sets and rough concepts are presented. A new approximation structure based on the topological near open sets is introduced. We debate the properties of the new approximation structure. We included an algorithm for detecting COVID-19 infection by its side effects. We believe that our approach will be helpful for any future detection.

Keywords: opological spaces; near open sets; rough set; information systems; COVID-19; reduct

MSC: 54A05; 54A10; 54C10; 54C15; 92C50

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1. Introduction

There is no human event that has been able to restrict the face of life on the earth in a significant and continuous manner. However, if we look at the emerging corona virus (COVID-19), which hardly has any weight, as specialists have said that the weight of all corona viruses that have infected millions in the world does not even reach two grams, it had a stronger effect on the world than atomic weapons. This is in relation to COVID-19, which killed and infected millions in the world. The issue is not only about the search for a cure, it is also about the aftershocks and effects that COVID-19 has left. Sooner or later, the signs appear, as the virus continues to spread in all countries of the world. With the increasing information available now about COVID-19, scientists are working on using modern tools and techniques in order to be able to analyze the numbers in a laboratory at the University of California and in cooperation with the Energy Lab. They have created new algorithms using mathematical programming tools and computers in order for the the analysis process for the COVID-19 epidemic, currently ongoing, to be more feasible and effective [1].

Scientists' efforts and responses to the COVID-19 pandemic: Clear fingerprints made by scientists and researchers in virology from all over the world in the face of the emerging COVID-19 pandemic, for which the world has joined together with all its capabilities to find a treatment or vaccine for the virus since its appearance in December 2019 in Wuhan, China and subsequent spread throughout the world, causing deaths and injuries by millions.

The appearance of COVID-19 has become a major public health concern due to its ability to cause many respiratory tract infections in people, and its ability to cause deaths. COVID-19 is transmitted from humans to humans in an incubation period of 2–10 days through droplets, contaminated hands, or contaminated surfaces. COVID-19 has emerged as a deadly virus. It has spread across the world, and there have been many mortalities. With each passing second, the death toll rises. COVID-19 affects people differently, and some people who contract it have mild to moderate symptoms and recover without needing to go to the hospital [2]. Accordingly, numerous researchers have published numerous articles on this pernicious virus (for instance, see the references [1,3–7]).

The rough set theory gives framework planners the capacity to deal with vulnerability. In the event that an ideal is not 'determinable' in a given knowledge base, rough sets can be 'surmised' for that information. From a clinical perspective, the characteristic worth limits are normally dubious [8].

A topology is a subfield of geometry known as rubber sheet geometry. Topology has numerous real-world applications and resolves several issues relating to continuity, either directly or indirectly. Its study does not depend on the dimension, i.e., an increase or decrease can happen without cutting. Using the neighborhood system, graphs are represented topologically and vice versa for some types of topologies. Recently, graphs, multisets, and rough sets have been used to represent structures such as the human heart [9] and medicine [10–12], which are useful in medicine, physics, and biology. Previously, we have used topology to study the similarity of DNA sequences and to identify mutations in genes, chromosomes, their locations, and amino acids that have changed [13]. We also used topology to study the recombination of DNA and to form a mathematical model for the recombination process [14]. Topological concepts can be used to build flexible mathematical models in biomathematics. Topological concepts play a vital role in the rough set, and may be useful in determining the causes of disease outbreaks and how to treat them. Moreover, they help in studying mutations that occur in genes and diseases, as well as in finding solutions.

We use the data set as an information system, and we use mathematical tools and studies to help the expert in decision making. The information systems of people and patients with COVID-19 can be classified via rough set theory and some topological structures. The significance of our paper is in studying the critical factors influencing the spread of COVID-19.

In this paper, we have demonstrated that rough topology can be used to solve problems in the real world. This technique has been employed in order to determine the determinants of the outbreak of the virus "COVID-19", which has been reported widely throughout the world. In this study, the rough topological model matches up well with that used by medical specialists in terms of clinical use. We seek to assist in identifying the symptoms that have the most impact on the disease's spread.

2. Preliminaries

A rough set is a novel mathematical technique for dealing with ambiguity in knowledge-based systems, market research, and information systems [15]. This concept has several applications in domains such as medical diagnosis and economics. Other rough set models were shown in [16–23]. Throughout this section, basic terminologies and definitions of the approximation-based rough set are presented. We also provide some primary concepts of the near open set.

Definition 1 ([24]). *Let $\mathfrak{U}$ be a universe of discourse, and R be an equivalence relation on U. Then, $\mathfrak{H} = (\mathfrak{U}, \mathfrak{R})$ is called an approximation structure. For any subset $X_1 \subseteq \mathfrak{U}$, the lower approximations (L_{app}) and upper approximations (U_{app}) are defined as:*

$$L_{app}(X_1) = \{x_1 \in \mathfrak{U} : [x_1]_{\mathfrak{R}} \subseteq X_1\},$$
$$U_{app}(X_1) = \{x_1 \in \mathfrak{U} : [x_1]_{\mathfrak{R}} \cap X_1 \neq \phi\},$$
$$B_R(X_1) = U_{app}(X_1) - L_{app}(X_1).$$

Proposition 1 ([24]). *If ($\mathfrak{U}$,R) is an approximation structure and $X_1, Y_1 \subseteq \mathfrak{U}$, then:*
$(i)\ L_{app}(X_1) \subseteq X_1 \subseteq U_{app}(X_1).$
$(ii)\ L_{app}(\phi) = U_{app}(\phi) = \phi and L_{app}(U) = U_{app}U(\phi) = U.$
$(iii)\ U_{app}(X_1 \cup Y_1) = U_{app}(X_1) \cup U_{app}(Y_1).$
$(iv)\ U_{app}(X_1 \cap Y_1) \subseteq U_{app}(X_1) \cap U_{app}(Y_1).$
$(v)\ L_{app}(X_1 \cup Y_1) \supseteq L_{app}(X_1) \cup L_{app}(Y_1).$

(vi) $L_{app}(X_1 \cap Y_1) = L_{app}(X_1) \cap L_{app}(Y_1)$.
(vii) *If* $X_1 \subseteq Y_1$ *then* $L_{app}(X_1) \subseteq L_{app}(Y_1)$ *and* $U_{app}(X_1) \subseteq U_{app}(Y_1)$.
(viii) $U_{app}(X_1^c) = ([L_{app}(X_1)]^c)$*and* $L_{app}(X_1^c) = ([U_{app}(X_1)]^c)$.
(ix) $U_{app}U_{app}(X_1) = L_{app}U_{app}(X_1) = U_{app}(X_1)$.
(x) $L_{app}L_{app}(X_1) = U_{app}L_{app}(X_1) = L_{app}(X_1)$.

Definition 2. *Let* $(\mathfrak{U}, \tau)$ *be a topological structure. Then,* $X_1 \subseteq \mathfrak{U}$ *is called:*

(i) α*-open [25] if* $X_1 \subseteq int(cl(int(X_1)))$.

(ii) γ*-open [26] if* $X_1 \subseteq cl(int(X_1)) \cup int(cl(X_1))$.

(iii) β*-open [27] if* $X_1 \subseteq cl(int(cl(X_1)))$.

The set of all β-open expressed by $\beta O(\mathfrak{U}, \tau)$ or $\beta O(\mathfrak{U})$, as well as $\alpha O(\mathfrak{U})$ and $\gamma O(\mathfrak{U})$.

3. Topological Near Open Sets' Approximation Structure

The necessity to represent subsets of a universe $\mathfrak{U}$ as a base for topology τ motivates the use of the topological rough set theory. Thus, if R is a binary relation on $\mathfrak{U}$, then $K_\tau = (\mathfrak{U}, R_\beta)$ is called a near open ($\mathfrak{No}$) approximation structure, where $\beta O(\mathfrak{U})$ is generated by taking R_β as a subbase for τ.

Definition 3. *Let* $(\mathfrak{U}, R_\beta)$ *be a* $\mathfrak{No}$ *approximation structure,* $X_1 \subseteq \mathfrak{U}$*. Then,* $\underline{R}_\beta(X_1) = \bigcup\{G \in \beta O(\mathfrak{U}) : G \subseteq X_1\}$, $\overline{R}_\beta(X_1) = \bigcap\{F \in \beta C(\mathfrak{U}) : X_1 \subseteq F\}$ *are called* $\mathfrak{No}$ L_{app} *(resp.* $\mathfrak{No}U_{app}$*), where* $G_{X_1\beta} \in \beta O(\mathfrak{U}), X_1 \in G_{X_1\beta}$.

Moreover, $POS_\beta(X_1) = \underline{R}_\beta(X_1)$ is called the topological positive region of X_1, $NEG_\beta(X_1) = \mathfrak{U} - \overline{R}_\beta(X_1)$ is called the topological nagative region of X_1 and $BND_\beta(X_1) = \overline{R}_\beta(X_1) - \underline{R}_\beta(X_1)$ is called the topological boundary region of X_1. The topological accuracy measure is defined as $\alpha_\beta(X_1) = \frac{|\underline{R}_\beta(X_1)|}{|\overline{R}_\beta(X_1)|}$, where $|\quad|$ represents the cardinality and $X_1 \neq \phi$.

The decision-making process in practical applications requires mathematical steps (upper and lower approximations, as well as boundary and accuracy).

Example 1. *Let* $\mathfrak{U} = \{q_1, q_2, q_3, q_4\}$, $R = \{(q_1, q_1), (q_1, q_2), (q_2, q_4), (q_3, q_4), (q_4, q_1), (q_4, q_2), (q_4, q_4)\}$ *and* $X_1 = \{q_1, q_2, q_3\}$*. Thus,* $q_1R = \{q_1, q_2\}$,$q_2R = q_3R = \{q_4\}$, $q_4R = \{q_1, q_2, q_4\}$ *and* $\tau = \{\mathfrak{U}, \phi, \{q_1, q_2\}, \{q_4\}, \{q_1, q_2, q_4\}\}$*. Therefore,* $\beta O(\mathfrak{U}) = \{\{q_1\}, \{q_2\}, \{q_3\}, \{q_4\}, \{q_1, q_2\}, \{q_1, q_3\}, \{q_1, q_4\}, \{q_2, q_3\}, \{q_2, q_4\}, \{q_3, q_4\}, \{q_1, q_2, q_3\}, \{q_1, q_2, q_4\}, \{q_1, q_3, q_4\}, \{q_2, q_3, q_4\}\}$*. Then,* $\underline{R}_\beta(X_1) = \{q_1, q_2, q_3\}$ *and* $\overline{R}_\beta(X_1) = \mathfrak{U}$*. The accuracy measure* $\alpha_\beta(X_1) = \frac{3}{4}$.

Proposition 2. *Let* $(\mathfrak{U}, R_\beta)$ *be a* $\mathfrak{No}$ *approximation structure,* $X_1, Y_1 \subseteq \mathfrak{U}$*. Then:*

(i) X_1 *is* $\mathfrak{No}$ *roughly a bottom, included in* Y_1 *if* $\underline{R}_\beta(X_1) \subseteq \underline{R}_\beta(Y_1)$;

(ii) X_1 *is* $\mathfrak{No}$ *roughly a top, included in* Y_1 *if* $\overline{R}_\beta(X_1) \subseteq \overline{R}_\beta(Y_1)$;

(iii) X_1 *is* $\mathfrak{No}$ *roughly included in* Y_1 *if* $\underline{R}_\beta(X_1) \subseteq \underline{R}_\beta(Y_1)$ *and* $\overline{R}_\beta(X_1) \subseteq \overline{R}_\beta(Y_1)$.

Definition 4. *Let* $(\mathfrak{U}, R_\beta)$ *be a* $\mathfrak{No}$ *approximation structure,* $X_1, Y_1 \subseteq \mathfrak{U}$*. Then:*

(i) R_β*-definable (*β*-exact) if* $\underline{R}_\beta(X_1) = \overline{R}_\beta(X_1)$ *or* $BND_\beta(X_1) = \phi$;

(ii) $\mathfrak{No}$ *rough if* $\underline{R}_\beta(X_1) \neq \overline{R}_\beta(X_1)$ *or* $BND_\beta(X_1) \neq \phi$.

Definition 5. *Let* $(\mathfrak{U}, R_\beta)$ *be a* $\mathfrak{No}$ *approximation structure,* $X_1, Y_1 \subseteq \mathfrak{U}$*. Then,* X_1, Y_1 *are:*

(i) $\mathfrak{No}$ *roughly equal to a top if* $\underline{R}_\beta(X_1) = \underline{R}_\beta(Y_1)$;

(ii) $\mathfrak{No}$ *roughly equal to a bottom if* $\overline{R}_\beta(X_1) = \overline{R}_\beta(Y_1)$;

(iii) $\mathfrak{No}$ *roughly equal if* $\underline{R}_\beta(X_1) = \underline{R}_\beta(Y_1)$ *and* $\overline{R}_\beta(X_1) = \overline{R}_\beta(Y_1)$.

Proposition 3. *If $(\mathfrak{U}, R_\beta)$ is a $\mathfrak{No}$ approximation structure, $X_1, Y_1 \subseteq \mathfrak{U}$, then,*

(i) Every exact set in $\mathfrak{U}$ is $\mathfrak{No}$ exact;

(ii) Every $\mathfrak{No}$ rough set in $\mathfrak{U}$ is rough.

Proof.

(i) Suppose that X_1 is an exact set in $\mathfrak{U}$. Thus, $BND(X_1) = BND_\beta(X_1) = \phi$ and $X_1 = \underline{R}_\beta(X_1) = \overline{R}_\beta(X_1)$. Therefore, X_1 is $\mathfrak{No}$ exact.

(ii) Suppose that $X_1 \subseteq \mathfrak{U}$ is $\mathfrak{No}$ rough. Hence, $BND_\beta(X_1) \neq \phi$ and $X \neq \underline{R}_\beta(X_1) \neq \overline{R}_\beta(X_1)$. Thus, X_1 is a rough set in $\mathfrak{U}$. □

Definition 6. *Let $(\mathfrak{U}, R_\beta)$ be a $\mathfrak{No}$ approximation structure, $X_1 \subseteq \mathfrak{U}$. Then, X_1 is:*

(i) Roughly R_β-definable, if $\underline{R}_\beta(X_1) \neq \phi$ and $\overline{R}_\beta(X_1) \neq \mathfrak{U}$;

(ii) Internally R_β-undefinable, if $\underline{R}_\beta(X_1) = \phi$ and $\overline{R}_\beta(X_1) \neq \mathfrak{U}$;

(iii) Externally R_β-undefinable, if $\underline{R}_\beta(X_1) \neq \phi$ and $\overline{R}_\beta(X_1) = \mathfrak{U}$;

(iv) Totally R_β-undefinable, if $\underline{R}_\beta(X_1) = \phi$ and $\overline{R}_\beta(X_1) = \mathfrak{U}$.

Example 2. *Let $\mathfrak{U} = \{x_1, x_2, x_3, x_4\}$ and $\mathfrak{U}/R = \{\{x_1, x_4\}, \{x_3\}, \{x_2, x_3, x_4\}\}$ be a base of topology defined on $\mathfrak{U}$. Take $\{X_1 = \{x_3, x_4\}\}$, then $\underline{R}_\beta(X_1) = X_1 \neq \phi$ and $\overline{R}_\beta(X_1) = \{x_2, x_3, x_4\}\} \neq \mathfrak{U}$. Therefore,$X_1$ is Roughly R_β-definable.*

Proposition 4. *Let $(\mathfrak{U}, R_\beta)$ be a $\mathfrak{No}$ approximation structure, $X_1, Y_1 \subseteq \mathfrak{U}$. Then,*

(i) $\underline{R}_\beta(X_1) \subseteq X_1 \subseteq \overline{R}_\beta(X_1)$;

(ii) $\underline{R}_\beta(\phi) = \overline{R}_\beta(\phi) = \phi$, $\underline{R}_\beta(X_1) = \overline{R}_\beta(X_1) = X_1$;

(iii) If $X_1 \subseteq Y_1$, then $\underline{R}_\beta(X_1) \subseteq \underline{R}_\beta(Y_1)$ and $\overline{R}_\beta(X_1) \subseteq \overline{R}_\beta(Y_1)$;

(iv) $\underline{R}_\beta(X_1) \cup \underline{R}_\beta(Y_1) = \underline{R}_\beta(X_1 \cup Y_1)$;

(v) $\overline{R}_\beta(X_1 \cap Y_1) = \overline{R}_\beta(X_1) \cap \overline{R}_\beta(Y_1)$;

(vi) $\underline{R}_\beta(X_1^c) = (\overline{R}_\beta(X_1))^c$,$\overline{R}_\beta(X_1^c) = (\underline{R}_\beta(X_1))^c$;

(vii) $\underline{R}_\beta(\underline{R}_\beta(X_1)) = \underline{R}_\beta(X_1)$, $\overline{R}_\beta(\overline{R}_\beta(X_1)) = \overline{R}_\beta(X_1)$.

Proof.

(i) Suppose that $x_1 \in \underline{R}_\beta(X_1)$. Then, $x_1 \in \bigcup\{G \in \beta O(\mathfrak{U}) : G \subseteq X_1\}$ and $\exists G_0 \in \beta O(\mathfrak{U})$ where $x_1 \in G_0 \subseteq X_1$. Therefore, $\underline{R}_\beta(X_1) \subseteq X_1$. Let $x_1 \in X_1$. Then,$x_1 \in F$ by the definition of the $\mathfrak{No}$ upper approximation $\forall F \in \beta C(\mathfrak{U})$. Thus, $X_1 \subseteq \overline{R}_\beta(X_1)$,

(ii) Obvious.

(iii) Assume that, $x_1 \in \underline{R}_\beta(X_1)$, $X_1 \subseteq Y_1$. Therefore, $x_1 \in \underline{R}_\beta(Y_1)$ by the definition of the $\mathfrak{No}$ upper approximation. Moreover, let $X_1 \notin \overline{R}_\beta(Y_1)$, hence $X_1 \notin \overline{R}_\beta(Y_1) = \bigcap\{F \in \beta C(\mathfrak{U}) : Y_1 \subseteq F\}$, then there exists $Y_1 \subseteq F \in \beta C(\mathfrak{U})$ and $X_1 \notin F$. This leads to $\exists F \in \beta C(\mathfrak{U})$, $X_1 \subseteq Y_1 \subseteq F$ and $X_1 \notin F$ and $X_1 \notin \overline{R}_\beta(X_1) = \bigcap\{F \in \beta C(\mathfrak{U}) : X_1 \subseteq F\}$. Hence, $X_1 \notin \overline{R}_\beta(X_1)$ and $\overline{R}_\beta(X_1) \subseteq \overline{R}_\beta(Y_1)$.

(iv) Since $X_1 \subseteq X_1 \cup Y_1$, $Y_1 \subseteq X_1 \cup Y_1$, then $\underline{R}_\beta(X_1) \subseteq \underline{R}_\beta(X_1 \cup Y_1)$,$\underline{R}_\beta(Y_1) \subseteq \underline{R}_\beta(X_1 \cup Y_1)$. Thus, $\underline{R}_\beta(X_1) \cup \underline{R}_\beta(Y_1) \subseteq \underline{R}_\beta(X_1 \cup Y_1)$. Let $x_1 \in \underline{R}_\beta(X_1 \cup Y_1)$, then $x_1 \in \bigcup\{G \in \beta O(U) : G \subseteq X_1 \cup Y_1\}$. Thus, $\exists G_0 \in \beta O(\mathfrak{U})$ where $x_1 \in G_0 \subseteq X_1 \cup Y_1$. There exists three cases:
Case (1) If $G_0 \subset X_1, X_1 \in X_1$, thus $x_1 \in \underline{R}_\beta(X_1)$.
Case (2) If $G_0 \cap X_1 = \phi$, then $G_0 \subset Y$ and $x_1 \in G_0$, so $x_1 \in \underline{R}_\beta(X_1)$.
Case (3) If $G_0 \cap X_1 \neq \phi$, where $x_1 \in G_0$ and G_0 is the β-open set, therefore $x_1 \in \beta C(X_1)$, and hence $x_1 \in \underline{R}_\beta(X_1)$.
From three cases, $x_1 \in \underline{R}_\beta(X_1) \cup \underline{R}_\beta(Y_1)$.

(v) Similar to (iv);

(vi) Let $x_1 \in \underline{R}_\beta(X_1^c)$. Then, $x_1 \in \bigcup\{G \in \beta O(U) : G \subseteq X_1^c\}$. So, $\exists G_0 \in \beta O(\mathfrak{U})$ such that $x_1 \in G_0 \subseteq X_1^c$. Then, there exists G_0^c, such that $X_1 \subseteq G_0^c$ and $X_1 \notin G_0^c$, $G_0^c \in \beta C(\mathfrak{U})$. Hence, $X_1 \notin \overline{R}_\beta(X_1)$. Thus, $x_1 \in (\overline{R}_\beta(X_1))^c$ and $\underline{R}_\beta(X_1^c) = (\overline{R}_\beta(X_1))^c$. Moreover, we can prove that $\overline{R}_\beta(X_1^c) = (\underline{R}_\beta(X_1))^c$.

(vii) Since $\underline{R}_\beta(X_1) = \bigcup\{G \in \beta O(\mathfrak{U}) : G \subseteq X_1\}$. Therefore, $\underline{R}_\beta(\underline{R}_\beta(X_1)) = \bigcup\{\bigcup\{G \in \beta O(\mathfrak{U}) : G \subseteq X_1\}\} = \bigcup\{G \in \beta O(\mathfrak{U}) : G \subseteq X_1\} = \underline{R}_\beta(X_1)$. $\overline{R}_\beta(\overline{R}_\beta(X_1)) = \overline{R}_\beta((\underline{R}_\beta(X_1^c))^c) = (\underline{R}_\beta((\underline{R}_\beta(X_1^c))^c))^c = (\underline{R}_\beta(X_1^c))^c = (\overline{R}_\beta(X_1^c))^c = \overline{R}_\beta(X_1)$. □

Example 3. *Consider* $\mathfrak{U} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ *and* $\mathfrak{U}/R = \{\{x_3\}, \{x_1, x_2, x_5\}\}$ *is a base of topology defined on* $\mathfrak{U}$ *. If* $\{X_1 = \{x_3, x_4, x_5\}\}$, $\underline{R}_\beta(X_1) = \{x_3, x_5\}$ *and* $\overline{R}_\beta(X_1) = \{x_3, x_4, x_5\}\}$. *Therefore,* $\underline{R}_\beta(X_1) \subseteq X_1$ *and* $X_1 \subseteq \overline{R}_\beta(X_1)$.

4. COVID-19 in Terms of Topological $\mathfrak{N}\mathfrak{o}$

Fever, fatigue, and a dry hack are the most well-known symptoms of COVID-19 infection. Torment and throbs, nasal blockage, migraine, conjunctivitis, sore throat, the runs, loss of taste or smell, a rash and discoloration of the fingers or toes are some of the less common adverse effects that a few people may experience. These manifestations are typically mild and begin gradually. Only a few people are able to be contained without mild symptoms. Symptoms may vary from one country to another and change from common to mild or strong symptoms and may become a fleeting symptom. With the occurrence of mutations in COVID-19, new symptoms appear and differ from one country to another in their severity, and other symptoms disappear. However, there are symptoms such as fever, fatigue, and dehydration that persist despite these mutations [8].

In this application, we analyze the data of a group of patients. They demonstrated a group of different symptoms. Their data was in the following Table 1, where high temperature, breathing difficulty, physical strain, sore throat, and lack of smell are represented by the symbols $\mathfrak{A}_1, \mathfrak{A}_2, \mathfrak{A}_3, \mathfrak{A}_4, \mathfrak{A}_5$, respectively. 1 refers to "Yes", 0 refers to "No", + refers to "Positive", and − refers to "Negative".

Table 1. The side effects of COVID-19 infection.

Patients	$\mathfrak{A}_1$	$\mathfrak{A}_2$	$\mathfrak{A}_3$	$\mathfrak{A}_4$	$\mathfrak{A}_5$	*Decision*
$\mathfrak{p}_1$	1	1	1	1	1	+
$\mathfrak{p}_2$	1	1	1	1	0	+
$\mathfrak{p}_3$	1	1	1	0	1	−
$\mathfrak{p}_4$	1	1	1	0	0	−
$\mathfrak{p}_5$	1	1	0	1	1	−
$\mathfrak{p}_6$	1	1	0	1	0	−
$\mathfrak{p}_7$	1	1	0	0	1	−
$\mathfrak{p}_8$	1	1	0	0	0	−
$\mathfrak{p}_9$	1	0	1	1	1	−
$\mathfrak{p}_{10}$	1	0	1	1	0	−
$\mathfrak{p}_{11}$	1	0	1	0	1	−
$\mathfrak{p}_{12}$	1	0	1	0	0	−
$\mathfrak{p}_{13}$	1	0	0	1	1	−
$\mathfrak{p}_{14}$	1	0	0	1	0	−
$\mathfrak{p}_{15}$	1	0	0	0	1	−
$\mathfrak{p}_{16}$	1	0	0	0	0	−
$\mathfrak{p}_{17}$	0	1	1	1	1	+
$\mathfrak{p}_{18}$	0	1	1	1	0	+
$\mathfrak{p}_{19}$	0	1	1	0	1	−
$\mathfrak{p}_{20}$	0	1	1	0	0	−

Table 1. *Cont.*

Patients	$\mathfrak{A}_1$	$\mathfrak{A}_2$	$\mathfrak{A}_3$	$\mathfrak{A}_4$	$\mathfrak{A}_5$	*Decision*
$\mathfrak{p}_{21}$	0	1	0	1	1	$-$
$\mathfrak{p}_{22}$	0	1	0	1	0	$-$
$\mathfrak{p}_{23}$	0	1	0	0	1	$-$
$\mathfrak{p}_{24}$	0	1	0	0	0	$-$
$\mathfrak{p}_{25}$	0	0	1	1	1	$-$
$\mathfrak{p}_{26}$	0	0	1	1	0	$-$
$\mathfrak{p}_{27}$	0	0	1	0	1	$-$
$\mathfrak{p}_{15}$	1	0	0	0	1	$-$
$\mathfrak{p}_{28}$	0	0	1	0	0	$-$
$\mathfrak{p}_{29}$	0	0	0	1	1	$-$
$\mathfrak{p}_{30}$	0	0	0	1	0	$-$
$\mathfrak{p}_{31}$	0	0	0	0	1	$-$
$\mathfrak{p}_{32}$	0	0	0	0	0	$-$

Based on the information system of COVID-19 in Table 1 and the $\mathfrak{No}$ approximation structure, we will discover and predict disease-causing factors and disease detection using a new algorithm.

Algorithm of the Side Effects of COVID-19 Infection

Step 1: Input $\mathfrak{U}$ is the universe of discourse, R is the set of condition attributes, and C is the decision attribute.
Step 2: Find Pawlak's L_{app}, U_{app} and the boundary of any set $X_1 \subseteq U$.
Step 3: Remove any attribute $A_i \in R$, take $U/R - A_i$ as a base for topology, and find the set $\beta O(\mathfrak{U})$.
Step 4: Calculate $\beta - L_{app}$, $\beta - U_{app}$ and the boundary of the set X_1.
Step 5: If the boundary of X_1 in Step 2 and Step 4 are the same, then $\mathfrak{A}_{\mathrm{i}}$ is a superfluous attribute.
Step 6: Repeat Step 3, Step 4 and Step 5 for all condition attributes and find the reduct (R).

Continued from Table 1:
Step 1: Let $\mathfrak{U} = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4, \mathfrak{p}_5, \mathfrak{p}_6, \mathfrak{p}_7, \mathfrak{p}_8, \mathfrak{p}_9\}$ be the set of patients, $R = \{\mathfrak{A}_1, \mathfrak{A}_2, \mathfrak{A}_3, \mathfrak{A}_4, \mathfrak{A}_5\}$ be the condition attributes, $C = \{+, -\}$ be the decision attributes, and $X_1 = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_{17}, \mathfrak{p}_{18}\}$ be the set of patients having positive results. Then, $U/R = \{\{\mathfrak{p}_1\}, \{\mathfrak{p}_2\}, \{\mathfrak{p}_3\}, \{\mathfrak{p}_4\}, \{\mathfrak{p}_5\}, \{\mathfrak{p}_6\}, \{\mathfrak{p}_7\}, \{\mathfrak{p}_8\}, \{\mathfrak{p}_9\}, \{\mathfrak{p}_{10}\}, \{\mathfrak{p}_{11}\}, \{\mathfrak{p}_{12}\}, \{\mathfrak{p}_{13}\}, \{\mathfrak{p}_{14}\}, \{\mathfrak{p}_{15}\}, \{\mathfrak{p}_{16}\}, \{\mathfrak{p}_{17}\}, \{\mathfrak{p}_{18}\}, \{\mathfrak{p}_{19}\}, \{\mathfrak{p}_{20}\}, \{\mathfrak{p}_{21}\}, \{\mathfrak{p}_{22}\}, \{\mathfrak{p}_{23}\}, \{\mathfrak{p}_{24}\}, \{\mathfrak{p}_{25}\}, \{\mathfrak{p}_{26}\}, \{\mathfrak{p}_{27}\}, \{\mathfrak{p}_{28}\}, \{\mathfrak{p}_{29}\}, \{\mathfrak{p}_{30}\}, \{\mathfrak{p}_{31}\}, \{\mathfrak{p}_{32}\}\}$.
Step 2: Pawlak's L_{app} and U_{app} of X_1 is: $\underline{R}(X_1) = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_{17}, \mathfrak{p}_{18}\}$, $\overline{R}(X_1) = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_{17}, \mathfrak{p}_{18}\}$, $BN_R(X_1) = \phi$.
Step 3: Case (i) Remove the attribute $\mathfrak{A}_1$, then $U/R - (\mathfrak{A}_1) = \{\{\mathfrak{p}_1, \mathfrak{p}_{17}\}, \{\mathfrak{p}_2, \mathfrak{p}_{18}\}, \{\mathfrak{p}_3, \mathfrak{p}_{19}\}, \{\mathfrak{p}_4, \mathfrak{p}_{20}\}, \{\mathfrak{p}_5, \mathfrak{p}_{21}\}, \{\mathfrak{p}_6, \mathfrak{p}_{22}\}, \{\mathfrak{p}_7, \mathfrak{p}_{23}\}, \{\mathfrak{p}_8, \mathfrak{p}_{24}\}, \{\mathfrak{p}_9, \mathfrak{p}_{25}\}, \{\mathfrak{p}_{10}, \mathfrak{p}_{26}\}, \{\mathfrak{p}_{11}, \mathfrak{p}_{27}\}, \{\mathfrak{p}_{12}, \mathfrak{p}_{28}\}, \{\mathfrak{p}_{13}, \mathfrak{p}_{29}\}, \{\mathfrak{p}_{14}, \mathfrak{p}_{30}\}, \{\mathfrak{p}_{15}, \mathfrak{p}_{31}\}, \{\mathfrak{p}_{16}, \mathfrak{p}_{32}\}\}$ is a base for topology; we can deduce the set $\beta O(\mathfrak{U})$.
Step 4: The $\beta - L_{app}$, $\beta - U_{app}$ and boundary of X_1 are: $\underline{R}_{\beta}(X_1) = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_{17}, \mathfrak{p}_{18}\}$, $\overline{R}_{\beta}(X_1) = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_{17}, \mathfrak{p}_{18}\}$ and $BN_{\beta}(X_1) = \phi$.
Step 5: Since $BN_{\beta}(X_1) = \phi = BN_R(X_1) = \phi$, then $\mathfrak{A}_1$ is a superfluous attribute which means it is not necessary for patients having positive results.
Step 6: Case (ii) Remove the attribute $\mathfrak{A}_2$, then $U/R - (\mathfrak{A}_2) = \{\{\mathfrak{p}_1, \mathfrak{p}_9\}, \{\mathfrak{p}_2, \mathfrak{p}_{10}\}, \{\mathfrak{p}_3, \mathfrak{p}_{11}\}, \{\mathfrak{p}_6, \mathfrak{p}_{14}\}, \{\mathfrak{p}_7, \mathfrak{p}_{15}\}, \{\mathfrak{p}_6, \mathfrak{p}_{22}\}, \{\mathfrak{p}_7, \mathfrak{p}_{23}\}, \{\mathfrak{p}_8, \mathfrak{p}_{24}\}, \{\mathfrak{p}_9, \mathfrak{p}_{25}\}, \{\mathfrak{p}_9, \mathfrak{p}_{25}\}, \{\mathfrak{p}_{10}, \mathfrak{p}_{26}\}, \{\mathfrak{p}_{11}, \mathfrak{p}_{27}\}, \{\mathfrak{p}_4, \mathfrak{p}_{12}\}, \{\mathfrak{p}_5, \mathfrak{p}_{13}\}\{\mathfrak{p}_8, \mathfrak{p}_{16}\}, \{\mathfrak{p}_{17}, \mathfrak{p}_{25}\}, \{\mathfrak{p}_{18}, \mathfrak{p}_{26}\}, \{\mathfrak{p}_{19}, \mathfrak{p}_{27}\}, \{\mathfrak{p}_{20}, \mathfrak{p}_{28}\}, \{\mathfrak{p}_{21}, \mathfrak{p}_{29}\}, \{\mathfrak{p}_{22}, \mathfrak{p}_{30}\}, \{\mathfrak{p}_{23}, \mathfrak{p}_{31}\}, \{\mathfrak{p}_{24}, \mathfrak{p}_{32}\}\}$. Therefore, $\underline{R}_{\beta}(X_1) = \phi$, $\overline{R}_{\beta}(X_1) = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_9, \mathfrak{p}_{10}, \mathfrak{p}_{17}, \mathfrak{p}_{18}, \mathfrak{p}_{25}, \mathfrak{p}_{26}\}$ and $BN_{\beta}(X_1) = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_9, \mathfrak{p}_{10}, \mathfrak{p}_{17}, \mathfrak{p}_{18}, \mathfrak{p}_{25}, \mathfrak{p}_{26}\} \neq BN_R(X_1)$.

Case (iii) Remove the attribute $\mathfrak{A}_3$, then $U/R-(\mathfrak{A}_3)=\{\{\mathfrak{p}_1,\mathfrak{p}_5\},\{\mathfrak{p}_2,\mathfrak{p}_6\},\{\mathfrak{p}_3,\mathfrak{p}_7\},\{\mathfrak{p}_4,\mathfrak{p}_8\},\{\mathfrak{p}_9,\mathfrak{p}_{13}\},\{\mathfrak{p}_{10},\mathfrak{p}_{14}\},\{\mathfrak{p}_{11},\mathfrak{p}_{15}\},\{\mathfrak{p}_{12},\mathfrak{p}_{16}\},\{\mathfrak{p}_{17},\mathfrak{p}_{21}\},\{\mathfrak{p}_{18},\mathfrak{p}_{22}\},\{\mathfrak{p}_{19},\mathfrak{p}_{23}\},\{\mathfrak{p}_{20},\mathfrak{p}_{24}\},\{\mathfrak{p}_{25},\mathfrak{p}_{29}\},\{\mathfrak{p}_{26},\mathfrak{p}_{30}\},\{\mathfrak{p}_{27},\mathfrak{p}_{31}\},\{\mathfrak{p}_{28},\mathfrak{p}_{32}\}\}$. Hence, $\underline{R}_\beta(X_1)=\phi$, $\overline{R}_\beta(X_1)=\{\mathfrak{p}_1,\mathfrak{p}_5,\mathfrak{p}_2,\mathfrak{p}_6,\mathfrak{p}_{17},\mathfrak{p}_{21},\mathfrak{p}_{18},\mathfrak{p}_{22}\}$ and $BN_\beta(X_1)=\{\mathfrak{p}_1,\mathfrak{p}_5,\mathfrak{p}_2,\mathfrak{p}_6,\mathfrak{p}_{17},\mathfrak{p}_{21},\mathfrak{p}_{18},\mathfrak{p}_{22}\}\neq BN_R(X_1)$.
Case (iv) Remove the attribute $\mathfrak{A}_4$, then $U/R-(\mathfrak{A}_4)=\{\{\mathfrak{p}_1,\mathfrak{p}_3\},\{\mathfrak{p}_2,\mathfrak{p}_4\},\{\mathfrak{p}_5,\mathfrak{p}_7\},\{\mathfrak{p}_6,\mathfrak{p}_8\},\{\mathfrak{p}_9,\mathfrak{p}_{11}\},\{\mathfrak{p}_{10},\mathfrak{p}_{12}\},\{\mathfrak{p}_{13},\mathfrak{p}_{15}\},\{\mathfrak{p}_{14},\mathfrak{p}_{16}\},\{\mathfrak{p}_{17},\mathfrak{p}_{19}\},\{\mathfrak{p}_{18},\mathfrak{p}_{20}\},\{\mathfrak{p}_{21},\mathfrak{p}_{23}\},\{\mathfrak{p}_{22},\mathfrak{p}_{24}\},\{\mathfrak{p}_{25},\mathfrak{p}_{27}\},\{\mathfrak{p}_{26},\mathfrak{p}_{28}\},\{\mathfrak{p}_{29},\mathfrak{p}_{31}\},\{\mathfrak{p}_{30},\mathfrak{p}_{32}\}\}$. Hence, $\underline{R}_\beta(X_1)=\phi$, $\overline{R}_\beta(X_1)=\{\mathfrak{p}_1,\mathfrak{p}_3,\mathfrak{p}_2,\mathfrak{p}_4,\mathfrak{p}_{17},\mathfrak{p}_{19},\mathfrak{p}_{18},\mathfrak{p}_{20}\}$ and $BN_\beta(X_1)=\{\mathfrak{p}_1,\mathfrak{p}_3,\mathfrak{p}_2,\mathfrak{p}_4,\mathfrak{p}_{17},\mathfrak{p}_{19},\mathfrak{p}_{18},\mathfrak{p}_{20}\}\neq BN_R(X_1)$.
Case (v) Remove the attribute $\mathfrak{A}_5$, then $\mathfrak{U}/R-(\mathfrak{A}_5)=\{\{\mathfrak{p}_1,P_2\},\{\mathfrak{p}_3,\mathfrak{p}_4\},\{\mathfrak{p}_5,\mathfrak{p}_6\},\{\mathfrak{p}_7,\mathfrak{p}_8\},\{\mathfrak{p}_9,\mathfrak{p}_{10}\},\{\mathfrak{p}_{11},\mathfrak{p}_{12}\},\{\mathfrak{p}_{13},\mathfrak{p}_{14}\},\{\mathfrak{p}_{15},\mathfrak{p}_{16}\},\{\mathfrak{p}_{17},\mathfrak{p}_{18}\},\{\mathfrak{p}_{19},\mathfrak{p}_{20}\},\{\mathfrak{p}_{21},\mathfrak{p}_{22}\},\{\mathfrak{p}_{23},\mathfrak{p}_{24}\},\{\mathfrak{p}_{25},\mathfrak{p}_{26}\},\{\mathfrak{p}_{27},\mathfrak{p}_{28}\},\{\mathfrak{p}_{29},\mathfrak{p}_{30}\},\{\mathfrak{p}_{31},\mathfrak{p}_{32}\}\}$. Hence, $\underline{R}_\beta(X_1)=\{\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_{17},\mathfrak{p}_{18}\}$, $\overline{R}_\beta(X_1)=\{\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_{17},\mathfrak{p}_{18}\}$ and $BN_\beta(X_1)=\phi$.
Therefore, reduct$(\mathfrak{R})=\{\mathfrak{A}_2,\mathfrak{A}_3,\mathfrak{A}_4\}$. Similarly, if $\mathfrak{Y}$ is the set of patients having a negative result, then again reduct $(\mathfrak{R})=\{\mathfrak{A}_2,\mathfrak{A}_3,\mathfrak{A}_4\}$.

Remark 1. *From the previous application, we conclude that only the symptoms that make up the reduct confirm the presence of the disease; therefore, appropriate preventive measures must be taken, given a positive situation.*

5. Conclusions

Rough topology is a useful mathematical tool in the process of decision making for many problems in daily life. Within this paper, we established a new combination between rough approximation and some topological concepts. Some properties of the new approximation structure are discussed. We design an algorithm depending on our approach for detecting COVID-19 disease. The paper concluded that breathing difficulty, physical strain, and sore throat, which are represented by the symbols $\mathfrak{A}_2,\mathfrak{A}_3$ and $\mathfrak{A}_4$, respectively, are closely connected to the disease (COVID-19). Through future work, we will investigate the challenges associated with human blood circulation, study the relations between diseases such as COVID-19, SARS..., etc., study mutations, repair mutations, make novelty mutations in plants and animals, and treat diseases.

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