

# Editorial: Overview and Some New Directions

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## 1. Overview of the Published Papers

The Special Issue contains eleven accepted and published submissions to a Special Issue of the MDPI journal *Axioms* on the subject of “Nonlinear Dynamical Systems with Applications”. In this volume, the invited authors have submitted their latest results on nonlinear dynamical systems and related various applications. All papers have been accepted after a rigorous reviewing process.

In their study [1], Sarem H. Hadi, Maslina Darus and Alina Alb Lupaş considered a class of Janowski-type  $(p, q)$ -convex harmonic functions involving a generalized  $q$ -Mittag-Leffler function. This research aims to present a linear operator  $\mathcal{L}_{p,q}^{\rho,\sigma,\mu} f$  by utilizing the  $q$ -Mittag-Leffler function and to introduce the subclass of harmonic  $p, q$ -convex functions  $\mathcal{HT}_{p,q}(\vartheta, \mathcal{W}, \mathcal{V})$  related to the Janowski function. For the harmonic  $p$ -valent functions  $f$  class, they investigated the harmonic geometric properties, such as coefficient estimates, convex linear combination, extreme points, and Hadamard product. Finally, the closure property was derived via using the subclass  $\mathcal{HT}_{p,q}(\vartheta, \mathcal{W}, \mathcal{V})$ , and the theory of fractional  $q$ -calculus operators in geometric function theory was developed.

In another study [2], Barić, Josipa provided a research article on the Levinson’s functional in time-scale settings. In their paper, a Levinson functional on time scales was introduced using the integral inequality of Levinson’s type in the terms of  $\Delta$ -integral for convex (concave) functions on time-scale sets, and the relevant properties of Levinson functionals were obtained, such as superadditivity and monotonicity. Then, the authors defined some new types of functionals using weighted generalized and power means on time scales, and proved their properties. These can be employed in future works to obtain refinements and converses of known integral inequalities on time scales.

The research of Gunasekaran Nallaselli, Arul Joseph Gnanaprakasam, Gunaseelan Mani, Ozgur Ege, Dania Santina and Nabil Mlaiki in reference [3] involves a study on fixed-point techniques under the  $\alpha$ - $F$ -convex contraction with an application. In this paper, they consider several classes of mappings related to the class of  $\alpha$ - $\Sigma$ -contraction mappings by introducing a convexity condition and establish some fixed-point theorems for such mappings in complete metric spaces. The result extends and generalizes the well-known results on  $\alpha$ -admissible and convex contraction mappings, and many others in the existing literature. An illustrative example is also provided to exhibit the utility of their main results. Finally, they derive the existence and uniqueness of a solution to an integral equation to support their main result and provide a numerical example to validate the application of their obtained results. In this paper, the authors extend and generalize their main theorem into an  $\alpha$ - $\Sigma$ -convex contraction of seven possible values (with rational type) in complete metric spaces inspired and motivated by previous research. Examples and applications to integral equations are provided to illustrate the usability of their obtained results.

The authors of reference [4], Xiaolan Yuan and Yusheng Zhou, produce a design of state-dependent switching rules for second-order switched linear systems revisited. For switched systems, differential equations are used to describe the dynamic behavior of the



**Citation:** Zeng, S.; Migórski, S.; Liu, Y. Editorial: Overview and Some New Directions. *Axioms* **2023**, *12*, 553. <https://doi.org/10.3390/axioms12060553>

Received: 1 June 2023

Accepted: 2 June 2023

Published: 4 June 2023



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continuous characteristic, which are marked as subsystems. A piecewise constant function is adopted to describe the discrete characteristic, which is referred to as a switching rule. An important part of hybrid systems, switched systems have been exhibited in many practical fields. This paper focuses on the asymptotic stability of second-order switched linear systems with positive real part conjugate complex roots for each subsystem. Compared with available studies, a more appropriate state-dependent switching rule is designed to stabilize a switched system with the phase trajectories of two subsystems rotating outward in the same direction or the opposite direction. Finally, several numerical examples are used to illustrate the effectiveness and superiority of the proposed method.

In the research presented in reference [5], Xinyue Zhu, Wei Li and Xueping Luo considered stability for a class of differential set-valued inverse variational inequalities in finite dimensional spaces. In this paper, the authors introduce and study a new class of differential set-valued inverse variational inequalities in finite dimensional spaces. By applying a result to differential inclusions involving an upper semi-continuous set-valued mapping with closed convex values, the authors first prove the existence of Carathéodory weak solutions for differential set-valued inverse variational inequalities. Then, using the existence result, the authors establish the stability for the differential set-valued inverse variational inequality problem when the constraint set and the mapping are perturbed by two different parameters. The closedness and continuity of Carathéodory weak solutions with respect to the two different parameters are obtained.

Some novel conditions for the stability results for a class of fractional-order quasi-linear impulsive integro-differential systems with multiple delays are discussed by Mathiyalagan Kalidass, Shengda Zeng and Mehmet Yavuz [6]. First, the existence and uniqueness of mild solutions for the considered system are discussed using contraction mapping theorem. Then, novel conditions for the Mittag-Leffler stability (MLS) of the considered system are established by using well-known mathematical techniques, and further, the two corollaries are deduced, which provides some new results. Finally, an example is provided to illustrate the applications of the results. The Mittag-Leffler stability of a fractional-order system (FOS) has not been fully investigated, which motivated the authors of the present study. Thus, in this study, the existence and uniqueness of solutions and MLS analysis of the impulsive quasi-linear FOS with multiple time delays are established using the well-known fixed point theorems and Mittag-Leffler approach. Furthermore, the main contribution of this paper lies in deriving new stability conditions for the fractional-order quasi-linear system with nonlocal conditions, multiple time delays and impulses. Novel conditions for the Mittag-Leffler stability of FOSs are established. The existence and uniqueness of mild solutions for the FOS are discussed with help of the contraction mapping principle. Finally, an example is provided to show the applicability of the results.

For the study presented in reference [7], Yunru Bai, Leszek Gasinski, and Nikolaos S. Papageorgiou consider nonlinear eigenvalue problems for the Dirichlet  $(p, 2)$ -Laplacian. The authors consider a nonlinear eigenvalue problem driven by the Dirichlet  $(p, 2)$ -Laplacian. The parametric reaction is a Carathéodory function, which exhibits  $(p - 1)$ -sublinear growth as  $x \rightarrow \infty$  and as  $x \rightarrow 0^+$ . Using variational tools and truncation and comparison techniques, the authors prove a bifurcation-type theorem describing the spectrum as  $\lambda > 0$  varies. The authors also prove the existence of a smallest positive eigenfunction for every eigenvalue. Finally, the authors indicate how the result can be extended to  $(p, q)$ -equations ( $q \neq 2$ ).

Considering the importance of the nutrient-phytoplankton, in reference [8], Ruizhi Yang, Liye Wang and Dan Jin consider Hopf bifurcation analysis of a diffusive nutrient-phytoplankton model with time delay. One of the most complex and difficult problems in water pollution treatment is the prevention and control of algal bloom. Due to the complexity of the pollution source and the difficulty factor of material removal, a lot of energy is required, but it is not very effective. Therefore, scientists are searching for better methods to prevent and cure algal bloom, especially using mathematical models, in order to find reasonable prevention and cure measures. In this paper, the authors studied a

nutrient-phytoplankton model with time delay and diffusion term. The authors studied the Turing instability, local stability, and the existence of Hopf bifurcation. Some formulas are obtained to determine the direction of the bifurcation and the stability of periodic solutions by the central manifold theory and normal form method. Finally, the conclusion is verified through numerical simulation.

New results on the Darboux transformation and  $N$ -soliton solutions of Gerdjikov–Ivanov equation on a time–space scale equation are presented by Huanhe Dong, Xiaoqian Huang, Yong Zhang, Mingshuo Liu and Yong Fang in [9]. The Gerdjikov–Ivanov (GI) equation is one type of derivative nonlinear Schrödinger equation that is widely used in quantum field theory, nonlinear optics, weakly nonlinear dispersion water waves and other fields. In this paper, the coupled GI equation on a time–space scale is deduced from Lax pairs and the zero curvature equation on a time–space scale, which can be reduced to the classical and the semi-discrete GI equation by considering different time–space scales. Furthermore, the Darboux transformation (DT) of the GI equation on a time–space scale is constructed via gauge transformation. Finally,  $N$ -soliton solutions of the GI equation are provided through applying its DT, which is expressed by the Cayley exponential function. One-soliton solutions are obtained at three different time–space scales ( $\mathbb{X} = \mathbb{R}$ ,  $\mathbb{X} = \mathbb{C}$ ,  $\mathbb{X} = \mathbb{K}_p$ ).

The research of Dumitru Motreanu is presented in [10]. The result concerns two Dirichlet boundary value problems whose differential operators in the principal part exhibit a lack of ellipticity and contain a convection term (depending on the solution and its gradient). They are driven by a degenerated  $(p, q)$ -Laplacian with weights and a competing  $(p, q)$ -Laplacian with weights, respectively. The notion of competing  $(p, q)$ -Laplacians with weights is considered for the first time. The author presents existence and approximation results that hold under the same set of hypotheses for the convection term for both problems. The proofs are based on weighted Sobolev spaces, Nemytskij operators, a fixed-point argument and finite dimensional approximation. A detailed example illustrates the effective applicability of the results.

New results are obtained concerning the global directed dynamic behaviors of a Lotka–Volterra competition–diffusion–advection system by Lili Chen, Shilei Lin and Yanfeng Zhao in [11]. Motivated by the aforementioned studies, the authors investigate the problem of the global directed dynamic behaviors of a Lotka–Volterra advection system between two organisms in heterogeneous environments, where the two organisms are competing for different fundamental resources, their advection and diffusion strategies follow the dispersal towards a positive distribution, and the functions of inter-specific competition ability are variable. This paper investigates the problem of the global directed dynamic behaviors of a Lotka–Volterra competition–diffusion–advection system between two organisms in heterogeneous environments. The two organisms not only compete for different basic resources, but also the advection and diffusion strategies follow the dispersal towards a positive distribution. By virtue of the principal eigenvalue theory, the linear stability of the co-existing steady state is established. Furthermore, the classification of dynamical behaviors is shown by utilizing the monotone dynamical system theory. This work can be seen as a further development of a competition–diffusion system.

## 2. Conclusions

The eleven papers published in the Special Issue on “Nonlinear Dynamical Systems with Applications” concern a broad range of subjects regarding Janowski-Type  $(p, q)$ -convex harmonic functions, Levinson’s functional, fixed-point techniques, dynamical systems, differential set-valued inverse variational inequalities, fractional-order quasi-linear impulsive integro-differential systems, nonlinear eigenvalue problems, the nutrient–phytoplankton problem, Gerdjikov–Ivanov equation and dynamic behaviors. Researchers interested in nonlinear dynamical systems with applications and related topics will find interesting insights and inspiring results in this volume.

**Conflicts of Interest:** The authors declare no conflict of interest.

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