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A Two-Stage Large Group Decision-Making Method Based on a Self-Confident Double Hierarchy Interval Hesitant Fuzzy Language

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Abstract: With the development of the cloud computing era, the decision-making environment and algorithm models have become increasingly complex, and traditional decision-making methods have been unable to meet the needs of large group decision-making (LGDM) problems. Firstly, in order to solve this problem, the concept of double hierarchy interval hesitant fuzzy language (DHIHFL) is proposed. Compared with the traditional double hierarchy hesitant fuzzy language (DHHFL), it contains all elements from the lower limit to the upper limit and more comprehensively characterizes the hesitation of language information. Secondly, for LGDM problems, a self-confident double hierarchy interval hesitant fuzzy language (SC-DHIHFL) is developed, and the integration of self-confident degree can better enrich the evaluation information and promote the achievement of group consensus. Thirdly, a new two-stage LGDM method is proposed. The first stage is clustering and grouping and reaching consensus within the group, and the second stage is the integration of LGDM information. The two-stage method contains novel methods such as expert clustering algorithm, subjective and objective comprehensive weight, consensus degree, and deviation weight considering minority opinions. Finally, the proposed LGDM consensus method is applied to a practical LGDM problem, and the effectiveness is verified by comparative analysis with existing methods.

Keywords: large group decision-making (LGDM); double hierarchy hesitant fuzzy language (DHHFL); self-confident double hierarchy interval hesitant fuzzy language (SC-DHIHFL); two-stage method; fuzzy theory; consensus model; clustering algorithm; subjective and objective comprehensive weight

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1. Introduction

With the development of the cloud computing era, the decision-making environment has undergone profound changes. Large group decision-making (LGDM) has become a common decision-making mode, widely used in government, enterprise, education, and other fields [1]. LGDM can reduce the blind spots of single thinking, consider problems from multiple perspectives and multi-thinking, and improve the quality of decision-making. However, LGDM is challenging, and as the number of participants increases, participants are usually 20 or more, LGDM problems tend to become more complex [2]. It is difficult for individuals involved in LGDM to communicate and cooperate effectively, and there are often constant differences, which consume a lot of energy and time. Therefore, how to optimize the decision-making process of large groups and improve the quality of decision-making, the consensus model of LGDM has become a hot and difficult issue in current research [3].

For the optimization problem of LGDM, scholars have carried out research from two aspects: the language expression model and the decision-making method. First of all, in LGDM problems, it is particularly important for experts to describe language information reasonably. Zadeh [4] proposed the fuzzy language method, which can use natural language to express the qualitative decision-making information of experts, and provides research ideas for decision-making problems. However, when a single language term is used to represent the value of a language variable, sometimes it cannot accurately express the true views of experts. To solve this problem, Rodríguez et al. [5] proposed the concept of a hesitant fuzzy language set; through text-free grammar and transformation function, the expert's decision-making language is transformed into an operable hesitant fuzzy language set. Hesitant fuzzy language provides a new and powerful tool to represent the qualitative decision-making information of experts, which attracts more and more scholars' research interest and produces many new research results. E.g., many scholars have extended fuzzy sets and proposed different forms of fuzzy sets such as intuitionistic fuzzy sets [6], interval intuitionistic fuzzy sets [7], and hesitant fuzzy sets [8]. In order to enable fuzzy language to express opinions and information better, many scholars have improved the fuzzy language method and proposed different language expression models, such as the subjunctive language model [9], the interval binary model [10], etc. These language expression models use a single number to express or use a combination of numbers and language, which still has limitations in fully expressing complex information and is prone to information loss. E.g., when evaluating a project, experts may give an opinion of "almost perfect." Traditional language models can only describe "perfect" but cannot accurately express "almost" such degree words or adjectives. In order to describe such more complex and precise language information, Gou et al. [11] proposed a double hierarchy hesitant fuzzy linguistic term set (DHHFLTS) to express complex language information through a double hierarchy linguistic term set (DHLTS). The second hierarchy linguistic term set (LTS) is the first hierarchy LTS. Linguistic features or detailed complements for each linguistic term in the term set. Some scholars apply traditional decision-making methods to double hierarchy hesitant fuzzy language (DHHFL) environments, such as TOPSIS [12], ORESTE [13], TODIM [14], ELECTRE [15], DEMATEL [16], and other methods.

In order to better solve LGDM problems, scholars have continuously improved the methods in the process of LGDM, and a relatively complete LGDM theory and method system has been formed [17], e.g., different weight determination methods have been studied, mainly subjective [18], objective [19] and combined subjective and objective methods [20]. Groups are clustered according to different preferences, decision information [21], bionic algorithm [22], and other directions have been studied. Aiming at the differences in the behavior of decision-making subjects, non-cooperative behavior, minority opinions [23], conflict behavior [24], and other aspects are considered for research. For the risks in the LGDM process, research studies are carried out from the aspects of risk preference [25] and psychological behavior [26]. The research field of DHHFL is still in its infancy [27], so there are relatively few studies on LGDM methods based on DHHFL.

At present, most LGDM methods based on DHHFL only consider one or two aspects of the LGDM system, which lacks universality. Firstly, for the advantages and limitations of DHHFL, the concept of double hierarchy interval hesitant fuzzy language (DHIHFL) is proposed in this study, which can describe the hesitation of language information more comprehensively and reduce the loss of language information. Secondly, considering the differences in the behavior of the decision-making subject, self-confidence is integrated on the basis of the DHHFL. Self-confidence is dynamic. When a person's opinion is close to the group's opinion, his confidence will increase, and when it deviates from the group's opinion, it will decrease. At the same time, self-confidence will have an important impact on the process of reaching a consensus in LGDM. Thirdly, based on the advantages of the two-stage approach [28], a new two-stage LGDM method is proposed based on a self-confident double hierarchy interval hesitant fuzzy language (SC-DHIHFL), which is divided into two stages. In the first stage, the experts are clustered and grouped based on the similarity measure, and then the consensus within the group is achieved, including the determination method of subjective and objective comprehensive weight, consensus degree, information identification, adjustment, etc. In the second stage, the intra-group information and inter-group information are integrated. Finally, the inter-large-group decision-making information is obtained, including methods such as considering the deviation weight of minority opinions and the integrated preference coefficient. Finally, the proposed method is applied to an actual LGDM problem, compared and verified by the application in practice with the existing methods, and then the future development direction and application prospects are discussed.

2. Preliminaries

2.1. Double Hierarchy Linguistic Term Set

When experts need to express evaluation information for decision problems, such as 'a little low', single hierarchy linguistic terms cannot describe them completely and accurately. Therefore, Gou et al. [11] proposed a double hierarchy linguistic term set (DHLTS), which consists of two completely independent linguistic term sets (LTS).

Definition 1 ([11]). *Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ and $O = \{o_k | k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}$ be the first hierarchy and second hierarchy LTS, respectively, and they are fully independent. Then a DHLTS, denoted as S_O , is shown as the following mathematic form:*

$$S_O = \left\{ s_{t<o_k>} \middle| t = -\tau, \ldots, -1, 0, 1, \ldots, \tau;\ k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma \right\} \tag{1}$$

where we call $s_{t<o_k>}$ the double hierarchy linguistic term (DHLT), when the first hierarchy term is s_t , the second hierarchy linguistic term is o_k.

Let $S = \{s_{-3} = none, s_{-2} = very\ low, s_{-1} = low, s_0 = medium, s_1 = high, s_2 = very\ high, s_3 = perfect\}$ and $O = \{o_{-2} = far\ from, o_{-1} = a\ little, o_0 = just\ right, o_1 = much, o_2 = entirely\}$ be the first hierarchy and second hierarchy LTS, respectively, as shown in Figure 1.

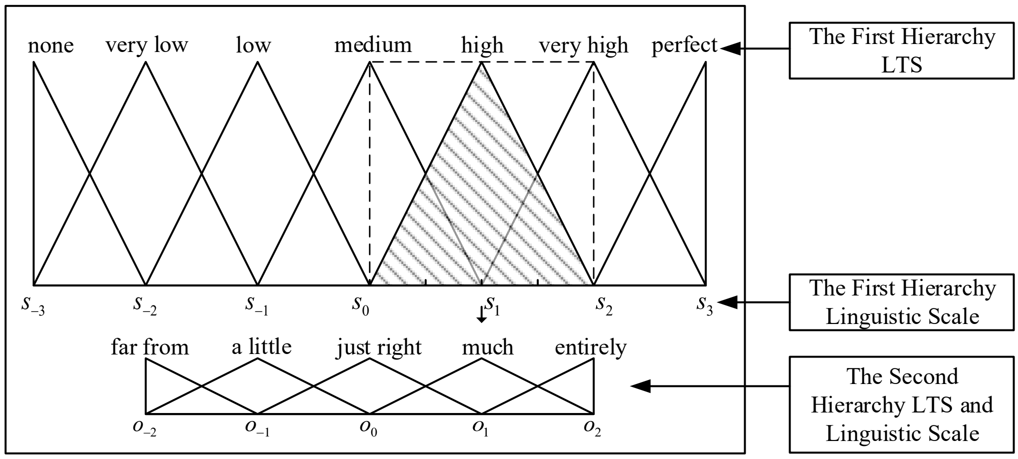

Figure 1. Double hierarchy linguistic term set.

Obviously, we can only express single linguistic terms with DHLTS, but not complex linguistic terms such as 'between much high and a little perfect'. Therefore, Gou et al. [11] developed S_o as hesitant fuzzy linguistic information and developed a double hierarchy hesitant fuzzy linguistic term set (DHHFLTS).

2.2. Double Hierarchy Hesitant Fuzzy Linguistic Term Set

Definition 2 ([11]). *Let X be a fixed set and $S_O = \{s_{t<o_k>} | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau;$ k $= -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}$ be a DHLTS. H_{S_O} is a finite ordered set of DHLT in S_O. A DHHFLTS on X, H_{S_O} is in the mathematical form of*

$$H_{S_O} = \{< x_i, h_{s_o}(x_i) > | x_i \in X\} \tag{2}$$

where $h_{s_o}(x_i)$ is a subset of S_O, indicating that the possible membership of element $x_i \in X$ to set H_{S_O}, as follows:

$$h_{S_O}(x_i) = \left\{ s_{\phi_l<o_{\varphi_l}>}(x_i) \middle| s_{\phi_l<o_{\varphi_l}>} \in S_O; l = 1, 2, \ldots, L; \phi_l = -\tau, \ldots, -1, 0, 1, \ldots, \tau; \varphi_l = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma \right\} \tag{3}$$

where L be the number of DHLTs in $h_{s_o}(x_i)$, and $s_{\phi_l<o_{\varphi_l}>}(x_i)(l = 1, \ldots, L)$ in each $h_{s_o}(x_i)$ be the continuous terms in S_O. For convenience, we call $h_{s_o}(x_i)$ the double hierarchy hesitant fuzzy linguistic element (DHHFLE), representing the possible degree of the linguistic variable x_i to S_O.

Definition 3 ([26]). *Let $\overline{S}_O = \{s_{t<o_k>} | t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$ be a continuous DHLTS, $h_{S_O} = \left\{ s_{\phi_l<o_{\varphi_l}>} \middle| s_{\phi_l<o_{\varphi_l}>} \in \overline{S}_O; l = 1, 2, .., L; \phi_l = [-\tau, \tau]; \varphi_l = [-\varsigma, \varsigma] \right\}$ be a DHHFLE, and let $h_\gamma = \{\gamma_l | \gamma_l \in [0, 1]; l = 1, 2, \ldots, L\}$ be a set of numerical scales with L being the number of linguistic terms in h_{S_O}. Then the equivalent conversion between the subscript $\phi_l < \varphi_l >$ of the DHLT $s_{\phi_l<o_{\varphi_l}>}$ and the real number γ_l can be realized to each other by the following functions f and f^{-1}, respectively:*

$$f : [-\tau, \tau] \times [-\varsigma, \varsigma] \to [0, 1],\ f(\phi_l, \varphi_l) = \frac{\varphi_l + (\tau + \phi_l)\varsigma}{2\varsigma\tau} = \gamma_l \tag{4}$$

$$\begin{aligned} f^{-1} : [0, 1] \to [-\tau, \tau] \times [-\varsigma, \varsigma], f^{-1}(\gamma_l) &= [2\tau\gamma_l - \tau] < \varsigma(2\tau\gamma_l - \tau - [2\tau\gamma_l - \tau]) > \\ &= [2\tau\gamma_l - \tau] + 1 < \varsigma(2\tau\gamma_l - \tau - [2\tau\gamma_l - \tau] - 1) > \end{aligned} \tag{5}$$

where [ ] is a rounding operation.

2.3. The Self-Confident Double Hierarchy Interval Hesitant Fuzzy Language

The DHHFLTS uses $\{s_{1<t_1>}, s_2, s_{3<t_{-1}>}\}$ to describe 'between much high and a little perfect,' but some term elements such as 'a little very high $s_{2<t_{-1}>}$', 'much very high $s_{2<t_1>}$', etc., are missing from the set. The DHIHFL uses $[s_{0<t_1>}, s_{2<t_{-1}>}]$ to express the evaluation information 'between much high and a little perfect', which includes all elements from the lower limit to the upper limit and can describe the hesitation of language information in a more comprehensive and detailed manner.

Definition 4. *Let $\overline{S}_O = \{s_{t<o_k>} | t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$ be a continuous DHLTS, $h_{S_O} = \left\{ s_{\phi_l<o_{\varphi_l}>} \middle| s_{\phi_l<o_{\varphi_l}>} \in \overline{S}_O; l = 1, 2, \ldots, L; \phi_l = [-\tau, \tau]; \varphi_l = [-\varsigma, \varsigma] \right\}$ be a DHHFLE. Then the DHIHFL can be expressed in the following mathematical form:*

$$[h^-_{S_O}, h^+_{S_O}] = \left[\min\left\{ s_{\phi_l<o_{\varphi_l}>} \middle| s_{\phi_l<o_{\varphi_l}>} \in \overline{S}_O \right\}, \max\left\{ s_{\phi_l<o_{\varphi_l}>} \middle| s_{\phi_l<o_{\varphi_l}>} \in \overline{S}_O \right\} \right] \tag{6}$$

where $[h^-_{S_O}, h^+_{S_O}]$ contains all elements from the lower limit $h^-_{S_O}$ to the upper limit $h^+_{S_O}$. Let $[h^-_{S_O}, h^+_{S_O}]$ be a DHIHFL, SC be the semantic value of self-confidence, then $R = ([h^-_{S_O}, h^+_{S_O}], SC)$ can be called an SC-DHIHFL.

Definition 5. *Let* $\overline{S}_O = \{s_{t<o_k>} | t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$ *be a continuous DHLTS,* $h_{S_O} = \left\{ s_{\phi_l<o_{\varphi_l}>} \middle| s_{\phi_l<o_{\varphi_l}>} \in \overline{S}_O; l = 1, 2, \ldots, L; \phi_l = [-\tau, \tau]; \varphi_l = [-\varsigma, \varsigma] \right\}$ *be a DHHFLE. Then we call*

$$E(h_{S_O}) = \frac{1}{L}\sum_{i=1}^{L} f(s_{\phi_l<o_{\varphi_l}>}) \quad (7)$$

the linguistic expected value of DHHFLE.

Definition 6 ([26]). *Let* $S = \{1, 2, \ldots, N\}$ *be the numerical set to express experts' self-confident degrees, where self-confident semantics have N levels. The 7-point numerical set is represented in this study as the self-confidence of experts, where the meaning of each element can be shown in* Table 1 *, as follows:*

Table 1. Detailed information about the 7-point numerical set.

Numerical Value	Semantic Meaning
1	Extremely low self-confidence
2	Very low self-confidence
3	Low self-confidence
4	Medium self-confidence
5	High self-confidence
6	Very high self-confidence
7	Extremely high self-confidence

3. A Consensus Model for Large Group Decision-Making Based on a Self-Confident Double Hierarchy Interval Hesitant Fuzzy Language

With the advent of the era of big data, the amount of data and the complexity of the model for decision-making problems are increasing day by day, GDM problems have gradually evolved into LGDM problems, and traditional decision-making methods are no longer competent. Specifically, compared with GDM, the most obvious feature of LGDM problems is that the number of experts is at least 20, including the alternative set $A = \{A_1, A_2, \ldots, A_n\}(n \geq 3)$ and the expert set $E = \{e^1, e^2, \ldots, e^m\}(m \geq 20)$ [17]. First of all, each expert gives the corresponding decision-making language information according to their own knowledge and understanding of the alternatives, which is transformed into SC-DHIHFL. Then, a consensus model for LGDM based on SC- DHIHFL is proposed in this study, including the expert clustering algorithm based on a similarity measure, the determination method of subjective and objective comprehensive weight, and consensus degree, as follows.

3.1. The Expert Clustering Algorithm Based on Similarity Measure

In the context of LGDM, the number of experts involved is huge, and there is a certain degree of heterogeneity among experts. Therefore, experts with similar decision-making information need to be divided into a group, and the key to effective grouping is how to consider the similarity measure between expert decision-making information. First, an improved expert clustering algorithm based on similarity measures is proposed in this study, and the distance measure and similarity measure between experts are defined. Then, the consideration criteria of expert grouping are considered in the clustering process, which is different from other clustering methods. Finally, the threshold change rate is introduced to determine the appropriate grouping results. The specific steps of the expert clustering algorithm are as follows:

Step 1: The similarity measure $sd(e^a, e^b) = \frac{1}{1+d(e^a, e^b)}$ between any two experts (e^a, e^b) $(a, b = 1, 2, \ldots, m)$ is constructed. Where $d(e^a, e^b)$ is the distance measure between any two experts (e^a, e^b), which can be obtained as follows:

$$d(e^a, e^b) = \frac{1}{n}\sum_{j=1}^{n} \sqrt{(h_{aj}^- - h_{bj}^-)^2 + (h_{aj}^+ - h_{bj}^+)^2} \; (j = 1, 2, \ldots, n) \quad (8)$$

For the convenience of expression, $[h_{aj}^-, h_{aj}^+]$ is the abbreviated form of the numerical form $[E(h_{S_{O_{aj}}}^-), E(h_{S_{O_{aj}}}^+)]$ after the equivalent transformation of the language information of the expert e^a for the alternative A_j. $d(e^a, e^b)$ satisfies the following properties: ① $0 \le d(e^a, e^b) \le 1$. ② $d(e^a, e^b) = 0$ if and only if $[h_{aj}^-, h_{aj}^+] = [h_{bj}^-, h_{bj}^+]$. ③ $d(e^a, e^b) = d(e^b, e^a)$. ④ $d(e^a, e^a) = 0$.

Step 2: A similarity measure matrix $SD = \left(sd(e^a, e^b)\right)_{m \times m} (a, b = 1, 2, \ldots, m)$ between experts is constructed. The upper triangular elements (except diagonal elements) in the similarity measure matrix SD are sorted in descending order, denoted as $\Phi_1 \ge \Phi_2 \ge \cdots \ge \Phi_k \cdots \ge \Phi_{m(m-1)/2}$, and recorded as the grouping threshold where $1 \le k \le m(m-1)/2$.

Step 3: Let the expert pair $\Phi_1, \Phi_2, \ldots, \Phi_{m(m-1)/2}$ corresponding to (e^a, e^b) be $\mathrm{E}_1, \mathrm{E}_2, \ldots, \mathrm{E}_{m(m-1)/2}$, respectively. Stepwise clustering is performed from E_1 to $\mathrm{E}_{m(m-1)/2}$, and each group after clustering is denoted as G_φ $(\varphi = 1, 2, \ldots, g; g \le m)$, where G_φ is represented as a set of experts. When considering $\mathrm{E}_i (1 \le i \le m(m-1)/2)$, if $G_\varphi \cap \mathrm{E}_i \ne \varnothing$ and E_i are classified into G_φ, and any pair of experts in the group satisfies $(e^a, e^b) \in \{\mathrm{E}_1, \mathrm{E}_2, \ldots, \mathrm{E}_i\}$, then the relevant experts are divided into a group.

It can be seen that any expert has a certain degree of similarity with other experts in the group. In other words, the similarity measure between the expert and an expert in the group is maintained at a higher level than in other groups. Finally, when considering $\mathrm{E}_{m(m-1)/2}$, there is $G_\varphi = \{e^1, e^2, \ldots, e^m\}$, i.e., when considering the last expert pairing, all experts are classified into the same group.

Example 1: It is already known that there exists a group $G_1 = \{e^1, e^2\}$, and expert pairs $\mathrm{E}_5 = (e^2, e^3)$. When considering expert pair $\mathrm{E}_{10} = (e^1, e^3)$, if e^3 is not included in other groups, then expert e^3 is included in group G_1 after considering E_{10}. Any expert pair in the group satisfies $(e^a, e^b) \in \{\mathrm{E}_1, \mathrm{E}_2, \ldots, \mathrm{E}_{10}\} (a, b = 1, 2, 3, a < b)$, so after considering E_{10}, there is $G_1 = \{e^1, e^2, e^3\}$.

Step 4: Let K_p be the threshold change rate, and $\mathrm{K}_p = (\Phi_{p-1} - \Phi_p)/(n_{p-1} - n_p)$. Where Φ_{p-1} and Φ_p are the grouping thresholds of the p-1-th and p-th clustering, respectively, n_p and n_{p-1} are the number of groups after the p-th and p-1-th clustering, respectively, and $n_p \ge 2$. Let $\mathrm{K}_\mu = \max_p \{\mathrm{K}_p\}$, it is considered that the grouping threshold Φ_μ after the μ-th clustering is the best grouping threshold, and it is the best grouping result of clustering by a large group of experts. Let the number of groups be $\eta = n_\mu$, and each group set is denoted as G_φ $(\varphi = 1, 2, \ldots, \eta)$.

3.2. The Determination Method of Subjective and Objective Comprehensive Weight

For LGDM problems in complex environments, it is usually necessary to comprehensively consider the decision-making objects and their attributes, so multiple experts are required to participate in the ranking of alternatives. However, due to the limitations of expert knowledge and the bias of cognition, the weight value of each expert needs to be adjusted to reflect its importance in the decision-making process. In this study, based on the characteristics of SC-DHIHFL, a method for determining the comprehensive weight of the subject and object is proposed. For the expert $e^\beta (1 \le \beta \le P_\varphi)$ in the group $G_\varphi (\varphi = 1, 2, \ldots, \eta)$, the weight determination method is as follows:

Step 1: The objective weight ow^β of the expert e^β is determined. Information entropy can measure the uncertainty of information. If the information entropy of expert decision-making information is smaller, it means that the uncertainty of the expert decision-making information is smaller, and the expert expresses his thoughts with language information more objectively and should be given a higher weight. The information entropy of expert e^β can be obtained:

$$H(e^\beta) = -\frac{1}{\ln n} \sum_{j=1}^{n} \frac{h_{\beta j}{}^+ + h_{\beta j}{}^-}{\sum_{j=1}^{n} (h_{\beta j}{}^+ + h_{\beta j}{}^-)} \ln \frac{h_{\beta j}{}^+ + h_{\beta j}{}^-}{\sum_{j=1}^{n} (h_{\beta j}{}^+ + h_{\beta j}{}^-)} \tag{9}$$

According to the information entropy, the objective weight of expert e^β is determined, which can be calculated as follows:

$$ow^\beta = \frac{1 - H(e^\beta)}{\sum_{\beta=1}^{P_\varphi} (1 - H(e^\beta))} \tag{10}$$

Step 2: The subjective weight sw^β of the expert e^β is determined. The subjective weight of the expert is defined according to the hesitation and self-confidence information of the expert's language information. The smaller the hesitation of the expert's decision-making information, the higher the confidence. From a subjective point of view, it should be given a higher weight, which can be calculated as follows:

$$sw^\beta = \frac{1}{2} \times \left(\frac{\sum_{j=1}^{n} SC_{\beta j}}{\sum_{j=1}^{n} \sum_{\beta=1}^{P_\varphi} SC_{\beta j}} + \frac{\sum_{j=1}^{n} \left(1 - ({h_{\beta j}}^{+} - {h_{\beta j}}^{-})\right)}{\sum_{j=1}^{n} \sum_{\beta=1}^{P_\varphi} \left(1 - ({h_{\beta j}}^{+} - {h_{\beta j}}^{-})\right)} \right) \tag{11}$$

Step 3: The comprehensive weight w^β of the expert e^β is determined. If only objective weights are used, there may be an expert who has influenced the information of the entire group, and the opinions of other experts have not been fully considered. If only subjective weights are used, there is little difference in the weights among experts, and it is difficult to distinguish the importance among experts. Considering the objective weight and the subjective weight comprehensively, the advantages of the two methods can be combined, and the defects of the single method can be overcome at the same time. The subjective and objective comprehensive parameter λ is defined, then the expert comprehensive weight can be determined as follows:

$$w^\beta = \lambda \times ow^\beta + (1 - \lambda) \times sw^\beta \tag{12}$$

where $0 \le \lambda \le 1$. Generally, let $\lambda = 0.5$, then, the comprehensive weight is the arithmetic mean of objective weight and subjective weight as follows: $w^\beta = (ow^\beta + sw^\beta)/2$.

3.3. The Consensus Degrees

For LGDM problems, the consensus degree can quantify the level of consensus among groups. The main purpose of the consensus model is to judge whether the group decision-making information needs to be adjusted through the preset consensus threshold. If the group consensus is lower than the threshold, it is necessary to guide experts to modify the decision-making information. In this study, based on the similarity measure between experts, a method for calculating the consensus degree of the group is proposed. The consensus degree ocd of the group G_φ is calculated as follows:

$$ocd = \frac{2}{P_\varphi (P_\varphi - 1)} \sum_{a=1}^{P_\varphi - 1} \sum_{b=a+1}^{P_\varphi} sd(e^a, e^b) \tag{13}$$

4. A Two-Stage Large Group Decision-Making Method Based on a Self-Confident Double Hierarchy Interval Hesitant Fuzzy Language

Aiming at the LGDM consensus problem in the DHHFL environment, a two-stage LGDM method is proposed in this study. The first stage: Firstly, the experts in the large group are grouped and clustered based on the similarity measure, and then the consensus degree of each group is calculated to determine whether a consensus is reached. For groups that do not reach a consensus, the experts who need to adjust the language information are identified, then the decision-making information is adjusted according to the adjustment rules, and finally, the consensus within the group is achieved. The second stage: Firstly, the group that first reached a consensus is integrated with the decision-making information of the experts in the group, and the inter-group decision-making information representing the group is obtained. Then the inter-group decision-making information is integrated using the preference coefficient, and finally, the inter-large-group decision-making information representing the large group can be obtained. The specific steps of the two-stage LGDM method are as follows:

4.1. Reaching a Consensus within the Group

In the first stage, each group reaches a consensus within the group, and the specific decision-making steps are as follows:

Step 1: According to Formula (4), the decision-making language information of each group of experts is input and converted.

Step 2: According to the expert clustering algorithm based on a similarity measure, all experts are clustered into $\eta(2 \le \eta \le m)$ groups.

Step 3: The consensus threshold is set to $\Re_{ocd} = 0.90$. According to (13), the overall consensus degree ocd is calculated, and it is judged whether the consensus has been reached. If $ocd \ge \Re_{ocd}$, the group reaches a consensus and proceeds to step 6. Otherwise, go to step 4, to identify decision makers and adjust evaluation information and self-confidence.

Step 4: According to the determination method of subjective and objective comprehensive weight, the comprehensive weight of each expert in the group is calculated.

Step 5: Experts who need to adjust language information are identified, and their decision-making information is adjusted according to adjustment rules. Then go to step 3.

Identification rules: Experts who need to adjust the linguistic information are recognized, and an identification rule is defined. In the R round, the evaluation information of the expert on the plan is compared with the group adjustment direction under the plan, and the proximity measure $pm_{\beta j}$ is defined to measure the closeness between the expert and the group as follows: $pm_{\beta j} = 1 - \frac{\left|h_{\beta j}^{-(r)} - T_{G_\varphi}^{(r)}(h_j^-)\right| + \left|h_{\beta j}^{+(r)} - T_{G_\varphi}^{(r)}(h_j^+)\right|}{2}$, where the group adjustment direction $\left[T_{G_\varphi}^{(r)}(h_j^-), T_{G_\varphi}^{(r)}(h_j^+)\right]$ of evaluation information is calculated as follows: $\left[T_{G_\varphi}^{(r)}(h_j^-), T_{G_\varphi}^{(r)}(h_j^+)\right] = \left[\sum_{\beta=1}^{P_\varphi} (w^\beta \times h_{\beta j}^{-(r)}), \sum_{\beta=1}^{P_\varphi} (w^\beta \times h_{\beta j}^{+(r)})\right]$.

The overall proximity measure ocd is sorted in ascending order, denoted as $\kappa_1 \le \cdots \le \kappa_c \le \cdots \le \kappa_v \le \cdots \le \kappa_{P_\varphi \times n}$, where $c = [P_\varphi \times n/4]$, $v = P_\varphi \times n - [P_\varphi \times n/4]$ and [ ] is a rounding operation. Let $\xi_{pm}^- = \kappa_c$ and $\xi_{pm}^+ = \kappa_v$ be recorded as the upper threshold of proximity measure and the lower threshold of proximity measure, respectively.

Adjustment rules: ① If $pm_{\beta j} \le \xi_{pm}^-$, it means that there is a large misunderstanding between the expert evaluation information of the alternative J and the group, which will hinder the reaching of group consensus, so it is necessary to reduce the expert's confidence and then adjust the evaluation information. ② If $pm_{\beta j} \ge \xi_{pm}^+$, it means that there is a great help between the expert evaluation information of the alternative J and the group, which will promote the reaching of group consensus, so it is necessary to improve the expert's confidence. ③ If $\xi_{pm}^- \le pm_{\beta j} \le \xi_{pm}^+$, it means that there is no great influence between the expert evaluation information of alternative J and the group, so there is no need to adjust the expert's confidence, but the evaluation information needs to be adjusted.

When an expert needs to adjust the self-confidence of alternative J, the adjustment rules are to increase or decrease the self-confidence of one level at a time. The specific adjustment rules are expressed as follows: $scd_{\beta j}^{(r)} = scd_{\beta j}^{(r-1)} \pm \frac{1}{N-1}$, where $scd = \frac{SC-1}{N-1}$. Obviously, if the 7-point numerical set is used to represent the self-confidence of experts, the adjusted semantic value cannot exceed 7 or be lower than 1, namely $0 \le scd_{\beta j} \le 1$. When an expert needs to make adjustments to the evaluation information of alternative J, self-confidence needs to be considered at the same time. The specific adjustment rules are as follows: $h_{\beta j}^{-(r)} = h_{\beta j}^{-(r-1)} + \left(T_{G_\varphi}^{(r)}(h_j^-) - h_{\beta j}^{-(r-1)}\right) \times \left(1 - (SC_{\beta j}^{(r)} - 1)/(N-1)\right)$, $h_{\beta j}^{+(r)} = h_{\beta j}^{+(r-1)} + \left(T_{G_\varphi}^{(r)}(h_j^+) - h_{\beta j}^{+(r-1)}\right) \times \left(1 - (SC_{\beta j}^{(r)} - 1)/(N-1)\right)$.

Step 6: According to Formula (5), the numerical value is converted into language information, and the language information of the group is output.

4.2. The Information Integration for Large Group Decision-Making

In the second stage, the decision-making information of the experts in the group that has reached a consensus is integrated to obtain the inter-group decision-making information representing the group, and then the decision-making information between the groups is integrated using the preference coefficient to obtain the inter-large-group decision-making information representing the large group. The specific LGDM information integration steps are as follows:

Step 1: The group that has reached a consensus is integrated with expert decision-making information within the group, and the integration formula is as follows: $\left[E_{G_\varphi}\left(h_j^-\right), E_{G_\varphi}\left(h_j^+\right)\right] = \left[\sum_{\beta=1}^{P_\varphi}\left(w^\beta \times h_{\beta j}^-\right), \sum_{\beta=1}^{P_\varphi}\left(w^\beta \times h_{\beta j}^+\right)\right]$.

Step 2: In LGDM, the decision-making information of experts in different groups is different, so the weighted integration of decision-making information between groups needs to be considered. Generally speaking, the greater the proportion of experts in the group, the greater its influence on the decision-making results, and the greater the weight of the group. The weight of the group can be calculated as follows: $w_{G_\varphi}^{maj} = \frac{P_\varphi}{\sum_{\varphi=1}^{\eta} P_\varphi}$.

Step 3: Although the proportion of the number of experts in the group to the total number of experts can reflect the degree of influence of the decision-making information of the experts in the group on the final decision-making result, it is also necessary to consider the inconsistent information in the group in reality. Therefore, the deviation degree of decision-making information between groups is further considered in the definition of weight, and the calculation formula of deviation weight is defined in this study. The greater the mean ratio of the degree of deviation between a certain group and other groups, the greater the weight of this group. This definition method can better reflect the inconsistent information in the group so as to consider the decision-making problem more comprehensively. The weight calculation formula is as follows: $w_{G_\varphi}^{\min} = \frac{DE_{G_\varphi}}{\sum_{\varphi=1}^{\eta} DE_{G_\varphi}}$, where $DE_{G_\varphi} = \frac{\sum_{\beta=1}^{P_\varphi}\sum_{j=1}^{n} pm_{\beta j}}{P_\varphi \times n}$ represents the deviation degree of the group G_φ.

Step 4: In the process of integrating decision information between groups, the preference coefficient is set to ϑ, and the group integration weight is defined as follows: $w_{G_\varphi} = \vartheta w_{G_\varphi}^{maj} + (1-\vartheta) w_{G_\varphi}^{\min}$. When the actual problem focuses on the opinions of the majority of experts, take $\vartheta > 0.5$. When the actual problem needs to focus on reflecting the disagreement, take $\vartheta < 0.5$. Generally, if there are no special circumstances, take $\vartheta = 0.5$.

Step 5: The two-stage LGDM information integration formula is:

$$\left[E\left(h_j^-\right), E\left(h_j^+\right)\right] = \left[\sum_{\varphi=1}^{\eta}\left(w_{G_\varphi} \times E_{G_\varphi}\left(h_j^-\right)\right), \sum_{\varphi=1}^{\eta}\left(w_{G_\varphi} \times E_{G_\varphi}\left(h_j^+\right)\right)\right] \tag{14}$$

4.3. The Two-Stage Large Group Decision-Making Method

Based on the above algorithm and discussion, the specific decision-making steps of the two-stage LGDM method based on the SC-DHIHFL are as follows:

Step 1: The decision object (alternative scheme) set $A = \{A_1, A_2, \ldots, A_n\}(n \geq 3)$ and the decision preference coefficient ϑ are given according to experience. Then experts are selected, and a large group of experts is formed as $E = \left\{e^1, e^2, \ldots, e^m\right\}(m \geq 20)$.

Step 2: According to the degree of understanding of the decision-making object and its own knowledge reserve, the large group of experts gives language evaluation information and self-confidence, and then the language information is transformed into SC-DHIHFL information.

Step 3: According to the decision-making language information of all experts, expert groups are grouped using the expert clustering algorithm.

Step 4: In the first stage, a consensus is reached within each group using the decision-making method. The consensus degree ocd of each group is calculated if $ocd \geq \Re_{ocd}$, then the group has reached a consensus; otherwise, the evaluation information and self-confidence need to be adjusted.

Step 5: In the second stage, large group decision information is integrated using the decision-making method. Firstly, the expert decision information in the same group is integrated, then the inter-group decision-making information is obtained, and finally, the inter-large-group decision-making information is acquired.

Step 6: According to the inter-large-group decision information, the decision objects are sorted. According to the two-stage LGDM method and the ranking results of the decision-making objects after the comprehensive discussion of experts, the optimal decision-making object is selected by the decision-making department. The formula for calculating the expected value of the scheme A_j is as follows:

$$E(A_j) = \frac{E(h_j^{-}) + E(h_j^{+})}{2} \tag{15}$$

The flowchart of the two-stage LGDM method is shown in Figure 2.

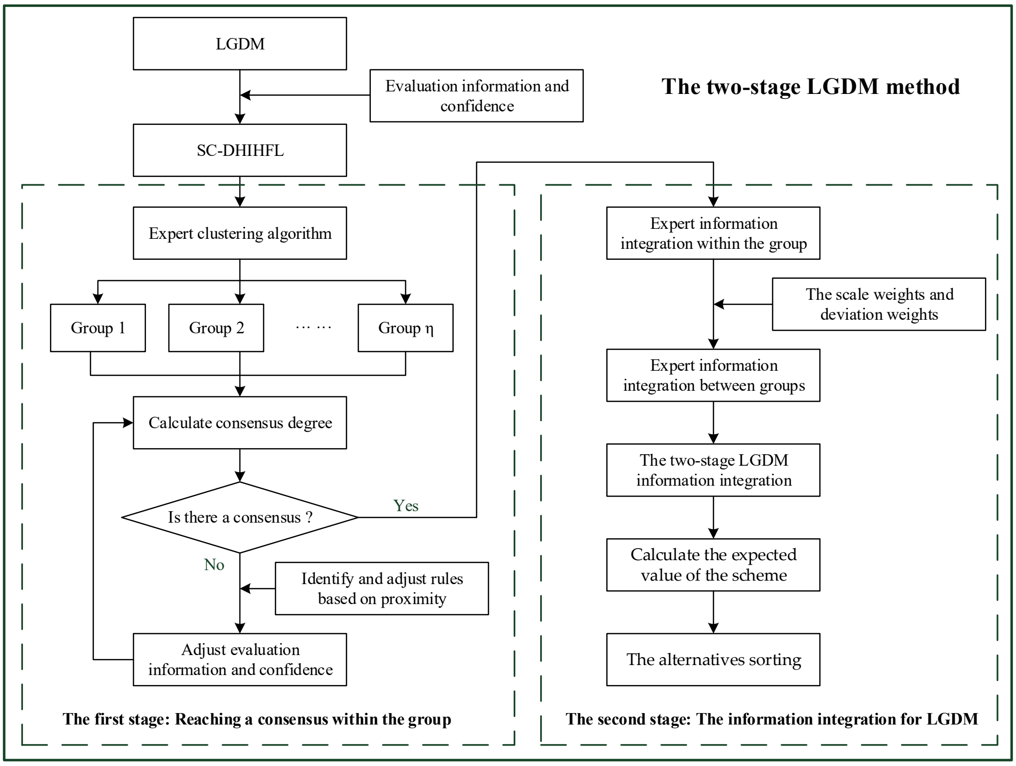

Figure 2. The flowchart of the two-stage LGDM method.

5. The Case Study

When the emergency plan is formulated and selected, the opinions of various professional fields and related departments must be fully considered to determine the best plan. An effective decision-making command center is responsible for coordinating the resources of various departments and experts. The center will collect, integrate, analyze, and evaluate relevant information and data so that responses and decisions can be made quickly. A hypothetical example: a public emergency in an area, and some students in a primary school suddenly vomited. The relevant departments set up an expert group $E = \{e^1, e^2, \cdots, e^{20}\}$ to formulate emergency measures and consider the plan from three aspects: timeliness, feasibility, and public opinion guidance. The expert group initially formulated four emergency plans, namely: A_1, relevant government organizations asked the school to notify the parents of the students and asked the parents to take the students to the hospital, and the school would bear the cost. A_2, relevant government organizations and schools immediately sent the relevant students to the hospital, waiting for the school or parents to come up with a solution. A_3, relevant government organizations and schools sent the relevant students to the hospital immediately, and the government department paid the expenses and held the responsible persons of the school accountable. At the same time, they actively communicated with students and parents to discuss solutions. A_4, relevant government organizations asked the school to send the relevant students to the hospital immediately and asked the school to provide a solution. Now it is necessary for the expert group to select a scheme and reach a consensus. The first level of the language evaluation scale is: "none, very low, low, medium, high, very high, excellent"; the second level of the language evaluation scale is: "far from, just a little, a little, just right, much, extremely, completely." All experts make evaluation information on these plans, e.g., the evaluation information made by expert e^1 on the three indicators of plan A_1 is: "between low and high," "between very medium and extremely medium," "between just a little low and very high," which has a low self-confidence score. The specific decision-making steps are as follows:

Step 1: Based on the experience and knowledge acquired by the decision-making group, the index weight is set to $w = (1/3, 1/3, 1/3)$, and the decision preference coefficient is $\vartheta = 0.5$.

Step 2: With DHIHFL, the evaluation information is converted into decision-making language information. The decision-making language information of expert e^1 is shown in Figure 3. The evaluation information of all experts on the alternatives can be found in Table A1 in Appendix A.

$$e^1 = \begin{pmatrix} [s_{-1<o_0>}, s_{1<o_0>}] & [s_{0<o_1>}, s_{0<o_2>}] & [s_{-1<o_2>}, s_{1<o_1>}] & 3 \\ [s_{-1<o_2>}, s_{-1<o_3>}] & [s_{-2<o_2>}, s_{-2<o_3>}] & [s_{-2<o_2>}, s_{-2<o_3>}] & 5 \\ s_{1<o_1>} & [s_{0<o_3>}, s_{1<o_3>}] & [s_{2<o_{-1}>}, s_{3<o_{-1}>}] & 6 \\ [s_{-1<o_{-1}>}, s_{-1<o_0>}] & [s_{-1<o_2>}, s_{0<o_2>}] & [s_{-3<o_2>}, s_{0<o_1>}] & 2 \end{pmatrix}$$

Figure 3. The decision-making language information of expert e^1.

Step 3: The large group is grouped using the expert clustering algorithm, and the result after grouping is $G_1 = \{e^1, e^{12}, e^{16}, e^{17}, e^{18}, e^{19}\}$, $G_2 = \{e^2, e^3, e^4, e^7, e^9, e^{13}, e^{14}, e^{20}\}$, $G_3 = \{e^6, e^8, e^{10}, e^{15}\}$, $G_4 = \{e^5, e^{11}\}$.

Step 4: In the first stage, let $\Re_{ocd} = 0.90$, and a consensus is reached in each group according to the decision-making method.

Step 5: In the second stage, LGDM information is integrated using the decision-making method. The decision information after the integration of each group is shown in Table 2 as follows.

Table 2. The decision information after integration of each group.

Group	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$	$\mathbf{A_4}$
G_1	[0.479, 0.631]	[0.417, 0.568]	[0.588, 0.769]	[0.366, 0.548]
G_2	[0.595, 0.774]	[0.590, 0.758]	[0.642, 0.741]	[0.675, 0.873]
G_3	[0.192, 0.466]	[0.360, 0.608]	[0.653, 0.787]	[0.558, 0.800]
G_4	[0.056, 0.500]	[0.369, 0.395]	[0.408, 0.649]	[0.350, 0.410]

In the process of integration of decision-making information between groups, the weights of each group after calculation are 0.28, 0.33, 0.22, and 0.18. Finally, the inter-large-group decision-making information is [0.378, 0.618], [0.452, 0.609], [0.589, 0.743], [0.507, 0.686].

Step 6: According to Formula (15), the expected value of each plan is 0.498, 0.530, 0.666, 0.596. The plan is sorted as $A_3 \succ A_4 \succ A_2 \succ A_1$.

In order to compare the effectiveness and feasibility of the decision-making method in this study, it is compared with the group decision-making method in the following five cases, and the comparison results are shown in Table 3.

- The self-confident single hierarchy interval hesitant fuzzy language (SC-SHIHFL). The second hierarchy LTS is deleted, and only the first hierarchy LTS is retained;
- The static self-confident double hierarchy interval hesitant fuzzy language (SSC-DHIHFL). Self-confidence is fixed, i.e., the value of self-confidence will not change with the group decision-making process;
- Double hierarchy hesitant fuzzy language based on TOPSIS [12] (DHHFL-TOPSIS). The DHHFL-TOPSIS method ranks alternatives by measuring their closeness to an idealized goal;
- Double hierarchy hesitant fuzzy language based on MULTIMOORA [11] (DHHFL-MULTIMOORA). The DHHFL-MULTIMOORA method analyzes the pros and cons of the scheme through three dimensions;
- Double hierarchy hesitant fuzzy language based on consensus model handling minority opinions and non-cooperative behaviors in LGDM [23] (DHHFL-Consensus Model in LGDM).

Table 3. Methods comparison.

Number	Methods	Sort
Method 1	SC-DHIHFL	$A_3 \succ A_4 \succ A_2 \succ A_1$
Method 2	SC-SHIHFL	$A_3 \succ A_4 = A_2 \succ A_1$
Method 3	SSC-DHIHFL	$A_3 \succ A_2 \succ A_4 \succ A_1$
Method 4	DHHFL-TOPSIS	$A_3 \succ A_2 \succ A_4 \succ A_1$
Method 5	DHHFL-MULTIMOORA	$A_3 \succ A_4 \succ A_2 \succ A_1$
Method 6	DHHFL-Consensus Model in LGDM	$A_3 \succ A_4 \succ A_2 \succ A_1$

According to Table 3, compared with Method 1, the expected value and ranking of Scheme 2 and Scheme 4 in Method 2 are equal, indicating that the single hierarchy language cannot distinguish the evaluation value of some similar schemes. Methods 3 and 4 ranked Scheme 2 and Scheme 4 differently from other methods, perhaps because they did not adjust expert confidence based on group information or did not take minority opinions into account. Compared with Methods 5 and 6, it is found that the ranking results of the method in this study are the same as those of these two literature studies. This result verifies the effectiveness and feasibility of the decision-making method proposed in this study. In addition, when considering the situation of increasing the number of participating decision makers to 100, compared with Method 5, the method proposed in this study adds an expert clustering algorithm, which is more suitable for the actual situation and makes the result of integrated aggregation more scientific. Compared with Method 6, the method proposed in this study has a faster and more efficient process of reaching a consensus.

6. Conclusions

First of all, the DHIHFL is proposed in this study and incorporates the concept of self-confidence, which can more comprehensively describe the hesitation of language information and the integrity of rich evaluation information. Compared with the traditional DHHFL, it has certain advantages and application value. Secondly, for LGDM problems, a new and effective two-stage LGDM method is proposed based on SC-DHIHFL. Finally, it is applied to the selection of emergency plans for public emergencies. The results show that the DHIHFL can describe language information more accurately than the single hierarchy language. A consensus was quickly reached, and a comparative analysis was carried out with some existing methods, which verified the effectiveness and feasibility of this method.

With the continuous development of information technology and the continuous expansion of applications, the advantages of DHHFL will become more obvious, and it will become one of the important decision-making tools. E.g., in the field of intelligent transportation, the uncertainty and ambiguity of information such as vehicles and road conditions are strong. Traditional fuzzy language and mathematical methods can no longer solve the problem well, and DHHFL can help AI better understand and process information to improve the accuracy and reliability of traffic decisions. At the same time, in terms of LGDM, the DHHFL model will be more and more widely used in government decision-making, enterprise management and social organization, and other fields. With the continuous development of big data and cloud computing technology, decision-making problems will become more and more complex and diverse, requiring more flexible and diversified decision-making tools to solve. DHHFL can be combined with other methods, such as machine learning, data mining, etc., to improve further decision-making efficiency and accuracy and better deal with uncertainty and complexity in LGDM. The proposal and application of DHHFL open up a new research direction for LGDM and has broad application prospects and development potential in the future.

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Appendix A

Table A1. The evaluation information of all experts on the alternatives.

Expert	A_1	A_2	A_3	A_4
e^1	$([s_{-1<o_2>}, s_{0<o_3>}],3)$	$([s_{-1<o_0>}, s_{-1<o_1>}],5)$	$([s_{1<o_1>}, s_{1<o_3>}],6)$	$([s_{-1<o_{-1}>}, s_{0<o_0>}],2)$
e^2	$([s_{-1<o_{-1}>}, s_{0<o_1>}],2)$	$([s_{0<o_{-1}>}, s_{2<o_0>}],3)$	$([s_{1<o_{-1}>}, s_{2<o_{-2}>}],2)$	$([s_{1<o_0>}, s_{1<o_1>}],2)$
e^3	$([s_{0<o_{-2}>}, s_{0<o_2>}],5)$	$([s_{1<o_2>}, s_{2<o_0>}],4)$	$([s_{1<o_2>}, s_{1<o_3>}],2)$	$([s_{0<o_0>}, s_{2<o_0>}],3)$
e^4	$([s_{2<o_{-1}>}, s_{2<o_2>}],5)$	$([s_{0<o_{-2}>}, s_{1<o_2>}],4)$	$([s_{0<o_0>}, s_{1<o_0>}],2)$	$([s_{2<o_{-2}>}, s_{3<o_{-1}>}],3)$
e^5	$([s_{-2<o_{-2}>}, s_{-1<o_3>}],2)$	$([s_{-2<o_1>}, s_{-1<o_{-1}>}],1)$	$([s_{0<o_{-1}>}, s_{1<o_3>}],2)$	$(s_{0<o_0>},4)$
e^6	$([s_{-1<o_0>}, s_{1<o_2>}],4)$	$([s_{0<o_{-2}>}, s_{1<o_{-2}>}],6)$	$([s_{1<o_0>}, s_{2<o_0>}],4)$	$([s_{1<o_0>}, s_{2<o_0>}],4)$
e^7	$([s_{0<o_3>}, s_{1<o_3>}],4)$	$([s_{0<o_{-1}>}, s_{1<o_{-1}>}],5)$	$([s_{2<o_{-3}>}, s_{2<o_{-2}>}],3)$	$([s_{1<o_0>}, s_{3<o_{-1}>}],2)$
e^8	$([s_{-2<o_0>}, s_{-1<o_1>}],3)$	$([s_{-1<o_1>}, s_{0<o_1>}],3)$	$([s_{1<o_0>}, s_{1<o_3>}],6)$	$([s_{0<o_{-3}>}, s_{1<o_{-1}>}],3)$
e^9	$(s_{1<o_1>},5)$	$([s_{1<o_2>}, s_{2<o_0>}],6)$	$([s_{1<o_1>}, s_{1<o_2>}],4)$	$([s_{2<o_{-2}>}, s_{3<o_{-1}>}],6)$
e^{10}	$([s_{-2<o_0>}, s_{0<o_0>}],2)$	$([s_{0<o_{-3}>}, s_{1<o_2>}],2)$	$(s_{1<o_1>},2)$	$([s_{2<o_{-1}>}, s_{3<o_{-1}>}],5)$
e^{11}	$([s_{-2<o_{-2}>}, s_{-1<o_3>}],5)$	$(s_{0<o_0>},3)$	$([s_{0<o_{-2}>}, s_{1<o_{-2}>}],6)$	$([s_{-2<o_1>}, s_{-2<o_3>}],5)$
e^{12}	$([s_{0<o_{-1}>}, s_{1<o_0>}],2)$	$([s_{0<o_2>}, s_{1<o_1>}],6)$	$([s_{0<o_{-1}>}, s_{1<o_{-2}>}],6)$	$([s_{0<o_{-2}>}, s_{1<o_0>}],5)$
e^{13}	$([s_{1<o_2>}, s_{2<o_0>}],5)$	$(s_{1<o_0>},6)$	$([s_{1<o_0>}, s_{2<o_0>}],4)$	$([s_{1<o_1>}, s_{2<o_3>}],4)$
e^{14}	$([s_{-1<o_0>}, s_{1<o_1>}],5)$	$([s_{1<o_0>}, s_{2<o_2>}],4)$	$([s_{1<o_{-2}>}, s_{1<o_{-1}>}],2)$	$(s_{1<o_1>},4)$
e^{15}	$([s_{-2<o_0>}, s_{-1<o_0>}],6)$	$([s_{-1<o_0>}, s_{0<o_2>}],2)$	$([s_{1<o_{-2}>}, s_{1<o_0>}],3)$	$([s_{0<o_{-2}>}, s_{1<o_2>}],2)$
e^{16}	$([s_{0<o_1>}, s_{0<o_2>}],4)$	$([s_{1<o_{-1}>}, s_{1<o_3>}],5)$	$([s_{1<o_{-2}>}, s_{1<o_{-1}>}],3)$	$([s_{0<o_{-3}>}, s_{1<o_0>}],4)$
e^{17}	$([s_{0<o_1>}, s_{1<o_1>}],6)$	$([s_{-2<o_0>}, s_{-1<o_3>}],2)$	$([s_{1<o_{-2}>}, s_{1<o_3>}],5)$	$([s_{0<o_{-1}>}, s_{0<o_0>}],6)$
e^{18}	$([s_{-1<o_2>}, s_{0<o_0>}],6)$	$([s_{0<o_0>}, s_{1<o_{-1}>}],3)$	$([s_{1<o_{-2}>}, s_{2<o_{-1}>}],3)$	$([s_{-1<o_{-1}>}, s_{-1<o_2>}],4)$
e^{19}	$([s_{-1<o_1>}, s_{1<o_{-1}>}],4)$	$([s_{0<o_{-2}>}, s_{0<o_1>}],4)$	$([s_{1<o_{-2}>}, s_{1<o_0>}],5)$	$([s_{0<o_{-1}>}, s_{0<o_3>}],4)$
e^{20}	$([s_{1<o_0>}, s_{1<o_3>}],6)$	$([s_{0<o_{-2}>}, s_{1<o_{-2}>}],2)$	$([s_{0<o_0>}, s_{1<o_0>}],5)$	$([s_{0<o_0>}, s_{1<o_3>}],2)$

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