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Coefficient Estimates of New Families of Analytic Functions Associated with q-Hermite Polynomials

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Abstract: In this paper, we introduce two new subclasses of bi-univalent functions using the q-Hermite polynomials. Furthermore, we establish the bounds of the initial coefficients v_2, v_3, and v_4 of the Taylor–Maclaurin series and that of the Fekete–Szegö functional associated with the new classes, and we give the many consequences of our findings.

Keywords: q-convolution operator; coefficient estimates; q-Hermite; bi-univalent functions; Babalola operator

MSC: 05A30; 30C45; 11B65; 47B38

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1. Introduction and Preliminaries

Let $\mathcal{M}(\mathbb{U})$ denote the class of analytic functions in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}.$$

Let $\mathcal{A}$ be the subclass of $\mathcal{M}(\mathbb{U})$ whose functions f satisfy the normalization condition given by

$$f(0) = f'(0) - 1 = 0,$$

that is, each function f in $\mathcal{A}$ can be represented by the following Taylor–Maclaurin series expansion:

$$f(z) = z + \sum_{k=2}^{\infty} v_k z^k \qquad (z \in \mathbb{U}). \tag{1}$$

Moreover, let $\mathcal{S}$ be the subclass of $\mathcal{A}$ whose functions are univalent in $\mathbb{U}$. The Koebe one-quarter theorem ensures that the image of $\mathbb{U}$ under every $f \in \mathcal{S}$ contains a disk of radius $1/4$.

It is known that every function $f \in \mathcal{S}$ has an inverse f^{-1} defined by

$$f^{-1}(f(z)) = z \quad (z \in \mathbb{U}),$$

and

$$f^{-1}(f(\omega)) = \omega \quad \left(|\omega| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right),$$

where

$$f^{-1}(\omega) = g(\omega) = \omega - v_2\omega^2 + \left(2v_2^2 - v_3\right)\omega^3 - \left(5v_2^3 - 5v_2v_3 + v_4\right)\omega^4 + \cdots. \tag{2}$$

A function is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1).

Lewin [1] investigated the class Σ and showed that $|v_2| < 1.51$. Subsequently, Brannan and Clunie [2] conjectured that $|v_2| < \sqrt{2}$. Netanyahu [3], on the other hand, showed that

$$\max_{f\in\Sigma}|v_2| = \frac{4}{3}.$$

The Taylor–Maclaurin coefficients $|v_n|$ $(n \geq 3,\ n \in \mathbb{N})$ in (1) are still unknown, and it is an open problem.

Similar to the subclasses $\mathcal{S}^*(\zeta)$ and $\mathcal{K}(\zeta)$ of the starlike and convex functions of the order ζ $(0 \leq \zeta < 1)$, respectively, that we are familiar with, Brannan and Taha [4] gave two subclasses of Σ, which are called $\mathcal{S}^*_\Sigma(\zeta)$ and $\mathcal{K}_\Sigma(\zeta)$ of the bi-starlike functions and bi-convex functions of the order ζ $(0 \leq \zeta < 1)$, respectively. It should be remarked here that, in their pioneering work, Srivastava et al. [5] actually revived the study of analytic and bi-univalent functions in recent years.

Moreover, for two analytic functions s_1 and s_2, the function s_1 is called subordinated to the function s_2, denoted as

$$s_1(z) \prec s_2(z) \qquad (z \in \mathbb{U}),$$

if there is an analytic function w in $\mathbb{U}$ with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1,$$

such that

$$s_1(z) = s_2(w(z)).$$

If the function $s_2 \in \mathcal{S}$, then

$$s_1(z) \prec s_2(z) \Leftrightarrow s_1(0) = s_2(0) \quad \text{and} \quad s_1(\mathbb{U}) \subset s_2(\mathbb{U}).$$

In 2008, Babalola [6] defined the operator $\mathfrak{I}_n^\sigma : \mathcal{A} \longrightarrow \mathcal{A}$ as

$$\mathfrak{I}_m^\tau f(z) = (\nu_\tau * \nu_{\tau,m}^{-1} * f)(z) \tag{3}$$

where

$$\nu_{\tau,m}(z) = \frac{z}{(1-z)^{\tau-(m-1)}}, \quad \tau - (m-1) > 0, \quad \nu_\tau = \nu_{\tau,0}$$

and $\nu_{\tau,m}^{-1}$ is such that

$$(\nu_{\tau,m} * \nu_{\tau,m}^{-1})(z) = \frac{z}{1-z} \quad (\tau, m \in \mathcal{N}).$$

Let $f \in \mathcal{A}$, then (3) is equivalent to

$$\mathfrak{I}_m^\tau f(z) = z + \sum_{j=2}^{\infty}\left[\frac{[\tau+j-1]!}{\tau!}\frac{[\tau-m]!}{[\tau+j-m-1]!}\right]v_j z^j. \tag{4}$$

The q-derivative operator $\mathfrak{D}_q$ of a function was introduced and researched by Jackson [7,8].

$$\mathfrak{D}_q f(z) = \frac{f(qz)-f(z)}{z(q-1)} = z^{-1}\left\{z + \sum_{k=2}^{\infty}[k]_q v_k z^k\right\} \tag{5}$$

and $\mathfrak{D}_q f(0) = f'(0)$. In particular, $f(z) = z^k$ for k is a positive integer, the q-derivative of $f(z)$ is given by

$$\mathfrak{D}_q z^k = \frac{(zq)^k - z^k}{z(q-1)} = [k]_q z^{k-1}, \tag{6}$$

$$\lim_{q\longrightarrow 1} [k]_q = \lim_{q\longrightarrow 1} \frac{q^k - 1}{q-1} = k. \tag{7}$$

For function $f(z)$ given by (1) and $g(z)$ given by

$$g(z) = z + \sum_{k=2}^{\infty} c_k z^k$$

the convolution of $f(z)$ and $g(z)$ is defined by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} v_k c_k z^k = (g * f)(z).$$

Let

$$\mathfrak{D}_q(z) = \frac{z}{(1-qz)(1-z)} = z + (1+e_1)z^2 + (1+e_1+e_2)z^3 + \cdots = z + \sum_{k=2}^{\infty} [k]_e z^k \tag{8}$$

where

$$[k]_e = 1 + e_1 + e_2 + \cdots + e_{k-1}, \quad e_k = q^k. \tag{9}$$

See [9,10] for additional information on q-derivative theories.

Quantum (or q-) calculus is a strong instrument for investigating a wide range of analytic functions, and it has sparked new research in mathematics and other fields. The first time it was used in the context of univalent functions was by Srivastava [11]. Many academics have studied q-calculus and its many applications due to the usefulness of q-analysis in mathematics and other areas. With the help of certain higher-order q-derivative operators, Khan et al. [12] constructed and analyzed a number of subclasses of q-starlike functions. Shi et al. (see also [13]) created a novel subclass of multivalent q-starlike Janowski functions using the q-differential operator. A variety of adequate requirements as well as some other noteworthy characteristics were investigated in both articles [12,14].

Because of the large range of applications and the usefulness of q-operators above fundamental operators, many scholars have looked into q-calculus in depth. Furthermore, Srivastava's recently published survey-cum-expository review study [15–17] is useful for academics and scholars studying these topics.

The q-Hermite polynomial was first introduced by Rogers [18] (see also [19,20]) and is usually defined by means of their generating function as follows

$$B_k(s|q) = \sum_{k=0}^{\infty} H_k(x;q) \frac{t^k}{(q;q)_k} = \prod_{k=0}^{\infty} \frac{1}{1 - 2xtq^k + t^2 q^{2k}} \qquad (0 < q < 1).$$

The q-derivative of the q-Hermite polynomial is

$$\mathfrak{D}_q\{B_{k+1}(s|q)\} = [k]_q B_k(s|q). \tag{10}$$

Moreover, Ismail et al. [18] were able to define the recursion relation as

$$tB_k(s|q) = B_{k+1}(s|q) + [k]_q B_{k-1}(s|q) \tag{11}$$

with

$$B_0(s|q) = 1 \quad \text{and} \quad B_{-1}(s|q) = 0.$$

Also from (11), we have

$$B_1(s|q) = s$$
$$B_2(s|q) = s^2 - 1$$
$$B_3(s|q) = s^3 - (2+q)s$$
$$B_4(s|q) = s^4 - (3+2q+q^2)s^2 + (1+q+q^2).$$

Remark 1. *It is clear that*

$$B_k(s|q=1) = B_{c_k}(s)$$

is the Hermite polynomials. Moreover, when

$$B_k(s|q=0) = U_k(s/2),$$

we have Chebyshev polynomials of the first kind, and they are defined by the recursion relation,

$$2sU_k(s) = U_{k-1}(s) + U_{k+1}(s) \tag{12}$$

with

$$U_0(s) = 1 \quad \text{and} \quad U_{-1}(s) = 0.$$

Next, we define the q-Babalola convolution operator which will be used throughout this paper.

Definition 1. *Let $f \in A$. Denote by $\Im_{\gamma,q} f(z)$ the q-Babalola convolution operator defined by*

$$\Im_{\gamma,q} f(z) = (\nu_{\tau,q} * \nu_{\gamma,q}^{(-1)} * f)(z) \tag{13}$$

where

$$\nu_{\gamma,q} = \frac{z}{(1-qz)^{\gamma}(1-z)}, \qquad \gamma > -1 \quad \text{and} \quad \nu_{\gamma,q}^{(-1)}$$

is such that

$$(\nu_{\gamma,q} * \nu_{\gamma,q}^{(-1)})(z) = \frac{z}{1-z}.$$

Hence,

$$\Im_{\gamma,q} f(z) = z + \sum_{k=2}^{\infty} \frac{[k]_e^{\sigma}}{[k]_e^{\gamma}} \upsilon_k z^k = z + \sum_{k=2}^{\infty} (k]_e^{\gamma} \upsilon_k z^k. \tag{14}$$

where

$$(k]_e^{\gamma} = \frac{1 + e_1(\tau) + e_2(\tau) \cdots e_{k-1}(\tau)}{1 + e_1(\gamma) + e_2(\gamma) \cdots e_{k-1}(\gamma)}$$

and

$$e_{k-1}(\tau) = \frac{(\tau+k-2)!}{(\tau-1)!} \frac{q^{k-1}}{(k-1)!}, \quad e_{k-1}(\gamma) = \frac{(\gamma+k-2)!}{(\gamma-1)!} \frac{q^{k-1}}{(k-1)!}.$$

Remark 2. *It is easily seen that, upon setting $q \longrightarrow 1-$, the extended Babalola convolution operator $\Im_{\gamma,q} f(z)$ reduces to the Babalola convolution operator $\Im_{\sigma}^{m} f(z)$ which was introduced and studied by Babalola [6]. For $m = \tau = 1$, the extended Babalola convolution operator $\Im_{\gamma,q} f(z)$ reduces to the q-derivative operator introduced and studied by Jackson [7,8]. Moreover, if $m = \tau$ and $q \longrightarrow 1-$, we have the Ruscheweyh's operator [21].*

Consider the univalent normalized functions of the kind (1); the Fekete–Szegö functional $|\upsilon_3 - \varphi\upsilon_2^2|$ has a long history in geometric function theory. The authors in [22] disproved Paley's conjecture and Littlewood's that the coefficients of odd univalent functions are confined by unity in 1933. Since then, the functional has gotten much attention, especially in subclasses of the family of univalent functions. This problem appears to have piqued the interest of scholars in recent years (see, for example, [23,24]).

We know that the q-Hermite polynomials and q-convolution operators still have not been studied with bi-univalent functions. The main goal of this paper is to start looking at the properties of the bi-univalent functions that are connected to q-Hermite polynomials and the q-convolution operator. In this study, the initial coefficient estimates for the Fekete–Szegö problem of analytic and bi-univalent functions are determined using the q-Hermite polynomial expansions and the q-convolution operator.

In Definition 2, we describe a class of convex bi-univalent functions that are defined by the q-convolution operator and linked to the q-Hermite polynomial.

Definition 2. *Let $N(z,s,q)$ be defined as follows:*

$$N(z,s,q) = \sum_{k=2}^{\infty} B_k(s|q) z^k. \tag{15}$$

A function $f \in \Sigma$ given by (1) is said to be in the class $\Gamma_\Sigma^q(s,\gamma,\tau)$, if the following conditions are satisfied:

$$1 + \frac{z\mathfrak{D}_q^2(\mathfrak{I}_{\gamma,q}f(z))}{\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f(z))} \prec N(z,s,q) \tag{16}$$

and

$$1 + \frac{\omega\mathfrak{D}_q^2(\mathfrak{I}_{\gamma,q}f^{-1}(\omega))}{\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f^{-1}(\omega))} \prec N(\omega,s,q). \tag{17}$$

Where $s \in \left(\frac{1}{2},1\right), 0 < q < 1, z \in \mathbb{U}, \omega \in \mathbb{U}, \gamma = \tau - m > -1.$

In Definition 3, we describe a class of starlike bi-univalent functions that are defined by the q-convolution operator and linked to the q-Hermite polynomial.

Definition 3. *Let $N(z,s,q)$ be defined as follows:*

$$N(z,s,q) = \sum_{k=2}^{\infty} B_k(s|q) z^k. \tag{18}$$

A function $f \in \Sigma$ given by (1) is said to be in the class $\Pi_\Sigma^q(s,\gamma,\tau)$, if the following conditions are satisfied:

$$\frac{z\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f(z))}{\mathfrak{I}_{\gamma,q}f(z)} \prec N(z,s,q) \tag{19}$$

and

$$\frac{\omega\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f^{-1}(\omega))}{\mathfrak{I}_{\gamma,q}f^{-1}(\omega)} \prec N(\omega,s,q). \tag{20}$$

We must recall the following lemma in order to arrive at our primary conclusions.

As is usually the case, we let $\mathcal{P}$ be the family of functions $p(z) = 1 + p_1z + p_2z^2 + \ldots$ regular with positive real part, for $z \in \mathbb{U}$.

Lemma 1 ([25])**.** *Let $\varphi(z) \in \mathcal{P}$, then*

$$|p_j| \le 2 \qquad (j \in \mathcal{N}).$$

2. Coefficient Estimates for the Class $\Gamma_\Sigma^q(s,\gamma,\tau)$

The initial coefficient bounds of the class $\Gamma_\Sigma^q(s,\gamma,\tau)$ of bi-univalent functions are investigated in this section.

Theorem 1. *Let $f \in \Gamma_{\Sigma}^{q}(s,\gamma,\tau)$. Then,*

$$|v_2| \le \sqrt{\Psi_1(s,q,\gamma)}, \tag{21}$$

$$|v_3| \le \frac{s^2}{[2]_b^2((2]_e^\gamma)^2} + \frac{s}{4[2]_b[3]_b(3]_e^\gamma}, \tag{22}$$

and

$$|v_4| \le \frac{s^3(2(1+e_1)[3]_e(3]_e^\gamma - [2]_e^2((2]_e^\gamma)^2)}{[2]_e^3[4]_e(4]_e^\gamma((2]_e^\gamma)^2} - \frac{s^2(2e_1[2]_e(2]_e^\gamma(3]_e^\gamma + 5[4]_e(4]_e^\gamma)}{2[2]_e^3[4]_e(2]_e^\gamma(3]_e^\gamma(4]_e^\gamma} + \frac{s^3 - 2s - 2qs - 4}{[2]_e[4]_e(4]_e^\gamma}$$

where

$$\Psi_1(s,q,\gamma) = \frac{s^3}{|s^2([2]_e[3]_e(3]_e^\gamma - [2]_e^2((2]_e^\gamma)^2) - [2]_e^2((2]_e^\gamma)^2(s^2-s-1)|}. \tag{23}$$

Proof. Let $f \in \Sigma$ be given by (1) be in the class $\Gamma_{\Sigma}^{q}(s,\gamma,\tau)$. Then,

$$1 + \frac{z\mathfrak{D}_q^2(\mathfrak{I}_{\gamma,q}f(z))}{\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f(z))} = N(d(z),s,q) \tag{24}$$

and

$$1 + \frac{z\mathfrak{D}_q^2(\mathfrak{I}_{\gamma,q}f^{-1}(\omega))}{\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f^{-1}(\omega))} = N(\varpi(\omega),s,q), \tag{25}$$

Let $\varrho, \eta \in \mathcal{P}$ be defined as

$$\varrho(z) = \frac{1+d(z)}{1-d(z)} = 1 + \varrho_1 z + \varrho_2 z^2 + \varrho_3 z^3 + \cdots \Rightarrow d(z) = \frac{\varrho(z)-1}{\varrho(z)+1}, \quad (z \in \mathbb{U}) \tag{26}$$

and

$$\eta(\omega) = \frac{1+\varpi(\omega)}{1-\varpi(\omega)} = 1 + \eta_1\omega + \eta_2\omega^2 + \eta_3\omega^3 + \cdots \Rightarrow \varpi(\omega) = \frac{\eta(\omega)-1}{\eta(\omega)+1}, \quad (\omega \in \mathbb{U}). \tag{27}$$

It follows that from (26) and (27) that

$$d(z) = \frac{1}{2}\left[\varrho_1 z + \left(\varrho_2 - \frac{\varrho_1^2}{2}\right)z^2 + \left(\varrho_3 - \varrho_1\varrho_2 + \frac{\varrho_1^3}{4}\right)z^3 + \cdots\right] \tag{28}$$

and

$$\varpi(\omega) = \frac{1}{2}\left[\eta_1\omega + \left(\eta_2 - \frac{\eta_1^2}{2}\right)\omega^2 + \left(\eta_3 - \eta_1\eta_2 + \frac{\eta_1^3}{4}\right)\omega^3 + \cdots\right]. \tag{29}$$

From (28) and (29), applying $N(z,s,q)$ as given in (18), we see that

$$N(d(z),s,q) = 1 + \frac{B_1(s|q)}{2}\varrho_1 z + \left[\frac{B_1(s|q)}{2}\left(\varrho_2 - \frac{\varrho_1^2}{2}\right) + \frac{B_2(s|q)}{4}\varrho_1^2\right]z^2 + \left[\frac{B_1(s|q)}{2}\left(\varrho_3 - \varrho_1\varrho_2 + \frac{\varrho_1^3}{4}\right) + \frac{B_2(s|q)}{2}\varrho_1\left(\varrho_2 - \frac{\varrho_1^2}{2}\right) + \frac{B_3(s|q)}{8}\varrho_1^3\right]z^3 + \cdots$$

and

$$N(\varpi(\omega),s,q)=1+\frac{B_1(s|q)}{2}\eta_1\omega+\left[\frac{B_1(s|q)}{2}\left(\eta_2-\frac{\eta_1^2}{2}\right)+\frac{B_2(s|q)}{4}\eta_1^2\right]\omega^2$$
$$+\left[\frac{B_1(s|q)}{2}\left(\eta_3-\eta_1\eta_2+\frac{\eta_1^3}{4}\right)+\frac{B_2(s|q)}{2}\eta_1\left(\eta_2-\frac{\eta_1^2}{2}\right)+\frac{B_3(s|q)}{8}\eta_1^3\right]\omega^3+\cdots. \tag{30}$$

It follows from (24), (30), and (25), we have

$$[2]_e(2]_e^{\gamma}v_2=\frac{B_1(s|q)}{2}\varrho_1, \tag{31}$$

$$[2]_e[3]_e(3]_e^{\gamma}v_3-[2]_e^2((2]_e^{\gamma})^2v_2^2=\frac{B_1(s|q)}{2}\left(\varrho_2-\frac{\varrho_1^2}{2}\right)+\frac{B_2(s|q)}{4}\varrho_1^2, \tag{32}$$

$$[3]_e[4]_e(4]_e^{\gamma}v_4-[2]_e[3]_e(2]_e^{\gamma}(3]_e^{\gamma}e_1v_2v_3+[2]_e^3((2]_e^{\gamma})^3v_2^3 \tag{33}$$
$$=\frac{B_1(s|q)}{2}\left(\varrho_3-\varrho_1\varrho_2+\frac{\varrho_1^3}{4}\right)+\frac{B_2(s|q)}{2}\varrho_1\left(\varrho_2-\frac{\varrho_1^2}{2}\right)+\frac{B_3(s|q)}{8}\varrho_1^3,$$

$$-[2]_e(2]_e^{\gamma}v_2=\frac{B_1(s|q)}{2}\eta_1, \tag{34}$$

$$(2[2]_e[3]_e(3]_e^{\gamma}-[2]_e^2((2]_e^{\gamma})^2)v_2^2-[2]_e[3]_e(3]_e^{\gamma}v_3=\frac{B_1(s|q)}{2}\left(\eta_2-\frac{\eta_1^2}{2}\right)+\frac{B_2(s|q)}{4}\eta_1^2, \tag{35}$$

$$(2(2+e_1)[2]_e[3]_e(2]_e^{\gamma}(3]_e^{\gamma}-5[3]_e[4]_e(4]_e^{\gamma}-[2]_e^3((2]_e^{\gamma})^2)v_2^3+(5[3]_e[4]_e(4]_e^{\gamma}$$
$$+[2]_e[3]_e(2]_e^{\gamma}(3]_e^{\gamma}e_1)v_2v_3-[3]_e[4]_e(4]_e^{\gamma}v_4=\frac{B_1(s|q)}{2}\left(\eta_3-\eta_1\eta_2+\frac{\eta_1^3}{4}\right)$$
$$+\frac{B_2(s|q)}{2}\eta_1\left(\eta_2-\frac{\eta_1^2}{2}\right)+\frac{B_3(s|q)}{8}\eta_1^3. \tag{36}$$

Adding (31) and (34), we have

$$\varrho_1=-\eta_1,\quad \varrho_1^2=\eta_1^2\quad\text{and}\quad \varrho_1^3=-\eta_1^3 \tag{37}$$

and

$$v_2^2=\frac{(B_1(s|q))^2(\varrho_1^2+\eta_1^2)}{8[2]_e^2((2]_e^{\gamma})^2}. \tag{38}$$

Moreover, adding (32) and (35) and applying (37) yields

$$4v_2^2[[2]_e[3]_e(3]_e^{\gamma}-[2]_b^2((2]_e^{\gamma})^2]=B_1(s|q)(\varrho_2+\eta_2)-\eta_1^2(B_1(s|q)-B_2(s|q)). \tag{39}$$

Applying (37) in (38) gives

$$\eta_1^2=\frac{4[2]_e^2((2]_e^{\gamma})^2v_2^2}{(B_1(s|q))^2}. \tag{40}$$

Putting (40) into (39) and with some calculations, we have

$$|v_2|^2=\left|\frac{(B_1(s|q))^3(\varrho_2+\eta_2)}{4[[2]_e[3]_e(3]_e^{\gamma}-[2]_b^2((2]_e^{\gamma})^2](B_1(s|q))^2+4[2]_e^2((2]_e^{\gamma})^2(B_1(s|q)-B_2(s|q))}\right|.$$

Applying triangular inequality and Lemma 1, we have

$$|v_2| \leq \sqrt{\Psi_1(s,q,\gamma)}. \tag{41}$$

Subtracting (35) from (32) and with some calculations, we have

$$v_3 = v_2^2 + \frac{B_1(s|q)[\varrho_2 - \eta_2]}{4[2]_e[3]_e(3)_e^{\gamma}} \tag{42}$$

$$v_3 = \frac{(B_1(s|q))^2\varrho_1^2}{4[2]_e^2((2]_e^{\gamma})^2} + \frac{B_1(s|q)[\varrho_2 - \eta_2]}{4[2]_e[3]_e(3]_e^{\gamma}}. \tag{43}$$

Applying triangular inequality and Lemma 1, we have

$$|v_3| \leq \frac{s^2}{[2]_b^2((2]_e^{\gamma})^2} + \frac{s}{4[2]_b[3]_b(3]_e^{\gamma}}. \tag{44}$$

Subtracting (36) from (33), we have

$$\begin{aligned}
2[2]_e[4]_e(4]_e^{\gamma} v_4 = {} & \frac{(2(1+e_1)[3]_e(3]_e^{\gamma} - [2]_e^2((2]_e^{\gamma})^2)(B_1(s|q))^3\varrho_1^3}{4[2]_e^2((2]_e^{\gamma})^2} \\
& - \frac{(2e_1[2]_e(2]_e^{\gamma}(3]_e^{\gamma} + 5[4]_e(4]_e^{\gamma})(B_1(s|q))^2\varrho_1(\varrho_2 - \eta_2)}{8[2]_e^2(2]_e^{\gamma}(3]_e^{\gamma}} \\
& + \frac{B_1(s|q)(\varrho_3 - \eta_3)}{2} + \frac{[B_2(s|q) - B_1(s|q)]\varrho_1(\varrho_2 + \eta_2)}{2} \\
& + \frac{(B_1(s|q) - 2B_2(s|q) + B_3(s|q))\varrho_1^3}{4}.
\end{aligned} \tag{45}$$

Applying triangular inequality and Lemma 1, we have

$$\begin{aligned}
|v_4| \leq {} & \frac{s^3(2(1+e_1)[3]_e(3]_e^{\gamma} - [2]_e^2((2]_e^{\gamma})^2)}{[2]_e^3[4]_e(4]_e^{\gamma}((2]_e^{\gamma})^2} - \frac{s^2(2e_1[2]_e(2]_e^{\gamma}(3]_e^{\gamma} + 5[4]_e(4]_e^{\gamma})}{2[2]_e^3[4]_e(2]_e^{\gamma}(3]_e^{\gamma}(4]_e^{\gamma}} \\
& + \frac{s^3 - 2s - 2qs - 4}{[2]_e[4]_e(4]_e^{\gamma}}.
\end{aligned}$$

□

3. Coefficient Estimates for the Class $\Pi_{\Sigma}^{q}(s,\gamma,\tau)$

The initial coefficient bounds of the class $\Pi_{\Sigma}^{q}(s,\gamma,\tau)$ of bi-univalent functions are investigated in this section.

Theorem 2. *Let $f \in \Pi_{\Sigma}^{q}(s,\gamma,\tau)$. Then,*

$$|v_2| \leq \sqrt{X_1(s,q,\gamma)}, \tag{46}$$

$$|v_3| \leq \frac{s^2}{e_1^2((2]_e^{\gamma})^2} + \frac{s}{(e_1+e_2)(3]_e^{\gamma}}, \tag{47}$$

and

$$\begin{aligned}
|v_4| \leq {} & \frac{s^3((2]_e^{\gamma}(3]_e^{\gamma}(4e_1+2e_2) - 2((2]_e^{\gamma})^3 e_1 - 10(e_1+e_2+e_3)(4]_e^{\gamma})}{2(e_1+e_2+e_3)((2]_e^{\gamma})^3(4]_e^{\gamma}e_1^3} \\
& - \frac{5s^2}{2(2]_e^{\gamma}(3]_e^{\gamma}e_1(e_1+e_2)} + \frac{s^3 - 2s - 2qs - 4}{(e_1+e_2+e_3)(4]_e^{\gamma}}
\end{aligned}$$

where

$$X_1(s,q,\gamma) = \frac{2s^3}{|s^2\{2(e_1+e_2)(3]_e^\gamma + (2]_e^\gamma - ((2]_e^\gamma)^2(1+2e_1)\} - 2((2]_e^\gamma)^2 e_1^2(s^2-s-1)|}. \tag{48}$$

Proof. Let $f \in \Sigma$ be given by (1) be in the class $\Pi_\Sigma^q(s,\gamma,\tau)$. Then,

$$\frac{z\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f(z))}{\mathfrak{I}_{\gamma,q}f(z)} = N(d(z),s,q) \tag{49}$$

and

$$\frac{\omega\mathfrak{D}_q(\mathfrak{I}_{\gamma,q}f^{-1}(\omega))}{\mathfrak{I}_{\gamma,q}f^{-1}(\omega)} = N(\varpi(\omega),s,q). \tag{50}$$

Let $\varrho,\eta \in \mathcal{P}$ be defined by

$$\varrho(z) = \frac{1+d(z)}{1-d(z)} = 1+\varrho_1(z)+\varrho_2 z^2+\varrho_3 z^3+\cdots \Rightarrow d(z) = \frac{\varrho(z)-1}{\varrho(z)+1}, \quad (z \in \mathbb{U}) \tag{51}$$

and

$$\eta(\omega) = \frac{1+\varpi(\omega)}{1-\varpi(\omega)} = 1+\eta_1(\omega)+\eta_2\omega^2+\eta_3\omega^3+\cdots \Rightarrow \varpi(\omega) = \frac{\eta(\omega)-1}{\eta(\omega)+1}, \quad (\omega \in \mathbb{U}). \tag{52}$$

It follows that from (51) and (52) that

$$d(z) = \frac{1}{2}\left[\varrho_1 z + \left(\varrho_2 - \frac{\varrho_1^2}{2}\right)z^2 + \left(\varrho_3 - \varrho_1\varrho_2 + \frac{\varrho_1^3}{4}\right)z^3 + \cdots\right] \tag{53}$$

and

$$\varpi(\omega) = \frac{1}{2}\left[\eta_1\omega + \left(\eta_2 - \frac{\eta_1^2}{2}\right)\omega^2 + \left(\eta_3 - \eta_1\eta_2 + \frac{\eta_1^3}{4}\right)\omega^3 + \cdots\right]. \tag{54}$$

From (53) and (54), applying $N(z,s,q)$ as given in (18), we see that

$$\begin{aligned} N(d(z),s,q) = {} & 1 + \frac{B_1(s|q)}{2}\varrho_1 z + \left[\frac{B_1(s|q)}{2}\left(\varrho_2 - \frac{\varrho_1^2}{2}\right) + \frac{B_2(s|q)}{4}\varrho_1^2\right]z^2 \\ & + \left[\frac{B_1(s|q)}{2}\left(\varrho_3 - \varrho_1\varrho_2 + \frac{\varrho_1^3}{4}\right) + \frac{B_2(s|q)}{2}\varrho_1\left(\varrho_2 - \frac{\varrho_1^2}{2}\right) + \frac{B_3(s|q)}{8}\varrho_1^3\right]z^3 + \cdots \end{aligned}$$

and

$$\begin{aligned} N(\varpi(\omega),s,q) = {} & 1 + \frac{B_1(s|q)}{2}\eta_1\omega + \left[\frac{B_1(s|q)}{2}\left(\eta_2 - \frac{\eta_1^2}{2}\right) + \frac{B_2(s|q)}{4}\eta_1^2\right]\omega^2 \\ & + \left[\frac{B_1(s|q)}{2}\left(\eta_3 - \eta_1\eta_2 + \frac{\eta_1^3}{4}\right) + \frac{B_2(s|q)}{2}\eta_1\left(\eta_2 - \frac{\eta_1^2}{2}\right) + \frac{B_3(s|q)}{8}\eta_1^3\right]\omega^3 + \cdots. \end{aligned} \tag{55}$$

It follows from (49), (55), and (50), we have

$$(2]_e^\gamma e_1 v_2 = \frac{B_1(s|q)}{2}\varrho_1, \tag{56}$$

$$(e_1+e_2)(3]_e^\gamma v_3-((2]_e^\gamma)^2 e_1 v_2^2=\frac{B_1(s|q)}{2}\left(\varrho_2-\frac{\varrho_1^2}{2}\right)+\frac{B_2(s|q)}{4}\varrho_1^2, \tag{57}$$

$$\begin{aligned}&(e_1+e_2+e_3)(4]_e^\gamma v_4-(2e_1+e_2)(2]_e^\gamma(3]_e^\gamma v_2v_3+e_1((2]_e^\gamma)^3 v_2^3\\&\quad=\frac{B_1(s|q)}{2}\left(\varrho_3-\varrho_1\varrho_2+\frac{\varrho_1^3}{4}\right)+\frac{B_2(s|q)}{2}\varrho_1\left(\varrho_2-\frac{\varrho_1^2}{2}\right)+\frac{B_3(s|q)}{8}\varrho_1^3,\end{aligned} \tag{58}$$

$$-(2]_e^\gamma e_1 v_2=\frac{B_1(s|q)}{2}\eta_1, \tag{59}$$

$$2(3]_e^\gamma(e_1+e_2)v_2^2-e_1((2]_e^\gamma)^2v_2^2-(e_1+e_2)(3]_e^\gamma v_3=\frac{B_1(s|q)}{2}\left(\eta_2-\frac{\eta_1^2}{2}\right)+\frac{B_2(s|q)}{4}\eta_1^2, \tag{60}$$

$$\begin{aligned}&((2]_e^\gamma(3]_e^\gamma(4e_1+2e_2)-5(4]_e^\gamma(e_1+e_2+e_3)-((2]_e^\gamma)^2e_1)v_2^3-((2]_e^\gamma(3]_e^\gamma(2e_1+e_2)\\&+5(4]_e^\gamma(e_1+e_2+e_3))v_2v_3-(4]_e^\gamma(e_1+e_2+e_3)v_4=\frac{B_1(s|q)}{2}\left(\eta_3-\eta_1\eta_2+\frac{\eta_1^3}{4}\right)\\&+\frac{B_2(s|q)}{2}\eta_1\left(\eta_2-\frac{\eta_1^2}{2}\right)+\frac{B_3(s|q)}{8}\eta_1^3.\end{aligned} \tag{61}$$

Adding (56) and (59), we have

$$\varrho_1=-\eta_1,\quad \varrho_1^2=\eta_1^2\quad\text{and}\quad \varrho_1^3=-\eta_1^3 \tag{62}$$

and

$$v_2^2=\frac{(B_1(s|q))^2(\varrho_1^2+\eta_1^2)}{8((2]_e^\gamma)^2e_1^2}. \tag{63}$$

Moreover, adding (57) and (60) and applying (62) yields

$$2v_2^2\{2(e_1+e_2)(3]_e^\gamma+(2]_e^\gamma-((2]_e^\gamma)^2(1+2e_1)\}=B_1(s|q)(\varrho_2+\eta_2)-\eta_1^2(B_1(s|q)-B_2(s|q)). \tag{64}$$

Applying (62) in (63) gives

$$\eta_1^2=\frac{4((2]_e^\gamma)^2e_1^2v_2^2}{(B_1(s|q))^2}. \tag{65}$$

Putting (65) into (64) and with some calculations, we have

$$|v_2|^2=\left|\frac{(B_1(s|q))^3(\varrho_2+\eta_2)}{\begin{array}{r}2[2(e_1+e_2)(3]_e^\gamma+(2]_e^\gamma-((2]_e^\gamma)^2(1+2e_1)](B_1(s|q))^2\\+4((2]_e^\gamma)^2e_1^2(B_1(s|q)-B_2(s|q))\end{array}}\right|.$$

Applying triangular inequality and Lemma 1, we have

$$|v_2|\leq\sqrt{X_1(s,q,\gamma)}. \tag{66}$$

Subtracting (60) from (57) and with some calculations, we have

$$v_3=v_2^2+\frac{B_1(s|q)[\varrho_2-\eta_2]}{4(e_1+e_2)(3]_e^\gamma} \tag{67}$$

$$v_3 = \frac{(B_1(s|q))^2\varrho_1^2}{4e_1^2((2]_e^\gamma)^2} + \frac{B_1(s|q)[\varrho_2 - \eta_2]}{4(e_1+e_2)(3]_e^\gamma}. \tag{68}$$

Applying triangular inequality and Lemma 1, we have

$$|v_3| \le \frac{s^2}{e_1^2((2]_e^\gamma)^2} + \frac{s}{(e_1+e_2)(3]_e^\gamma}. \tag{69}$$

Subtracting (61) from (58), we have

$$\begin{aligned} 2(e_1+e_2+e_3)(4]_e^\gamma v_4 = {} & \frac{(3]_e^\gamma(4e_1+2e_2)(B_1(s|q))^3\varrho_1^3}{8e_1^2((2]_e^\gamma)^2} - \frac{(B_1(s|q))^3\varrho_1^3}{4e_1^2} \\ & - \frac{5(4]_e^\gamma(e_1+e_2+e_3)(B_1(s|q))^3\varrho_1^3}{4((2]_e^\gamma)^3e_1^3} \\ & - \frac{5(4]_e^\gamma(e_1+e_2+e_3)(B_1(s|q))^2\varrho_1(\varrho_2-\eta_2)}{8(2]_e^\gamma(3]_e^\gamma e_1(e_1+e_2)} \\ & + \frac{B_1(s|q)(\varrho_3-\eta_3)}{2} + \frac{[B_2(s|q)-B_1(s|q)]\varrho_1(\varrho_2+\eta_2)}{2} \end{aligned} \tag{70}$$

$$+ \frac{(B_1(s|q)-2B_2(s|q)+B_3(s|q))\varrho_1^3}{4}. \tag{71}$$

Applying triangular inequality and Lemma 1, we have

$$\begin{aligned} |v_4| \le {} & \frac{s^3((2]_e^\gamma(3]_e^\gamma(4e_1+2e_2) - 2((2]_e^\gamma)^3e_1 - 10(e_1+e_2+e_3)(4]_e^\gamma)}{2(e_1+e_2+e_3)((2]_e^\gamma)^3(4]_e^\gamma e_1^3} \\ & - \frac{5s^2}{2(2]_e^\gamma(3]_e^\gamma e_1(e_1+e_2)} + \frac{s^3-2s-2qs-4}{(e_1+e_2+e_3)(4]_e^\gamma}. \end{aligned}$$

□

4. Fekete–Szego Inequalities for the Function Class $\Gamma_\Sigma^q(s,\gamma,\tau)$

Theorem 3. *Let $f \in \Gamma_\Sigma^q(s,\gamma,\tau)$. Then, for some $\varphi \in \mathbb{R}$,*

$$\left|v_3 - \varphi v_2^2\right| \le \begin{cases} 2|1-\varphi|\Psi_1(s,q,\gamma) & \left(|1-\varphi| \ge \frac{s}{[2]_b[3]_b(3]_e^\gamma\Psi_1(s,q,\gamma)}\right) \\ \frac{2s}{[2]_b[3]_b(3]_e^\gamma} & \left(|1-\varphi| \le \frac{s}{[2]_b[3]_b(3]_e^\gamma\Psi_1(s,q,\gamma)}\right), \end{cases}$$

where

$$\Psi_1(s,q,\gamma) = \frac{s^3}{|s^2([2]_e[3]_e(3]_e^\gamma - [2]_e^2((2]_e^\gamma)^2) - [2]_e^2((2]_e^\gamma)^2(s^2-s-1)|}. \tag{72}$$

Proof. From (42), we have

$$v_3 - \varphi v_2^2 = v_2^2 + \frac{B_1(s|q)[\varrho_2-\eta_2]}{4[2]_e[3]_e(3]_e^\gamma} - \varphi v_2^2.$$

By triangular inequality, we have

$$|v_3 - \varphi v_2^2| \le \frac{s}{[2]_e[3]_e(3]_e^\gamma} + |1-\varphi|\Psi_1(s,q,\gamma). \tag{73}$$

Suppose

$$|1-\varphi|\Psi_1(s,q,\gamma) \ge \frac{s}{[2]_e[3]_e(3]_e^\gamma}$$

then we have

$$|v_3 - \varphi v_2^2| \leq 2|1-\varphi|\Psi_1(s,q,\gamma) \tag{74}$$

where

$$|1-\varphi| \geq \frac{s}{[2]_e[3]_e(3]_e^{\gamma}\Psi_1(s,q,\gamma)}$$

and suppose

$$|1-\varphi|\Psi_1(s,q,\gamma) \leq \frac{s}{[2]_e[3]_e(3]_e^{\gamma}}$$

then we have

$$|v_3 - \delta v_2^2| \leq \frac{2s}{[2]_e[3]_e(3]_e^{\gamma}}$$

where

$$|1-\varphi| \leq \frac{s}{[2]_e[3]_e(3]_e^{\gamma}\Psi_1(s,q,\gamma)}$$

and $\Psi_1(s,q,\gamma)$ is given in (72). □

5. Fekete–Szego Inequalities for the Function Class $\Pi_{\Sigma}^{q}(s,\gamma,\tau)$

Theorem 4. *Let $f \in \Pi_{\Sigma}^{q}(s,\gamma,\tau)$. Then, for some $\varphi \in \mathbb{R}$,*

$$\left|v_3 - \varphi v_2^2\right| \leq \begin{cases} 2|1-\varphi|X_1(s,q,\gamma) & \left(|1-\varphi| \geq \frac{s}{(e_1+e_2)(3]_e^{\gamma}X_1(s,q,\gamma)}\right) \\ \frac{2s}{(e_1+e_2)(3]_e^{\gamma}} & \left(|1-\varphi| \leq \frac{s}{(e_1+e_2)(3]_e^{\gamma}X_1(s,q,\gamma)}\right), \end{cases}$$

where

$$X_1(s,q,\gamma) = \frac{2s^3}{|s^2\{2(e_1+e_2)(3]_e^{\gamma} + (2]_e^{\gamma} - ((2]_e^{\gamma})^2(1+2e_1)\} - 2((2]_e^{\gamma})^2 e_1^2(s^2-s-1)|}. \tag{75}$$

Proof. From (67), we have

$$v_3 - \varphi v_2^2 = v_2^2 + \frac{B_1(s|q)[\varrho_2 - \eta_2]}{4(e_1+e_2)(3]_e^{\gamma}} - \varphi v_2^2.$$

By triangular inequality, we have

$$|v_3 - \varphi v_2^2| \leq \frac{s}{(e_1+e_2)(3]_e^{\gamma}} + |1-\varphi|X_1(s,q,\gamma). \tag{76}$$

Suppose

$$|1-\varphi|X_1(s,q,\gamma) \geq \frac{s}{(e_1+e_2)(3]_e^{\gamma}}$$

then we have

$$|v_3 - \varphi v_2^2| \leq 2|1-\varphi|X_1(s,q,\gamma) \tag{77}$$

where

$$|1-\varphi| \geq \frac{s}{(e_1+e_2)(3]_e^{\gamma}X_1(s,q,\gamma)}$$

and suppose

$$|1-\varphi|\Psi_1(s,q,\gamma) \leq \frac{s}{(e_1+e_2)(3]_e^{\gamma}}$$

then we have

$$|v_3 - \delta v_2^2| \leq \frac{2s}{(e_1+e_2)(3]_e^{\gamma}}$$

where

$$|1-\varphi| \leq \frac{s}{(e_1+e_2)(3]_e^{\gamma} X_1(s,q,\gamma)}$$

and $X_1(s,q,\gamma)$ is given in (75). $\square$

6. Conclusions

As we mentioned earlier, q-calculus is a vital tool for understanding a large class of analytic functions and its applications. Several useful results related to the q-version of the starlike function and the q-derivative, bi-univalent functions, for instance, were provided in [26–31]. In recent decades, the orthogonal polynomials and special functions have played an essential role in mathematics, physics, engineering, and other research disciplines. In our current analysis, we used q-Hermite polynomials and q-convolution operators and systematically defined two new subclasses of bi-univalent functions, which was primarily prompted by the recent research cited in this paper. We then obtained several significant findings, such as bonds for the initial coefficients of v_2, v_3, and v_4 of the Taylor–Maclaurin series and the Fekete–Szegö functional results for our established function classes.

Moreover, to have more new theorems under the present examinations, new generalizations and applications can be explored with some positive and novel outcomes in various fields of science, mainly in geometric function theory. These recent surveys will be presented in the future research work being processed by the authors of the present paper.

However, the purported trivial (p,q)-calculus extension was clearly demonstrated to be a relatively insignificant variation of the classical q-calculus, the extra parameter p being redundant or superfluous (see, for details, [17], p. 340, and [32], pp. 1511–1512). This observation by Srivastava (see [17,32]) will indeed also apply to any future attempt to produce the rather straightforward (p,q)-variants of the results which we have presented in this paper.

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