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Article

Some Common Fixed-Circle Results on Metric Spaces

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Abstract: Recently, the fixed-circle problems have been studied with different approaches as an interesting and geometric generalization. In this paper, we present some solutions to an open problem CC: what is (are) the condition(s) to make any circle $C_{\varpi_0,\sigma}$ as the common fixed circle for two (or more than two) self-mappings? To do this, we modify some known contractions which are used in fixed-point theorems such as the Hardy–Rogers-type contraction, Kannan-type contraction, etc.

Keywords: metric spaces; fixed circle; common fixed circle

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1. Introduction

In the recent past, the fixed-circle problem has been introduced as a new geometric generalization of fixed-point theory. After that, some solutions to this problem have been investigated using various techniques (for example, see [1–8], and the references therein). In addition, in [1], the following open problem was given:

Let $(X, \mathfrak{D})$ be a metric space and $C_{\varpi_0,\sigma} = \{\varpi \in X : \mathfrak{D}(\varpi, \varpi_0) = \sigma\}$ be any circle on X.

Open Problem CC: What is (are) the condition(s) to make any circle $C_{\varpi_0,\sigma}$ as the common fixed circle for two (or more than two) self-mappings?

Let ξ and g be two self-mappings on a set X. If $\xi\varpi = g\varpi = \varpi$ for all $\varpi \in C_{\varpi_0,\sigma}$, then $C_{\varpi_0,\sigma}$ is called a common fixed circle of the pair (ξ, g) (see [9] for more details).

Some solutions were given for this open problem (for example, see [8,9]). To obtain new solutions, in this paper, we define new contractions for the pair (ξ, g) and prove new common fixed-circle results on metric spaces. Before moving on to the main results, we recall the following.

Throughout this article, we denote by $\mathbb{R}$ the set of all real numbers and by $\mathbb{R}_+$ the set of all positive real numbers.

Let ξ and g be self-mappings on a set X. If $\xi\varpi = g\varpi = w$ for some ϖ in X, then ϖ is called a coincidence point of ξ and g, w is called a point of coincidence of ξ and g.

Let $C(\xi, g) = \{\varpi \in X : \xi\varpi = g\varpi = \varpi\}$ denote the set of all common fixed-points of self-mappings ξ and g.

In [10], Wardowski introduced the following family of functions to obtain a new type of contraction called $\mathcal{F}$-contraction.

Let $\mathbb{F}$ be the family of all mappings $\mathcal{F} : \mathbb{R}_+ \to \mathbb{R}$ that satisfy the following conditions:

($\mathcal{F}1$) $\mathcal{F}$ is strictly increasing, that is, for all $a, b \in \mathbb{R}_+$ such that $a < b$ implies that $\mathcal{F}(a) < \mathcal{F}(b)$;

($\mathcal{F}2$) For every sequence $\{a_n\}_{n\in N}$ of positive real numbers, $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} \mathcal{F}(a_n) = -\infty$ are equivalent;

$(\mathcal{F}3)$There exists $k \in (0,1)$ such that $\lim_{a \to 0^+} a^k \mathcal{F}(a) = 0$.

Some examples of functions that confirm the conditions $(\mathcal{F}1)$, $(\mathcal{F}2)$, and $(\mathcal{F}3)$ are as follows:

- $\mathcal{F}(a) = \ln(a)$;
- $\mathcal{F}(a) = \ln(a) + a$;
- $\mathcal{F}(a) = \ln\left(a^2 + a\right)$;
- $\mathcal{F}(a) = -\frac{1}{\sqrt{a}}$ (see [10] for more details).

Definition 1. *[10] Let $(X, \mathfrak{D})$ be a metric space, $\mathcal{F} \in \mathbb{F}$ and $\xi : X \to X$. The mapping ξ is called an $\mathcal{F}$-contraction if there exists $\tau > 0$ such that*

$$\tau + \mathcal{F}(\mathfrak{D}(\xi\varpi, \xi v)) \leq \mathcal{F}(\mathfrak{D}(\varpi, v))$$

for all $\varpi, v \in X$ satisfying $\mathfrak{D}(T\varpi, Tv) > 0$.

2. Main Results

In this section, we prove new common fixed-circle theorems on metric spaces. For this purpose, we modify some well-known contractions such as the Wardowski-type contraction [10], Nemytskii–Edelstein-type contraction [11,12], Banach-type contraction [13], Hardy–Rogers-type contraction [14], Reich-type contraction [15], Chatterjea-type contraction [16], and Kannan-type contraction [17].

At first, we introduce the following new contraction type for two mappings to obtain some common fixed-circle results on metric spaces.

Definition 2. *Let $(X, \mathfrak{D})$ be a metric space and ξ, g be two self-mappings on X. If there exist $\tau > 0, \mathcal{F} \in \mathbb{F}$ and $\varpi_0 \in X$ such that*

$$\tau + \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) \leq \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi))$$

for all $\varpi \in X$ satisfying $\min\{\mathfrak{D}(\varpi, \xi\varpi), \mathfrak{D}(\varpi, g\varpi)\} > 0$, then the pair (ξ, g) is called a Wardowski-type $\mathcal{F}_{\xi g}$-contraction.

Notice that the point ϖ_0 mentioned in Definition 2 must be a common fixed-point of the mappings ξ and g. In fact, if ϖ_0 is not a common fixed-point of ξ and g, then we have $\mathfrak{D}(\varpi_0, \xi\varpi_0) > 0$ and $\mathfrak{D}(\varpi_0, g\varpi_0) > 0$. Hence, we obtain

$$\min\{\mathfrak{D}(\varpi_0, \xi\varpi_0), \mathfrak{D}(\varpi_0, g\varpi_0)\} > 0 \Longrightarrow \tau + \mathcal{F}(\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)) \leq \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi_0)).$$

This gives a contradiction since the domain of $\mathcal{F}$ is $(0, \infty)$. As a result, we receive the following proposition as a consequence of Definition 2.

Proposition 1. *Let $(X, \mathfrak{D})$ be a metric space. If the pair (ξ, g) is a Wardowski-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$, then we have $\xi\varpi_0 = g\varpi_0 = \varpi_0$.*

Using this new type contraction, we give the following fixed-circle theorem.

Theorem 1. *Let $(X, \mathfrak{D})$ be a metric space and the pair (ξ, g) be a Wardowski-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$. Define the number σ by*

$$\sigma = \inf\{\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) : \varpi \neq \xi\varpi, \varpi \neq g\varpi, \varpi \in X\}. \quad (1)$$

Then, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g). Especially, ξ and g fix every circle $C_{\varpi_0,r}$ where $r < \sigma$.

Proof. We distinguish two cases.

Case 1: Let $\sigma = 0$. Clearly, $C_{\varpi_0,\sigma} = \{\varpi_0\}$ and by Proposition 1, we see that $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g).

Case 2: Let $\sigma > 0$ and $\varpi \in C_{\varpi_0,\sigma}$. If $\xi\varpi \neq \varpi$ and $g\varpi \neq \varpi$, then by (1), we have $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) \geq \sigma$. Hence, using the Wardowski-type $\mathcal{F}_{\xi g}$-contraction property and the fact that $\mathcal{F}$ is increasing, we obtain

$$\begin{array}{rcl}\mathcal{F}(\sigma) & \leq & \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) \\ & \leq & \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi)) - \tau \\ & < & \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi)) \\ & = & \mathcal{F}(\sigma)\end{array}$$

This gives a contradiction. Therefore, we have $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) = 0$, that is, $\varpi = \xi\varpi$ and $\varpi = g\varpi$. As a consequence, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g).

Now, we show that ξ and g also fix any circle $C_{\varpi_0,r}$ with $r < \sigma$. Let $\varpi \in C_{\varpi_0,r}$ and suppose that $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) > 0$. With the Wardowski-type $\mathcal{F}_{\xi g}$-contraction property, we have

$$\begin{array}{rcl}\mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) & \leq & \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi)) - \tau \\ & < & \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi)) \\ & = & \mathcal{F}(r).\end{array}$$

Since $\mathcal{F}$ is increasing, then we find

$$\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) < \mathfrak{D}(\varpi_0, \varpi) < r < \sigma.$$

However, $\sigma = \inf\{\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) : \varpi \neq \xi\varpi, \varpi \neq g\varpi, \varpi \in X\}$, so this gives a contradiction. Thus, $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) = 0$ and $\varpi = \xi\varpi = g\varpi$. Hence, $C_{\varpi_0,r}$ is a common fixed circle of the pair (ξ, g). □

Example 1. *Let* $X = \left\{0, 1, -e, e, e-1, e+1, -e^2, e^2, e^2-1, e^2+1, e^2-e, e^2+e\right\}$ *with usual metric. Define* $\xi, g : X \to X$ *by*

$$\xi\varpi = \begin{cases} 1, & \varpi = 0 \\ \varpi, & otherwise \end{cases}$$

and

$$g\varpi = \begin{cases} e-1, & \varpi = 0 \\ \varpi, & otherwise \end{cases}.$$

Take $\mathcal{F}(a) = \ln(a) + a$, $a > 0, \tau = e$ *and* $\varpi_0 = e^2$. *Thus, the pair* (ξ, g) *is a Wardowski-type* $\mathcal{F}_{\xi g}$*-contraction. For* $\varpi = 0$, *we have*

$$\begin{array}{rcl}\min\{\mathfrak{D}(\varpi, \xi\varpi), \mathfrak{D}(\varpi, g\varpi)\} & = & \min\{\mathfrak{D}(0,1), \mathfrak{D}(0, e-1)\} \\ & = & \min\{1, e-1\} \\ & = & 1 > 0\end{array}$$

In addition, we can easily see that the following inequality is satisfied:

$$\begin{array}{rcl}\tau + \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) & \leq & \mathcal{F}(\mathfrak{D}(\varpi_0, \varpi)) \\ e + \mathcal{F}(1 + e - 1) & \leq & \mathcal{F}\left(e^2\right) \\ e + \ln e + e & \leq & \ln e^2 + e^2 \\ 2e + 1 & \leq & 2 + e^2\end{array}$$

With Theorem (1), we obtain

$$\sigma = \inf\{\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) : \varpi \neq \xi\varpi, \varpi \neq g\varpi, \varpi \in X\} = \inf\{1 + e - 1\} = e$$

and ξ, g fix the circle $C_{e^2,e} = \{e^2 - e, e^2 + e\}$. Notice that ξ and g fix also the circle $C_{e^2,1} = \{e^2 - 1, e^2 + 1\}$.

The converse of Theorem 1 fails. The following example confirms this statement.

Example 2. *Let $(X, \mathfrak{D})$ be a metric space with any point $\varpi_0 \in X$. Define the self-mappings ξ and g as follows:*

$$\xi\varpi = \begin{cases} \varpi, & \mathfrak{D}(\varpi, \varpi_0) \leq \mu \\ \varpi_0, & \mathfrak{D}(\varpi, \varpi_0) > \mu \end{cases}$$

and

$$g\varpi = \begin{cases} \varpi, & \mathfrak{D}(\varpi, \varpi_0) \leq \mu \\ \varpi_0, & \mathfrak{D}(\varpi, \varpi_0) > \mu \end{cases},$$

for all $\varpi \in X$ with any $\mu > 0$. Then, it can be easily checked that the pair (ξ, g) is not a Wardowski-type $\mathcal{F}_{\xi g}$-contraction for the point ϖ_0 but ξ and g fix every circle $C_{\varpi_0,r}$ where $r \leq \mu$.

Example 3. *Let $\mathbb{C}$ be the set of complex numbers, $(\mathbb{C}, \mathfrak{D})$ be the usual metric space, and define the self-mappings $\xi, g : \mathbb{C} \to \mathbb{C}$ as follows:*

$$\xi\varpi = \begin{cases} \varpi, & |\varpi - 2| < e \\ \varpi + \frac{1}{2}, & |\varpi - 2| \geq e \end{cases}$$

and

$$g\varpi = \begin{cases} \varpi, & |\varpi - 2| < e \\ \varpi - \frac{1}{2}, & |\varpi - 2| \geq e \end{cases},$$

for all $\varpi \in C$. We have $\sigma = \inf\{\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) : \varpi \neq \xi\varpi, \varpi \neq g\varpi, \varpi \in C\}$. Thus, the pair (ξ, g) is a Wardowski-type $\mathcal{F}_{\xi g}$-contraction with $\mathcal{F} = ln(a), \tau = \ln e$ and $\varpi_0 = 2 \in C$. Obviously, the number of common fixed circles of ξ and g is infinite.

Definition 3. *If there exist $\tau > 0, \mathcal{F} \in \mathcal{F}$ and $\varpi_0 \in X$ such that for all $\varpi \in X$ the following holds:*

$$\tau + \mathcal{F}(\mathfrak{D}(\xi\varpi, \varpi) + \mathfrak{D}(g\varpi, \varpi)) < \mathcal{F}(\mathfrak{D}(\varpi, \varpi_0))$$

with $\min\{\mathfrak{D}(\xi\varpi, \varpi), \mathfrak{D}(g\varpi, \varpi)\} > 0$, then the pair (ξ, g) is called a Nemytskii–Edelstein-type $\mathcal{F}_{\xi g}$-contraction.

Proposition 2. *Let $(X, \mathfrak{D})$ be a metric space. If the pair (ξ, g) is a Nemytskii-Edelstein-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$, then we have $\xi\varpi_0 = g\varpi_0 = \varpi_0$.*

Proof. It can be easily proved from the similar arguments used in Proposition 1. □

Theorem 2. *Let the pair (ξ, g) be a Nemytskii–Edelstein-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$ and σ be defined as in (1). Then, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g). Especially, ξ and g fix every circle $C_{\varpi_0,r}$ where $r < \sigma$.*

Proof. It can be easily seen from the proof of Theorem 1. □

In addition, we inspire the classical Banach contraction principle to give the following definition:

Definition 4. *If there exist $\tau > 0$, $\mathcal{F} \in \mathcal{F}$ and $\varpi_0 \in X$ such that for all $\varpi \in X$, the following holds:*

$$\tau + \mathcal{F}(\mathfrak{D}(\xi\varpi, \varpi) + \mathfrak{D}(g\varpi, \varpi)) \leq \mathcal{F}(\eta\mathfrak{D}(\varpi, \varpi_0))$$

with $\min\{\mathfrak{D}(\xi\varpi, \varpi), \mathfrak{D}(g\varpi, \varpi)\} > 0$ where $\eta \in [0, 1)$, then the pair (ξ, g) is called a Banach-type $\mathcal{F}_{\xi g}$-contraction.

Proposition 3. *Let $(X, \mathfrak{D})$ be a metric space. If the pair (ξ, g) is a Banach-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$, then we have $\xi\varpi_0 = g\varpi_0 = \varpi_0$.*

Proof. It can be easily proved from the similar arguments used in Proposition 1. □

Theorem 3. *Let the pair (ξ, g) be a Banach-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$ and σ be defined as in (1). Then $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g). Especially, ξ and g fix every circle $C_{\varpi_0,r}$ where $r < \sigma$.*

Proof. It can be easily seen from the proof of Theorem 1. □

If we consider Example 1, then the pair (ξ, g) is both a Nemytskii–Edelstein-type $\mathcal{F}_{\xi g}$-contraction and a Banach-type $\mathcal{F}_{\xi g}$-contraction with $\mathcal{F}(a) = \ln(a) + a$, $a > 0, \tau = e$, $\varpi_0 = e^2$ and so ξ, g have two common fixed circles $C_{e^2,e}$ and $C_{e^2,1}$.

We introduce the notion of Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction.

Definition 5. *Let $(X, \mathfrak{D})$ be a metric space and ξ, g be two self-mappings on X. The pair (ξ, g) is called a Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction if there exist $\tau > 0$ and $\mathcal{F} \in \mathcal{F}$ such that*

$$\tau + \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) \leq \mathcal{F}\begin{pmatrix} \alpha\mathfrak{D}(\varpi, \varpi_0) + \beta\mathfrak{D}(\varpi, \xi\varpi) \\ +\gamma\mathfrak{D}(\varpi, g\varpi) + \delta\mathfrak{D}(\varpi_0, \xi\varpi_0) + \eta\mathfrak{D}(\varpi_0, g\varpi_0) \end{pmatrix} \quad (2)$$

holds for any $\varpi, \varpi_0 \in X$ with $\min\{\mathfrak{D}(\varpi, \xi\varpi), \mathfrak{D}(\varpi, g\varpi)\} > 0$, where $\alpha, \beta, \gamma, \delta, \eta$ are nonnegative numbers, $\alpha \neq 0$ and $\alpha + \beta + \gamma + \delta + \eta \leq 1$.

Proposition 4. *If the pair (ξ, g) is a Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$, then we have $\xi\varpi_0 = g\varpi_0 = \varpi_0$.*

Proof. Suppose that $\xi\varpi_0 \neq \varpi_0$ and $g\varpi_0 \neq \varpi_0$. From the definition of the Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction with $\min\{\mathfrak{D}(\varpi_0, \xi\varpi_0), \mathfrak{D}(\varpi_0, g\varpi_0)\} > 0$, we obtain

$$\begin{aligned} \tau + \mathcal{F}(\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)) &\leq \mathcal{F}\begin{pmatrix} \alpha\mathfrak{D}(\varpi_0, \varpi_0) + \beta\mathfrak{D}(\varpi_0, \xi\varpi_0) \\ +\gamma\mathfrak{D}(\varpi_0, g\varpi_0) + \delta\mathfrak{D}(\varpi_0, \xi\varpi_0) + \eta\mathfrak{D}(\varpi_0, g\varpi_0) \end{pmatrix} \\ &= \mathcal{F}((\beta + \delta)\mathfrak{D}(\varpi_0, \xi\varpi_0) + (\gamma + \eta)\mathfrak{D}(\varpi_0, g\varpi_0)) \\ &< \mathcal{F}(\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)) \end{aligned}$$

a contradiction because of $\tau > 0$. Thus, we have $\xi\varpi_0 = g\varpi_0 = \varpi_0$. □

Using Proposition 4, we rewrite the condition (2) as follows:

$$\tau + \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi), \mathfrak{D}(\varpi, g\varpi)) \leq \mathcal{F}(\alpha\mathfrak{D}(\varpi, \varpi_0) + \beta\mathfrak{D}(\varpi, \xi\varpi) + \gamma\mathfrak{D}(\varpi, g\varpi))$$

with $\min\{\mathfrak{D}(\varpi, \xi\varpi), \mathfrak{D}(\varpi, g\varpi)\} > 0$ where α, β, γ are nonnegative numbers, $\alpha \neq 0$ and $\alpha + \beta + \gamma \leq 1$.

Using this inequality, we present the following fixed-circle result.

Theorem 4. *Let the pair (ξ, g) be a Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$ and σ be defined as in (1). If $\beta = \gamma$, then $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g). In addition, ξ and g fix every circle $C_{\varpi_0,r}$ with $r < \sigma$.*

Proof. We distinguish two cases.

Case 1: Let $\sigma = 0$. Clearly, $C_{\varpi_0,\sigma} = \{\varpi_0\}$ and by Proposition 4, we see that $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g).

Case 2: Let $\sigma > 0$ and $\varpi \in C_{\varpi_0,\sigma}$. Using the Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contractive property and the fact that $\mathcal{F}$ is increasing, we have

$$\begin{aligned}
\mathcal{F}(\sigma) &\leq \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) \\
&\leq \mathcal{F}(\alpha\mathfrak{D}(\varpi, \varpi_0) + \beta\mathfrak{D}(\varpi, \xi\varpi) + \gamma\mathfrak{D}(\varpi, g\varpi)) - \tau \\
&< \mathcal{F}(\alpha\sigma + \beta(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi))) \\
&< \mathcal{F}((\alpha + \beta)(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi))) \\
&< \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)).
\end{aligned}$$

This gives a contradiction. Therefore, $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) = 0$ and so $\xi\varpi = \varpi = g\varpi$. As a result, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ, g).

Now, we show that ξ and g also fix any circle $C_{\varpi_0,r}$ with $r < \sigma$. Let $\varpi \in C_{\varpi_0,r}$ and suppose that $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) > 0$. By the Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction, we have

$$\begin{aligned}
\mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)) &\leq \mathcal{F}(\alpha\mathfrak{D}(\varpi, \varpi_0) + \beta\mathfrak{D}(\varpi, \xi\varpi) + \gamma\mathfrak{D}(\varpi, g\varpi)) - \tau \\
&< \mathcal{F}(\alpha\mathfrak{D}(\varpi, \varpi_0) + \beta\mathfrak{D}(\varpi, \xi\varpi) + \gamma\mathfrak{D}(\varpi, g\varpi)) \\
&< \mathcal{F}(\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi))
\end{aligned}$$

a contradiction. So, we obtain $\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi) = 0$ and $\xi\varpi = \varpi = g\varpi$. Thus, $C_{\varpi_0,r}$ is a common fixed circle of the pair (ξ, g). □

Remark 1. *If we take $\alpha = 1$ and $\beta = \gamma = \delta = \eta = 0$ in Definition 5, then we obtain the concept of a Wardowski-type $\mathcal{F}_{\xi g}$-contractive mapping.*

Now, we give the concept of a Reich-type $\mathcal{F}_{\xi g}$-contraction as follows.

Definition 6. *If there exist $\tau > 0$, $\mathcal{F} \in \mathcal{F}$ and $\varpi_0 \in X$ such that for all $\varpi \in X$, the following holds*:

$$\tau + \mathcal{F}(\mathfrak{D}(\xi\varpi, \varpi) + \mathfrak{D}(g\varpi, \varpi)) \leq \mathcal{F}\begin{pmatrix} \alpha\mathfrak{D}(\varpi, \varpi_0) + \beta[\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)] \\ +\gamma[\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)] \end{pmatrix} \tag{3}$$

with $\min\{\mathfrak{D}(\xi\varpi, \varpi), \mathfrak{D}(g\varpi, \varpi)\} > 0$, *where $\alpha + \beta + \gamma < 1$, $\alpha \neq 0$ and $\alpha, \beta, \gamma \in [0, \infty)$. Then, the pair (ξ, g) is called a Reich-type $\mathcal{F}_{\xi g}$-contraction on X.*

Proposition 5. *If the pair (ξ, g) is a Reich-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0 \in X$, then we have $\xi\varpi_0 = \varpi_0 = g\varpi_0$.*

Proof. Assume that $\xi\varpi_0 \neq \varpi_0$ and $g\varpi_0 \neq \varpi_0$. From the definition of the Reich-type $\mathcal{F}_{\xi g}$-contraction with $\min\{\mathfrak{D}(\varpi_0, \xi\varpi_0), \mathfrak{D}(\varpi_0, g\varpi_0)\} > 0$, we get

$$\begin{aligned}
\tau + \mathcal{F}(\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)) &\leq \mathcal{F}\begin{pmatrix} \alpha\mathfrak{D}(\varpi_0, \varpi_0) + \beta[\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)] \\ +\gamma[\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)] \end{pmatrix} \\
&= \mathcal{F}((\beta + \gamma)[\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0)]) \\
&< \mathcal{F}(\mathfrak{D}(\varpi_0, \xi\varpi_0) + \mathfrak{D}(\varpi_0, g\varpi_0))
\end{aligned}$$

a contradiction because of $\tau > 0$. Then, we have $\xi\varpi_0 = \varpi_0 = g\varpi_0$. □

Using Proposition 5, we rewrite the condition (3) as follows:

$$\tau + \mathcal{F}(\mathfrak{D}(\xi\varpi, \varpi) + \mathfrak{D}(g\varpi, \varpi)) \leq \mathcal{F}(\alpha\mathfrak{D}(\varpi, \varpi_0) + \beta[\mathfrak{D}(\varpi, \xi\varpi) + \mathfrak{D}(\varpi, g\varpi)])$$

with $\min\{\mathfrak{D}(\xi\varpi,\varpi),\mathfrak{D}(g\varpi,\varpi)\} > 0$ where $\alpha+\beta<1$, $\alpha\neq 0$ and $\alpha,\beta\in[0,\infty)$.

Using this inequality, we obtain the following common fixed-circle result.

Theorem 5. *Let the pair (ξ,g) be a Reich-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0\in X$ and σ be defined as in (1). Then, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). Especially, ξ and g fix every circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$.*

Proof. We distinguish two cases.

Case 1: Let $\sigma=0$. Clearly, $C_{\varpi_0,\sigma}=\{\varpi_0\}$ and by Proposition 5, we see that $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g).

Case 2: Let $\sigma>0$ and $\varpi\in C_{\varpi_0,\sigma}$. This case can be easily seen since

$$\begin{aligned}\mathcal{F}(\sigma) &\leq \mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\\ &\leq \mathcal{F}((\alpha+\beta)[\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)])\\ &< \mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)).\end{aligned}$$

Consequently, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). Especially, ξ and g fix every circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$. □

To obtain, some new common fixed-circle results, we define the following contractions.

Definition 7. *If there exist $\tau>0$, $\mathcal{F}\in\mathcal{F}$ and $\varpi_0\in X$ such that for all $\varpi\in X$, the following holds:*

$$\tau+\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\leq\mathcal{F}(\eta[\mathfrak{D}(\xi\varpi,\varpi_0)+\mathfrak{D}(g\varpi,\varpi_0)])$$

with $\min\{\mathfrak{D}(\xi\varpi,\varpi),\mathfrak{D}(g\varpi,\varpi)\}>0$ where $\eta\in\left(0,\frac{1}{3}\right)$, then the pair (ξ,g) is called a Chatterjea-type $\mathcal{F}_{\xi g}$-contraction.

Proposition 6. *If the pair (ξ,g) is a Chattereja-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0\in X$, then we have $\xi\varpi_0=\varpi_0=g\varpi_0$.*

Proof. From the similar arguments used in Proposition 4, it can be easily proved. □

Theorem 6. *Let the pair (ξ,g) be a Chatterjea-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0\in X$ and σ be defined as in (1). Then, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). Especially, ξ and g fix every circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$.*

Proof. We distinguish two cases.

Case 1: Let $\sigma=0$. Clearly, $C_{\varpi_0,\sigma}=\{\varpi_0\}$ and by Proposition 6, we see that $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g).

Case 2: Let $\sigma>0$ and $\varpi\in C_{\varpi_0,\sigma}$. Using the Chatterjea-type $\mathcal{F}_{\xi g}$-contractive property, the fact that $\mathcal{F}$ is increasing, and the triangle inequality property of metric function d, we have

$$\begin{aligned}\mathcal{F}(\sigma) &\leq \mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\\ &\leq \mathcal{F}(\eta[\mathfrak{D}(\xi\varpi,\varpi_0)+\mathfrak{D}(g\varpi,\varpi_0)])-\tau\\ &\leq \mathcal{F}(\eta[\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(\varpi,\varpi_0)+\mathfrak{D}(g\varpi,\varpi)+\mathfrak{D}(\varpi,\varpi_0)])\\ &= \mathcal{F}(\eta[2\mathfrak{D}(\varpi,\varpi_0)+[\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)]])\\ &= \mathcal{F}(3\eta[\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)])\\ &< \mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)).\end{aligned}$$

This gives a contradiction. Thus, $\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)=0$, that is, $\xi\varpi=\varpi=g\varpi$. As a result, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). By the similar arguments used in the proof of Theorem 1, ξ and g also fix any circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$. □

Definition 8. *If there exist $\tau>0$, $\mathcal{F}\in\mathcal{F}$ and $\varpi_0\in X$ such that for all $\varpi\in X$ the following holds*:

$$\tau+\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\leq\mathcal{F}(\eta[\mathfrak{D}(\varpi,\xi\varpi_0)+\mathfrak{D}(\varpi,g\varpi_0)])\tag{4}$$

with $\min\{\mathfrak{D}(\xi\varpi,\varpi),\mathfrak{D}(g\varpi,\varpi)\}>0$ *where* $\eta\in\left(0,\frac{1}{2}\right)$*, then the pair (ξ,g) is called a Kannan-type $\mathcal{F}_{\xi g}$-contraction.*

Proposition 7. *If the pair (ξ,g) is a Kannan-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0\in X$, then we have $\xi\varpi_0=\varpi_0=g\varpi_0$.*

Proof. From the similar arguments used in Proposition 4, it can be easily obtained. □

Theorem 7. *Let the pair (ξ,g) be a Kannan-type $\mathcal{F}_{\xi g}$-contraction with $\varpi_0\in X$ and σ be defined as in (1). Then, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). Especially, ξ and g fix every circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$.*

Proof. We distinguish two cases.

Case 1: Let $\sigma=0$. Clearly, $C_{\varpi_0,\sigma}=\{\varpi_0\}$ and by Proposition 7, we see that $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g).

Case 2: Let $\sigma>0$ and $\varpi\in C_{\varpi_0,\sigma}$. Using the Kannan-type $\mathcal{F}_{\xi g}$-contractive property, the fact that $\mathcal{F}$ is increasing, and the triangle inequality property of metric function d, we have

$$\begin{aligned}\mathcal{F}(\sigma)&\leq\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\\&\leq\mathcal{F}(\eta[\mathfrak{D}(\varpi,\xi\varpi_0)+\mathfrak{D}(\varpi,g\varpi_0)])-\tau\\&\leq\mathcal{F}(\eta[\mathfrak{D}(\varpi,\varpi_0)+\mathfrak{D}(\varpi,\varpi_0)])\\&\leq\mathcal{F}(2\eta\sigma)\\&<\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)).\end{aligned}$$

This gives a contradiction. Thus, $\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi)=0$, that is, $\xi\varpi=\varpi=g\varpi$. As a result, $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). By similar arguments used in the proof of Theorem 1, ξ and g also fix any circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$. □

Now, we present an illustrative example of our obtained results.

Example 4. *Let $X=\left\{1,2,e^2,e^2-1,e^2+1\right\}$ be the metric space with the usual metric. Let us define the self-mappings $\xi,g:X\longrightarrow X$ as*

$$\xi\varpi=\begin{cases}2, & \varpi=1\\ \varpi, & otherwise\end{cases}$$

and

$$g\varpi=\begin{cases}2, & \varpi=1\\ \varpi, & otherwise\end{cases},$$

for all $\varpi\in X$.

The pair (ξ,g) is a Hardy–Rogers-type $\mathcal{F}_{\xi g}$-contraction with $\mathcal{F}=lna+a$, $\tau=0.01$, $\alpha=\beta=\gamma=\frac{1}{4}$ and $\varpi_0=e^2$. Indeed, we get

$$\min\{\mathfrak{D}(\varpi,\xi\varpi),\mathfrak{D}(\varpi,g\varpi)\}=\min\{\mathfrak{D}(1,2),\mathfrak{D}(1,2)\}=1>0$$

for $\varpi = 1$ and we get

$$\begin{aligned}\alpha\mathfrak{D}(\varpi,\varpi_0)+\beta\mathfrak{D}(\varpi,\xi\varpi)+\gamma\mathfrak{D}(\varpi,g\varpi) &= \frac{1}{4}\Big[\mathfrak{D}\Big(1,e^2\Big)+\mathfrak{D}(1,2)+\mathfrak{D}(1,2)\Big]\\ &= \frac{1}{4}\Big[e^2-1+1+1\Big]\\ &= \frac{e^2+1}{4}.\end{aligned}$$

Then, we have

$$\begin{aligned}\tau+\mathcal{F}(\mathfrak{D}(\varpi,\xi\varpi)+\mathfrak{D}(\varpi,g\varpi)) &= 0.01+\ln 2+2\\ &\leq \mathcal{F}\left(\frac{e^2+1}{4}\right)\\ &= \ln\Big(e^2+1\Big)-\ln 4+\frac{e^2+1}{4}.\end{aligned}$$

The pair (ξ, g) is a Reich-type $\mathcal{F}_{\xi g}$-contraction with $\mathcal{F} = \ln a$, $\tau = \ln\big(e^2+1\big) - \ln 6$, $\alpha = \beta = \frac{1}{3}$ and $\varpi_0 = e^2$. Indeed, we get

$$\min\{\mathfrak{D}(\varpi,\xi\varpi),\mathfrak{D}(\varpi,g\varpi)\} = \min\{\mathfrak{D}(1,2),\mathfrak{D}(1,2)\} = 1 > 0$$

for $\varpi = 1$ and we have

$$\begin{aligned}\alpha\mathfrak{D}(\varpi,\varpi_0)+\beta[\mathfrak{D}(\varpi,\xi\varpi)+\mathfrak{D}(\varpi,g\varpi)] &= \frac{1}{3}\mathfrak{D}\Big(1,e^2\Big)+\frac{1}{3}[\mathfrak{D}(1,2)+\mathfrak{D}(1,2)]\\ &= \frac{e^2+1}{3}.\end{aligned}$$

Then, we obtain

$$\begin{aligned}\tau+\mathcal{F}(\mathfrak{D}(\varpi,\xi\varpi)+\mathfrak{D}(\varpi,g\varpi)) &= \ln\Big(e^2+1\Big)-\ln 6+\ln 2\\ &\leq \mathcal{F}\left(\frac{e^2+1}{3}\right)\\ &= \ln(e^2+1)-\ln 3.\end{aligned}$$

The pair (ξ, g) is both a Chatterjea-type $\mathcal{F}_{\xi g}$-contractions and a Kannan-type $\mathcal{F}_{\xi g}$-contraction with $\mathcal{F} = lna$, $\tau = \ln\big(e^2-2\big) - \ln 4$, $\eta = \frac{1}{4}$ and $\varpi_0 = e^2$. Indeed, for Chatterjea-type $\mathcal{F}_{\xi g}$-contractions, we get

$$\min\{\mathfrak{D}(\varpi,\xi\varpi),\mathfrak{D}(\varpi,g\varpi)\} = \min\{\mathfrak{D}(1,2),\mathfrak{D}(1,2)\} = 1 > 0$$

for $\varpi = 1$ and we have

$$\begin{aligned}\eta[\mathfrak{D}(\varpi_0,\xi\varpi)+\mathfrak{D}(\varpi_0,g\varpi)] &= \frac{1}{4}\Big[\mathfrak{D}\Big(e^2,2\Big)+\mathfrak{D}\Big(e^2,2\Big)\Big]\\ &\leq \frac{1}{4}\Big[2(e^2-2)\Big]\\ &= \frac{e^2-2}{2}.\end{aligned}$$

Then, we obtain

$$\begin{aligned}\tau+\mathcal{F}(\mathfrak{D}(\varpi,\xi\varpi)+\mathfrak{D}(\varpi,g\varpi)) &= \ln\left(e^2-2\right)-\ln 4+\ln 2\\ &\leq \mathcal{F}\left(\frac{e^2-2}{2}\right)\\ &= \ln\left(e^2-2\right)-\ln 2.\end{aligned}$$

For Kannan-type $\mathcal{F}_{\xi g}$-contractions, we have

$$\min\{\mathfrak{D}(\varpi,\xi\varpi),\mathfrak{D}(\varpi,g\varpi)\}=\min\{\mathfrak{D}(1,2),\mathfrak{D}(1,2)\}=1>0$$

for $\varpi=1$ and we have

$$\begin{aligned}\eta[\mathfrak{D}(\varpi,\xi\varpi_0)+\mathfrak{D}(\varpi,g_0)] &= \frac{1}{4}\left[\mathfrak{D}\left(1,e^2\right)+\mathfrak{D}\left(1,e^2\right)\right]\\ &\leq \frac{1}{4}\left[2(e^2-1)\right]\\ &= \frac{e^2-1}{2}.\end{aligned}$$

Then, we obtain

$$\begin{aligned}\tau+\mathcal{F}(\mathfrak{D}(\varpi,\xi\varpi)+\mathfrak{D}(\varpi,g\varpi)) &= \ln\left(e^2-2\right)-\ln 4+\ln 2\\ &\leq \mathcal{F}\left(\frac{e^2-1}{2}\right)\\ &= \ln\left(e^2-1\right)-\ln 2.\end{aligned}$$

Consequently, ξ and g fix the circle $C_{e^2,1}=\{e^2-1,e^2+1\}$.

If we combine the notions of Banach-type $\mathcal{F}_{\xi g}$-contractions, Chatterjea-type $\mathcal{F}_{\xi g}$-contractions, and Kannan-type $\mathcal{F}_{\xi g}$-contractions, then we get the following corollary. This corollary can be considered as Zamfirescu-type common fixed-circle result [18].

Corollary 1. *Let $(X,\mathfrak{D})$ be a metric space, $\xi,g:X\longrightarrow X$ be two self-mappings and σ be defined as in (1). If there exist $\tau>0$, $\mathcal{F}\in\mathcal{F}$ and $\varpi_0\in X$ such that for all $\varpi\in X$, at least one of the followings holds*:

(1) $\tau+\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\leq\mathcal{F}(\alpha\mathfrak{D}(\varpi,\varpi_0))$,

(2) $\tau+\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\leq\mathcal{F}(\beta[\mathfrak{D}(\xi\varpi,\varpi_0)+\mathfrak{D}(g\varpi,\varpi_0)])$,

(3) $\tau+\mathcal{F}(\mathfrak{D}(\xi\varpi,\varpi)+\mathfrak{D}(g\varpi,\varpi))\leq\mathcal{F}(\gamma[\mathfrak{D}(\varpi,\xi\varpi_0)+\mathfrak{D}(\varpi,g\varpi_0)])$,

with $\min\{\mathfrak{D}(\xi\varpi,\varpi),\mathfrak{D}(g\varpi,\varpi)\}>0$ *where* $0\leq\alpha<1$, $0\leq\beta,\gamma<\frac{1}{2}$, *then $C_{\varpi_0,\sigma}$ is a common fixed circle of the pair (ξ,g). Especially, ξ and g fix every circle $C_{\varpi_0,\rho}$ with $\rho<\sigma$.*

Proof. It is obvious. □

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