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Group Acceptance Sampling Plan Based on New Compounded Three-Parameter Weibull Model

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Abstract: In this study, we introduce a new compounded model called the complementary Bell–Weibull model and use it to address the problem of a group acceptance sampling plan predicted on a truncated life test. The median lifespan is used as a quality index to obtain the design constraints, namely sample size and approval number, under a predefined consumerś risk and test termination period. Additionally, two real data applications are presented, and unknown parameters are estimated using the maximum likelihood approach.

Keywords: acceptance sampling; complementary Bell–Weibull; compounding; quality parameter

MSC: 62N05; 60E05; 11K06; 94A20

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1. Introduction

Technology development spurs industry innovation and accelerates the production process. A 100% inspection cannot be highlighted in the bulk of statistical quality control experiments due to time and monetary limitations. Consequently, numerous statistical destructive tests and sampling distributions are used to evaluate the product's quality and to make the quality control experiment as effective as possible. Acceptance sampling plans (ASP) have gained some popularity in the area of quality control and lifetime testing. In ASP, the decision to approve or disapprove a lot is made based on a life test or inspection that involves selecting a random sample from the lot. The ASP is a strategy that typically outlines the criteria for lot dispositioning as well as the appropriate sample size that should be used. As a result, the ASP sets the quantity of units, say, n, that would be used for screening as well as the acceptance number c; hence, if there are no more than c failures out of n items, the lot is approved, but otherwise, it is discarded. The chances that a poor lot will be approved and a fine lot will be denied are the consumer and producer risks, respectively. Each sampling plan often includes information on the risks to consumers and producers. The chances that a poor lot will be accepted and a good lot will be rejected, respectively, depend on the ASP. The associated hazards to consumers and producers are often included with every acceptance sampling plan. A group acceptance sampling plan (GASP) was developed by [1] and is merely an extension of the standard acceptance sampling plan. It is a variable sampling plan for sudden death testing that adopts a Weibull distribution.

Several studies related to GASP are documented in statistical literature when a lifetime of a product follows a certain distribution. Here, we mention a few. Readers are referred to Tripathi et al. [2], Rao and Suseela [3] for designing a GASP using exponentiated an inverted Weibull distribution. Yagiter et al. [4] used a compounded Weibull exponential distribution for designing a GASP for a time-truncated life test. Almarsshi et al. [5] designed a GASP based on the four-parameter Marshal–Olkin–Kumaraswamy exponential distribution taking the median as a quality parameter. Aslam et al. [6] used an extended exponential distribution for designing a time-truncated GASP. Aslam et al. [7] designed a GASP for a resubmitted lot based on a Burr-12 distribution, and Aslam et al. [8] proposed a design of a GASP when a lifetime of a product follows a generalized Pareto distribution.

As a rival or predecessor to the Poisson distribution, Castellares et al. [9] launched the discrete Bell distribution, which displays many fascinating characteristics, including a single parameter distribution and pertaining to the one-parameter exponential family of distribution, as well as the Poisson distribution. The readers are referred to [10,11] for detailed discussion and formulation along with motivation for the compounding approach. They looked at the fact that the Poisson model cannot be embedded into the Bell model, but the Bell model also approaches the Poisson distribution for modest values of the parameter. Additionally, the Bell model is infinitely scalable and has a higher variance than the mean that can be used to get over the zero vertex and over dispersion problems that arises with count data. These properties of the Bell model inspired us to create a generalised class of distributions by compounding and comparing its mathematical and empirical properties to those of the compounding Poisson-G class and its specialised models. The readers are directed to [10,11] for in-depth analysis and formulation along with motivation for the compounding approach.

Complementary risk models are important in many practical areas such as industrial reliability, quality control, biomedical studies and actuarial sciences. Generally, in reliability analysis, the models only determine the maximum components' lifetime values among all risks in parallel system. However, component failure is dependent upon several risk factors, and there is no such information regarding which factor is more prone to or responsible for component failure [12]. The proposed model origin started on a complementary risk problem base in the presence of latent risks.

The manuscript is structured as follows: CBell-G is illustrated in Section 2 by taking the Weibull distribution as a special model. Section 3 is based on designing a GASP when a lifetime of a product follows a complementary Bell–Weibull (CBellW) model. Section 4 provides a discussion and an illustrative example. Section 5 contains real data applications. Finally, the manuscript is concluded in Section 6 with a few summarising comments.

2. Complementary Bell-G with Motivation

The challenge of complementary risk in the presence of latent risks prompted the development of the proposed model. This section contained the physical motivation for the new family, consider a system that consists of N independent subsystems that are all operating at the same time and where N is a truncated Bell random variable with probability mass function

$$P(N=n)=\frac{\lambda^n e^{1-e^{\lambda}}B_n}{n!\left[1-e^{1-e^{\lambda}}\right]}\;;\quad n=1,2,3\cdots \tag{1}$$

Here y_i denotes the life of the ith subsystem, and θ parallel units constitute the subsystem. If every component of a parallel structure fails, the entire system crumbles. For a series system, on the other hand, the failure of any subsystem results in the total destruction of the system. The failure time of the components for the ith subsystem is i.i.d. Then, the following is the conditional cumulative distribution function of $t|N$, where $T=\max(Y_1,\ldots Y_N)$

$$F(t|N)=\mathbb{P}[\max(Y_1,\ldots Y_N)<t|N]=\left[\mathbb{P}^{\theta}(T_{n,n}\leq t)\right]^N=\left[G(t,\tau)^{\theta}\right]^N. \tag{2}$$

The T unconditional cumulative distribution function corresponds to (2) as

$$F(t)=\sum_{n=1}^{\infty}[F(t|N)]\mathbb{P}(N=n)=\sum_{n=1}^{\infty}\left[G(t,\tau)^{\theta}\right]^N\mathbb{P}(N=n) \tag{3}$$

If $Z[G(t;\tau),\theta]=G(t;\tau)^{\theta}$ and N is a zero truncated Bell (ZTBell) random vairable with the probability mass function given in (1), the cumulative distribution function and density function of the CBell-G is defined as with $\theta=1$.

$$F(t) = \frac{\exp\left[e^{\lambda G(t)} - 1\right] - 1}{\exp\left[e^{\lambda} - 1\right] - 1}. \tag{4}$$

The probability density function corresponding to (4) is given by (5)

$$f(t) = \frac{\lambda\, g(t)\, \exp[\lambda\, G(t)]\, \exp\left[e^{\lambda G(t)} - 1\right]}{\exp\left[e^{\lambda} - 1\right] - 1}. \tag{5}$$

The CBellG family of a distribution's pth quantile function is given by

$$t_p = G^{-1}\left\{\lambda^{-1}\, \ln\left[1 + \ln\left\{1 + \mathrm{p}[\exp(e^{\lambda} - 1) - 1]\right\}\right]\right\}. \tag{6}$$

Complementary Bell Weibull Distribution

Here, we consider the Weibull distribution as a baseline model with a cumulative distribution function and probability density function, respectively, given as $G(t) = 1 - \exp\left[-(t/\alpha)^{\beta}\right]$ and $g(t) = \frac{\beta}{\alpha^{\beta}}(t)^{\beta-1} \exp\left[-(t/\alpha)^{\beta}\right]$, where $\alpha > 0$ is the scale and $\beta > 0$ is the shape parameters. Then, the cumulative distribution function and probability density function of the proposed CBellW model, respectively, are given by

$$F(t) = \frac{\exp\left[e^{\lambda\left\{1-\exp\left[-(t/\alpha)^{\beta}\right]\right\}} - 1\right] - 1}{\exp\left[e^{\lambda} - 1\right] - 1}. \tag{7}$$

and

$$\begin{aligned} f_{CBellW}(t) \;=\; & \frac{\lambda\beta}{\alpha^{\beta}}(t)^{\beta-1} \exp\left[-(t/\alpha)^{\beta}\right] \exp\left[\lambda\left\{1 - \exp\left[-(t/\alpha)^{\beta}\right]\right\}\right] \times \\ & \exp\left[e^{\lambda\left\{1-\exp\left[-(t/\alpha)^{\beta}\right]\right\}} - 1\right]\left\{\exp\left[e^{\lambda} - 1\right] - 1\right\}^{-1}. \end{aligned} \tag{8}$$

The pth quantile of the Weibull distribution can be written as $t_p = \alpha[-\ln(1-p)]^{1/\beta}$. The pth quantile function for CBellW model can be written using Equation (6)

$$t_p = \alpha\left[-\ln\left(1 - \left\{\lambda^{-1}\, \ln\left[1 + \ln\left\{1 + \mathrm{p}[\exp(e^{\lambda} - 1) - 1]\right\}\right]\right\}\right)\right]^{1/\beta} \tag{9}$$

3. Designing of GASP under CBellW Model

The sampling approach is said to save time and money. Under different sampling schemes, specific quality control testing is done before approving or rejecting a lot. This section is built around an example of a GASP that works under the premise that an item's lifespan distribution follows a CBellW model with known parameters λ and θ and the cumulative distribution function in Equation (7). Consider a GASP in which a random sample of size n is selected, distributed, and kept on life screening for r items for each group for a predetermined period of time. The reader is referred to Aslam et al. [6] and Khan and Alqarni [13] for a concise explanation of GASP and application to actual data. The GASP cut down on both time and the expense. Numerous lifetime conventional and expanded models are used ([5,6,14–16]) in constructing the GASP by incorporating the quality index as mean or median; generally, the median is recommended for skewed distributions [6].

The GASP is merely a part of the ordinary sampling plan (OSP), i.e., the GASP approaches the OSP by substituting $r = 1$, and hence $n = g$ [17]. The sample size of the considered lot will be $n = r \times g$ using GASP, which is derived from various forms. First of all, we choose g and assign predetermined r items to every group. Secondly, we choose c and t_0 which stands for the investigation's approval number and the time, respectively. Thirdly, we run the experiment concurrently for g groups and note how many groups failed.

Finally, a decision is made regarding whether to approve or disapprove the lot. If no more than c failures occur in any one group, the lot is approved; otherwise, it is denied. The following expression provides a lot's acceptance probability:

$$p_{a(p)} = \left[\sum_{i=0}^{c} \binom{r}{i} p^i [1-p]^{r-i}\right]^g, \tag{10}$$

where p represents the likelihood that an item in the group would expire before t_o and is derived by substituting (9) in (7).

$$m = \alpha\left[-\ln\left(1-\left\{\lambda^{-1}\ln\left[1+\ln\left\{1+\mathrm{p}[\exp(e^{\lambda}-1)-1]\right\}\right]\right\}\right)\right]^{1/\beta}$$

$$\text{Let } \zeta = \left[-\ln\left(1-\left\{\lambda^{-1}\ln\left[1+\ln\left\{1+\mathrm{p}[\exp(e^{\lambda}-1)-1]\right\}\right]\right\}\right)\right]^{1/\beta}$$

Now, by replacing $\alpha = m/\zeta$ and $t = a_1 m_0$ in Equation (7), the probability of failure can be expressed as

$$F(t) = \frac{\exp\left[e^{\lambda\left\{1-\exp\left[-\left(\frac{a_1\zeta}{r_2}\right)^{\beta}\right]\right\}}-1\right]-1}{\exp\left[e^{\lambda}-1\right]-1} \tag{11}$$

From Equation (11), for selected β and λ, p can be computed when a_1 and r_2 are predetermined , where $r_2 = m/m_0$. Here, we consider the two failing probabilities denoted as p_1 and p_2 related to the consumer risk and producer risk, respectively. For a given particular value of the constraints θ and λ, r_2, a_1, β and γ, we need to determine the value of c and g that meet the following two equations concurrently:

$$p_{a\left(p_1|\frac{m}{m_0}=r_1\right)} = \left[\sum_{i=0}^{c} \binom{r}{i} p_1^i [1-p_1]^{r-i}\right]^g \leq \beta, \tag{12}$$

and

$$p_{a\left(p_2|\frac{m}{m_0}=r_2\right)} = \left[\sum_{i=0}^{c} \binom{r}{i} p_2^i [1-p_2]^{r-i}\right]^g \geq 1-\gamma. \tag{13}$$

where r_1 and r_2 stands for the average ratio between consumer and producer risk, and Equations (12) and (13) represent the failure probability to be used in the expression

$$p_1 = \frac{\exp\left[e^{\lambda\left\{1-\exp\left[-(a_1\zeta)^{\beta}\right]\right\}}-1\right]-1}{\exp\left[e^{\lambda}-1\right]-1}, \tag{14}$$

and

$$p_2 = \frac{\exp\left[e^{\lambda\left\{1-\exp\left[-\left(\frac{a_1\zeta}{r_2}\right)^{\beta}\right]\right\}}-1\right]-1}{\exp\left[e^{\lambda}-1\right]-1} \tag{15}$$

4. Discussion with Illustrative Examples

Tables 1 and 2 showed the architectural constraints under GASP at various values of λ (1.25, 1.50) and taking r (5, 10). The analysis revealed that reducing β (consumerś risk) tends to increase the number of groups. Moreover, the numbers of groups rapidly declined as r_2 increased. However, after a certain point, the probability of accepting a lot is increases with constant g and c. The Tables 1 and 2 also show the impact of a_1 (0.5, 1) and revealed that when β = 0.25, a_1 = 0.5, r_2 = 2, λ = 1.25, and for r = 5, there should be a required 1235 (247 × 5 = 1237) number of units to be put on the life test, while on the other hand, when r

increases to 10, a total number of 510 units are required to put on life test. Here, 10 groups would be preferable. For the considered GASP, by using median lifespan as a quality index, under the CBellW model, the value of OC increases and the number of groups decreases as the true median life increases. This phenomenon is also explained graphically in Figure 1. It can be observed from Figure 1 that as the true median lifetime increases, the g and c tends to decrease, while the operating characteristic (OC) values tends to increase gradually. Hence, at those points, the lot under consideration will be accepted. Here, in Figure 1, it would be preferable to accept the lot for $r = 10$ as the minimum number of groups will be tested as compared to $r = 5$. From Table 1, when β = 0.01, a_1 = 1, λ = 1.25, and r = 10.

r_2	2	4	6	8
g	10	3	2	2
c	5	3	2	2
$P(a)$	0.9513	0.9839	0.9874	0.9938

Here, we illustrate the results of given in Tables 1 and 2 by considering an example; readers are referred to [5,16]. Let the lifespan of a ball bearing be put to a test following the CBellW model, with λ = 1.25. The mean prescribed life of the ball bearing is 2000 cycles. The consumer and the producer face 25% and 5% risk when the mean lifetime is 3000 and 5000 cycles, respectively. If an investigator wants to run an experiment of 1000 cycles with 10 units in each group to see if the ball bearing mean life is longer than the specified life, for this framework, we have m_0 = 3000 cycles, λ = 1.50, a_1 = 0.5, β = 0.25, r = 10, r_1 = 1, producer risk = 0.05 and r_2 = 4. Furthermore, from Table 2, we have $g = 8$ with an approval number, i.e., c = 1. This implies that 40 units should be drawn, with 5 units distributed to all of the 8 groups. If no more than 1 unit expires in all of these groups before 1000 cycles, the mean of the ball bearing will be statistically guaranteed to be higher than the stated life. Here, if an investigator wants to check the hypothesis that ball bearings have a life span of 5000 cycles but a true mean life of four times that, he can test 8 groups of 5 units each; if higher than 1 unit fail in 1000 cycles, as $a_1 = 0.5$, and the mean life length is in thousands of cycles, the investigator will conclude that the life is higher than 5000 cycles with 95% level of certainty. Consequently, the lot under investigation ought to be approved.

Table 1. GASP under CBellW model, $\beta = 1$ and $\lambda = 1.25$.

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
β	r_2	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$
0.25	2	247	3	0.9670	7	3	0.9614	51	4	0.9803	3	5	0.9851
	4	5	1	0.9548	1	1	0.9537	4	2	0.9876	1	2	0.9680
	6	5	1	0.9815	1	1	0.9823	2	1	0.9688	1	2	0.9919
	8	5	1	0.9900	1	1	0.9908	2	1	0.9828	1	2	0.9627
0.1	2	–	–	–	73	4	0.9817	84	4	0.9678	5	5	0.9753
	4	44	2	0.9872	4	2	0.986	7	2	0.9784	1	2	0.9680
	6	9	1	0.9669	2	1	0.9649	3	1	0.9536	1	2	0.9919
	8	9	1	0.9820	2	1	0.9817	3	1	0.9743	1	1	0.9627
0.05	2	–	–	–	95	4	0.9763	110	4	0.958	7	5	0.9656
	4	58	2	0.9832	5	2	0.9825	9	2	0.9723	2	3	0.9916
	6	11	1	0.9597	2	1	0.9649	9	2	0.9926	2	2	0.9839
	8	11	1	0.9781	2	1	0.9817	4	1	0.9659	1	1	0.9627
0.01	2	–	–	–	146	4	0.9638	992	5	0.9771	10	5	0.9513
	4	88	2	0.9746	7	2	0.9756	14	2	0.9572	3	3	0.9839
	6	88	2	0.9935	7	2	0.9944	14	2	0.9885	2	2	0.9874
	8	17	1	0.9663	3	1	0.9727	14	2	0.9954	2	2	0.9938

Remark: Hyphens (–) are inserted in required cells for a large sample size.

Table 2. GASP under CBellW model, $\beta = 1$ and $\lambda = 1.50$.

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
β	r_2	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$
0.25	2	52	2	0.9527	7	3	0.9845	27	3	0.9792	2	4	0.9803
	4	8	1	0.9763	1	1	0.9804	3	1	0.9624	1	2	0.9906
	6	8	1	0.9911	1	1	0.9937	3	1	0.9855	1	1	0.9739
	8	8	1	0.9953	1	1	0.9970	3	1	0.9923	1	1	0.9873
0.1	2	–	–	–	12	3	0.9735	45	3	0.9656	3	4	0.9706
	4	13	1	0.9618	2	1	0.9612	4	1	0.9502	1	2	0.9906
	6	13	1	0.9855	2	1	0.9874	4	1	0.9807	1	1	0.9739
	8	13	1	0.9924	2	1	0.9940	4	1	0.9898	1	1	0.9873
0.05	2	–	–	–	15	3	0.967	59	3	0.9552	4	4	0.961
	4	17	1	0.9504	2	1	0.9612	16	2	0.9905	2	2	0.9813
	6	17	1	0.9811	2	1	0.9874	5	1	0.9759	1	1	0.9739
	8	17	1	0.9901	2	1	0.9940	5	1	0.9873	1	1	0.9873
0.01	2	–	–	–	146	4	0.9889	463	4	0.9796	5	4	0.9515
	4	171	1	0.9910	7	2	0.9935	24	2	0.9858	2	2	0.9813
	6	26	1	0.9713	3	1	0.9812	8	1	0.9617	2	2	0.9964
	8	26	1	0.9849	3	1	0.9911	8	1	0.9797	2	1	0.9748

Remark: Hyphens (–) are inserted in required cells for a large sample size.

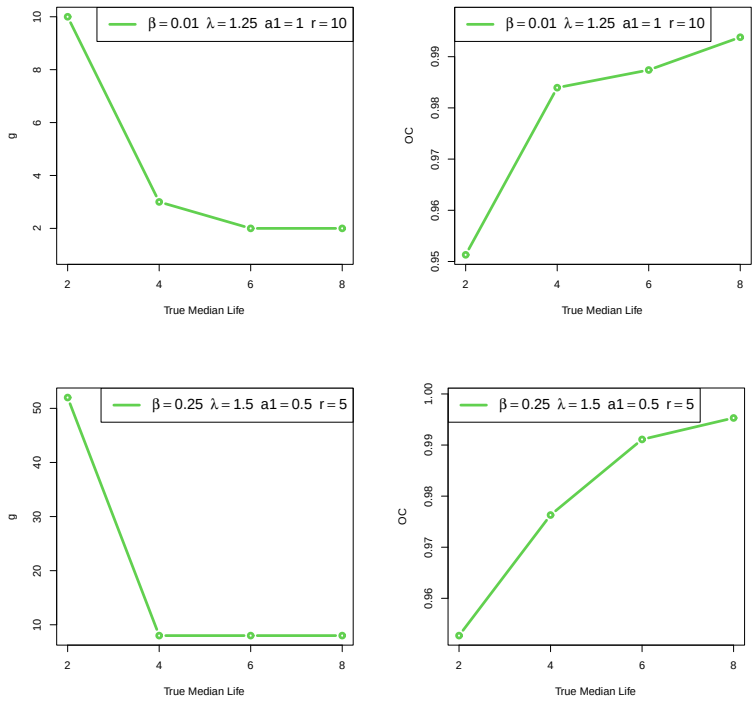


Figure 1. Graphical illustration of g and OC for some parametric values taken from Tables 1 and 2, respectively.

5. Application

From a practical prospective, here, we take up two real data applications given below in Table 3 along with descriptive statistics for both data sets. The data sets are right skewed, which indicates that the using median life as a quality parameter is a good option. The first data set contains 50 observations of the breaking strength of stress of carbon fibre in Gba units. Almarashi et al. [5] recently innovated a GASP for the Marshall–Olkin–Kumaraswamy exponential distribution by using the first data set. The second data set contains the lifetime of fatigue fractures of Kevlar 373/epoxy data based on 76 observed values extracted from [18].

Table 3. Real data along-with descriptive summary.

Data-1									
1.120	0.170	0.640	4.320	1.220	0.370	1.160	1.420	0.090	1.670
0.130	0.250	0.080	0.040	2.350	0.200	0.780	0.340	1.020	0.170
1.760	2.390	0.500	1.350	3.360	0.450	0.900	2.920	6.530	1.620
7.460	3.190	2.490	1.400	7.490	0.570	0.140	0.630	5.230	0.710
0.680	0.120	0.090	3.470	5.930	1.820	4.200	7.290	3.130	3.410
Descriptive Summary									
n	**Min**	**Max**	**Q1**	**Q3**	**Mean**	**Median**	**SD**	**S**	**K**
50	0.04	7.49	0.39	3.078	1.975	1.19	2.115767	1.3253	0.8032
Data-2									
0.0251	0.0886	0.0891	0.2501	0.3113	0.3451	0.4763	0.565	0.5671	0.6566
0.6748	0.6751	0.6753	0.7696	0.8375	0.8391	0.8425	0.8645	0.8851	0.9113
0.912	0.9836	1.0483	1.0596	1.0773	1.1733	1.257	1.2766	1.2985	1.3211
1.3503	1.3551	1.4595	1.488	1.5728	1.5733	1.7083	1.7263	1.746	1.763
1.7746	1.8275	1.8375	1.8503	1.8808	1.8878	1.8881	1.9316	1.9558	2.0048
2.0408	2.0903	2.1093	2.133	2.21	2.246	2.2878	2.3203	2.347	2.3513
2.4951	2.526	2.9911	3.0256	3.2678	3.4045	3.4846	3.7433	3.7455	3.9143
4.8073	5.4005	5.4435	5.5295	6.5541	9.096				
Descriptive Summary									
n	**Min**	**Max**	**Q1**	**Q3**	**Mean**	**Median**	**SD**	**S**	**K**
76	0.0251	9.096	0.9048	2.2959	1.95924	1.73615	1.573981	1.97956	5.16079

Table 4 represents the maximum likelihood estimates with the standard error (SE), Kolmogorov–Smirnov (K-S) and p-value under the fitted CBellW model. The K-S test demonstrated that the maximum distance between actual and fitted values under the CBellW model are 0.0776 and 0.0834 with p-values of 0.9241 and 0.6356, respectively.

Table 4. Summary of fitted CBellW model for the data sets.

	$\hat{\alpha}$	**SE**	$\hat{\beta}$	**SE**	$\hat{\lambda}$	**SE**	**K-S**	p**-Value**
Data-1	0.0201	0.0374	0.2916	0.0800	2.0975	0.4556	0.0776	0.9241
Data-2	0.4255	0.2385	0.6734	0.1358	1.6316	0.3002	0.0834	0.6356

The graphical illustration including the plot of estimated probability density function, cumulative distribution function, hazard rate function, Kaplan–Meier curves and total time on test (TTT) of the data sets are presented in Figures 2 and 3 respectively. The estimated probability density function and cumulative distribution function show good agreement between the actual and fitted values under the CBellW model. Based on the estimated parameters values under the maximum likelihood approach, Tables 5 and 6 show the GASP when the life of a product follows a CBellW model showing minimum g and c. The results are consistent as in Tables 1 and 2.

Table 5. GASP under CBellW model, $\hat{\beta} = 0.6734$ and $\hat{\lambda} = 1.6316$.

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
β	r_2	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$
0.25	2	186	3	0.9684	44	4	0.984	38	4	0.9805	3	5	0.9781
	4	5	1	0.9504	3	2	0.9875	4	2	0.9857	1	2	0.9626
	6	5	1	0.9787	1	1	0.9805	2	1	0.9642	1	2	0.9907
	8	5	1	0.9878	1	1	0.9899	2	1	0.9792	1	1	0.9591
0.1	2	–	–	–	73	4	0.9735	62	4	0.9684	5	5	0.9638
	4	36	2	0.9879	4	3	0.9833	6	2	0.9786	1	2	0.9626
	6	8	1	0.9661	2	1	0.9613	6	2	0.9939	1	2	0.9907
	8	8	1	0.9806	2	1	0.9798	3	1	0.9690	1	1	0.9591
0.05	2	–	–	–	95	4	0.9657	81	4	0.9589	–	–	–
	4	47	2	0.9842	5	2	0.9792	8	2	0.9716	2	3	0.9895
	6	10	1	0.9578	2	1	0.9613	8	2	0.9918	2	2	0.9815
	8	10	1	0.9758	2	1	0.9798	3	1	0.9690	1	1	0.9591
0.01	2	–	–	–	–	–	–	625	5	0.978	–	–	–
	4	72	2	0.9759	7	2	0.9710	12	2	0.9577	3	3	0.9815
	6	72	2	0.9934	7	2	0.9935	12	2	0.9878	2	2	0.9843
	8	15	1	0.9639	3	1	0.9699	12	2	0.9947	1	2	0.9928

Remark: hyphens (–) are inserted in required cells for a large sample size.

Table 6. GASP under CBellW model, $\hat{\beta} = 0.2916$ and $\hat{\lambda} = 2.0975$.

		$r = 5$						$r = 10$					
		$a_1 = 0.5$			$a_1 = 1$			$a_1 = 0.5$			$a_1 = 1$		
β	r_2	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$	g	c	$p(a)$
0.25	2	–	–	–	–	–	–	–	–	–	–	–	–
	4	8	2	0.9668	7	3	0.9802	4	3	0.9790	2	4	0.9741
	6	8	2	0.9904	3	2	0.9703	2	2	0.9764	1	3	0.9845
	8	3	1	0.9618	3	2	0.9874	2	2	0.9898	1	2	0.9625
0.1	2	–	–	–	–	–	–	–	–	–	–	–	–
	4	72	3	0.9876	12	3	0.9662	6	3	0.9686	4	3	0.9615
	6	13	2	0.9845	4	2	0.9605	3	2	0.9648	2	3	0.9692
	8	13	2	0.9936	4	2	0.9833	3	2	0.9847	1	2	0.9725
0.05	2	–	–	–	–	–	–	–	–	–	–	–	–
	4	93	3	0.9840	15	3	0.9580	7	3	0.9635	7	5	0.9864
	6	17	2	0.9798	5	2	0.9509	4	2	0.9533	2	3	0.9692
	8	17	2	0.9917	5	2	0.9791	4	2	0.9797	2	3	0.9894
0.01	2	–	–	–	–	–	–	–	–	–	–	–	–
	4	143	3	0.9755	146	4	0.9848	28	4	0.9855	10	5	0.9806
	6	26	2	0.9693	23	3	0.9869	11	3	0.9882	3	3	0.9542
	8	26	2	0.9873	7	2	0.9709	5	2	0.9747	3	3	0.9842

Remark: hyphens (–) are inserted in required cells for a large sample size.

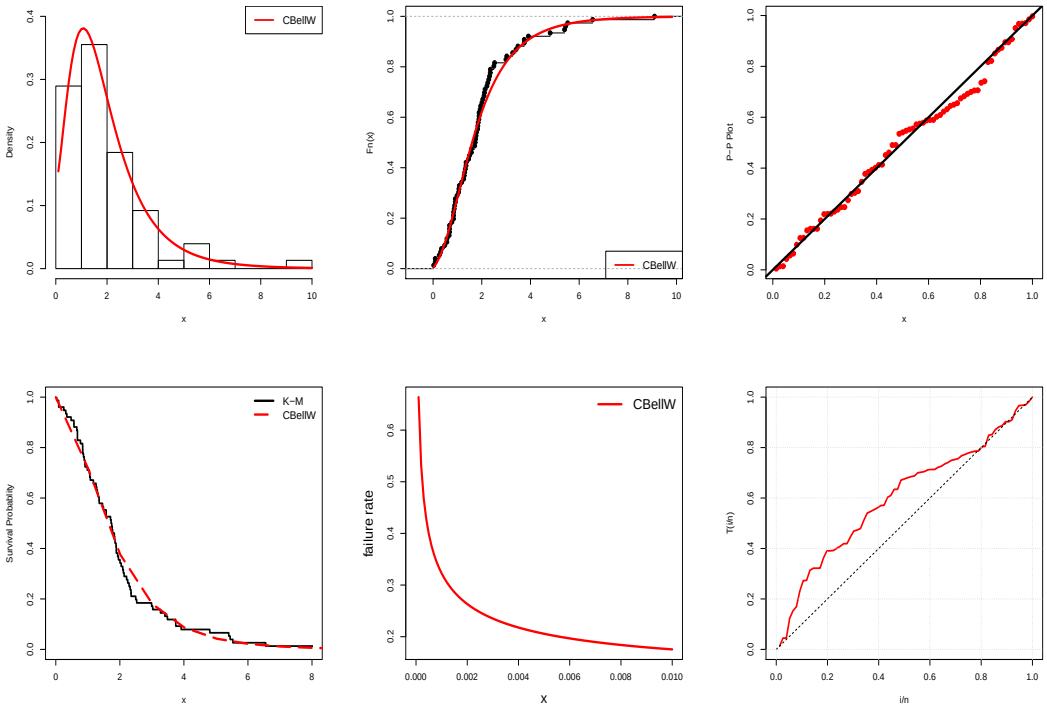


Figure 2. Graphical illustration of estimated plots under CBellW model of data-1.

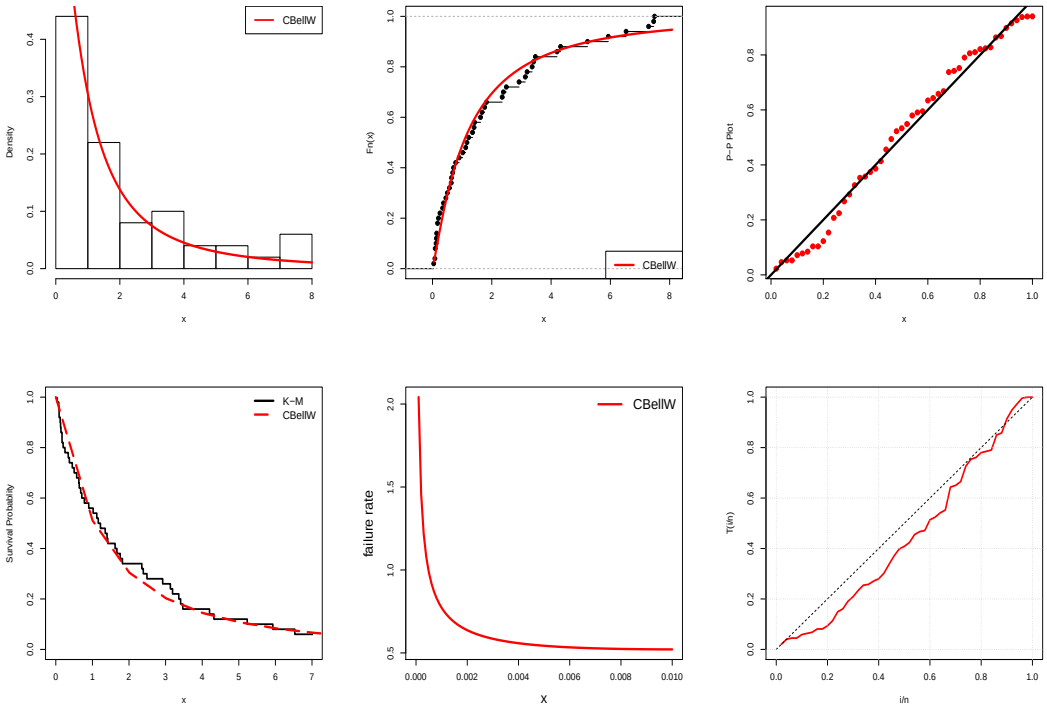


Figure 3. Graphical illustration of estimated plots under CBellW model of data-2.

6. Conclusions

In this article, we proposed a new compounded model called complementary Bell–G. The proposed family holds the fascinating properties of a Bell distribution with a single parameter. Furthermore, a special model called Bell–Weibull is presented. Additionally, a GASP is designed for when the life of a product follows the CBellW model. The median

lifetime is used as a quality index to obtain the design constraints, namely sample size and acceptance number, under a predefined consumer's risk and test termination period. For a suggested approach, as true median life rises, the number of groups drops, but the OC value rises. These results are consistent with the recent studies by [5,19].

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