

Article

A New Discrete Mycorrhiza Optimization Nature-Inspired Algorithm

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Abstract: This paper presents the discrete version of the Mycorrhiza Tree Optimization Algorithm (MTOA), using the Lotka–Volterra Discrete Equation System (LVDES) formed by the Predator–Prey, Cooperative and Competitive Models. The Discrete Mycorrhizal Optimization Algorithm (DMOA) is a stochastic metaheuristic that integrates randomness in its search processes. These algorithms are inspired by nature, specifically by the symbiosis between plant roots and a fungal network called the Mycorrhizal Network (MN). The communication in the network is performed using chemical signals of environmental conditions and hazards, the exchange of resources, such as Carbon Dioxide (CO_2) that plants perform through photosynthesis to the MN and to other seedlings or growing plants. The MN provides water (H_2O) and nutrients to plants that may or may not be of the same species; therefore, the colonization of plants in arid lands would not have been possible without the MN. In this work, we performed a comparison with the CEC-2013 mathematical functions between MTOA and DMOA by conducting Hypothesis Tests to obtain the efficiency and performance of the algorithms, but in future research we will also propose optimization experiments in Neural Networks and Fuzzy Systems to verify with which methods these algorithms perform better.

Keywords: discrete; optimization; metaheuristic; mycorrhiza; plants; Lotka–Volterra (LV) Discrete Equation System



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1. Introduction

In the literature, multiple definitions of optimization can be found, and can be summarized as the search for the best probable solutions for a given problem. In [1], the desired concepts and objectives represent obtaining the best solution [2]. When integrating processes, optimization is an effective and powerful tool. The objective function being optimized quantifies the level of the solution being sought [3]. The constraints are actually the limitations of the system model in the search process. Consequently, in optimization we maximize or minimize the value of the objective function subject to constraints imposed on the decision variables [4].

The proposal presented in this work is a new optimization algorithm inspired by nature, where the roots of plants and the Mycorrhiza Network (MN) play the main role, i.e., the symbiosis that occurs between these two protagonists is modeled through LVDES.

The principal contribution of this research is to model the symbiosis between plant roots and the Mycorrhizal Fungal Network (MN) with the Lotka–Volterra Discrete Equation System (LVDES), unlike the previous research [5] for which we used the Lotka–Volterra Continuous Equation System (LVCES). We believe that this algorithm can provide more flexibility and operability in the handling of the parameters and that it adapts to iterative processes.

Many articles were developed using the Lotka–Volterra system of equations, and some investigations followed the direction of continuous equations such as the works of Atena Ghasemabadi and Mohammad Hossein Rahmani Doust in 2019, “Investigating the dynamics of Lotka–Volterra model with disease in the prey and predator species” [6]; James

P. O'Dwyer in 2018, "Whence Lotka–Volterra? Conservation laws and integrable systems in ecology" [7]; Sundarapandian Vaidyanathan in 2015, who studied the "Lotka–Volterra Two-Species Mutualistic Biology Models and Their Ecological Monitoring" [8]; other LV discrete equations were explored in the investigations of Qamar Din in 2013, while the dynamics of a discrete Lotka–Volterra model can be found in [9]; Li Xu, Lianjun Zou, Zhongxiang Chang, Shanshan Lou, Xiangwei Peng, and Guang Zhang in 2014, presented a "Bifurcation in a Discrete Competition System" in [10]; Li Xu, Shanshan Lou, Panqi Xu, and Guang Zhang in 2018, studied "Feedback Control and Parameter Invasion for a Discrete Competitive Lotka–Volterra System", in [11], etc. From 1925, when Alfred Lotka independently published 'Elements of Physical Biology' and in 1926 when Vito Volterra published 'Variations and fluctuations in the number of individuals in cohabiting animal species' [12,13], their works became conceptual ground for modern theoretical ecology and many researchers have been inspired in their work to pursue new fields of research.

The Mycorrhiza Tree Optimization Algorithm (MTOA) [14] is the antecedent of the Discrete Mycorrhiza Optimization Algorithm (DMOA) presented in this work, in which both algorithms have the same inspiration, they use the Lotka–Volterra System of Equations and the fundamental difference between both is that the DMOA uses the discrete LV equations, which provides a radical change in the convergence and optimization process of the algorithm.

This article is organized as follows: In Section 1, we discuss optimization algorithms inspired by nature and also refer to the inspiration that gave rise to this DMOA algorithm. Section 2 shows the symbiotic relationship between the Mycorrhiza Network (MN) and plants that are the foundation of this algorithm. Section 3 shows the Discrete Mycorrhiza Optimization Algorithm (DMOA). Section 4 shows the results of the experiments performed with the DMOA and MTOA [14] algorithms and the hypothesis tests that were performed between these two methods. Section 5 describes the conclusions that resulted from the experiments conducted between these algorithms.

2. Mycorrhiza Network and Plants

Mycorrhizal Networks (MNs), shown in Figure 1, are fungal hyphae or filaments that connect the roots of at least two plants that may or may not be of the same species. These networks are very important ecologically because they facilitate the transfer of resources between plants such as carbon and nitrogen, colonization, and substantially improve plant regeneration, as shown in Figure 1. In these MNs there are "donor" trees, which are old trees also referred to as "mother trees", as shown in Figure 2, which donate all their resources (carbon, nitrogen, potassium, sulfur, and other nutrients) to the seedlings or growing plant "recipients" and it has been proven that they have a higher degree of survival due to this generous fact of nature; furthermore, it has been proven that the MN are essential for the regeneration of forests and the colonization and survival of plants in arid areas. In this exchange of resources, because of their height, trees (large plants) can perform better photosynthesis due to their exposure to the sun and their main contribution is carbon, while fungi contribute mainly water and some nutrients such as nitrogen, phosphorus, potassium, etc., to the MN. The complete life cycle of MN fungi involves recognition, communication and the establishment of symbiosis as shown in Figure 2 between fungi and plant roots [15]. Figure 3 shows the topology of the MN with the links between the different plants. The nodes of the trees and according to their diameter are represented by circles with different shades of green or yellow and increase their shade in relation to age, while the lines between trees represent their linkage with Euclidean distances. The thickness of the line increases with the number of links between pairs of plants [16].

Thousands of mycorrhizae form in the subsoil of a forest with the roots of plants and different types of fungi [17]. Many plants share fungi whose mycelia connect the roots of different plants, as shown in Figure 3, resulting in the formation of mycorrhizal networks. Mycorrhizal networks of established plants can colonize other nearby plants and seedlings and create fungal pathways that allow plants to exchange carbon, nutrients

or water [18–23]. In forest dynamics, mycorrhizal networks are recognized for their role in plant establishment, survival, growth and competitive ability [24–29]. The importance of mycorrhizal networks for seedling success in relation to other biotic or abiotic constraints is not well understood and may vary with growth stage, climatic and site conditions [30]. Mycorrhizal networks are considered to be the most important form of forest colonization; even when mycorrhizal networks are disrupted by soil disturbance (climatic factors such as snow, wind, ice, floods, fires, logging, animals, etc.), mycelial fragments retain inoculum potential, and the network can rapidly reform [31]. Fungal colonization of naturally regenerating seedlings occurs through mycorrhizal networks and through inoculum dispersed by wind, soil, or mammals, and the role of mycorrhizal networks decreases with an increasing severity of disturbance and loss of residual trees [32]. When disturbances are very severe, plants die, the forest floor is consumed and biomass is reduced. Recolonization requires other sources such as spores carried by air, soil, or mammals, and fungal hyphae found deeper in the soil [33,34].

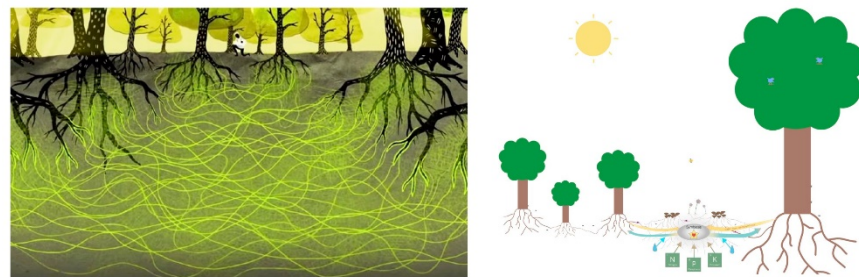


Figure 1. MN shows how communication and resource exchange between plants and fungi takes place, and Reciprocal resource exchange between plants through MN, carbon (CO_2) transferred by trees in exchange for water (H_2O), nitrogen (N), phosphorus (P), sulfur (S), and other nutrients provided by the Mycorrhiza Network (MN).

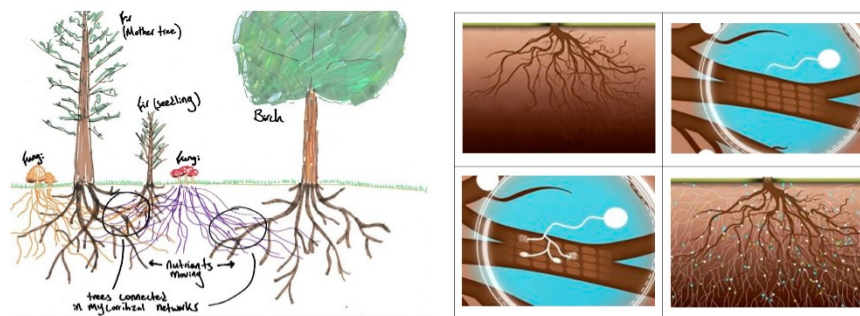


Figure 2. Other plants integrated in the MN, and Symbiosis between plant roots and fungi.

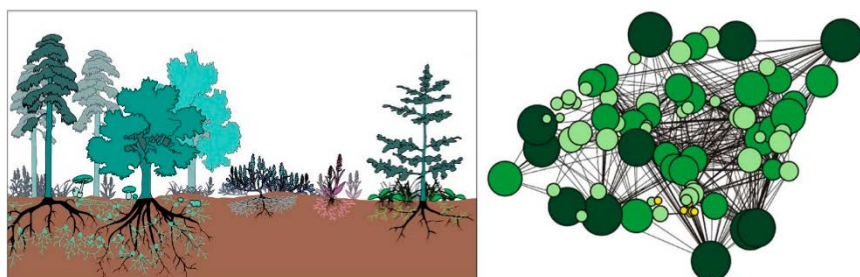


Figure 3. MN involves plants of different species and sizes as well as shrubs, and the Topology MN.

3. Discrete Mycorrhized Optimization Algorithm (DMOA)

Through photosynthesis, trees produce carbon-rich sugars (CO_2) which they exchange with fungi that form a symbiotic association with plant roots. In the Mycorrhizae Network, exchange with plants consists mainly of water (H_2O), nitrogen (N), phosphorus (P), sulfur (S) and other nutrients. Mycorrhizae have also been found to connect plants to each other.

This algorithm, as mentioned above, is inspired by the symbiotic relationship between plant roots and the mycorrhizal fungal network. Research has shown that the following processes occur in these associations, which are the foundations on which this algorithm is based [16,35]:

- (A) Plant defense against external stress and danger situations by means of biochemical signals via the MN.
 - (B) Cooperation in the ecosystem with the exchange of resources (carbon, water, nitrogen, phosphorus, and other nutrients) through the MN.
 - (C) Competition for resources between the various plants and the MN. These processes are modeled in the algorithm by the Lotka–Volterra system of discrete equations [36–38].
1. Predator-Prey Defense Model: Defensive behavior against predators that can be insects or animals, for the survival of the entire habitat (plants and fungi).
 2. Resource Exchange Cooperative Model: Exchange of resources, such as carbon, water, nitrogen, phosphorus, and other nutrients to other plants and the fungal network through the MN.
 3. Colonization Competitive Model: In any ecosystem, resources are limited and all living beings compete to obtain them. In a forest, colonization occurs by the MN, where the different plants and fungi compete for the resources provided by all the members connected by the Mycorrhiza.

In this research, the initial population for both x and y were different and were obtained by random solutions. With these populations, we found the best fitness solution. With the best solutions found, we used the Lotka–Volterra Cooperative Equation for parameters a and d ; the result will influence one of the two LV Equations (Predator-Prey or Competitive), depending on the random diversity result. Diversity is controlled by the probability d [1,2] to choose one of the two Lotka–Volterra equations to try to simulate what happens in an ecosystem.

To overcome the stagnation of the algorithm at local minima, this is controlled by the epochs where the two populations are renewed after each cycle of 30 iterations.

In the Prey–Predator Model, trees are constantly subjected to abiotic stresses (external factors) that exert an influence on plants, which are as follows: water (water stress), salts (salt stress), temperature (heat stress), anoxia (lack of oxygen), oxidative stress, heavy metals, environmental pollutants and atmospheric pollutants. Both the trees and the mycorrhizal network, through chemical signals, are alerted to the danger of animals, insects or any other threat. The Cooperative Model, in which the resources of the entire ecosystem are shared as a way of survival of the entire habitat, the large trees supply the network and other plants with CO_2 because with their height they can perform photosynthesis, and the Mycorrhizae Network supplies the trees and other plants that are connected to the network, mainly with water and other nutrients such as phosphorus, nitrogen, potassium and zinc, among others. The Competitive Model simulates the colonization of the forest, the expansion of the Mycorrhizae network, and the inclusion of other plants in the ecosystem. This process of Defense, Cooperation and Competition ensures the survival of the entire habitat.

3.1. DMOA Pseudocode

Algorithm 1 describes the pseudocode of the DMOA Algorithm with two populations (plants and MN) and the following three biological operators: Defense (Predator-Prey Model), Resource Exchange (Cooperative Model), and Colonization (Competitive Model) represented by the LVDES.

Algorithm 1. Discrete Mycorrhiza Optimization Algorithm (DMOA).

Objective min or max $f(x)$, $x = (x_1, x_2, \dots, x_d)$
Define parameters (a, b, c, d, e, f, x, y)
Initialize a population of n plants and mycorrhiza with random solutions
*Find the best solution **fit** in the initial population*
while $(t < \text{maxIter})$
 for $i = 1:n$ (for n plants and Micorrhiza population)
 $X_p = \text{abs}(\text{FitA})$
 $X_m = \text{abs}(\text{FitB})$
 end for
 $a = \text{minor}X_p$
 $d = \text{minor}X_m$
 Apply (LV-Cooperative Model)
 $x_i^{t+1} = x^t(a - by^t)$
 $y_i^{t+1} = y^t(-c + dx^t)$
 if $x_i < y_i$
 $x^t = x_i$
 else
 $x^t = y_i$
 end if
 rand $([1 \ 2])$
 if $(\text{rand} = 1)$
 Apply (LV-Predator-Prey Model)
 $x_i^{t+1} = x^t(a - bx^t - cy^t)$
 $y_i^{t+1} = y^t(d - ex^t - fy^t)$
 else
 Apply (LV-Competitive Model)
 $x_i^{t+1} = x^t(a - bx^t + cy^t)$
 $y_i^{t+1} = y^t(d + ex^t - fy^t)$
 end if
 Evaluate new solutions.
 If the new solutions are better, the best new solutions are updated.
 Find the current best **fit** solution.
end while

3.2. DMOA Flowchart

The flowchart of the DMO algorithm is shown in Figure 4 where the biological operators of defense, resource exchange and colonization model the symbiosis between plants and MN. The flowchart illustrates how the information flows in the algorithm.

3.3. Lotka–Volterra Discrete Equation System

Discrete models governed by difference equations are more suitable than continuous models when reproductive generations last only one reproductive season (non-overlapping generations) [5,39]. The Discrete Lotka–Volterra models shown in Table 1 have many applications in applied sciences. These models were initially developed in mathematical biology, then their studies were extended to other areas [40–43].

Equations and Rational Difference Equations can be found in [44–49]. Additionally, other authors discussed the dynamics of Rational Difference Equations [50–62].

The mathematical description of Discrete Equations (1) and (2) are part of the Lotka–Volterra System of Discrete Equations for the Predator–Prey–Defense Model, where the parameters a, b, d , and g are positive constants, and x_i and y_i represent the initial conditions of the population for the two species and are positive real numbers [37,63].

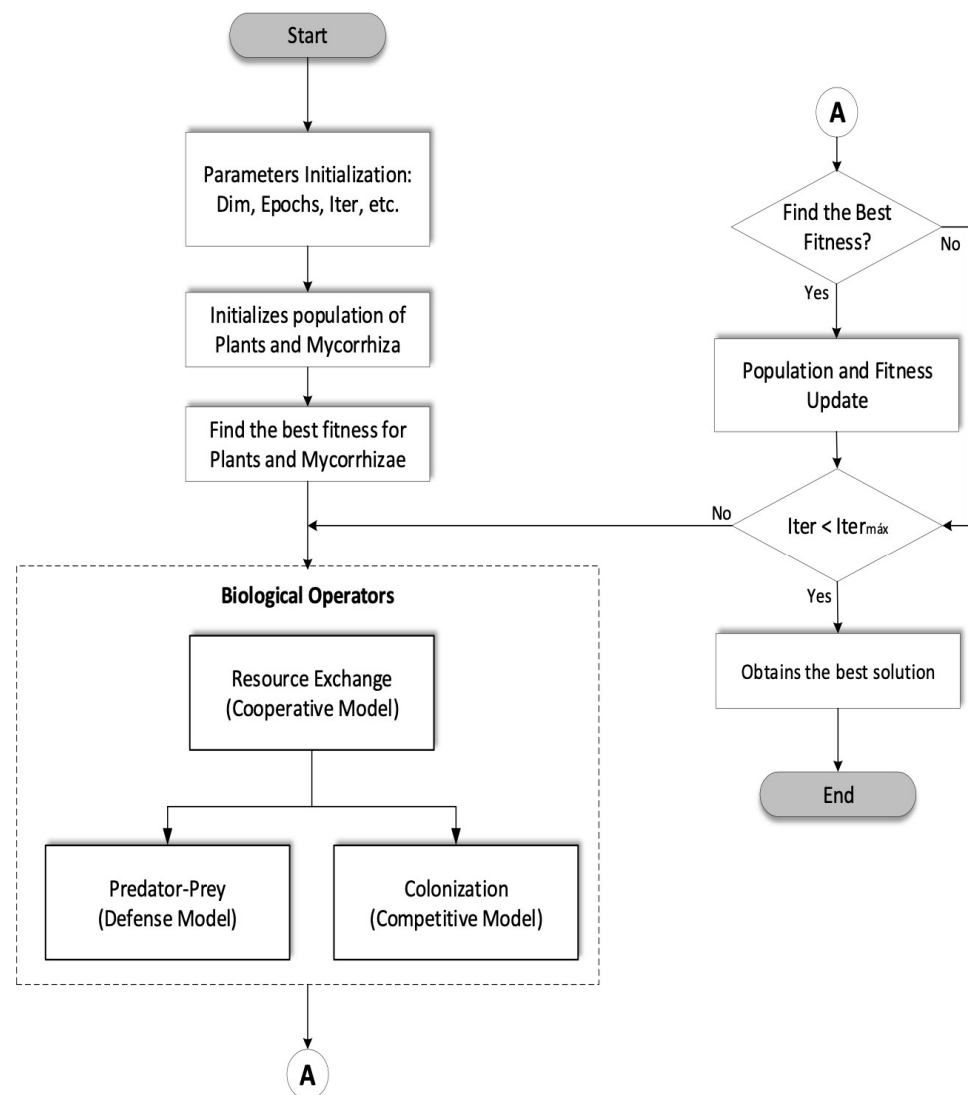


Figure 4. Flowchart of Discrete Mycorrhiza Optimization Algorithm—DMOA.

Table 1. Lotka–Volterra Discrete Equation System.

Model	Discrete Equations
Predator-Prey (Defense)	$x_i^{t+1} = ax_i(1 - x_i) - bx_iy_i \quad (1)$ $y_i^{t+1} = dx_iy_i - gy_i \quad (2)$
Competitive (Colonization)	$x_i^{t+1} = \frac{ax_i - bx_iy_i}{(1 + gx_i)} \quad (3)$ $y_i^{t+1} = \frac{dy_i - ex_iy_i}{(1 + hy_i)} \quad (4)$
Cooperative (Resource Exchange)	$x_i^{t+1} = \frac{ax_i - bx_iy_i}{(1 - gx_i)} \quad (5)$ $y_i^{t+1} = \frac{dy_i + ex_iy_i}{(1 + hy_i)} \quad (6)$

Discrete Equations (3) and (4) are the Cooperative Model of Lotka–Volterra (Resource Exchange) for two species, where the parameters a, b, d, e, g , and h are positive constants, and x_i and y_i represent the initial conditions of the population for the two species and are positive real numbers [36].

Discrete Equations (5) and (6) represent the Lotka–Volterra Competitive Model-Colonization for two species, where the parameters a, b, d, e, g , and h are positive constants,

and x_i and y_i are the populations for each of the species and are positive real numbers. Each of the parameters of the aforementioned equations are described below [9,36].

The main differences among the three biological operators are as follows:

1. The six equations are different and each pair of equations corresponding to each biological operator model the conditions under which said operators work.
2. The Cooperative model infers randomly in one of the two biological operators of either Defense and Competitive, to try to simulate what can happen in a living and changing ecosystem over time.

The inspiration of the MTOA Algorithm is the symbiosis between the Trees and the Mycorrhizal Network and uses the System of Continuous Equations of Lotka–Volterra. The algorithm, DMOA is inspired by the symbiosis between the Plants and the Mycorrhizal Network and tries to simulate in part what happens over time in this ecosystem. There is no modification of the algorithm, with the difference being the use of the Lotka–Volterra System of Discrete Equations. In the MTOA algorithm, the best fitness values are decided by taking the lowest result of the three biological operators. In the DMOA algorithm, the result of the cooperative operator is randomly inferred through the parameter x_i (grow rates of populations x at time t) in one of the two biological operators of Predator–Prey or Competitive, resulting in the best fitness; we used this parameter because in the experimentation it provided us with better results for the desired purpose.

3.4. DMOA Parameters

Table 2 lists the parameters of population, growth rate, dimensions, epochs, iterations, etc., that were used in the experiments with DMOA and MTOA algorithms.

Table 2. DMOA and MTOA Algorithm Parameters.

Description	Parameter	Value
Population x at time t	x_i^{t+1}	
Population y at time t	y_i^{t+1}	
Grow rates of populations x at time t	x_i	
Grow rates of populations y at time t	y_i	
time	t	
Population growth rate x	a	0.01
Influence of population x on itself	b	0.02
Influence of population y on population x	g	0.06
Population growth rate y	d	0
Influence of population x on population y	e	1.70
Influence of population y on itself	h	0.09
Initial population in x	x	0.0002
Initial population in y	y	0.0006
Population size	Population	20
Number of populations	Populations	2
Dimensions size	Dimensions	30, 50, 100
Number of epochs	Epochs	30
Iterations size	Iterations	30, 50, 100, 500
In the absence of population $x = 0$, In the absence of population $y = 0$		
a, b, c, d, e and f —are positive constants		

Parameter settings: a —Population growth rate x . d —Population growth rate y . x_i —Population growth rates x at time t . y_i —Population growth rates y at time t . Population size. Dimensions size. Number of epochs. Iteration size.

The parameters a and d result from the populations for x and y , which change in each iteration followed by the smallest values being selected. These results are assigned to the population growth rates x_i and y_i of the discrete Lotka–Volterra equations, while the other parameters such as population size, dimensions size, number of epochs, and iteration size are positive integers that are initially defined and usually have the following values: 20, (30, 50, 50, 100, 200), 20, and 30, respectively, however there is flexibility to play with other data to achieve better results.

4. Experimental Results

Table 3 shows the 36 mathematical functions of the CEC-2013 competition that we used to carry out the experiments. Table 1 shows the name of the functions, their range and their nature (Unimodal or Multimodal) and Figure 5 shows the corresponding graphs of a sample of the functions for illustration purpose.

Table 3. 36 Mathematical Functions CEC-2013.

F _n	Function	Range	Nature
F1	Sphere	[−5.12, 5.12]	U
F2	Rosenbrok	[42]	U
F3	Griewank	[−600, 600]	M
F4	Rastrigin	[−5.12, 5.12]	M
F5	Ackley	[−32.768, 32.768]	M
F6	Dixon-Price	[42]	U
F7	Michalewicz	[0, π]	M
F8	Powell	[15]	U
F9	RHE—Rotate Hyper Ellipsoid	[−65.536, 65.536]	U
F10	Schwefel	[−500, 500]	M
F11	Styblinski-Tang	[15]	U
F12	SDP—Sum Different Powers	[1]	M
F13	Sum Squares	[42]	U
F14	Trid	[− d^2 , d^2]	U
F15	Zakharov	[42]	U
F16	Bukin No 6	[−15, −5]	U
F17	Cross-in Tray	[42]	M
F18	Drop-Wave	[−5.12, 5.12]	M
F19	Eggholder	[−5.12, 5.12]	M
F20	Beale	[−4.5, 4.5]	U
F21	Holder-Table	[42]	M
F22	Branin	[42]	M
F23	Levy	[42]	M
F24	Levy 13	[42]	M
F25	Schaffer 2	[−100, 100]	M
F26	Schaffer 4	[−100, 100]	M
F27	Shubert	[42]	M
F28	Bohachevsky 1	[−100, 100]	M
F29	Bohachevsky 2	[−100, 100]	M
F30	Bohachevsky 3	[−100, 100]	M
F31	Booth	[42]	U
F32	Matyas	[42]	U
F33	Mccormick	[−1.5, 4]	U
F34	Easom	[−100, 100]	U
F35	Goldstein-Price	[2]	M
F36	Three-Hump Camel	[15]	M

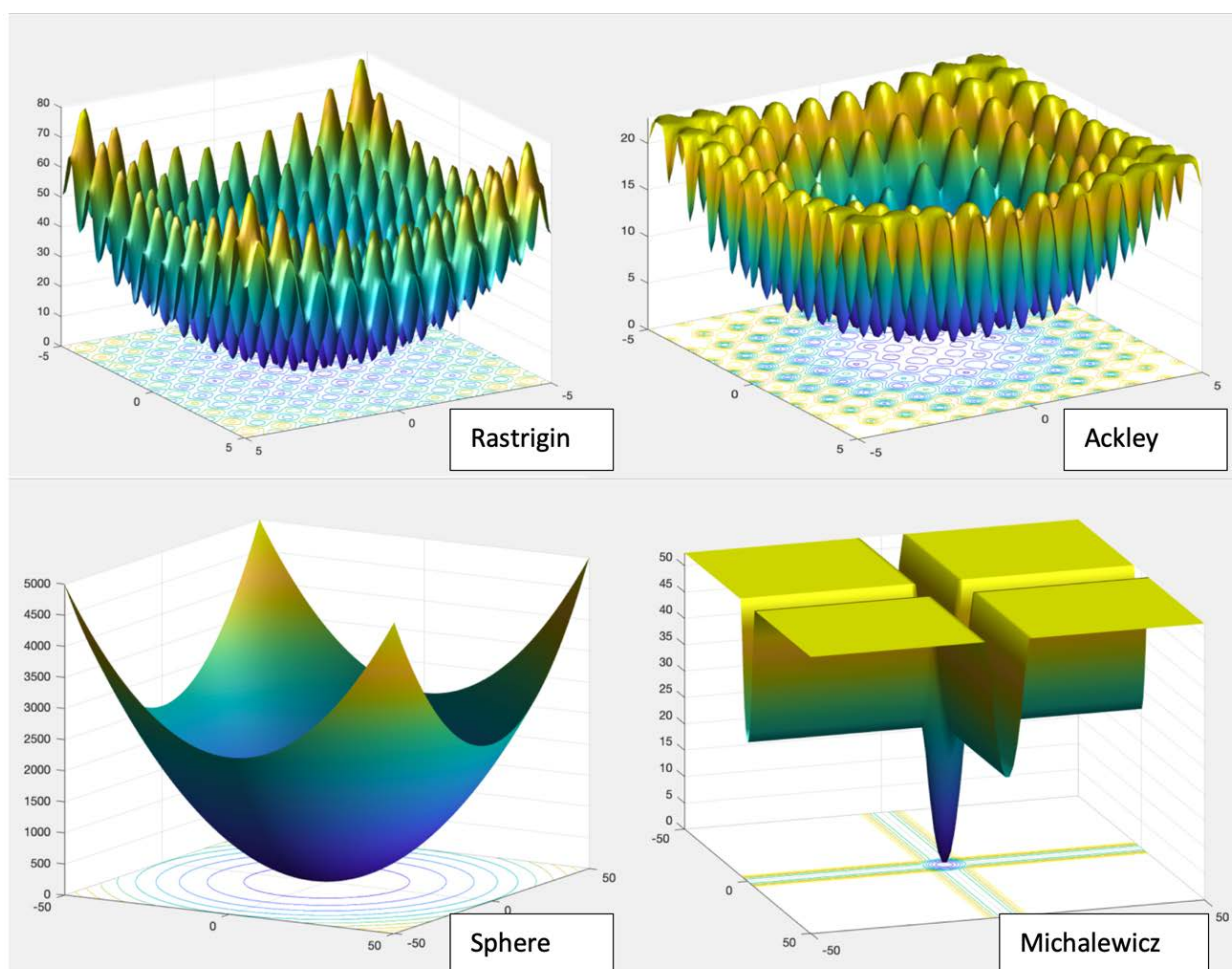


Figure 5. Example of benchmark mathematical functions.

4.1. Results of the MTOA Algorithm Experiments with CEC-2013

Tables 4–6 show the results of the experiments of the 36 mathematical functions of the CEC-2013. The tables show the means and standard deviations that were obtained for 30, 50, and 100 dimensions and for 30, 50, 100, and 500 iterations, respectively.

Figures 6–8 show the convergence of mathematical functions: Sphere, Ackley, and Rastrigin, and for 30, 50, and 100 dimensions for the MTOA and DMOA algorithms.

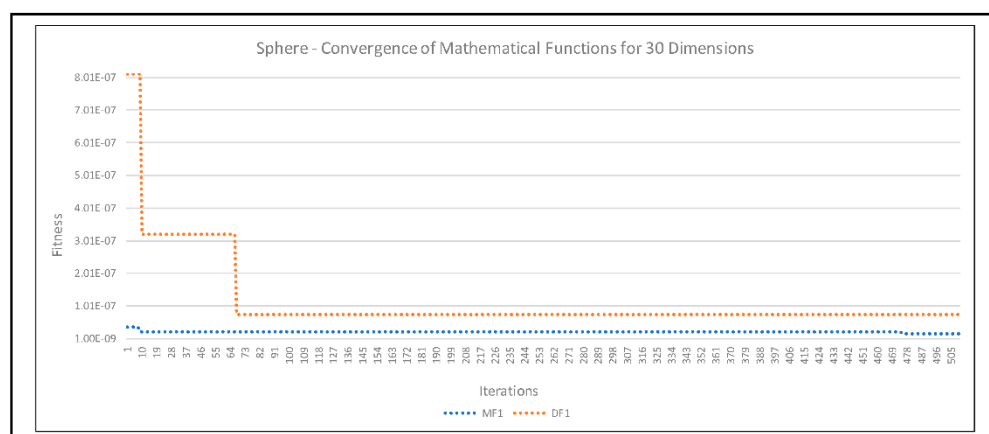


Figure 6. Sphere-Convergence of the DMOA and MTOA algorithms for 30 dimensions.

Table 4. MTOA: Results of the experiments for 30 dimensions.

Dim	30							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f1	4.72×10^{-8}	1.98×10^{-8}	3.28×10^{-8}	1.25×10^{-8}	2.34×10^{-8}	8.91×10^{-9}	8.11×10^{-9}	4.79×10^{-9}
f2	1.67×10^{-8}	1.06×10^{-8}	8.33×10^{-9}	6.77×10^{-9}	6.17×10^{-10}	3.78×10^{-10}	1.64×10^{-9}	1.39×10^{-9}
f3	1.29×10^{-8}	7.35×10^{-9}	4.32×10^{-9}	3.38×10^{-9}	2.67×10^{-10}	1.21×10^{-10}	1.44×10^{-9}	1.21×10^{-9}
f4	3.85×10^{-8}	1.69×10^{-8}	2.78×10^{-8}	1.16×10^{-8}	2.29×10^{-8}	1.04×10^{-8}	8.13×10^{-9}	4.42×10^{-9}
f5	5.62×10^{-8}	2.45×10^{-8}	3.26×10^{-8}	1.34×10^{-8}	4.30×10^{-8}	1.99×10^{-8}	6.58×10^{-9}	3.32×10^{-9}
f6	3.12×10^{-8}	1.88×10^{-8}	2.63×10^{-8}	9.75×10^{-9}	2.21×10^{-8}	9.88×10^{-9}	6.76×10^{-9}	4.01×10^{-9}
f7	3.51×10^{-8}	1.94×10^{-8}	2.51×10^{-8}	1.12×10^{-8}	1.55×10^{-8}	7.70×10^{-9}	5.70×10^{-9}	3.18×10^{-9}
f8	4.49×10^{-8}	2.02×10^{-8}	2.52×10^{-8}	1.17×10^{-8}	1.98×10^{-8}	9.02×10^{-9}	9.13×10^{-9}	2.87×10^{-9}
f9	3.88×10^{-8}	1.93×10^{-8}	2.27×10^{-8}	9.90×10^{-9}	1.97×10^{-8}	9.81×10^{-9}	6.95×10^{-9}	3.99×10^{-9}
f10	4.95×10^{-8}	1.86×10^{-8}	2.57×10^{-8}	1.37×10^{-8}	1.60×10^{-8}	9.72×10^{-9}	8.42×10^{-9}	4.41×10^{-9}
f11	3.91×10^{-8}	2.31×10^{-8}	3.20×10^{-8}	1.06×10^{-8}	1.98×10^{-8}	9.47×10^{-9}	8.11×10^{-9}	4.14×10^{-9}
f12	4.09×10^{-8}	2.16×10^{-8}	2.26×10^{-8}	1.18×10^{-8}	2.02×10^{-8}	1.09×10^{-8}	6.34×10^{-9}	3.72×10^{-9}
f13	4.03×10^{-8}	1.87×10^{-8}	2.40×10^{-8}	1.20×10^{-8}	2.02×10^{-8}	9.20×10^{-9}	8.02×10^{-9}	4.68×10^{-9}
f14	4.80×10^{-8}	2.22×10^{-8}	2.12×10^{-8}	1.09×10^{-8}	2.24×10^{-8}	1.03×10^{-8}	7.02×10^{-9}	3.78×10^{-9}
f15	4.41×10^{-8}	2.11×10^{-8}	2.21×10^{-8}	1.28×10^{-8}	2.50×10^{-8}	1.09×10^{-8}	9.58×10^{-9}	4.28×10^{-9}
f16	3.35×10^{-8}	1.96×10^{-8}	2.69×10^{-8}	1.07×10^{-8}	2.57×10^{-8}	1.09×10^{-8}	8.75×10^{-9}	4.63×10^{-9}
f17	1.96×10^{-8}	1.13×10^{-8}	9.04×10^{-9}	5.96×10^{-9}	5.60×10^{-10}	3.88×10^{-10}	1.73×10^{-9}	1.24×10^{-9}
f18	2.86×10^{-9}	2.62×10^{-9}	7.57×10^{-10}	5.65×10^{-10}	3.92×10^{-10}	3.13×10^{-10}	9.52×10^{-11}	1.13×10^{-10}
f19	3.46×10^{-8}	2.04×10^{-8}	2.48×10^{-8}	1.12×10^{-8}	2.57×10^{-8}	1.07×10^{-8}	9.63×10^{-9}	4.01×10^{-9}
f20	4.87×10^{-8}	2.29×10^{-8}	2.49×10^{-8}	1.18×10^{-8}	1.96×10^{-8}	8.25×10^{-9}	8.04×10^{-9}	4.75×10^{-9}
f21	1.75×10^{-8}	1.02×10^{-8}	8.60×10^{-9}	5.31×10^{-9}	1.88×10^{-7}	7.25×10^{-9}	2.95×10^{-9}	1.93×10^{-9}
f22	4.45×10^{-8}	1.95×10^{-8}	2.10×10^{-8}	1.03×10^{-8}	2.26×10^{-8}	1.03×10^{-8}	8.31×10^{-9}	4.30×10^{-9}
f23	3.60×10^{-8}	1.91×10^{-8}	2.92×10^{-8}	1.33×10^{-8}	2.19×10^{-8}	8.63×10^{-9}	1.00×10^{-8}	4.81×10^{-9}
f24	4.72×10^{-8}	2.30×10^{-8}	2.40×10^{-8}	1.10×10^{-8}	1.71×10^{-8}	9.74×10^{-9}	9.08×10^{-9}	4.24×10^{-9}
f25	4.09×10^{-9}	2.64×10^{-9}	2.48×10^{-9}	1.75×10^{-9}	2.48×10^{-8}	1.15×10^{-8}	1.77×10^{-10}	1.86×10^{-10}
f26	3.52×10^{-7}	1.50×10^{-8}	3.36×10^{-7}	2.03×10^{-8}	3.23×10^{-7}	2.82×10^{-8}	2.93×10^{-7}	2.68×10^{-8}
f27	5.13×10^{-8}	2.22×10^{-8}	2.22×10^{-8}	9.72×10^{-9}	1.80×10^{-8}	9.18×10^{-9}	8.15×10^{-9}	4.08×10^{-9}
f28	4.46×10^{-8}	2.27×10^{-8}	2.43×10^{-8}	1.15×10^{-8}	3.60×10^{-8}	1.57×10^{-8}	7.36×10^{-9}	4.10×10^{-9}
f29	4.19×10^{-8}	1.81×10^{-8}	2.68×10^{-8}	1.22×10^{-8}	3.10×10^{-8}	1.31×10^{-8}	8.07×10^{-9}	4.05×10^{-9}
f30	3.67×10^{-8}	1.99×10^{-8}	2.37×10^{-8}	1.25×10^{-8}	2.88×10^{-8}	1.21×10^{-8}	8.35×10^{-9}	4.71×10^{-9}
f31	4.73×10^{-8}	1.82×10^{-8}	2.52×10^{-8}	1.10×10^{-8}	2.09×10^{-8}	1.09×10^{-8}	8.10×10^{-9}	4.17×10^{-9}

Table 4. Cont.

Dim	30							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f32	1.57×10^{-8}	7.77×10^{-9}	1.07×10^{-8}	5.53×10^{-9}	5.20×10^{-8}	1.93×10^{-8}	1.06×10^{-8}	5.44×10^{-9}
f33	3.89×10^{-8}	2.05×10^{-8}	2.62×10^{-8}	1.12×10^{-8}	2.60×10^{-8}	1.30×10^{-8}	6.78×10^{-9}	4.07×10^{-9}
f34	3.72×10^{-24}	4.26×10^{-24}	4.98×10^{-25}	8.72×10^{-25}	2.92×10^{-31}	1.06×10^{-31}	6.35×10^{-27}	1.08×10^{-26}
f35	3.91×10^{-8}	1.85×10^{-8}	2.50×10^{-8}	1.11×10^{-8}	2.55×10^{-8}	1.03×10^{-8}	8.48×10^{-9}	4.75×10^{-9}
f36	4.06×10^{-8}	1.90×10^{-8}	2.17×10^{-8}	9.43×10^{-9}	1.81×10^{-8}	1.19×10^{-8}	7.27×10^{-9}	4.03×10^{-9}

Table 5. MTOA: Results of the experiments for 50 dimensions.

Dim	50							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f1	4.58×10^{-8}	2.02×10^{-8}	2.84×10^{-8}	1.65×10^{-8}	2.33×10^{-8}	1.04×10^{-8}	8.56×10^{-9}	4.23×10^{-9}
f2	1.72×10^{-8}	1.18×10^{-8}	1.00×10^{-8}	7.87×10^{-9}	2.70×10^{-9}	1.79×10^{-9}	8.26×10^{-10}	6.32×10^{-10}
f3	3.77×10^{-9}	3.30×10^{-9}	4.64×10^{-9}	3.76×10^{-9}	5.86×10^{-10}	2.81×10^{-10}	1.25×10^{-10}	9.09×10^{-11}
f4	3.71×10^{-8}	1.89×10^{-8}	2.74×10^{-8}	1.25×10^{-8}	2.16×10^{-8}	1.02×10^{-8}	7.36×10^{-9}	3.90×10^{-9}
f5	5.67×10^{-8}	2.47×10^{-8}	2.89×10^{-8}	1.15×10^{-8}	3.06×10^{-8}	1.13×10^{-8}	1.49×10^{-8}	5.59×10^{-9}
f6	4.69×10^{-8}	2.15×10^{-8}	3.22×10^{-8}	1.52×10^{-8}	2.42×10^{-8}	1.11×10^{-8}	7.82×10^{-9}	4.45×10^{-9}
f7	2.60×10^{-8}	1.70×10^{-8}	2.55×10^{-8}	1.18×10^{-8}	1.70×10^{-8}	8.06×10^{-9}	6.37×10^{-9}	3.01×10^{-9}
f8	4.10×10^{-8}	2.20×10^{-8}	3.21×10^{-8}	1.35×10^{-8}	2.43×10^{-8}	9.38×10^{-9}	5.95×10^{-9}	3.45×10^{-9}
f9	5.03×10^{-8}	1.91×10^{-8}	3.48×10^{-8}	1.40×10^{-8}	1.88×10^{-8}	9.42×10^{-9}	6.11×10^{-9}	4.01×10^{-9}
f10	3.91×10^{-8}	1.98×10^{-8}	2.87×10^{-8}	1.61×10^{-8}	2.46×10^{-8}	9.12×10^{-9}	6.57×10^{-9}	3.95×10^{-9}
f11	4.93×10^{-8}	2.28×10^{-8}	3.38×10^{-8}	1.38×10^{-8}	2.22×10^{-8}	1.10×10^{-8}	6.98×10^{-9}	3.57×10^{-9}
f12	4.93×10^{-10}	1.34×10^{-10}	1.50×10^{-10}	1.52×10^{-10}	2.96×10^{-10}	1.70×10^{-10}	9.63×10^{-12}	1.45×10^{-11}
f13	4.52×10^{-8}	2.09×10^{-8}	2.99×10^{-8}	1.52×10^{-8}	2.29×10^{-8}	9.79×10^{-9}	8.00×10^{-9}	3.95×10^{-9}
f14	4.85×10^{-8}	1.73×10^{-8}	3.54×10^{-8}	1.42×10^{-8}	2.37×10^{-8}	8.52×10^{-9}	9.64×10^{-9}	4.11×10^{-9}
f15	4.05×10^{-8}	2.32×10^{-8}	3.17×10^{-8}	1.52×10^{-8}	2.31×10^{-8}	1.04×10^{-8}	5.45×10^{-9}	4.34×10^{-9}
f16	4.09×10^{-8}	2.14×10^{-8}	2.78×10^{-8}	1.36×10^{-8}	2.41×10^{-8}	1.15×10^{-8}	6.95×10^{-9}	4.19×10^{-9}
f17	1.07×10^{-8}	8.19×10^{-9}	1.05×10^{-8}	8.30×10^{-9}	3.95×10^{-9}	2.34×10^{-9}	9.38×10^{-10}	6.17×10^{-10}
f18	3.10×10^{-9}	2.10×10^{-9}	1.63×10^{-9}	1.55×10^{-9}	4.06×10^{-10}	3.66×10^{-10}	1.09×10^{-10}	9.07×10^{-11}

Table 5. Cont.

Dim	50							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f19	3.93×10^{-8}	1.86×10^{-8}	3.81×10^{-8}	1.57×10^{-8}	1.94×10^{-8}	1.12×10^{-8}	7.40×10^{-9}	4.11×10^{-9}
f20	4.22×10^{-8}	2.30×10^{-8}	2.70×10^{-8}	1.21×10^{-8}	2.23×10^{-8}	1.06×10^{-8}	7.70×10^{-9}	4.55×10^{-9}
f21	4.12×10^{-8}	1.93×10^{-8}	1.65×10^{-8}	8.64×10^{-9}	1.80×10^{-8}	8.26×10^{-9}	1.61×10^{-9}	1.13×10^{-9}
f22	4.27×10^{-8}	1.91×10^{-8}	3.42×10^{-8}	1.54×10^{-8}	2.08×10^{-8}	1.06×10^{-8}	6.47×10^{-9}	4.02×10^{-9}
f23	5.26×10^{-8}	2.13×10^{-8}	3.10×10^{-8}	1.44×10^{-8}	2.06×10^{-8}	9.27×10^{-9}	8.40×10^{-9}	4.93×10^{-9}
f24	4.96×10^{-8}	1.99×10^{-8}	3.23×10^{-8}	1.55×10^{-8}	1.60×10^{-8}	9.57×10^{-9}	7.24×10^{-9}	3.80×10^{-9}
f25	1.57×10^{-8}	8.34×10^{-9}	1.87×10^{-9}	1.56×10^{-9}	1.57×10^{-8}	8.03×10^{-9}	2.44×10^{-9}	2.60×10^{-9}
f26	3.52×10^{-7}	1.61×10^{-8}	3.40×10^{-7}	1.99×10^{-8}	3.22×10^{-7}	2.80×10^{-8}	2.66×10^{-7}	1.78×10^{-8}
f27	4.09×10^{-8}	1.83×10^{-8}	2.45×10^{-8}	1.33×10^{-8}	1.81×10^{-8}	8.69×10^{-9}	6.69×10^{-9}	3.34×10^{-9}
f28	4.45×10^{-8}	2.16×10^{-8}	2.84×10^{-8}	1.41×10^{-8}	2.49×10^{-8}	1.04×10^{-8}	7.98×10^{-9}	3.79×10^{-9}
f29	4.95×10^{-8}	1.88×10^{-8}	3.17×10^{-8}	1.55×10^{-8}	2.58×10^{-8}	9.34×10^{-9}	7.54×10^{-9}	4.22×10^{-9}
f30	4.57×10^{-8}	2.00×10^{-8}	3.14×10^{-8}	1.49×10^{-8}	1.94×10^{-8}	9.71×10^{-9}	1.12×10^{-8}	4.68×10^{-9}
f31	4.31×10^{-8}	2.05×10^{-8}	3.02×10^{-8}	1.48×10^{-8}	2.11×10^{-8}	1.03×10^{-8}	8.47×10^{-9}	4.58×10^{-9}
f32	3.24×10^{-8}	1.52×10^{-8}	2.68×10^{-8}	4.44×10^{-9}	6.94×10^{-9}	3.62×10^{-9}	9.48×10^{-9}	4.10×10^{-9}
f33	5.17×10^{-8}	1.88×10^{-8}	2.71×10^{-8}	1.53×10^{-8}	2.25×10^{-8}	9.29×10^{-9}	7.88×10^{-9}	4.07×10^{-9}
f34	1.34×10^{-24}	2.27×10^{-24}	1.55×10^{-24}	1.95×10^{-24}	1.04×10^{-26}	2.98×10^{-26}	1.19×10^{-29}	1.99×10^{-29}
f35	3.64×10^{-8}	2.07×10^{-8}	3.08×10^{-8}	1.55×10^{-8}	2.22×10^{-8}	1.14×10^{-8}	8.68×10^{-9}	4.51×10^{-9}
f36	4.10×10^{-8}	2.34×10^{-8}	3.23×10^{-8}	1.61×10^{-8}	2.49×10^{-8}	1.04×10^{-8}	7.70×10^{-9}	4.00×10^{-9}

Table 6. MTOA: Results of the experiments for 100 dimensions.

Dim	100							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f1	4.63×10^{-8}	1.90×10^{-8}	3.02×10^{-8}	1.65×10^{-8}	2.12×10^{-8}	1.18×10^{-8}	7.00×10^{-9}	3.83×10^{-9}
f2	9.89×10^{-9}	7.36×10^{-9}	8.01×10^{-9}	5.84×10^{-9}	6.08×10^{-10}	3.60×10^{-10}	1.03×10^{-9}	5.81×10^{-10}
f3	3.15×10^{-9}	7.40×10^{-10}	5.75×10^{-9}	2.80×10^{-10}	5.05×10^{-9}	2.17×10^{-10}	2.99×10^{-9}	4.99×10^{-10}
f4	3.29×10^{-8}	1.80×10^{-8}	3.19×10^{-8}	1.64×10^{-8}	1.94×10^{-8}	8.33×10^{-9}	8.40×10^{-9}	3.98×10^{-9}
f5	6.71×10^{-8}	2.42×10^{-8}	3.23×10^{-8}	1.43×10^{-8}	2.49×10^{-8}	8.37×10^{-9}	1.16×10^{-7}	2.13×10^{-8}

Table 6. Cont.

Dim	100							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f6	3.62×10^{-8}	1.76×10^{-8}	3.31×10^{-8}	1.21×10^{-8}	2.13×10^{-8}	1.10×10^{-8}	6.82×10^{-9}	4.31×10^{-9}
f7	3.16×10^{-8}	1.58×10^{-8}	2.35×10^{-8}	1.07×10^{-8}	1.50×10^{-8}	7.20×10^{-9}	6.99×10^{-9}	3.73×10^{-9}
f8	3.79×10^{-8}	2.01×10^{-8}	2.42×10^{-8}	1.41×10^{-8}	1.90×10^{-8}	1.00×10^{-8}	7.87×10^{-9}	4.35×10^{-9}
f9	4.18×10^{-8}	2.19×10^{-8}	3.31×10^{-8}	1.58×10^{-8}	1.99×10^{-8}	1.01×10^{-8}	6.87×10^{-9}	4.61×10^{-9}
f10	4.75×10^{-8}	2.21×10^{-8}	3.18×10^{-8}	1.32×10^{-8}	2.05×10^{-8}	8.57×10^{-9}	8.17×10^{-9}	4.17×10^{-9}
f11	3.59×10^{-8}	2.10×10^{-8}	2.50×10^{-8}	1.40×10^{-8}	2.12×10^{-8}	1.05×10^{-8}	8.02×10^{-9}	3.98×10^{-9}
f12	0	0	0	0	0	0	0	0
f13	4.09×10^{-8}	1.83×10^{-8}	2.21×10^{-8}	1.12×10^{-8}	1.70×10^{-8}	1.06×10^{-8}	6.96×10^{-9}	4.36×10^{-9}
f14	4.36×10^{-8}	2.19×10^{-8}	3.37×10^{-8}	1.69×10^{-8}	1.95×10^{-8}	8.85×10^{-9}	8.99×10^{-9}	4.28×10^{-9}
f15	6.03×10^{-10}	2.63×10^{-10}	5.35×10^{-10}	1.80×10^{-10}	4.86×10^{-10}	1.69×10^{-10}	1.39×10^{-10}	1.44×10^{-10}
f16	4.04×10^{-8}	1.97×10^{-8}	3.20×10^{-8}	1.43×10^{-8}	1.85×10^{-8}	9.95×10^{-9}	9.20×10^{-9}	3.32×10^{-9}
f17	1.03×10^{-8}	8.56×10^{-9}	8.67×10^{-9}	5.12×10^{-9}	8.02×10^{-10}	3.94×10^{-10}	1.06×10^{-9}	5.57×10^{-10}
f18	2.01×10^{-9}	1.62×10^{-9}	1.01×10^{-9}	1.04×10^{-9}	5.64×10^{-10}	4.15×10^{-10}	6.24×10^{-11}	6.16×10^{-11}
f19	3.62×10^{-8}	1.68×10^{-8}	3.34×10^{-8}	1.54×10^{-8}	2.35×10^{-8}	1.06×10^{-8}	7.23×10^{-9}	3.54×10^{-9}
f20	4.87×10^{-8}	2.14×10^{-8}	2.85×10^{-8}	1.72×10^{-8}	2.44×10^{-8}	1.15×10^{-8}	7.85×10^{-9}	4.55×10^{-9}
f21	4.19×10^{-9}	2.89×10^{-9}	2.82×10^{-8}	1.48×10^{-8}	1.22×10^{-7}	4.49×10^{-9}	7.48×10^{-9}	3.66×10^{-9}
f22	3.91×10^{-8}	1.93×10^{-8}	3.09×10^{-8}	1.42×10^{-8}	2.21×10^{-8}	1.01×10^{-8}	8.31×10^{-9}	4.03×10^{-9}
f23	4.12×10^{-8}	1.86×10^{-8}	2.62×10^{-8}	1.42×10^{-8}	2.20×10^{-8}	1.03×10^{-8}	7.23×10^{-9}	3.76×10^{-9}
f24	4.59×10^{-8}	1.97×10^{-8}	3.20×10^{-8}	1.54×10^{-8}	1.90×10^{-8}	1.04×10^{-8}	6.63×10^{-9}	4.48×10^{-9}
f25	3.04×10^{-8}	1.17×10^{-8}	2.94×10^{-8}	1.02×10^{-8}	2.44×10^{-8}	1.32×10^{-8}	3.24×10^{-9}	2.92×10^{-9}
f26	3.47×10^{-7}	2.36×10^{-8}	3.40×10^{-7}	1.86×10^{-8}	3.19×10^{-7}	2.51×10^{-8}	2.63×10^{-7}	1.53×10^{-8}
f27	4.24×10^{-8}	1.92×10^{-8}	2.38×10^{-8}	1.24×10^{-8}	1.76×10^{-8}	9.79×10^{-9}	6.32×10^{-9}	4.13×10^{-9}
f28	4.06×10^{-8}	1.79×10^{-8}	3.44×10^{-8}	1.83×10^{-8}	1.75×10^{-8}	8.24×10^{-9}	8.55×10^{-9}	3.84×10^{-9}
f29	3.99×10^{-8}	2.07×10^{-8}	3.29×10^{-8}	1.51×10^{-8}	2.26×10^{-8}	1.03×10^{-8}	8.21×10^{-9}	4.25×10^{-9}
f30	4.37×10^{-8}	2.26×10^{-8}	2.76×10^{-8}	1.49×10^{-8}	2.13×10^{-8}	9.86×10^{-9}	8.63×10^{-9}	4.91×10^{-9}
f31	4.64×10^{-8}	1.81×10^{-8}	2.78×10^{-8}	1.52×10^{-8}	2.09×10^{-8}	1.01×10^{-8}	8.57×10^{-9}	4.66×10^{-9}
f32	3.95×10^{-8}	1.84×10^{-8}	2.35×10^{-8}	1.08×10^{-8}	1.99×10^{-8}	6.87×10^{-9}	3.11×10^{-8}	1.64×10^{-9}
f33	4.50×10^{-8}	2.20×10^{-8}	2.29×10^{-8}	1.49×10^{-8}	2.72×10^{-8}	1.21×10^{-8}	8.07×10^{-9}	4.96×10^{-9}
f34	9.61×10^{-26}	2.37×10^{-25}	6.93×10^{-28}	2.11×10^{-27}	5.46×10^{-31}	3.48×10^{-31}	1.61×10^{-30}	1.63×10^{-30}
f35	4.06×10^{-8}	2.30×10^{-8}	2.69×10^{-8}	1.42×10^{-8}	2.24×10^{-8}	1.07×10^{-8}	7.99×10^{-9}	4.38×10^{-9}
f36	4.02×10^{-8}	2.05×10^{-8}	3.31×10^{-8}	1.63×10^{-8}	2.02×10^{-8}	1.11×10^{-8}	7.76×10^{-9}	4.23×10^{-9}

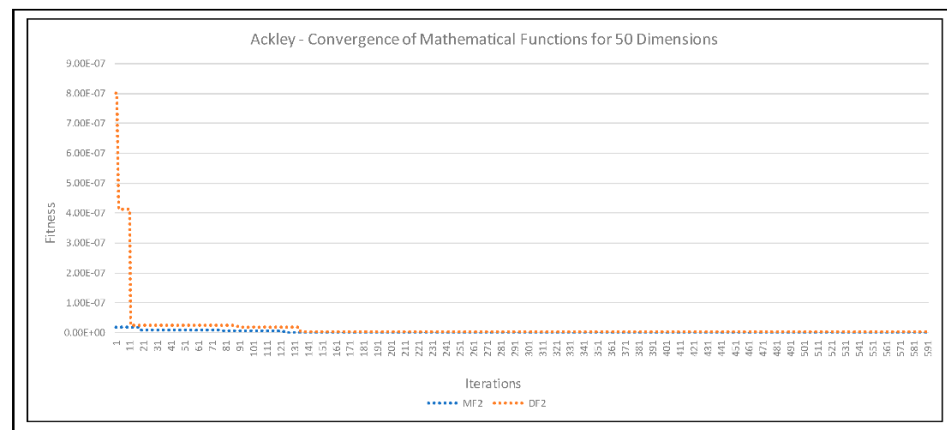


Figure 7. Ackley-Convergence of the DMOA and MTOA algorithms for 50 dimensions.

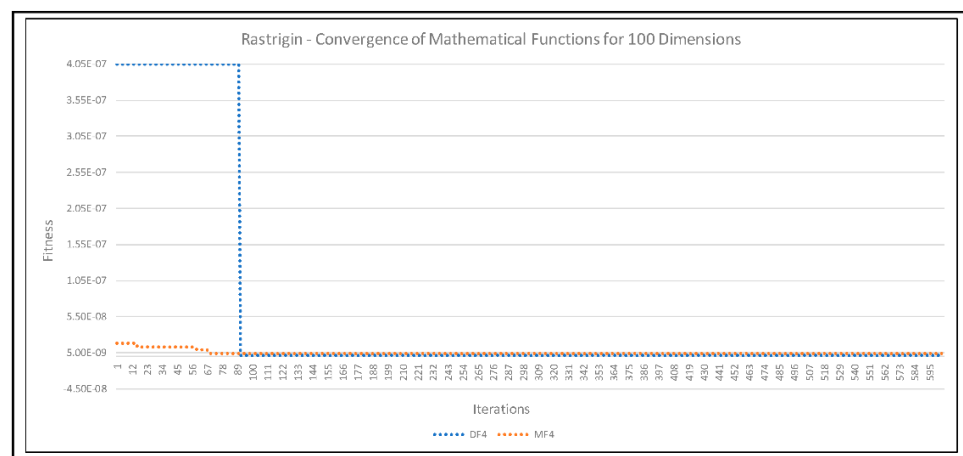


Figure 8. Rastrigin Convergence of the DMOA and MTOA algorithms for 100 dimensions.

4.2. Results of the DMOA Algorithm Experiments with CEC-2013

Tables 7–9 show the results of the experiments of the 36 mathematical functions of the CEC-2013 of the mean and standard deviation that were carried out for 30, 50 and 100 dimensions and for 30, 50, 100, and 500 iterations, respectively.

Figure 9 show the convergence of the following three mathematical functions: Sphere, Ackley, and Rosenbrock for 30, 50, and 100 dimensions for the DMOA and MTOA algorithms.

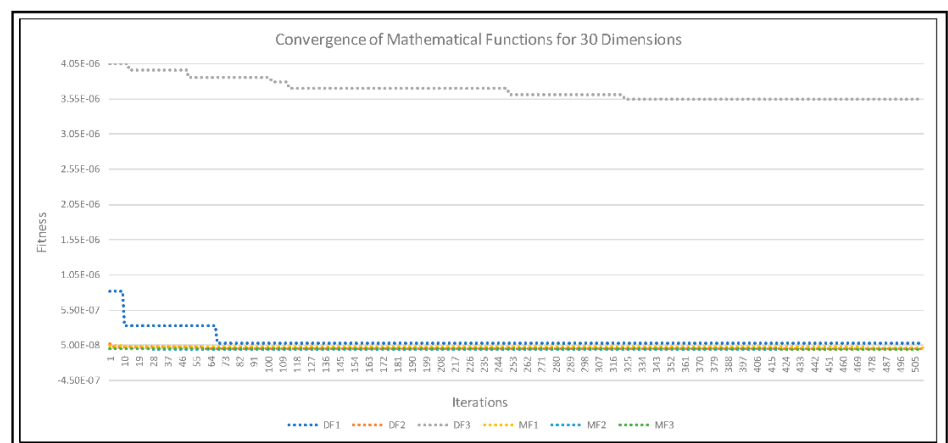


Figure 9. Convergence of the DMOA and MTOA algorithms for 30 dimensions, for the functions F1, F2, and F3.

Table 7. DMOA: Results of the experiments for 30 dimensions.

Dim	30							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f1	2.79×10^{-7}	2.36×10^{-7}	2.17×10^{-7}	1.79×10^{-7}	1.36×10^{-7}	9.36×10^{-8}	1.87×10^{-8}	1.35×10^{-8}
f2	6.51×10^{-8}	3.74×10^{-8}	4.63×10^{-8}	3.32×10^{-8}	1.83×10^{-8}	1.74×10^{-8}	2.69×10^{-9}	2.53×10^{-9}
f3	2.88×10^{-8}	2.40×10^{-8}	1.53×10^{-8}	1.15×10^{-8}	6.42×10^{-9}	6.60×10^{-9}	1.00×10^{-9}	8.92×10^{-10}
f4	3.34×10^{-7}	2.44×10^{-7}	1.90×10^{-7}	1.65×10^{-7}	9.94×10^{-8}	6.43×10^{-8}	1.61×10^{-8}	1.46×10^{-8}
f5	4.01×10^{-7}	3.75×10^{-7}	2.23×10^{-7}	1.89×10^{-7}	1.32×10^{-7}	1.06×10^{-7}	2.02×10^{-8}	1.78×10^{-8}
f6	3.32×10^{-7}	2.61×10^{-7}	2.08×10^{-7}	1.66×10^{-7}	9.40×10^{-8}	7.98×10^{-8}	1.82×10^{-8}	1.88×10^{-8}
f7	3.67×10^{-7}	2.84×10^{-7}	1.95×10^{-7}	1.55×10^{-7}	1.05×10^{-7}	8.18×10^{-8}	1.95×10^{-8}	1.53×10^{-8}
f8	2.34×10^{-7}	2.73×10^{-7}	1.67×10^{-7}	1.18×10^{-7}	1.14×10^{-7}	7.66×10^{-8}	1.89×10^{-8}	1.41×10^{-8}
f9	3.55×10^{-7}	2.65×10^{-7}	1.42×10^{-7}	1.59×10^{-7}	1.09×10^{-7}	9.83×10^{-8}	1.88×10^{-8}	1.66×10^{-8}
f10	2.67×10^{-7}	2.36×10^{-7}	2.61×10^{-7}	1.80×10^{-7}	1.07×10^{-7}	7.59×10^{-8}	2.04×10^{-8}	1.90×10^{-8}
f11	3.66×10^{-7}	2.91×10^{-7}	2.53×10^{-7}	1.61×10^{-7}	1.08×10^{-7}	7.56×10^{-8}	1.61×10^{-8}	1.51×10^{-8}
f12	3.00×10^{-7}	2.98×10^{-7}	2.03×10^{-7}	1.62×10^{-7}	1.14×10^{-7}	8.19×10^{-8}	1.65×10^{-8}	1.62×10^{-8}
f13	3.06×10^{-7}	2.80×10^{-7}	2.20×10^{-7}	1.38×10^{-7}	9.61×10^{-8}	7.50×10^{-8}	1.68×10^{-8}	1.31×10^{-8}
f14	3.49×10^{-7}	2.50×10^{-7}	1.60×10^{-7}	1.22×10^{-7}	1.13×10^{-7}	7.40×10^{-8}	2.14×10^{-8}	1.68×10^{-8}
f15	3.43×10^{-7}	2.77×10^{-7}	2.03×10^{-7}	1.67×10^{-7}	8.07×10^{-8}	5.44×10^{-8}	1.84×10^{-8}	1.62×10^{-8}
f16	2.87×10^{-7}	2.69×10^{-7}	2.13×10^{-7}	1.64×10^{-7}	1.17×10^{-7}	9.83×10^{-8}	1.87×10^{-8}	1.47×10^{-8}
f17	5.39×10^{-8}	4.18×10^{-8}	3.94×10^{-8}	3.06×10^{-8}	1.73×10^{-8}	1.34×10^{-8}	2.16×10^{-9}	1.89×10^{-9}
f18	2.20×10^{-9}	3.13×10^{-9}	9.75×10^{-10}	8.56×10^{-10}	4.31×10^{-10}	4.48×10^{-10}	2.40×10^{-11}	2.89×10^{-11}
f19	3.43×10^{-7}	2.52×10^{-7}	1.91×10^{-7}	1.24×10^{-7}	9.13×10^{-8}	7.10×10^{-8}	1.62×10^{-8}	1.39×10^{-8}
f20	3.49×10^{-7}	2.46×10^{-7}	1.51×10^{-7}	1.49×10^{-7}	8.26×10^{-8}	6.69×10^{-8}	1.53×10^{-8}	1.36×10^{-8}
f21	1.28×10^{-7}	1.05×10^{-7}	6.40×10^{-8}	6.31×10^{-8}	2.21×10^{-8}	2.66×10^{-8}	4.21×10^{-9}	4.46×10^{-9}
f22	3.21×10^{-7}	2.62×10^{-7}	1.92×10^{-7}	1.72×10^{-7}	8.15×10^{-8}	6.76×10^{-8}	1.64×10^{-8}	1.51×10^{-8}
f23	3.47×10^{-7}	2.65×10^{-7}	1.98×10^{-7}	1.84×10^{-7}	1.02×10^{-7}	6.45×10^{-8}	1.78×10^{-8}	1.38×10^{-8}
f24	3.37×10^{-7}	2.18×10^{-7}	2.15×10^{-7}	1.81×10^{-7}	7.28×10^{-8}	7.30×10^{-8}	2.17×10^{-8}	1.51×10^{-8}
f25	2.03×10^{-8}	2.47×10^{-8}	7.98×10^{-9}	1.06×10^{-8}	3.50×10^{-9}	3.14×10^{-9}	3.98×10^{-10}	3.72×10^{-10}
f26	1.08×10^{-4}	2.07×10^{-8}	1.08×10^{-4}	1.37×10^{-8}	1.08×10^{-4}	7.91×10^{-9}	1.08×10^{-4}	2.58×10^{-9}
f27	3.23×10^{-7}	3.01×10^{-7}	2.20×10^{-7}	1.90×10^{-7}	7.78×10^{-8}	6.82×10^{-8}	1.92×10^{-8}	1.66×10^{-8}
f28	2.68×10^{-7}	2.10×10^{-7}	2.42×10^{-7}	1.72×10^{-7}	9.09×10^{-8}	7.59×10^{-8}	2.48×10^{-8}	1.95×10^{-8}
f29	3.86×10^{-7}	3.05×10^{-7}	1.94×10^{-7}	1.74×10^{-7}	9.27×10^{-8}	7.64×10^{-8}	1.94×10^{-8}	1.50×10^{-8}
f30	4.48×10^{-7}	2.79×10^{-7}	1.88×10^{-7}	1.83×10^{-7}	1.14×10^{-7}	7.91×10^{-8}	1.64×10^{-8}	1.28×10^{-8}
f31	3.92×10^{-7}	2.27×10^{-7}	2.25×10^{-7}	1.54×10^{-7}	1.13×10^{-7}	9.11×10^{-8}	1.44×10^{-8}	1.51×10^{-8}

Table 7. Cont.

Dim	30							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f32	3.21×10^{-7}	2.57×10^{-7}	1.65×10^{-7}	1.42×10^{-7}	8.84×10^{-8}	7.41×10^{-8}	1.18×10^{-8}	1.22×10^{-8}
f33	3.44×10^{-7}	2.31×10^{-7}	2.11×10^{-7}	1.50×10^{-7}	1.28×10^{-7}	8.49×10^{-8}	1.55×10^{-8}	1.65×10^{-8}
f34	5.22×10^{-28}	2.25×10^{-28}	3.61×10^{-28}	2.22×10^{-28}	2.37×10^{-28}	1.60×10^{-28}	5.19×10^{-29}	3.51×10^{-29}
f35	3.60×10^{-7}	2.92×10^{-7}	2.44×10^{-7}	2.04×10^{-7}	1.12×10^{-7}	8.18×10^{-8}	1.73×10^{-8}	1.55×10^{-8}
f36	3.01×10^{-7}	2.52×10^{-7}	1.91×10^{-7}	1.54×10^{-7}	8.06×10^{-8}	6.31×10^{-8}	1.62×10^{-8}	1.54×10^{-8}

Table 8. DMOA: Results of the experiments for 50 dimensions.

Dim	50							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f1	4.24×10^{-7}	2.89×10^{-7}	1.78×10^{-7}	1.34×10^{-7}	1.09×10^{-7}	7.52×10^{-8}	1.84×10^{-8}	1.52×10^{-8}
f2	5.64×10^{-8}	4.83×10^{-8}	3.77×10^{-8}	2.84×10^{-8}	1.79×10^{-8}	1.27×10^{-8}	2.59×10^{-9}	2.55×10^{-9}
f3	2.25×10^{-8}	2.16×10^{-8}	2.27×10^{-8}	1.65×10^{-8}	1.22×10^{-8}	9.04×10^{-9}	1.03×10^{-9}	9.38×10^{-10}
f4	3.49×10^{-7}	3.02×10^{-7}	1.81×10^{-7}	1.40×10^{-7}	1.25×10^{-7}	7.72×10^{-8}	1.83×10^{-8}	1.64×10^{-8}
f5	3.15×10^{-7}	3.36×10^{-7}	3.22×10^{-7}	3.08×10^{-7}	1.54×10^{-7}	1.25×10^{-7}	2.79×10^{-8}	2.24×10^{-8}
f6	3.50×10^{-7}	2.92×10^{-7}	1.59×10^{-7}	1.38×10^{-7}	7.60×10^{-8}	6.45×10^{-8}	2.07×10^{-8}	1.55×10^{-8}
f7	3.32×10^{-7}	2.48×10^{-7}	2.23×10^{-7}	1.66×10^{-7}	8.10×10^{-8}	7.95×10^{-8}	1.69×10^{-8}	1.62×10^{-8}
f8	2.55×10^{-7}	2.18×10^{-7}	1.94×10^{-7}	1.45×10^{-7}	7.25×10^{-8}	6.91×10^{-8}	2.16×10^{-8}	1.66×10^{-8}
f9	3.64×10^{-7}	2.91×10^{-7}	2.26×10^{-7}	1.91×10^{-7}	1.16×10^{-7}	7.89×10^{-8}	1.71×10^{-8}	1.60×10^{-8}
f10	2.51×10^{-7}	2.07×10^{-7}	1.94×10^{-7}	1.64×10^{-7}	1.01×10^{-7}	7.44×10^{-8}	1.67×10^{-8}	1.36×10^{-8}
f11	2.39×10^{-7}	2.15×10^{-7}	1.87×10^{-7}	1.67×10^{-7}	9.65×10^{-8}	7.85×10^{-8}	1.66×10^{-8}	1.52×10^{-8}
f12	0	0	0	0	0	0	0	0
f13	2.97×10^{-7}	2.17×10^{-7}	1.84×10^{-7}	1.66×10^{-7}	9.41×10^{-8}	6.93×10^{-8}	1.62×10^{-8}	1.46×10^{-8}
f14	2.84×10^{-7}	2.35×10^{-7}	1.86×10^{-7}	1.37×10^{-7}	9.72×10^{-8}	7.04×10^{-8}	1.66×10^{-8}	1.47×10^{-8}
f15	3.69×10^{-7}	2.89×10^{-7}	2.04×10^{-7}	1.64×10^{-7}	9.23×10^{-8}	7.53×10^{-8}	1.75×10^{-8}	1.72×10^{-8}
f16	3.57×10^{-7}	2.54×10^{-7}	2.48×10^{-7}	1.88×10^{-7}	1.03×10^{-7}	7.70×10^{-8}	1.54×10^{-8}	1.42×10^{-8}
f17	6.91×10^{-8}	5.62×10^{-8}	3.47×10^{-8}	2.78×10^{-8}	1.66×10^{-8}	1.23×10^{-8}	3.68×10^{-9}	2.86×10^{-9}
f18	2.31×10^{-9}	3.42×10^{-9}	8.79×10^{-10}	8.52×10^{-10}	2.59×10^{-10}	2.89×10^{-10}	3.09×10^{-11}	4.05×10^{-11}
f19	3.60×10^{-7}	2.88×10^{-7}	2.45×10^{-7}	1.74×10^{-7}	9.82×10^{-8}	7.93×10^{-8}	1.80×10^{-8}	1.75×10^{-8}

Table 8. Cont.

Dim	50							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f20	3.89×10^{-7}	2.89×10^{-7}	2.28×10^{-7}	1.75×10^{-7}	1.03×10^{-7}	8.89×10^{-8}	1.45×10^{-8}	1.49×10^{-8}
f21	1.48×10^{-7}	1.07×10^{-7}	7.75×10^{-8}	6.17×10^{-8}	2.28×10^{-8}	2.48×10^{-8}	2.51×10^{-9}	3.17×10^{-9}
f22	2.64×10^{-7}	2.50×10^{-7}	1.68×10^{-7}	1.49×10^{-7}	9.59×10^{-8}	6.50×10^{-8}	1.63×10^{-8}	1.45×10^{-8}
f23	2.82×10^{-7}	2.02×10^{-7}	1.97×10^{-7}	1.66×10^{-7}	8.36×10^{-8}	5.98×10^{-8}	2.17×10^{-8}	1.48×10^{-8}
f24	3.33×10^{-7}	2.83×10^{-7}	1.95×10^{-7}	1.47×10^{-7}	7.44×10^{-8}	6.45×10^{-8}	1.91×10^{-8}	1.45×10^{-8}
f25	2.20×10^{-8}	2.46×10^{-8}	1.33×10^{-8}	1.53×10^{-8}	3.27×10^{-9}	4.29×10^{-9}	4.84×10^{-10}	4.15×10^{-10}
f26	1.08×10^{-4}	1.95×10^{-8}	1.08×10^{-4}	1.43×10^{-8}	1.08×10^{-4}	9.69×10^{-9}	1.08×10^{-4}	2.51×10^{-9}
f27	2.99×10^{-7}	2.46×10^{-7}	1.51×10^{-7}	9.83×10^{-8}	8.82×10^{-8}	6.57×10^{-8}	1.61×10^{-8}	1.61×10^{-8}
f28	3.15×10^{-7}	2.36×10^{-7}	1.99×10^{-7}	1.77×10^{-7}	1.29×10^{-7}	7.41×10^{-8}	2.22×10^{-8}	1.83×10^{-8}
f29	3.88×10^{-7}	2.88×10^{-7}	2.51×10^{-7}	1.88×10^{-7}	8.23×10^{-8}	7.30×10^{-8}	2.22×10^{-8}	1.94×10^{-8}
f30	3.66×10^{-7}	2.42×10^{-7}	2.36×10^{-7}	2.03×10^{-7}	1.04×10^{-7}	8.31×10^{-8}	2.26×10^{-8}	1.91×10^{-8}
f31	4.14×10^{-7}	3.22×10^{-7}	1.69×10^{-7}	1.44×10^{-7}	9.73×10^{-8}	7.94×10^{-8}	1.84×10^{-8}	1.63×10^{-8}
f32	3.37×10^{-7}	2.50×10^{-7}	1.88×10^{-7}	1.43×10^{-7}	1.20×10^{-7}	8.49×10^{-8}	1.99×10^{-8}	1.62×10^{-8}
f33	3.24×10^{-7}	2.59×10^{-7}	2.08×10^{-7}	1.33×10^{-7}	1.13×10^{-7}	8.01×10^{-8}	2.51×10^{-8}	1.64×10^{-8}
f34	7.10×10^{-28}	3.34×10^{-28}	3.81×10^{-28}	2.20×10^{-28}	2.82×10^{-28}	1.64×10^{-28}	5.98×10^{-29}	4.80×10^{-29}
f35	3.06×10^{-7}	2.48×10^{-7}	2.06×10^{-7}	1.67×10^{-7}	1.09×10^{-7}	8.47×10^{-8}	1.83×10^{-8}	1.65×10^{-8}
f36	4.33×10^{-1}	3.13×10^{-7}	1.90×10^{-7}	1.41×10^{-7}	7.89×10^{-8}	6.22×10^{-8}	2.21×10^{-8}	1.74×10^{-8}

Table 9. DMOA: Results of the experiments for 100 dimensions.

Dim	100							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f1	3.55×10^{-7}	3.12×10^{-7}	1.75×10^{-7}	1.88×10^{-7}	9.84×10^{-8}	8.69×10^{-8}	2.49×10^{-8}	2.02×10^{-8}
f2	6.51×10^{-8}	5.65×10^{-8}	2.93×10^{-8}	2.59×10^{-8}	1.66×10^{-8}	1.35×10^{-8}	2.38×10^{-9}	2.04×10^{-9}
f3	1.89×10^{-7}	1.55×10^{-7}	1.10×10^{-7}	1.04×10^{-7}	4.70×10^{-8}	3.89×10^{-8}	1.22×10^{-8}	9.77×10^{-9}
f4	3.54×10^{-7}	2.65×10^{-7}	1.91×10^{-7}	1.82×10^{-7}	9.54×10^{-8}	7.94×10^{-8}	2.25×10^{-8}	1.50×10^{-8}
f5	1.18×10^{-6}	9.47×10^{-7}	5.55×10^{-7}	4.44×10^{-7}	2.76×10^{-7}	2.37×10^{-7}	7.94×10^{-8}	5.61×10^{-8}
f6	3.49×10^{-7}	2.54×10^{-7}	1.92×10^{-7}	1.28×10^{-7}	1.11×10^{-7}	7.17×10^{-8}	1.57×10^{-8}	1.44×10^{-8}

Table 9. Cont.

Dim	100							
Iter	30		50		100		500	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD
f7	3.12×10^{-7}	2.81×10^{-7}	2.51×10^{-7}	1.90×10^{-7}	8.44×10^{-8}	7.15×10^{-8}	2.14×10^{-8}	1.41×10^{-8}
f8	3.79×10^{-7}	2.67×10^{-7}	1.81×10^{-7}	1.35×10^{-7}	9.76×10^{-8}	7.78×10^{-8}	1.55×10^{-8}	1.72×10^{-8}
f9	3.23×10^{-7}	2.34×10^{-7}	2.98×10^{-7}	1.84×10^{-7}	8.54×10^{-8}	7.36×10^{-8}	2.29×10^{-8}	1.76×10^{-8}
f10	2.84×10^{-7}	2.32×10^{-7}	1.79×10^{-7}	1.59×10^{-7}	1.02×10^{-7}	9.02×10^{-8}	1.99×10^{-8}	1.53×10^{-8}
f11	3.62×10^{-7}	3.09×10^{-7}	1.87×10^{-7}	1.75×10^{-7}	9.09×10^{-8}	8.45×10^{-8}	2.35×10^{-8}	1.59×10^{-8}
f12	0	0	0	0	0	0	0	0
f13	3.00×10^{-7}	2.52×10^{-7}	1.61×10^{-7}	1.32×10^{-7}	9.33×10^{-8}	8.23×10^{-8}	2.08×10^{-8}	1.67×10^{-8}
f14	2.81×10^{-7}	2.25×10^{-7}	2.05×10^{-7}	1.63×10^{-7}	1.09×10^{-7}	8.33×10^{-8}	2.35×10^{-8}	1.59×10^{-8}
f15	1.20×10^{-9}	1.73×10^{-9}	2.40×10^{-10}	9.13×10^{-10}	0	0	0	0
f16	2.97×10^{-7}	3.05×10^{-7}	1.15×10^{-7}	1.28×10^{-7}	8.98×10^{-8}	6.56×10^{-8}	1.71×10^{-8}	1.39×10^{-8}
f17	4.77×10^{-8}	4.51×10^{-8}	3.34×10^{-8}	2.90×10^{-8}	1.66×10^{-8}	1.72×10^{-8}	2.46×10^{-9}	2.03×10^{-9}
f18	2.60×10^{-09}	2.37×10^{-09}	9.563×10^{-10}	9.67×10^{-10}	3.35×10^{-10}	4.152×10^{-10}	2.42×10^{-11}	3.06×10^{-11}
f19	3.89×10^{-7}	3.17×10^{-7}	2.13×10^{-7}	1.73×10^{-7}	9.34×10^{-8}	8.40×10^{-8}	1.95×10^{-8}	1.60×10^{-8}
f20	1.95×10^{-7}	1.43×10^{-7}	2.25×10^{-7}	1.33×10^{-7}	8.72×10^{-8}	8.20×10^{-8}	1.67×10^{-8}	1.60×10^{-8}
f21	1.03×10^{-7}	8.05×10^{-8}	5.74×10^{-8}	6.44×10^{-8}	3.34×10^{-8}	2.55×10^{-8}	4.57×10^{-9}	3.79×10^{-9}
f22	2.54×10^{-7}	2.19×10^{-7}	1.99×10^{-7}	1.40×10^{-7}	8.83×10^{-8}	9.01×10^{-8}	1.57×10^{-8}	1.37×10^{-8}
f23	2.82×10^{-7}	2.03×10^{-7}	1.80×10^{-7}	1.69×10^{-7}	1.11×10^{-7}	1.03×10^{-7}	1.76×10^{-8}	1.68×10^{-8}
f24	2.44×10^{-7}	2.37×10^{-7}	2.03×10^{-7}	1.80×10^{-7}	9.99×10^{-8}	6.72×10^{-8}	1.98×10^{-8}	1.51×10^{-8}
f25	1.93×10^{-8}	2.48×10^{-8}	1.39×10^{-8}	1.43×10^{-8}	2.71×10^{-9}	2.42×10^{-9}	4.36×10^{-10}	3.45×10^{-10}
f26	1.08×10^{-4}	1.97×10^{-8}	1.08×10^{-4}	1.42×10^{-8}	1.08×10^{-4}	8.79×10^{-9}	1.08×10^{-4}	2.43×10^{-9}
f27	2.77×10^{-7}	2.67×10^{-7}	2.08×10^{-7}	1.18×10^{-7}	1.00×10^{-7}	8.68×10^{-8}	1.31×10^{-8}	1.27×10^{-8}
f28	3.77×10^{-7}	3.26×10^{-7}	2.12×10^{-7}	1.91×10^{-7}	1.10×10^{-7}	7.39×10^{-8}	1.59×10^{-8}	1.57×10^{-8}
f29	3.42×10^{-7}	2.58×10^{-7}	1.99×10^{-7}	1.44×10^{-7}	1.07×10^{-7}	9.12×10^{-8}	2.27×10^{-8}	1.62×10^{-8}
f30	3.07×10^{-7}	2.63×10^{-7}	2.24×10^{-7}	1.74×10^{-7}	8.61×10^{-8}	7.65×10^{-8}	1.86×10^{-8}	1.58×10^{-8}
f31	2.80×10^{-7}	2.53×10^{-7}	1.52×10^{-7}	1.27×10^{-7}	1.24×10^{-7}	7.30×10^{-8}	1.77×10^{-8}	1.90×10^{-8}
f32	2.87×10^{-7}	2.85×10^{-7}	2.25×10^{-7}	1.48×10^{-7}	9.29×10^{-8}	7.73×10^{-8}	1.09×10^{-8}	1.06×10^{-8}
f33	4.20×10^{-7}	2.74×10^{-7}	2.55×10^{-7}	1.67×10^{-7}	1.17×10^{-7}	9.39×10^{-8}	1.64×10^{-8}	1.53×10^{-8}
f34	4.93×10^{-28}	3.04×10^{-28}	4.35×10^{-28}	2.07×10^{-28}	2.78×10^{-28}	1.60×10^{-28}	7.02×10^{-29}	4.37×10^{-29}
f35	2.25×10^{-7}	2.39×10^{-07}	2.11×10^{-7}	1.81×10^{-7}	9.11×10^{-8}	8.29×10^{-8}	1.41×10^{-8}	1.55×10^{-8}
f36	2.88×10^{-7}	2.44×10^{-7}	1.95×10^{-7}	1.97×10^{-7}	1.19×10^{-7}	8.09×10^{-8}	2.36×10^{-8}	1.57×10^{-8}

Figure 10 show the convergence of the following three mathematical functions: Ackley, Rosenbrock, and Rastrigin for 30, 50, and 100 dimensions for the DMOA and MTOA algorithms.

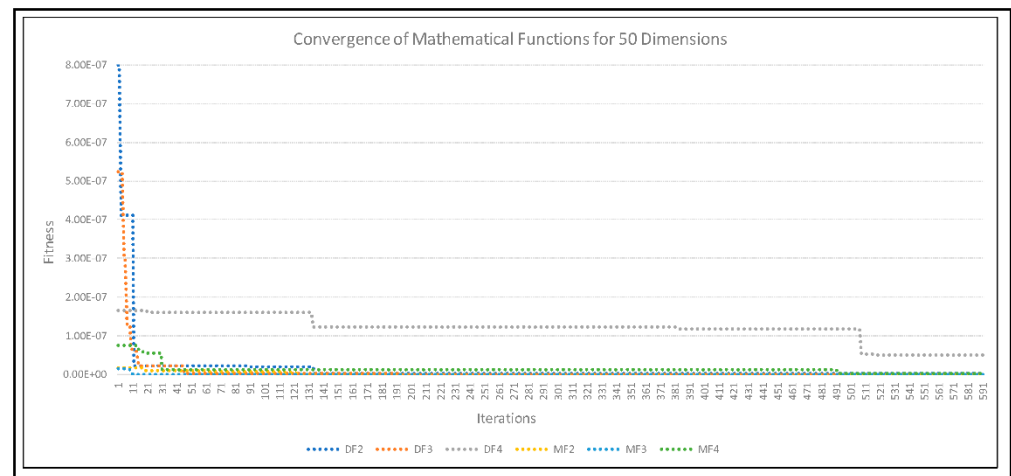


Figure 10. Convergence of the DMOA and MTOA algorithms for 50 dimensions, for the functions F2, F3, and F4.

Figure 11 show the convergence of the following three mathematical functions: Rosenbrock, Rastrigin, and Griewank for 30, 50, and 100 dimensions for the DMOA and MTOA algorithms.

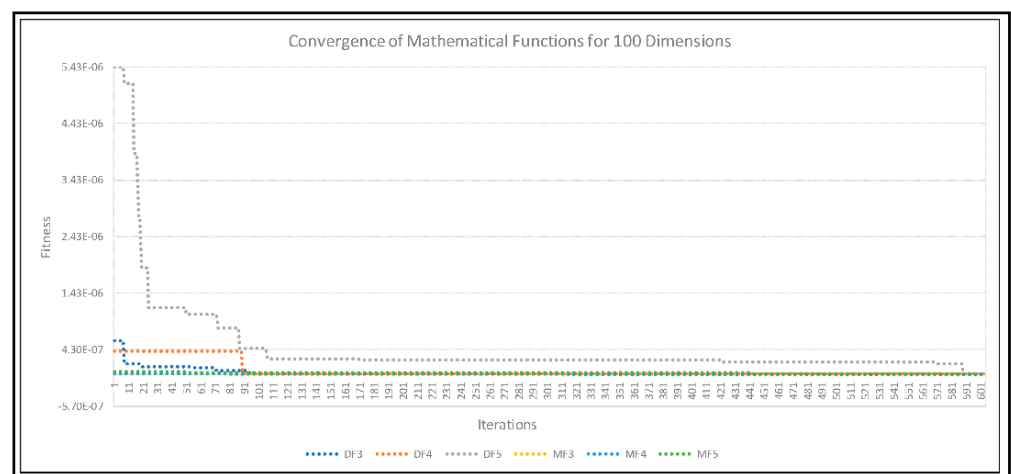


Figure 11. Convergence of the DMOA and MTOA algorithms for 100 dimensions, for the functions F3, F4, and F5.

4.3. Hypothesis Tests

All hypothesis tests were performed for two independent samples with 30 experiments each, where μ_1 , μ_2 , σ_1 , and σ_2 were known [64–67]. Table 10 shows the null and alternative hypothesis, while Equation (7) shows the Z test statistics formula and Table 11 shows the hypothesis test parameters.

Table 12 shows 108 hypothesis tests (z) that were performed on the MTOA and DMOA algorithms for 30, 50, and 100 dimensions and 30, 500, and 100 iterations, respectively. In most of the tests, in 92% of cases there was significant evidence that the MTOA algorithm is favorable over the DMOA algorithm, and the DMOA algorithm only performed better in eight tests (7.4%), in two functions for 30×30 , in 2 for 50×50 , and in four functions for 100×100 .

Table 10. Null and Alternative Hypothesis and Z Test Statistics Formula.

$H_0 : \mu_1 \geq \mu_2$
$H_a : \mu_1 < \mu_2 \text{ claim}$
$H_0 : DMOA$
$H_a : MTOA$
$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad (7)$

Table 11. Hypothesis Test Parameters.

$\bar{x}_1$ = Mean of sample 1
$\bar{x}_2$ = Mean of sample 2
σ_1 = Standard Deviation of sample 1
σ_2 = Standard Deviation of sample 2
n_1 = Number of sample data 1
n_2 = Number of sample data 2
$\mu_1 - \mu_2 = D_0$
$D_0 = 0$

Table 12. Results for the MTOA vs. DMOA hypothesis tests.

Dim/Iter	30 × 30	50 × 50	100 × 100
Func	MTOA vs. DMOA	MTOA vs. DMOA	MTOA vs. DMOA
f1	-5.35×10^0	-6.07×10^0	-4.82×10^0
f2	-6.81×10^0	-5.15×10^0	-6.50×10^0
f3	-3.47×10^0	-5.84×10^0	-5.90×10^0
f4	-6.61×10^0	-5.99×10^0	-5.22×10^0
f5	-5.02×10^0	-5.20×10^0	-5.79×10^0
f6	-6.29×10^0	-5.00×10^0	-6.77×10^0
f7	-6.38×10^0	-6.49×10^0	-5.29×10^0
f8	-3.78×10^0	-6.08×10^0	-5.49×10^0
f9	-6.51×10^0	-5.48×10^0	-4.83×10^0
f10	-5.02×10^0	-5.51×10^0	-4.94×10^0
f11	-6.13×10^0	-5.02×10^0	-4.48×10^0
f12	-4.74×10^0	5.41×10^0	Nan
f13	-5.20×10^0	-5.06×10^0	-5.03×10^0
f14	-6.58×10^0	-5.99×10^0	-5.84×10^0
f15	-5.90×10^0	-5.75×10^0	1.58×10^1
f16	-5.15×10^0	-6.40×10^0	-5.88×10^0
f17	-4.33×10^0	-4.57×10^0	-5.03×10^0
f18	8.86×10^{-1}	2.31×10^0	2.13×10^0
f19	-6.68×10^0	-6.47×10^0	-4.52×10^0
f20	-6.66×10^0	-6.28×10^0	-4.15×10^0
f21	-5.75×10^0	-5.36×10^0	1.87×10^1
f22	-5.76×10^0	-4.87×10^0	-4.00×10^0
f23	-6.41×10^0	-5.44×10^0	-4.70×10^0
f24	-7.25×10^0	-6.01×10^0	-6.51×10^0
f25	-3.58×10^0	-4.08×10^0	8.86×10^0
f26	-2.30×10^4	-2.40×10^4	-2.21×10^4
f27	-4.93×10^0	-6.97×10^0	-5.19×10^0
f28	-5.80×10^0	-5.28×10^0	-6.82×10^0
f29	-6.17×10^0	-6.37×10^0	-5.05×10^0
f30	-8.06×10^0	-5.51×10^0	-4.60×10^0

Table 12. Cont.

Dim/Iter	30 × 30	50 × 50	100 × 100
Func	MTOA vs. DMOA	MTOA vs. DMOA	MTOA vs. DMOA
f31	-8.31×10^0	-5.25×10^0	-7.67×10^0
f32	-6.50×10^0	-6.17×10^0	-5.15×10^0
f33	-7.20×10^0	-7.39×10^0	-5.21×10^0
f34	4.79×10^0	4.36×10^0	-9.46×10^0
f35	-6.02×10^0	-5.70×10^0	-4.51×10^0
f36	-5.64×10^0	-6.10×10^0	-6.66×10^0

NaN—Not a Number—For expressions like 0/0.

4.4. Compared with Others Methods

In Tables 13 and 14, a comparative study of the DMOA Algorithm with other methods of the mean and Standard Deviation is shown, including MTOA (Mycorrhiza Tree Optimization Algorithm) [14], Self-Defense (A New Bio-inspired Optimization Algorithm Based on the Self-defense Mechanism of Plants in Nature) [68], FA (FireFly Algorithm) [69], GSA (Gravitational Search Algorithm) [69], CS (Cuckoo Search) [69], GA (Genetic Algorithm) [69], DE (Differential Evolution) [70], HS (Harmony Search) [69], and GA (Genetic Algorithm) [69] for 30 and 50 dimensions. In Figures 12 and 13, we observe the behavior of the Standard Deviation of the DMOA Algorithm and the aforementioned methods for 30 and 50 dimensions and realize that in both behaviors the Mean and Standard Deviation of the DMOA were superior in most cases compared to the other methods.

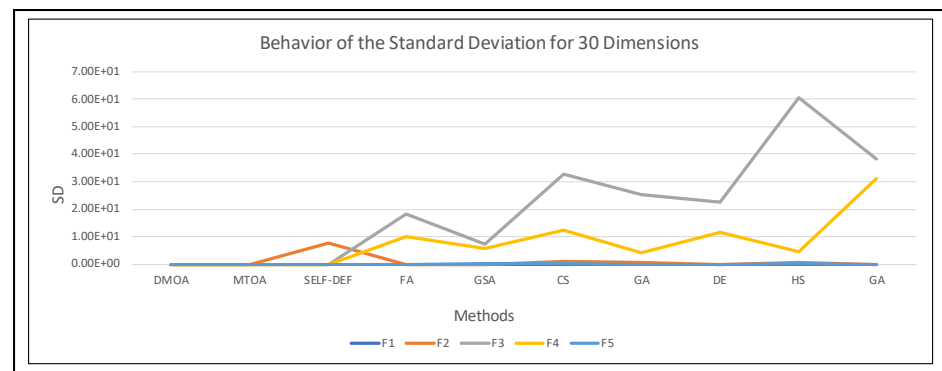


Figure 12. Behavior of the Standard Deviation of the DMOA Algorithm and the methods: MTOA, Self Defense, FA FireFly Algorithm, GSA, CS, GA, DE, HS, and GA for 30 dimensions.

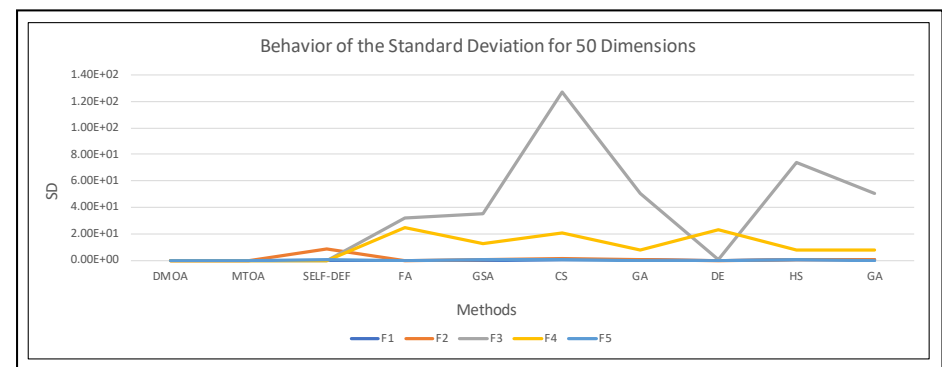


Figure 13. Behavior of the Standard Deviation of the DMOA Algorithm and the methods: MTOA, Self Defense, FA FireFly Algorithm, GSA, CS, GA, DE, HS, and GA for 50 dimensions.

Table 13. Comparison of the Mean and Standard Deviation of the DMOA Algorithm with other methods, for 30 dimensions.

30 Dimensions										
Method	DMOA		MTOA		SELF-DEFENSE		FA		GSA	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
F1	2.79×10^{-7}	2.36×10^{-7}	4.72×10^{-8}	1.98×10^{-8}	1.53×10^{-8}	6.42×10^{-8}	1.99×10^{-3}	5.13×10^{-4}	9.45×10^{-17}	3.81×10^{-17}
F2	6.51×10^{-8}	3.74×10^{-8}	1.67×10^{-8}	1.06×10^{-8}	4.98×10^0	7.54×10^0	1.06×10^{-2}	1.42×10^{-3}	7.16×10^{-9}	1.69×10^{-9}
F3	2.88×10^{-8}	2.40×10^{-8}	1.29×10^{-8}	7.35×10^{-9}	4.92×10^{-9}	4.02×10^{-9}	3.41×10^1	1.84×10^1	2.70×10^1	7.45×10^0
F4	3.34×10^{-7}	2.44×10^{-7}	3.85×10^{-8}	1.69×10^{-8}	4.22×10^{-4}	1.61×10^{-3}	3.71×10^1	9.92×10^0	2.63×10^1	5.69×10^0
F5	4.01×10^{-7}	3.75×10^{-7}	5.62×10^{-8}	2.45×10^{-8}	1.94×10^{-4}	6.99×10^{-4}	3.38×10^{-3}	1.59×10^{-3}	3.19×10^{-2}	3.06×10^{-2}
Method	CS		GA		DE		HS		GA	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
F1	2.13×10^{-4}	2.18×10^{-4}	4.20×10^{-3}	3.88×10^{-3}	1.59×10^{-3}	4.28×10^{-4}	3.66×10^{-2}	5.95×10^{-3}	9.28×10^{-5}	5.09×10^{-5}
F2	2.15×10^0	1.09×10^0	4.71×10^{-1}	5.22×10^{-1}	9.78×10^{-3}	1.31×10^{-3}	2.32×10^0	4.68×10^{-1}	2.62×10^{-3}	5.91×10^{-4}
F3	2.88×10^1	3.29×10^1	1.59×10^1	2.54×10^1	3.38×10^1	2.25×10^1	$1.02 \times 10^{+2}$	6.06×10^1	5.06×10^1	3.83×10^1
F4	5.34×10^1	1.22×10^1	1.31×10^1	4.13×10^0	3.72×10^1	1.17×10^1	2.89×10^1	4.68×10^0	8.76×10^1	3.13×10^1
F5	2.72×10^{-2}	5.86×10^{-2}	4.43×10^{-4}	5.20×10^{-4}	3.18×10^{-3}	1.57×10^{-3}	2.71×10^{-2}	2.34×10^{-2}	1.93×10^{-3}	3.33×10^{-3}

Table 14. Comparison of the Mean and Standard Deviation of the DMOA Algorithm with other methods, for 50 dimensions.

50 Dimensions										
Method	DMOA		MTOA		SELF-DEFENSE		FA		GSA	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
F1	4.24×10^{-7}	2.89×10^{-7}	4.58×10^{-8}	2.02×10^{-8}	1.78×10^{-7}	7.25×10^{-5}	1.15×10^{-2}	3.20×10^{-3}	4.54×10^{-16}	2.19×10^{-16}
F2	5.64×10^{-8}	4.83×10^{-8}	1.72×10^{-8}	1.18×10^{-8}	3.77×10^{-8}	8.79×10^0	2.49×10^{-2}	3.66×10^{-3}	7.27×10^{-2}	2.73×10^{-1}
F3	2.25×10^{-8}	2.16×10^{-8}	3.77×10^{-9}	3.30×10^{-9}	2.27×10^{-8}	5.90×10^{-5}	5.85×10^1	3.18×10^1	7.62×10^1	3.50×10^1
F4	3.49×10^{-7}	3.02×10^{-7}	3.71×10^{-8}	1.89×10^{-8}	1.81×10^{-7}	9.60×10^{-3}	7.45×10^1	2.47×10^1	5.07×10^1	1.26×10^1
F5	3.15×10^{-7}	3.36×10^{-7}	5.67×10^{-8}	2.47×10^{-8}	3.22×10^{-7}	5.97×10^{-2}	7.01×10^{-3}	2.04×10^{-3}	1.24×10^0	2.44×10^{-1}
Method	CS		GA		DE		HS		GA	
Func	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
F1	9.90×10^{-1}	7.89×10^{-1}	1.77×10^0	6.66×10^{-1}	7.19×10^{-3}	1.79×10^{-3}	3.54×10^{-1}	7.94×10^{-2}	1.77×10^0	6.66×10^{-1}
F2	4.84×10^0	1.32×10^0	1.73×10^0	3.15×10^{-1}	1.86×10^{-2}	2.05×10^{-3}	3.12×10^0	3.14×10^{-1}	1.73×10^0	3.15×10^{-1}
F3	$2.28 \times 10^{+2}$	$1.27 \times 10^{+2}$	2.96×10^1	5.07×10^1	4.80×10^1	7.41×10^{-1}	$2.04 \times 10^{+2}$	7.36×10^1	2.96×10^1	5.07×10^1
F4	$1.15 \times 10^{+2}$	2.07×10^1	3.70×10^1	7.63×10^0	7.81×10^1	2.27×10^1	8.26×10^1	8.04×10^0	3.70×10^1	7.63×10^0
F5	6.21×10^{-1}	2.13×10^{-1}	5.05×10^{-2}	1.61×10^{-2}	5.91×10^{-3}	1.09×10^{-3}	1.22×10^0	7.73×10^{-2}	5.05×10^{-2}	1.61×10^{-2}

4.5. Programming Environment

The language used in the programming of the DMOA algorithm in MATLAB R2019b and the equipment where the programming and experiments were carried out is a Desktop Computer Intel Core i5 4460S 2.90 GHz, RAM DDR3 16 Gb, Intel HD Graphics 4600, and Operating System Windows 10 Professional.

5. Conclusions

The MTOA and DMOA algorithms have the same inspiration in nature, derived from the symbiosis between the roots of the plants and the Mycorrhiza Network (MN). The main difference between both algorithms is the use of the Lotka–Volterra equations system; in the MTOA algorithm, we previously used continuous LV equations and in the DMOA algorithm we used the discrete LV equations that according to the literature have better handling, flexibility and results in the algorithms in the use of time.

In this research, 36 mathematical functions from CEC-2013 were used. All 36 experiments of the MTOA and DMOA algorithms were performed in 30, 50, and 100 dimensions, and for 30, 50, 100, and 500 iterations. Additionally, 108 hypothesis tests were performed for two samples and there was significant evidence that the DMOA algorithm was better in only eight tests, as shown in Table 9. The research will be furthered as experiments will be carried out in the future with Type-1 and Interval Type-2 Fuzzy Logic Systems (T1FLS–IT2FLS) in the adaptation and optimization of parameters, as well as in the optimization of neural networks and it is possible that each of these algorithms could be better suited to different fields, recalling the No Free Lunch Theorem (NFLT) of Optimization principle that “a general-purpose, universal optimization strategy is impossible” [71,72].

A comparative analysis of the behavior of the Mean and the Standard Deviation for 30 and 50 dimensions was carried out with the methods of MTOA (Mycorrhiza Tree Optimization Algorithm) [14], Self-Defense (A New Bio-inspired Optimization Algorithm Based on the Self-defense Mechanism of Plants in Nature) [68], FA (FireFly Algorithm) [69], GSA (Gravitational Search Algorithm) [69], CS (Cuckoo Search) [69], GA (Genetic Algorithm) [69], DE (Differential Evolution) [70], HS (Harmony Search) [70], and GA (Genetic Algorithm) [69] for five mathematical functions of Sphere, Ackley, Rosenbrock, Rastrigin and Griewank. The results we obtained revealed that the Mean and the Deviation of the DMOA standard were lower than in the other six methods and was only surpassed by the MTOA in the five functions, the Self-Defense in two functions (F1 and F3) and the GSA in two functions (F1 and F2). We can conclude that the DMOA is a competitive algorithm in relation to the other compared methods.

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