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How to Incorporate Preference Information in a Weight-Restricted DEA Model: A Straightforward Solution Applied in the Field of Economics, Based on Simos' Revised Method

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Abstract: Data envelopment analysis (DEA) is one the most successful techniques in the field of Operations Research. DEA is a non-parametric and objective approach for evaluating the relative efficiency of a set of decision-making units. The original DEA proposal contemplated the total freedom of variation of weights. This free variation may lead to situations with non-realistic weights and to the impossibility of incorporating the judgments of decision-makers. This work studies the links between multicriteria decision analysis (MCDA) and DEA by introducing weight restrictions in a DEA model using a methodology developed to obtain criteria weights in a MCDA context: the so-called Revised Simos' Procedure. The presented approach is suitable to be applied in the field of economics and management, being an intuitive and simple enough method for decision-makers who are not familiar with working with DEA models or multicriteria decision analysis. A classic example is presented, where the results found with this approach are compared with the results of other approaches which also use multicriteria decision analysis as a tool to obtain weight restrictions for a DEA model.

Keywords: data envelopment analysis; multicriteria decision analysis; weight restrictions; assurance regions; Simos' procedure

MSC: 62C99; 90B50

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Citation: De Vicente Oliva, M.A.; Romero-Ania, A. How to Incorporate Preference Information in a Weight-Restricted DEA Model: A Straightforward Solution Applied in the Field of Economics, Based on Simos' Revised Method. *Axioms* **2022**, *11*, 367. https://doi.org/10.3390/axioms11080367

Academic Editors: Goran Ćirović and Dragan Pamučar

Received: 1 July 2022
Accepted: 25 July 2022
Published: 27 July 2022

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1. Introduction

The novelty of this study lies in the fact that whereas previous research has extensively applied DEA and MCDA in the fields of economics and management [1–3], until now, it is the first time that a revised Simos' Procedure has been used to obtain weight restrictions in a data envelopment analysis (DEA) model.

The originality of this study is to propose a new application of an intuitive approach for incorporating preference information in a weight-restricted DEA model based on MCDA methodologies.

The main target of this research is to propose an intuitive MCDA-based methodology to incorporate preference information in a weight-restricted DEA model.

Data envelopment analysis is one the most successful techniques in the field of operations research. DEA is a data-oriented approach for evaluating the relative efficiency of a homogenous set of decision-making units (DMUs).

The original DEA proposal contemplated the total freedom of variation of weights, thus putting the evaluated DMU in the best possible light.

Nevertheless, it should be noticed that this full weight flexibility leads to three main problems: weakness in discriminating power, unreasonable weight assignment, and contradiction with the prior available information (or preferences) of the decision-maker (DM).

When the number of DMUs is not large enough, compared to the number of inputs and outputs, a great number of DMUs can become efficient; that is, in these situations, DEA has low discriminating power.

As for the unreasonable weight assignment, a DMU can become efficient due to the extremely low weights of some of the inputs or outputs, meaning that these inputs or outputs are ignored in practice. Finally, the complete flexibility of weights can contradict existing prior information or expert opinions.

A usual way to restrict weight flexibility is through the introduction of weight restrictions. General references about the use of weight restrictions in DEA can be found in [4,5]. The most usual ways of introducing weight restrictions are absolute weight restrictions [6], assurance regions [7], virtual weight restrictions [8], and cone-ratio models [9]. Different approaches to tackle the problem of the complete flexibility of weights are using regression analysis [10] or determining a common set of weights using an ideal decision-making unit [11].

It is important to note that as pointed out by [12], the introduction of weight restrictions in DEA models can constrain the problem in such a way that it becomes infeasible.

A multiple criteria decision making (MCDM) problem can be reformulated in terms of DEA by replacing DMU with alternatives, outputs with criteria to be maximized, and inputs with criteria to be minimized. These similarities between DEA and MCDM techniques have led some researchers to develop approaches that are combinations of DEA and MCDM models.

These approaches, by using the MCDM, allow for incorporating the preferences of the decision-maker through the restrictions to the weights of the criteria.

A large part of this research studies the possible contributions of the analytic hierarchy process (AHP) to establish restrictions on weights [13] presented an analytical process, in which the lower and upper limits of the virtual weight restrictions are determined by the AHP method.

AHP is used to obtain the relative weights among the outputs for each input, where the lower and upper limits are the minimum and maximum weights reached by this output through all the inputs.

As it has been shown by [14], another option is to integrate a DEA model with assurance regions and AHP. They considered the judgments of decision-makers who were familiar with the characteristics of the evaluated DMUs and used AHP to derive the weights of the criteria for each DM.

In that case, the maximum and minimum rates between the different weights proposed by each DM were used as the AR weights. Ref. [15] used the judgment matrix of MACBETH (Measuring Attractiveness by a Categorical Based Evaluation Technique) to generate the weights, taking the minimum and maximum of these values as the lower and upper bounds in the virtual weight restrictions DEA model.

According to [16], who used stability intervals based on PROMETHEE II (Preference Ranking Organization METHod for Enrichment of Evaluations) as absolute weight restrictions in DEA and the unicriterion net flow score of the PROMETHEE II matrix instead of the initial DEA evaluation matrix.

The relationship between TOPSIS (Technique for Order Preference by Similarity to the Ideal Solution) and DEA has been studied by [17]. They constructed two hypothetical DMUs and called them “the ideal DMU” and “the nadir DMU”.

These DMUs were used as reference points to evaluate a set of information technology (IT) investment strategies, taking into account their Euclidean distance from these reference points. It should be taken into consideration that the best relative efficiency of the fuzzy ideal DMU and the worst relative efficiency of the fuzzy nadir DMU were determined and combined to rank the DMUs.

A different approach for combining DEA and TOPSIS is used by [18].

In the first phase, DEA has been applied to shortlist the efficient DMUs having the desired characteristics (from the point of view of stakeholders) and then, the TOPSIS method is employed to rank those efficient DMUs while also identifying the best-performing DMU.

The use of used concepts from multi-attribute utility models with imprecise information in an additive DEA model has been applied by [19]. In a first phase, they convert inputs and outputs into utility functions which are aggregated with a weighted sum. In a second phase, DMU choose the weights that minimize the difference of utility with the best DMU.

On the other hand, ref. [20] proposed incorporating preference information in DEA by linking multiple criteria decision-making with an additive DEA model that takes into account criteria interactivity, by taking advantage of the Choquet multiple criteria preference aggregation model.

The literature review and theoretical background show a research gap that we decided to fill in this research: until now there has not been used a tool that has been exclusively designed to determine the weights for some multicriteria methods to introduce weight restrictions in DEA models.

Our paper also explores possible contributions of MCDA to DEA. The objective is to introduce weight restrictions that allow for the incorporation of the judgments of the decision-maker.

The particularity of our study is that the MCDA tool we use to build a restricted DEA is not a complete MCDA method (such as PROMETHEE) or a part of one (such as AHP or MACBETH), but a tool exclusively designed to determine the weights for some multicriteria methods.

Specifically, we use the revised Simos' procedure [21] to produce type I Assurance Regions for the DEA model.

The revised Simos' procedure is used to give an appropriate value to the weights of criteria in outranking methods such as ELECTRE or PROMETHEE.

From our point of view, the advantage of this approach is that we are using a tool to think about how to incorporate a priori information about weights with a method simple enough for a DM who is not necessarily familiarized with multicriteria decision aiding or making.

This procedure is very well-adapted to contexts where several sets of weights are needed and, so, to proposing bounds to the weights.

Therefore, this study raises the following research question:

Can MCDA techniques be applied to incorporate preference information into DEA models?

A priori, the most logical answer to the above research question is the following hypothesis: The revised Simos' method can be applied to find intervals of variation of input and output weights in a DEA model that reflect a priori information about weights.

The paper is organized as follows. Section 2 presents data and methods, which include the data source, the basic DEA formulations, an introduction to weight restrictions with Assurance Regions in DEA models, the revised Simos' procedure, and a subsection dedicated to explaining our proposed methodology including the proposed revised Simos' procedure clarifying how to use the revised Simos' procedure to obtain Type I Assurance Regions for DEA models (WR-RSP DEA).

Section 3 shows results comparison followed by a numerical example. Finally, Section 4 is devoted to drawing some conclusions.

2. Data and Methods

2.1. Data

The proposed revised Simos' procedure is based on an example, which uses data published by [22]. Data used in our research is available in Table 1, entitled "Normalized data of [22]".

2.2. Method: WR-RSP DEA Model

Figure 1 shows the graphical abstract of the theoretical framework and applied methodology.

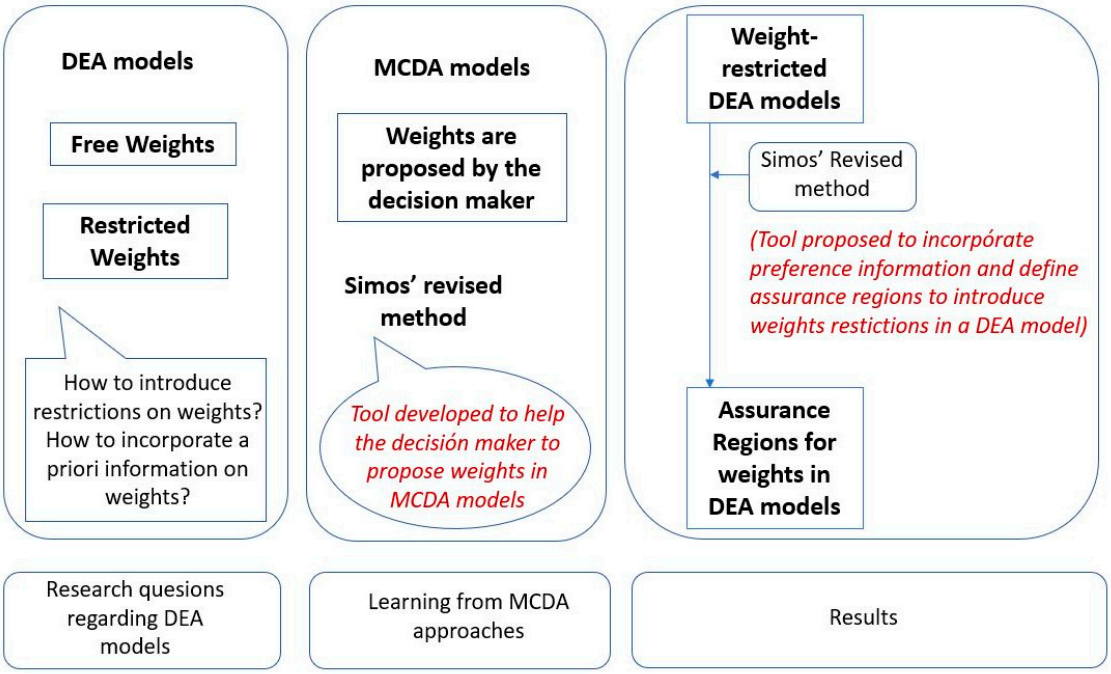


Figure 1. Graphical abstract of the theoretical framework and applied methodology.

2.2.1. Data Envelopment Analysis

The basic DEA model (called CCR) was proposed by [23]. DEA is a non-parametric and non-statistical performance assessment methodology to measure the relative efficiency of a set of homogeneous decision-making units. DEA models compute the efficiency score for a given DMU, compared with the rest of the DMUs of the set.

To do this, DEA considers inputs and outputs. The relative efficiency of a DMU is calculated as the ratio between the weighted sum of outputs to the weighted sum of inputs.

These weights are obtained through linear programming in such a way that the DMU maximizes its efficiency score.

Let E_0 be the efficiency score of the observed DMU_0. Let y_{j0}, $j = 1, \ldots, s$, be the outputs and x_{i0}, $i = 1, \ldots, m$, be the inputs used to compute the efficiency, and n be the total number of DMUs ($r = 1, \ldots, n$). Then, the relative efficiency of DMU_0 is calculated as:

$$E_0 = Max \frac{\sum_{j=1}^{s} u_j y_{j0}}{\sum_{i=1}^{m} v_i x_{i0}} \tag{1}$$

where u_j and v_j are non-negative weights. If we impose the condition that the efficiency must be less than or equal to 1, we have the classical CCR model under the constant return to scale (CRS) assumption:

$$E_0 = Max \sum_{j=1}^{s} u_j y_{j0},$$

$s.t.:$

$$\begin{gathered} \sum_{i=1}^{m} v_j x_{i0} = 1, \\ \sum_{j=1}^{s} u_j y_{jr} - \sum_{i=1}^{m} v_j x_{ir} \leq 0 \quad r = 1, \ldots, n, \\ u_j,\ v_j \geq 0 \quad \forall i, j. \end{gathered} \tag{2}$$

DEA solves n different LP problems for a set of n DMUs.

The CRS model assumes that if an efficient DMU increases its inputs by a constant factor, then its outputs are expected to increase by the same factor.

The above model is an input-oriented CRS (constant returns to scale) model. It is said to be input-oriented because it determines how to improve the inputs of a DMU to make it efficient as long as the outputs remain at the same level.

As it has been proposed by [24], a variable return to scale (VRS) version of the CCR model (referred to as the BCC model) for situations where the CRS assumption does not hold. For more detail about DEA models, see [25].

2.2.2. Weight Restrictions with Assurance Regions

The original DEA proposal presented in Section 2 contemplates the total freedom of variation of weights.

Nevertheless, this full weight flexibility sometimes leads to unreasonable weight assignments or contradiction with the prior available information (or preferences) of the decision-maker.

A DMU can become efficient due to the extremely low weights of some of the inputs or outputs, meaning that these inputs or outputs are ignored in practice. On the other hand, the complete flexibility of weights can contradict existing prior information or expert opinions.

A usual way to restrict weight flexibility is through the introduction of weight restrictions. The most usual ways of introducing weight restrictions are absolute weight restrictions [6], assurance regions [7], virtual weight restrictions [8], and cone-ratio models [9].

Assurance regions (AR) impose restrictions on the ratios between the weights of inputs and/or outputs. More specifically, Type I assurance regions consider lower and upper bounds on the ratio between the weights of either the inputs:

$$L^X_{i'i} \leq \frac{v_i}{v_{i'}} \leq U^X_{i'i} \quad i' = 1, \ldots, m;\ i = 1, \ldots, m \tag{3}$$

or the outputs:

$$L^Y_{j'j} \leq \frac{u_j}{u_{j'}} \leq U^Y_{j'j} \quad j' = 1, \ldots, s;\ j = 1, \ldots, s. \tag{4}$$

The CRS input-oriented DEA model with type I assurance regions is formulated as follows:

$$E_0 = Max \sum_{j=1}^{s} u_j y_{j0},$$

$s.t.$:

$$\begin{gathered} \textstyle\sum_{i=1}^{m} v_j x_{i0} = 1, \\ \textstyle\sum_{j=1}^{s} u_j y_{jr} - \sum_{i=1}^{m} v_j x_{ir} \leq 0 \quad r = 1, \ldots, n, \\ L^X_{i'i} \leq \frac{v_i}{v_{i'}} \leq U^X_{i'i} \quad i' = 1, \ldots, m;\ i = 1, \ldots, m, \\ L^Y_{j'j} \leq \frac{u_j}{u_{j'}} \leq U^Y_{j'j} \quad j' = 1, \ldots, s;\ j = 1, \ldots, s, \\ u_j,\ v_j \geq 0 \quad \forall i, j. \end{gathered} \tag{5}$$

The values of the bounds in Type I AR are dependent on the scaling of the inputs and outputs; that is, these bounds are sensitive to the units of measurement of the related factors [4].

Due to this, in our proposal, the original data are normalized by dividing each alternative data in one criterion by the criterion mean.

2.2.3. The Revised Simos' Procedure

The revised Simos' procedure is a revised version of the method proposed by [26]. This method allows any decision-maker, even those not necessarily familiar with the decision-making processes, to obtain parameters reflecting the relative importance of the criteria involved in a decision process.

The method is well-adapted when we have different sets of weights; for example, when we consider the preferences of different decision-makers or due to a lack of knowledge or uncertainty leading them to prefer to propose several possible values for weights (which may occur even with a single decision-maker).

Both the original and the revised version of Simos' method attribute intrinsic weights to the criteria.

We think that this is an interesting point, as the decision-maker usually considers the importance of the criteria independently of their scales. Nevertheless, it is important to take into account that the aggregation procedure used during the decision process must respect this characteristic.

Note that a weighted sum, for example, does not respect it. Therefore, it is not a DEA model.

For this reason, we use the revised Simos' method to obtain bounds for the weights of a DEA model (in this case, the inputs and outputs of the DEA model will play the role of criteria in the decision process) and normalize the data by dividing by the mean to obtain non-dimensional data.

In the following, we present an outline of the revised Simos' procedure. For complete details of the method, see [21].

Weight restrictions with assurance regions include collecting the information, determining the weights of criteria, determining the non-normalized weights, and determining the normalized weights.

For collecting the information, the revised Simos' procedure respects the way of collecting information proposed initially by Simos.

Collecting information is done through three steps:

1. The decision-maker receives a set of cards (as many as the criteria considered) with the name of the criteria written on the card. In addition, the decision-maker will receive a set of white cards as well;
2. The decision-maker is asked to rank the cards from least to most important. The first rank is Rank 1, the second is Rank 2, and so on. If some criteria are considered by the decision-maker having the same importance, they are considered *ex aequo* and are situated into the same Rank. So, in general, we will have packets of cards (subsets of *ex aequo*) reflecting a complete pre-order of the set of criteria;
3. At this point, the decision-maker has to think about the distance between two successive ranks (either formed by a single criterion or by *ex eaquos*). The weights or the criteria must reflect the differences of importance between successive ranks. We ask the decision-maker to introduce white cards between ranks to reflect this difference of importance: the greater the importance, the greater the number of white cards.

As mentioned in [21]:

4. No white card means that the criteria do not have the same weight and that the difference between the weights can be chosen as the unit for measuring the intervals between weights. Let this unit is denoted by u;
5. One white card means a difference of two times u;
6. Two white cards mean a difference of three times u, and so on.

For determining the weights of criteria with the revised Simos' procedure, once the information is collected, the algorithm of the revised Simos' procedure attributes a numerical value to weights of the criteria. For each criterion g_i, $i = 1, \ldots, n$, two weights are calculated:

a. First, the *non-normalized* weights associated to each *ex aequo* subset are calculated, according to its rank. Let these weights be $k(1)$, $\ldots$, $k(r), \ldots$, $k(n^*)$, where n^* is the total number of ranks. By convention, $k(1) = 1$.
b. Then, the *normalized* weights are calculated, such that $\sum_{i=1}^{n} k_i = 100$, where k_i is the normalized weight of the criterion g_i, $i = 1, \ldots, n$.

For determining the non-normalized weigh $k(r)$ [21], let e'_r be the number of white cards between the ranks r and $r+1$. Set

$$\begin{cases} e_r = e'_r + 1 \quad \forall r = 1, \ldots, n^* - 1 \\ e = \sum_{r=1}^{n^*-1} e_r \\ u = \frac{z-1}{e} \end{cases}, \tag{6}$$

where z is the ratio translating the information about how many times the last criterion is more important than the first one in the ranking.

This information has to be provided by the decision-maker. For u, retain six decimal places.

We define:

$$k(r) = 1 + u(e_0 + \cdots + e_{r-1}) \quad with\ e_0 = 0; \tag{7}$$

for these weights, retain only two decimal places. The *ex aequo* criteria must have the same weight $k(r)$.

For determining the normalized weights k_i [21], let g_i be a criterion of rank r and k'_i be the weight of this criterion in its non-normalized expression $k'_i = k(r)$.

Set

$$\begin{cases} K' = \sum_{i=1}^{n} k'_i \\ k^*_i = \frac{100}{K'} k'_i \end{cases}. \tag{8}$$

Then, k''_i is derived from k'_i by deleting some decimal figures. Three options are considered and are characterized by the value of w:

$w = 0$: takes into account no figures after the decimal point;

$w = 1$: takes into account one figure after the decimal point; and

$w = 2$: takes into account two figures after the decimal point.

The following result is obtained:

$$\begin{cases} K'' = \sum_{i=1}^{n} k''_i \leq 100 \\ \epsilon = 100 - K'' \leq 10^{-w} n \end{cases} \tag{9}$$

The value $v = 10^w \epsilon$ is an integer at most equal to n. Setting $k_i = k''_i + 10^{-w}$ for v suitably selected criteria and k''_i for the other $n - v$ criteria, we have $\sum_{i=1}^{n} k_i = 100$ with normalized weights, where k_i shows the required number of decimal places.

For details about how v is chosen, see [21].

2.2.4. How to Use the Revised Simos' Procedure to Obtain Type I Assurance Regions for DEA Models: WR-RSP DEA

As it has been previously mentioned in the introduction, full weight flexibility leads to three main problems: weakness in discrimination power, unreasonable weight assignment, and contradiction with prior available information (or preferences) of the decision-maker (DM).

A usual way to restrict weight flexibility is through the introduction of weight restrictions.

In this section, we explain how to use the revised Simos' procedure to produce weight restrictions for DEA models. Specifically, we will generate Type I Assurance Regions for a CCR input-oriented DEA model from the results obtained with the revised Simos' procedure.

Wong and Beasley proposed three different approaches for the utilization of virtual weights:

(a) Restrict only the observed DMU;
(b) Restrict all the DMUs; or
(c) Restrict the observed DMU and an "average" DMU.

These approaches can also be considered in the case of Assurance Regions. In our case, we will restrict the study only to the observed DMU.

Type I assurance regions consider lower and upper bounds on the ratio between the weights of either the inputs or the outputs. Here, we present the algorithm to obtain such bounds in the case of inputs (a similar algorithm can be implemented in the case of the outputs).

The algorithm has the following steps:

Step 1. Data are normalized by dividing each alternative data in one input/criterion by the input/criterion mean. This is done to avoid scaling problems.

Step 2. Provide the information required by the revised Simos' procedure:

Step 2.1. Rank the cards representing the criteria from the least important to the most important; if some criteria have the same importance (i.e., the same weight), build a subset of cards holding them together.

Step 2.2. Introduce white cards between two successive cards (or subsets of *ex aequo* cards) to represent the importance of two successive criteria.

Step 2.3. Propose two values for the parameter z (ratio between the weights of the most important criterion and the least important one in the ranking). We will call these two values z_{min} and z_{max}.

Step 3. Obtain the normalized weights with the revised Simos' procedure for z_{min} and for z_{max}. Let these weights be $w_{min,1}, \ldots, w_{min,m}$ and $w_{max,1}, \ldots, w_{max,m}$, respectively.

Step 4. For every pair of criteria (inputs) i_1, i_2 the ratio $\frac{v_{i_1}}{v_{i_2}}$ will have two bounds, L_{i_1,i_2} and U_{i_1,i_2}, such that:

$$L_{i_1,i_2} \leq \frac{v_{i_1}}{v_{i_2}} \leq U_{i_1,i_2}, \tag{10}$$

where the bounds are calculated, by using the normalized weights of the revised Simos' procedure with z_{min} and z_{max}, as:

$$L_{i_1,i_2} = \min_{k \in (z_{min}, z_{max})} \frac{w_{ki_1}}{w_{ki_2}};\ U_{i_1,i_2} = \max_{k \in (z_{min}, z_{max})} \frac{w_{ki_1}}{w_{ki_2}} \tag{11}$$

In multi-criteria decision methods, a sensitivity analysis is always carried out. Therefore, it seems consistent to do so also on the results obtained with the revised Simos' method.

This sensitivity analysis will traduce the possible doubts of hesitations of the decision-maker by choosing not two but several values for z, introducing different combinations of white cards between criteria or even establishing a different number of decimals in the solution by changing the value of w.

Let $k = 1, \ldots, p$ be the situations studied in the sensitivity analysis with variations of z, white cards, and w. Then, the Lower and Upper bounds are calculated as:

$$L_{i_1,i_2} = \min_{k=1,\ldots,p} \frac{w_{ki_1}}{w_{ki_2}};\ U_{i_1,i_2} = \max_{k=1,\ldots,p} \frac{w_{ki_1}}{w_{ki_2}}. \tag{12}$$

The main contribution of the authors is to propose an intuitive method to incorporate a priori information about the weights in a weight-restricted DEA, which can be applied in the field of economics and management.

The decision-maker can incorporate preferential information in a simple way by proposing assurance regions for the weights.

This is based on the revised Simos' procedure, which, starting from basic information and intuitive questions, generates variation intervals for the weights of the inputs and outputs. We will refer to this model as WR-RSP DEA (Weight-Restrictions based on Revised Simos' Procedure DEA model).

3. Results: Comparison and Numerical Example

In this section, we use a data set that was evaluated in the paper of [22] to apply the methodology we have proposed in this paper.

Table 1 lists the criteria (inputs and outputs) and the normalized data used to choose the location of a solid waste management system in Oulu, Finland.

As stated in Section 4, the first step should be to normalize the data by dividing by the mean.

The case has been discussed in [2], as it is an application of the ELECTRE-III method to a municipal solid waste management system problem in the Oulu district of Northern Finland.

The objective was to find the most reasonable solution for waste management. The following waste treatment methods were considered: landfilling, incineration, and composting.

All methods considered were in accordance with Finnish environmental legislation. One of the targets was to recover 50% of municipal waste, which included a target for the level of recycling (30%).

The problem has been studied by [22], who included 22 alternatives and eight criteria. The normalized data of [22] are shown in Table 1, and the criteria weights in [22] are shown in Table 2.

Table 1. Normalized data of [22].

	INPUTS					OUTPUTS		
	Cost	**Global**	**Health**	**Acidif.**	**Surface**	**Feasibility**	**Employees**	**Resource**
1	0.8945	1.0657	1.2419	0.9874	1.3701	0.7120	0.9112	0.5107
2	1.0717	1.0395	1.1726	0.9874	1.3947	0.5696	1.1716	0.8670
3	1.2435	0.9267	1.3663	1.0140	1.2147	0.5696	1.5621	1.4610
4	0.8031	1.0794	0.8382	0.9883	0.9059	1.2816	0.6509	0.5107
5	0.9626	1.0264	0.6628	0.9883	0.8262	0.9968	0.9112	0.8670
6	1.1372	0.9075	1.0197	1.0173	0.7853	0.9256	1.1716	1.4941
7	0.7908	1.0817	0.8116	0.9883	0.8793	1.2816	0.6509	0.5107
8	0.9299	1.0263	0.6403	0.9883	0.8037	0.9968	0.9112	0.8670
9	1.1426	0.8997	1.0217	1.0198	0.7628	0.9256	1.4320	1.5338
10	0.7895	1.0828	0.7607	0.9883	0.8282	1.2816	0.5858	0.5107
11	0.9381	1.0264	0.5955	0.9883	0.7566	0.9968	0.8462	0.8670
12	1.1426	0.8973	1.0176	1.0206	0.7382	0.9256	1.1065	1.5602
13	0.8113	1.0808	1.0197	0.9883	1.1002	1.2816	0.7811	0.5107
14	0.9667	1.0277	0.8198	0.9883	1	0.9968	1.1065	0.8670
15	1.1576	0.9143	1.3215	1.0173	1.1002	0.9256	1.3018	1.4941
16	0.8236	1.0808	1.0197	0.9883	1.1002	1.2816	0.7811	0.5107
17	1.0035	1.0277	0.8198	0.9883	1	0.9968	1.1065	0.8670
18	1.1876	0.9143	1.3215	1.0173	1.1002	0.9256	1.3018	1.4941
19	0.7895	1.0966	1.0095	0.9899	1.1411	1.2816	0.4556	0.5107
20	0.9476	1.0354	0.8647	0.9916	1.0941	0.8544	1.1716	0.8670
21	1.1276	0.8816	1.3276	1.0264	1.0491	0.9968	1.0414	1.6594
22	1.3390	0.8816	1.3276	1.0281	1.0491	0.9968	1.0414	1.6594

Table 2. Criteria weights in [22].

	INPUTS					OUTPUTS		
	Cost	**Global Effects**	**Health Effects**	**Acidificative Releases**	**Surface Water Dispersed Releases**	**Technical Reliability**	**Employees**	**Resource Recovery**
Weight	0.2700	0.0160	0.0960	0.0470	0.0900	0.2600	0.0500	0.1400
Normalized weights	0.5202	0.0308	0.1850	0.0906	0.1734	0.5778	0.1111	0.3111

The alternatives were obtained by combination of the following three factors in different municipalities of Oulu: co-operation level, treatment method, and number of treatment sites.

The criteria are classified into inputs and outputs:

- Inputs: Cost, Global (Global Effects), Health (Health Effects), Acidif. (Acidificative Re-leases), and Surface (Surface Water Dispersed Releases);
- Outputs: Feasibility (Technical Reliability), Employees, and Resource (Resource Recovery).

In the paper of [22], the weights were obtained by interviewing 113 experts (decision-makers). These weights are shown in Table 2. In the original article [22], all information concerning the construction of the criteria, alternatives, and the obtaining of performances and weights can be found. Minimizing criteria will be considered as inputs and maximizing criteria as outputs.

To develop the methodology proposed in this paper, and with the philosophy of considering a joint framework for DEA and multicriteria decision analysis, we used the Diviz software both for the DEA and for obtaining the weight restrictions from the revised Simos' method. Diviz is a software for modelling, processing, and sharing algorithmic workflows in MCDA (Multicriteria Decision Aid) which has recently incorporated other tools such as DEA. Interested readers can find more information in [27,28].

We began by trying to reproduce, as accurately as possible, the original weights of [22] with the revised Simos' method. We did this separately for inputs and outputs. We begin with the inputs.

The first step was to rank the cards representing the criteria from the least important to the most important:

$$i_2 < i_4 < i_5 < i_3 < i_1$$

Next, we introduced white cards between successive criteria. The ranks of the criteria, once we introduced the white cards, were as indicated in Table 3:

Table 3. Ranks of criteria inputs after introducing white cards.

Input/Criterion	Rank
i_2	1
i_4	2
i_5	4
i_3	4
i_1	11

The criteria ratio weight was set at $z = 16$ (the exact ratio was 16.875). Then, the weights proposed by the revised Simos' method were as shown in Table 4:

Table 4. Input weights for $z = 16$.

Input/Criterion	Weight
i_1	52.46
i_2	3.27
i_3	18.04
i_4	8.19
i_5	18.04

We can see that the values are very similar to those we wanted to simulate (i.e., those of [22]).

As mentioned in Section 4, we have to propose two values for the parameter z: z_{min} and z_{max}. In our example, we set $z_{min} = 13$ and $z_{max} = 19$.

The weights proposed by the revised Simos' method were as stated in Table 5 and in Table 6:

Table 5. Input weights for $z = 13$.

Input/Criterion	Weight
i_1	51.19
i_2	3.93
i_3	18.12
i_4	8.67
i_5	18.12

Table 6. Input weights for $z = 19$.

Input/Criterion	Weight
i_1	53.37
i_2	2.81
i_3	17.97
i_4	7.87
i_5	17.98

We used only the bounds for consecutive criteria (inputs). These bounds were:

$$2.21 \leq \frac{v_{i_4}}{v_{i_2}} \leq 2.8, \tag{13}$$

$$2.09 \leq \frac{v_{i_5}}{v_{i_4}} \leq 2.28,$$

$$1 \leq \frac{v_{i_3}}{v_{i_5}} \leq 1,$$

$$2.85 \leq \frac{v_{i_1}}{v_{i_3}} \leq 2.94$$

It is always advisable to do a sensitivity analysis and see how the weights vary. In this case, we studied the changes when $z = 11$ and $z = 22$. The changes observed were minimal, only affecting the decimals.

Consequently, we decided to keep the above bounds with 0 decimal places. Expressing these restrictions with integer numbers also helps and simplifies the decision-maker's thinking process. These were the final input restrictions:

$$2 \leq \frac{v_{i_4}}{v_{i_2}} \leq 3, \tag{14}$$

$$2 \leq \frac{v_{i_5}}{v_{i_4}} \leq 2,$$

$$1 \leq \frac{v_{i_3}}{v_{i_5}} \leq 1,$$

$$3 \leq \frac{v_{i_1}}{v_{i_3}} \leq 3.$$

Next, we generated the bounds for the outputs. As we did with the inputs, we began with the simulation of the original weights proposed by [22]. The ranking of the outputs, from the least important to the most important, was:

$$o_2 < o_3 < o_1.$$

After the decision-maker has established the ranking of the criteria (outputs, in this case), they are asked to introduce white cards between two successive cards (or subsets of *ex aequo* cards) to represent the relative importance of two successive criteria. The ranks assigned to the criteria after this process are shown in Table 7:

Table 7. Ranks of criteria outputs after introducing white cards.

Output/Criterion	Rank
o_1	1
o_2	3
o_3	6

In this case, we set $z = 5$ (the exact ratio was 5.2). The result that we have obtained is indicated in Table 8:

Table 8. Output weights for $z = 5$.

Output/Criterion	Weight
o_1	58.14
o_2	11.63
o_3	30.23

Once we checked the similarity of the result, we set $z_{min} = 3$ and $z_{max} = 7$. As it is shown in Tables 9 and 10, the weights resulting from the revised Simos' method were:

Table 9. Output weights for $z = 3$.

Output/Criterion	Weight
o_1	51.73
o_2	17.25
o_3	31.04

Table 10. Output weights for $z = 7$.

Output/Criterion	Weight
o_1	61.41
o_2	8.78
o_3	29.83

The bounds for consecutive outputs were:

$$1.8 \leq \frac{o_3}{o_2} \leq 3.4, \tag{15}$$

$$1.73 \leq \frac{o_1}{o_3} \leq 1.98;$$

or, considering integer bounds:

$$2 \leq \frac{o_3}{o_2} \leq 3, \tag{16}$$

$$2 \leq \frac{o_1}{o_3} \leq 2.$$

As with the inputs, we also carried out a sensitivity analysis studying the possible changes with variation of z.

We analyzed what happened when $z = 2$ and $z = 8$. In this case, there were significant changes. The minimum and the maximum (integer) ratios were:

$$1 \leq \frac{o_3}{o_2} \leq 4, \tag{17}$$

$$1 \leq \frac{o_1}{o_3} \leq 2$$

Thus, we had the assurance regions for the weights and could introduce these bounds into a CCR input-oriented DEA model.

We will now present a comparison of the results obtained with DEA depending on whether we introduce restrictions to the weights or not and which restrictions we introduce depending on the values obtained after applying the revised Simos' method to calculate the assurance regions.:

DEA without restrictions on the weights

DEA + RSMI: DEA model with assurance regions provided by the revised Simos' method after sensitivity analysis on inputs and outputs. That is to say, $z = 11$ and $z = 22$ in the Simos' method for the inputs obtaining equations [7] for the assurance regions for input values. For the outputs, we have $z = 2$ and $z = 8$ in the Simos' method obtaining equations [9] for the assurance regions of outputs.

DEA + RSMII: DEA model with assurance regions provided by revised Simos' method after sensitivity analysis only on inputs and without sensitivity analysis on outputs. That means $z = 11$ and $z = 22$ in the Simos' method for the inputs obtaining equations [7] for the assurance regions for input values. While for the outputs, we have $z = 3$ and $z = 7$ in the Simos' method obtaining equations [8] for the assurance regions of outputs

The comparison is shown in Table 11:

Table 11. Efficiency scores on DEA models with assurance regions from revised Simos' method.

DMU	DEA without Restrictions	DEA + RSMI	DEA + RSMII
dmu1	0.8438	0.5800	0.5542
dmu2	0.8702	0.6107	0.5584
dmu3	1	0.7783	0.6744
dmu4	1	0.9726	0.9699
dmu5	0.9921	0.9281	0.9114
dmu6	0.9832	0.9633	0.9259
dmu7	1	0.9910	0.9880
dmu8	1	0.9547	0.9370
dmu9	1	1	0.9712
dmu10	1	1	1
dmu11	1	0.9546	0.9380
dmu12	1	0.9884	0.9436
dmu13	1	0.9219	0.9141
dmu14	1	0.9015	0.8772
dmu15	0.9762	0.8748	0.8462
dmu16	1	0.9168	0.9090
dmu17	1	0.8867	0.8630
dmu18	0.9762	0.8631	0.8349
dmu19	1	0.8842	0.8813
dmu20	1	0.8315	0.7975
dmu21	1	0.9512	0.9023
dmu22	1	0.8672	0.8226

We can observe that there is a large difference between the efficiency values obtained with the DEA without restrictions on the weights and the other two models where assurance regions have been set for the values of the weights. The first thing that is striking is that the unconstrained DEA model generates a large number of efficient DMUs. While the other models have only 2 or 1 efficient DMUs. The solutions of the models DEA + RSMI and DEA + RSMII are quite similar.

Finally, Table 12 shows a comparison of the results obtained with the approach presented in this paper and the results obtained by [16], we call this model PIIWCCR (PROMETHEE II weighted CCR), and [15] on the same example:

Table 12. Comparative of efficiency scores on DEA models on the example of [22] and our approach.

DMU	DEA CCR without Restrictions	DEA + RSMI	DEA + RSMII	[15]	[16] (PIIWCCR)
dmu1	0.8438	0.5800	0.5542	0.6728	0.558
dmu2	0.8702	0.6107	0.5584	0.6076	0.358
dmu3	1	0.7783	0.6744	0.6218	0.283
dmu4	1	0.9726	0.9699	0.9834	0.983
dmu5	0.9921	0.9281	0.9114	0.9681	0.727
dmu6	0.9832	0.9633	0.9259	0.9388	0.384
dmu7	1	0.9910	0.9880	1	1
dmu8	1	0.9547	0.9370	0.9966	0.821
dmu9	1	1	0.9712	1	0.462
dmu10	1	1	1	1	1
dmu11	1	0.9546	0.9380	0.9958	0.858
dmu12	1	0.9884	0.9436	0.9423	0.425
dmu13	1	0.9219	0.9141	0.9972	0.993
dmu14	1	0.9015	0.8772	0.9692	0.552
dmu15	0.9762	0.8748	0.8462	0.8815	0.266
dmu16	1	0.9168	0.9090	0.9869	0.976
dmu17	1	0.8867	0.8630	0.9530	0.468
dmu18	0.9762	0.8631	0.8349	0.8700	0.266
dmu19	1	0.8842	0.8813	0.8838	0.991
dmu20	1	0.8315	0.7975	0.9009	0.534
dmu21	1	0.9512	0.9023	0.8942	0.281
dmu22	1	0.8672	0.8226	0.8251	0.281

Efficiencies obtained with models DEA + RSMI and DEA + RSMII and the one proposed on [15] are similar (or at least not very different). Results obtained with our proposal (DEA + RSMI and DEA + RSMII) are better because they have greater discriminatory power (there are less efficient DMUS).

The efficiencies produced by the PIIWCCR model are significantly different from the ones obtained with the other approaches.

4. Conclusions

We have proposed, in this work, the use of the Revised Simos' Procedure to generate assurance regions for the weights of a DEA model.

The restricted DEA model obtained showed a dramatic reduction in the number of efficient DMU's. This improvement in the discriminatory power of the DEA is done through a set of realistic weights, in which the judgments of decision-makers have been incorporated.

The particularity of this work is that the MCDA tool we used to build the restricted DEA model is not a complete MCDA method or a part of one (as other authors have done with PROMETHEE, AHP, or MACBETH), but is a tool exclusively designed to determine the weights for some multicriteria methods. From our point of view, the advantage of this approach is that we are using a tool to think about how to incorporate a priori information about weights with a method which is simple enough for a DM not necessarily familiarized with decision-making or assistance.

The procedure is very well-adapted to contexts where several sets of weights are needed and, so, to propose bounds on the weights.

The method was tested on a classical example presented by [22], and the results were compared with those obtained by other authors on the same example. The use of the Revised Simos' Procedure to obtain weight restrictions for a DEA model demonstrated improved discrimination power, much greater than that of the other proposed approaches, while allowing for the incorporation of the judgments of decision-makers and working with realistic weights.

The improved theoretical foundations presented in this research, have proven to be useful to be applied to the specific case of study of choosing a solid waste management system using multicriteria decision analysis [22]. As in the previous case study, the theoretical result proposed in this paper, based on using the Revised Simos' Procedure to obtain weight restrictions in a Data Envelopment Analysis model, is suitable to be applied in the field of economics and management, being an intuitive and simple enough method for decision-makers who are not familiar with working with DEA models or multicriteria decision analysis.

Therefore, our theoretical contribution opens new avenues of research in the decision support systems in general, and more specifically in the area of economics and management decision support systems in particular.

Author Contributions: Conceptualization, A.R.-A.; data curation, M.A.D.V.O.; formal analysis, M.A.D.V.O.; investigation, M.A.D.V.O. and A.R.-A.; methodology, M.A.D.V.O. and A.R.-A.; project administration, A.R.-A.; software, M.A.D.V.O.; supervision, A.R.-A.; validation; M.A.D.V.O.; visualization, M.A.D.V.O. and A.R.-A.; writing—original draft preparation M.A.D.V.O. and A.R.-A.; writing—review and editing, M.A.D.V.O. and A.R.-A. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data presented in this study are available in the tables included in this paper.

Conflicts of Interest: The authors declare no conflict of interest.

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