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# Soft $R\omega$-Open Sets and the Soft Topology of Soft $\delta_\omega$-Open Sets

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**Abstract:** The author devotes this paper to defining a new class of soft open sets, namely soft $R\omega$-open sets, and investigating their main features. With the help of examples, we show that the class of soft $R\omega$-open sets lies strictly between the classes of soft regular open sets and soft open sets. We show that soft $R\omega$-open subsets of a soft locally countable soft topological space coincide with the soft open sets. Moreover, we show that soft $R\omega$-open subsets of a soft anti-locally countable coincide with the soft regular open sets. Moreover, we show that the class of soft $R\omega$-open sets is closed under finite soft intersection, and as a conclusion, we show that this class forms a soft base for some soft topology. In addition, we define the soft $\delta_\omega$-closure operator as a new operator in soft topological spaces. Moreover, via the soft $\delta_\omega$-closure operator, we introduce soft $\delta_\omega$-open sets as a new class of soft open sets which form a soft topology. Moreover, we study the correspondence between soft $\delta_\omega$-open in soft topological spaces and $\delta_\omega$-open in topological spaces.

**Keywords:** soft regular-open sets; soft $\delta$-open sets; $R\omega$-open sets; $\delta_\omega$-open sets; soft $\omega$-regularity; soft generated soft topological spaces; soft induced topological spaces

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**Citation:** Al Ghour, S. Soft $R\omega$-Open Sets and the Soft Topology of Soft $\delta_\omega$-Open Sets. *Axioms* **2022**, *11*, 177. https://doi.org/10.3390/axioms11040177

Academic Editors: Salvador Hernandez and Sidney A. Morris

Received: 24 March 2022
Accepted: 13 April 2022
Published: 15 April 2022

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## 1. Introduction

Some of the problems that confront us in engineering, medicine, sociology, economics and other fields have their own uncertainties. Therefore, we are unable to deal with these problems by traditional methods. Several mathematical tools for dealing with uncertainties were introduced in [1–3] and others. In 1999, Molodtsov [4] introduced soft set theory as a mathematical tool for dealing with uncertainty.

General topology, as one of the important branches of mathematics, is the basis for other branches of topology such as geometric topology, algebraic topology, and differential topology. Soft topology as a new branch of topology that combines soft set theory and topology is introduced in [5]. Mathematicians then transferred many topological concepts to include soft topology in [6–23] and others, and substantial contributions can still be made.

Topologists have used closure and interior operators to give rise to several different new classes of sets. Some are a generalized form of open sets while a few others are the so-called regular sets. Researchers have discovered applications for these regular sets not only in mathematics but even in a variety of fields outside of mathematics [24–26].

Soft regular open sets and $\delta$-open sets are defined and investigated in [27,28], respectively.

The targets of this work are to scrutinize the behaviors of soft $R\omega$-open sets via soft topological spaces, to introduce the soft topology of soft $\delta_\omega$-open as a new soft topology, and to open the door to redefine and investigate some of the soft topological concepts such as soft compactness, soft correlation, soft class axioms, soft assignments, etc., via soft $R\omega$-open sets.

The author devotes this paper to defining a new class of soft open sets, namely soft $R\omega$-open sets, and investigating their main features. With the help of examples, we show that the class of soft $R\omega$-open sets lies strictly between the classes of soft regular open sets and soft open sets. We show that soft $R\omega$-open subsets of a soft locally countable soft topological space coincide with the soft open sets. Moreover, we show that soft $R\omega$-open subsets of a soft anti-locally countable coincide with the soft regular open sets. Moreover, we show that the class of soft $R\omega$-open sets is closed under finite soft intersection, and as a conclusion, we show that this class forms a soft base for some soft topology. In addition, we define the soft $\delta_\omega$-closure operator as a new operator in soft topological spaces. Furthermore, we use the soft $\delta_\omega$-closure operator to introduce soft $\delta_\omega$-open sets as a new class of sets and we prove that this class of sets forms a soft topology that coincides with the soft topology generated by soft $R\omega$-open sets as a soft base. Moreover, we study the correspondence between soft $\delta_\omega$-open in soft topological spaces and $\delta_\omega$-open in topological spaces.

The arrangement of this article is as follows:

In Section 2, we collect the main definitions and results that will be used in this research.

In Section 3, we define and investigate soft $R\omega$-open sets as a class of soft sets which lies strictly between the classes of soft regular open sets and soft open sets. We introduce several results regarding soft $R\omega$-open sets. In particular, we show that the class of soft $R\omega$-open sets forms a soft base for some soft topology. In addition, we study the correspondence between soft $R\omega$-open sets in soft topological spaces and $R\omega$-open sets in topological spaces.

In Section 4, we define the soft $\delta_\omega$-closure operator and we use it to define soft $\delta_\omega$-open sets. We study relationships between soft $\delta_\omega$-open sets and other types of soft open sets. Moreover, we show that the collection of soft $\delta_\omega$-open sets forms a soft topology. In addition, we study the correspondence between soft $\delta_\omega$-open sets in soft topological spaces and $\delta_\omega$-open sets in topological spaces.

In Section 5, we give some conclusions and possible future work.

## 2. Preliminaries

In this paper, we follow the notions and terminologies as they appear in [29,30]. Throughout this paper, topological space and soft topological space will be denoted by ST and STS, respectively. Let $(X, \xi, A)$ be an STS, $(W, \mu)$ be a TS, $M \in SS(Y, B)$, and $T \subseteq W$. Throughout this paper, $\xi^c$ will denote the collection of all soft closed sets of $(X, \xi, A)$, and $\mu^c$ will denote the collection of all closed sets of $(W, \mu)$, with $Cl_\xi(M)$, $Cl_\mu(T)$, $Int_\xi(M)$, $Int_\mu(T)$, and $Ext_\xi(M)$ denoting the soft closure of $M$ in $(X, \xi, A)$, the closure of $T$ in $(W, \mu)$, the soft interior of $M$ in $(X, \xi, A)$, the interior of $T$ in $(W, \mu)$, and the soft exterior of $M$ in $(X, \xi, A)$, respectively.

The following definitions and results will be used in the sequel:

**Definition 1.** *Let $(Y, \mu)$ be a TS and let $S \subseteq Y$. Then*

*a. Ref. [31] $S$ is called a regular-open set in $(Y, \mu)$ if $Int_\mu(Cl_\mu(S)) = S$. The family of all regular-open sets in $(Y, \mu)$ will be denoted by $RO(Y, \mu)$.*

*b. Ref. [31] $S$ is called a regular-closed set in $(Y, \mu)$ if $Y - S \in RO(Y, \mu)$. The family of all regular-closed sets in $(Y, \mu)$ will be denoted by $RC(Y, \mu)$.*

*c. Ref. [32] $S$ is called an $R\omega$-open set in $(Y, \mu)$ if $Int_\mu(Cl_{\mu_\omega}(S)) = S$. The family of all $R\omega$-open sets in $(Y, \mu)$ will be denoted by $R\omega O(Y, \mu)$.*

*d. Ref. [32] $S$ is called an $R\omega$-closed set in $(Y, \mu)$ if $Y - S \in R\omega O(Y, \mu)$. The family of all $R\omega$-closed sets in $(Y, \mu)$ will be denoted by $R\omega C(Y, \mu)$.*

**Definition 2.** *Let $(Y, \mu)$ be a TS and let $S \subseteq Y$. Then*

*a. Ref. [33] The $\delta$-closure of $S$ in $(Y, \mu)$ is denoted by $Cl^\mu_\delta(S)$ and defined as follows:*
*$y \in Cl^\mu_\delta(S)$ if and only if for each $S \in \mu$ with $y \in S$, we have $Int_\mu(Cl_\mu(S)) \cap S \neq \emptyset$.*

*b. Ref. [34] The $\delta_\omega$-closure of $S$ in $(Y, \mu)$ is denoted by $Cl^\mu_{\delta_\omega}(S)$ and defined as follows:*
*$y \in Cl^\mu_{\delta_\omega}(S)$ if and only if for each $S \in \mu$ with $y \in S$, we have $Int_\mu(Cl_{\mu_\omega}(S)) \cap S \neq \emptyset$.*

*c. Ref. [33] $S$ is called a $\delta$-closed set in $(Y, \mu)$ if $S = Cl^\mu_\delta(S)$.*

*d. Ref. [34] $S$ is called a $\delta_\omega$-closed set in $(Y, \mu)$ if $S = Cl^\mu_{\delta_\omega}(S)$.*

*e. Ref. [33] $S$ is called a $\delta$-open set in $(Y, \mu)$ if $Y - S$ is $\delta$-closed set in $(Y, \mu)$.*

*f. Ref. [34] $S$ is called a $\delta_\omega$-open set in $(Y, \mu)$ if $Y - S$ is $\delta_\omega$-closed set in $(Y, \mu)$.*

For any TS $(Y,\mu)$, denote the collection of all $\delta$-open sets (resp. $\delta_\omega$-open sets) in $(Y,\mu)$ by $\mu_\delta$ (resp. $\mu_{\delta_\omega}$).

**Theorem 1** ([34]). *Let $(Y,\mu)$ be a TS. Then $(Y,\mu_\delta)$ and $(Y,\mu_{\delta_\omega})$ are TSs with $\mu_\delta \subseteq \mu_{\delta_\omega} \subseteq \mu$.*

**Definition 3** ([27]). *Let $(Y,\sigma,B)$ be an STS and let $M \in SS(Y,B)$. Then*

*a. $M$ is called a soft regular-open set in $(Y,\sigma,B)$ if $Int_\sigma(Cl_\sigma(M)) = M$. The family of all soft regular-open sets in $(Y,\sigma,B)$ will be denoted by $RO(Y,\sigma,B)$.*

*b. $M$ is called a soft regular-closed set in $(Y,\sigma,B)$ if $1_B - M \in RO(Y,\sigma,B)$. The family of all soft regular-closed sets in $(Y,\sigma,B)$ will be denoted by $RC(Y,\sigma,B)$.*

**Definition 4** ([28]). *Let $(Y,\sigma,B)$ be an STS and let $K \in SS(Y,B)$.*

*a. The soft $\delta$-closure of $K$ in $(Y,\sigma,B)$ is denoted by $Cl^\sigma_\delta(K)$ and defined as follows:*
*$b_y \widetilde{\in} Cl^\sigma_\delta(K)$ if and only if for each $S \in \sigma$ with $b_y \widetilde{\in} S$, we have $Int_\sigma(Cl_\sigma(S)) \widetilde{\cap} K \neq 0_B$.*

*b. $K$ is called a soft $\delta$-closed set in $(Y,\sigma,B)$ if $K = Cl^\sigma_\delta(K)$.*

*c. $K$ is called a soft $\delta$-open set in $(Y,\sigma,B)$ if $1_B - K$ is a soft $\delta$-closed set in $(Y,\sigma,B)$.*

For any STS $(Y,\sigma,B)$, denote the collection of all soft $\delta$-open sets in $(Y,\sigma,B)$ by $\sigma_\delta$.

**Theorem 2** ([28]). *Let $(Y,\sigma,B)$ be an STS. Then $(Y,\sigma_\delta,B)$ is an STS with $\sigma_\delta \subseteq \sigma$.*

**Definition 5** ([35]). *An STS $(Y,\sigma,B)$ is soft $\omega$-regular if whenever $K \in \sigma$ and $b_y \widetilde{\in} K$, there exists $F \in \sigma$ such that $b_y \widetilde{\in} F \widetilde{\subseteq} Cl_{\sigma_\omega}(F) \widetilde{\subseteq} K$.*

### 3. Soft $R\omega$-Open Sets

In this section, we define soft $R\omega$-open sets as a new class of soft open sets. With the help of examples, we will show that the class of soft $R\omega$-open sets lies strictly between the classes of soft regular open sets and soft open sets. We will show that soft $R\omega$-open subsets of a soft locally countable soft topological space coincide with the soft open sets. Moreover, we will show that soft $R\omega$-open subsets of a soft anti-locally countable coincide with the soft regular open sets. Moreover, we will show that the class of soft $R\omega$-open sets is closed under finite soft intersection, and as a conclusion, we show that this class forms a soft base for some soft topology.

**Definition 6.** *Let $(Y,\sigma,B)$ be an STS and let $M \in SS(Y,B)$. Then*

*a. $M$ is called a soft $R\omega$-open set in $(Y,\sigma,B)$ if $Int_\sigma(Cl_{\sigma_\omega}(M)) = M$. The family of all soft $R\omega$-open sets in $(Y,\sigma,B)$ will be denoted by $R\omega O(Y,\sigma,B)$.*

*b. $M$ is called a soft $R\omega$-closed set in $(Y,\sigma,B)$ if $1_B - M \in R\omega O(Y,\sigma,B)$. The family of all soft $R\omega$-closed sets in $(Y,\sigma,B)$ will be denoted by $R\omega C(Y,\sigma,B)$.*

**Theorem 3.** *Let $(Y,\sigma,B)$ be an STS. Then $RO(Y,\sigma,B) \subseteq R\omega O(Y,\sigma,B) \subseteq \sigma$.*

**Proof.** To see that $RO(Y,\sigma,B) \subseteq R\omega O(Y,\sigma,B)$, let $M \in RO(Y,\sigma,B)$. Then $M = Int_\sigma(Cl_\sigma(M))$. Since $Cl_{\sigma_\omega}(M) \widetilde{\subseteq} Cl_\sigma(M)$, then $Int_\sigma(Cl_{\sigma_\omega}(M)) \widetilde{\subseteq} Int_\sigma(Cl_\sigma(M)) = M$. On the other hand, since $M \widetilde{\subseteq} Cl_{\sigma_\omega}(M)$ and $M \in \sigma$, then $M \widetilde{\subseteq} Int_\sigma(Cl_{\sigma_\omega}(M))$. It follows that $Int_\sigma(Cl_{\sigma_\omega}(M)) = M$. Hence, $M \in R\omega O(Y,\sigma,B)$. The inclusion $R\omega O(Y,\sigma,B) \subseteq \sigma$ is obvious. □

The following two examples will show that each of the inclusions in Theorem 3 cannot be replaced by equality, in general:

**Example 1.** *Consider $(\mathbb{R},\sigma,\mathbb{N})$ where $\sigma = \left\{0_{\mathbb{N}}, 1_{\mathbb{N}}, C_{[0,1]}\right\}$. Then $Int_\sigma\left(Cl_{\sigma_\omega}(C_{[0,1]})\right) = Int_\sigma(1_{\mathbb{N}}) = 1_{\mathbb{N}} \neq C_{[0,1]}$. Thus, $C_{[0,1]} \in \sigma - R\omega O(Y,\sigma,B)$.*

**Example 2.** *Consider* $(\mathbb{R}, \sigma, \mathbb{N})$ *where* $\sigma = \{0_{\mathbb{N}}, 1_{\mathbb{N}}, C_{\mathbb{Z}}\}$. *Then* $Int_{\sigma}(Cl_{\sigma_{\omega}}(C_{\mathbb{Z}})) = Int_{\sigma}(C_{\mathbb{Z}}) = C_{\mathbb{Z}}$ *and* $Int_{\sigma}(Cl_{\sigma}(C_{\mathbb{Z}})) = Int_{\sigma}(1_{\mathbb{N}}) = 1_{\mathbb{N}} \neq C_{\mathbb{Z}}$. *Thus,* $C_{[0,1]} \in R\omega O(Y, \sigma, B) - RO(Y, \sigma, B)$.

**Theorem 4.** *For any STS* $(Y, \sigma, B)$, $\sigma \cap (\sigma_{\omega})^{c} \subseteq R\omega O(Y, \sigma, B)$.

**Proof.** Let $M \in \sigma \cap (\sigma_{\omega})^{c}$. Since $M \in (\sigma_{\omega})^{c}$, then $Cl_{\sigma_{\omega}}(M) = M$, and so $Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) = Int_{\sigma}(M)$. Since $M \in \sigma$, then $Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) = Int_{\sigma}(M) = M$. Therefore, $M \in R\omega O(Y, \sigma, B)$. □

**Corollary 1.** *For any STS* $(Y, \sigma, B)$, $\sigma \cap CSS(Y, B) \subseteq R\omega O(Y, \sigma, B)$.

**Proof.** This follows from Theorem 4 of this paper and Theorem 2 (d) of [30]. □

**Theorem 5.** *For any soft locally countable STS* $(Y, \sigma, B)$, $R\omega O(Y, \sigma, B) = \sigma$.

**Proof.** By Theorem 3, $R\omega O(Y, \sigma, B) \subseteq \sigma$. To see that $\sigma \subseteq R\omega O(Y, \sigma, B)$, let $M \in \sigma$. Then $Int_{\sigma}(M) = M$. Since $(Y, \sigma, B)$ is soft locally countable, then by Corollary 5 of [30], $Cl_{\sigma_{\omega}}(M) = M$. Therefore, $Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) = Int_{\sigma}(M) = M$. Hence, $M \in R\omega O(Y, \sigma, B)$. □

**Theorem 6.** *For any soft anti-locally countable STS* $(Y, \sigma, B)$, $RO(Y, \sigma, B) = R\omega O(Y, \sigma, B)$.

**Proof.** By Theorem 3, $RO(Y, \sigma, B) \subseteq R\omega O(Y, \sigma, B)$. To see that $R\omega O(Y, \sigma, B) \subseteq RO(Y, \sigma, B)$, let $M \in R\omega O(Y, \sigma, B)$. Then $Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) = M$. Since $(Y, \sigma, B)$ is soft anti-locally countable, then by Theorem 14 of [30], $Cl_{\sigma_{\omega}}(M) = Cl_{\sigma}(M)$. Therefore, $Int_{\sigma}(Cl_{\sigma}(M)) = Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) = M$. Hence, $M \in RO(Y, \sigma, B)$. □

**Theorem 7.** *For any STS* $(Y, \sigma, B)$, $R\omega O(Y, \sigma_{\omega}, B) = RO(Y, \sigma_{\omega}, B)$

**Proof.** By Theorem 3, we have $RO(Y, \sigma_{\omega}, B) \subseteq R\omega O(Y, \sigma_{\omega}, B)$. To see that $R\omega O(Y, \sigma_{\omega}, B) \subseteq RO(Y, \sigma_{\omega}, B)$, let $M \in R\omega O(Y, \sigma_{\omega}, B)$. Then $Int_{\sigma_{\omega}}\left(Cl_{(\sigma_{\omega})_{\omega}}(M)\right) = M$. However, by Theorem 5 of [30], $(\sigma_{\omega})_{\omega} = \sigma_{\omega}$. Therefore, $Int_{\sigma_{\omega}}(Cl_{\sigma_{\omega}}(M)) = M$. Hence, $M \in RO(Y, \sigma_{\omega}, B)$. □

**Theorem 8.** *Let* $(Y, \sigma, B)$ *be an STS and let* $M, N \in R\omega O(Y, \sigma, B)$. *Then* $M \tilde{\cap} N \in R\omega O(Y, \sigma, B)$.

**Proof.** Let $M, N \in R\omega O(Y, \sigma, B)$. Then $Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) = M$ and $Int_{\sigma}(Cl_{\sigma_{\omega}}(N)) = N$. Since $M, N \in \sigma$, then $M \tilde{\cap} N \in \sigma$, and so $M \tilde{\cap} N = Int_{\sigma}(M \tilde{\cap} N) \tilde{\subseteq} Int_{\sigma}(Cl_{\sigma_{\omega}}(M \tilde{\cap} N))$. Conversely, since $Cl_{\sigma_{\omega}}(M \tilde{\cap} N) \subseteq Cl_{\sigma_{\omega}}(M) \tilde{\cap} Cl_{\sigma_{\omega}}(N)$, then

$$
\begin{aligned}
& Int_{\sigma}(Cl_{\sigma_{\omega}}(M \tilde{\cap} N)) \tilde{\subseteq} Int_{\sigma}(Cl_{\sigma_{\omega}}(M) \tilde{\cap} Cl_{\sigma_{\omega}}(N)) \\
= \; & Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) \tilde{\cap} Int_{\sigma}(Cl_{\sigma_{\omega}}(N)) \\
= \; & M \tilde{\cap} N.
\end{aligned}
$$

Therefore, $M \tilde{\cap} N = Int_{\sigma}(Cl_{\sigma_{\omega}}(M \tilde{\cap} N))$, and hence $M \tilde{\cap} N \in R\omega O(Y, \sigma, B)$. □

The following example will show that $R\omega O(Y, \sigma, B)$ need not be closed under finite soft unions:

**Example 3.** *Let* $Y = \mathbb{R}$, $\mu$ *be the usual topology on* $\mathbb{R}$, *and* $B$ *be any set of parameters. Let* $M = C_{(0,1)}$ *and* $N = C_{(1,2)}$. *Then* $M, N \in R\omega O(Y, \sigma, B)$, *while* $Int_{\sigma}(Cl_{\sigma_{\omega}}(M \tilde{\cup} N)) = Int_{\sigma}(C_{(0,2)}) = C_{(0,2)} \neq M \tilde{\cup} N$, *and hence* $M \tilde{\cup} N \notin R\omega O(Y, \sigma, B)$.

**Theorem 9.** *Let* $(Y, \sigma, B)$ *be an STS and let* $M \in SS(Y, B)$. *Then* $Int_{\sigma}(Cl_{\sigma_{\omega}}(M)) \in R\omega O(Y, \sigma, B)$.

**Proof.** Let $K = Int_\sigma(Cl_{\sigma_\omega}(M))$. Since $K = Int_\sigma(Cl_{\sigma_\omega}(M)) \widetilde{\subseteq} Cl_{\sigma_\omega}(M)$, then $Cl_{\sigma_\omega}(K) \widetilde{\subseteq} Cl_{\sigma_\omega}(M)$ and thus, $Int_\sigma(Cl_{\sigma_\omega}(K)) \widetilde{\subseteq} Int_\sigma(Cl_{\sigma_\omega}(M)) = K$. Moreover, since $K \in \sigma$, then $K = Int_\sigma(K) \widetilde{\subseteq}$
$Int_\sigma(Cl_{\sigma_\omega}(K))$. Therefore, $K = Int_\sigma(Cl_{\sigma_\omega}(K))$. Hence, $Int_\sigma(Cl_{\sigma_\omega}(M)) \in R\omega O(Y, \sigma, B)$. □

**Theorem 10.** *Let $(Y, \sigma, B)$ be an STS and let $M \in SS(Y, B)$. Then $M \in R\omega C(Y, \sigma, B)$ if and only if $M = Cl_\sigma(Int_{\sigma_\omega}(M))$.*

**Proof.** *Necessity.* Let $M \in R\omega C(Y, \sigma, B)$. Then $1_B - M \in R\omega O(Y, \sigma, B)$, and so $1_B - M = Int_\sigma(Cl_{\sigma_\omega}(1_B - M))$. Thus,

$$\begin{aligned} M &= 1_B - Int_\sigma(Cl_{\sigma_\omega}(1_B - M)) \\ &= 1_B - Ext_\sigma(1_B - Cl_{\sigma_\omega}(1_B - M)) \\ &= 1_B - (1_B - Cl_\sigma(1_B - Cl_{\sigma_\omega}(1_B - M))) \\ &= Cl_{\sigma_\omega}(1_B - Cl_{\sigma_\omega}(1_B - M)) \\ &= Cl_\sigma(Ext_{\sigma_\omega}(1_B - M)) \\ &= Cl_\sigma(Int_{\sigma_\omega}(M)). \end{aligned}$$

*Sufficiency.* Suppose that $M = Cl_\sigma(Int_{\sigma_\omega}(M))$. We are going to show that $1_B - M = Int_\sigma(Cl_{\sigma_\omega}(1_B - M))$.

As $M = Cl_\sigma(Int_{\sigma_\omega}(M))$, then

$$\begin{aligned} 1_B - M &= 1_B - Cl_\sigma(Int_{\sigma_\omega}(M)) \\ &= Ext_\sigma(Int_{\sigma_\omega}(M)) \\ &= Int_\sigma(1_B - Int_{\sigma_\omega}(M)) \\ &= Int_\sigma(1_B - Ext_{\sigma_\omega}(1_B - M)) \\ &= Int_\sigma(Cl_{\sigma_\omega}(1_B - M)). \end{aligned}$$

□

**Theorem 11.** *For any STS $(Y, \sigma, B)$, $\sigma^c \cap \sigma_\omega \subseteq R\omega C(Y, \sigma, B)$.*

**Proof.** Let $M \in \sigma^c \cap \sigma_\omega$. Since $M \in \sigma_\omega$, then $Int_{\sigma_\omega}(M) = M$, and so $Cl_\sigma(Int_{\sigma_\omega}(M)) = Cl_\sigma(M)$. Since $M \in \sigma^c$, then $Cl_\sigma(M) = M$. Hence, $Cl_\sigma(Int_{\sigma_\omega}(M)) = M$. Therefore, by Theorem 10, $M \in R\omega C(Y, \sigma, B)$. □

**Definition 7.** *A STS $(Y, \sigma, B)$ is called saturated if $T(b) \neq \emptyset$ for all $T \in \sigma - \{0_B\}$ and $b \in B$.*

**Theorem 12.** *Let $(Y, \sigma, B)$ be a saturated STS. Let $M \in \sigma$ and $K \in \sigma^c$. Then for each $b \in B$ we have*

*(a) $Cl_{\sigma_b}(M(b)) = (Cl_\sigma(M))(b)$.*
*(b) $Int_{\sigma_b}(K(b)) = (Int_\sigma(K))(b)$.*
*(c) $(Int_\sigma(Cl_\sigma(M)))(b) = Int_{\sigma_b}(Cl_{\sigma_b}(M(b)))$.*

**Proof.** (a) By Proposition 7 of [5], $Cl_{\sigma_b}(M(b)) \subseteq (Cl_\sigma(M))(b)$. To show that $(Cl_\sigma(M))(b) \subseteq Cl_{\sigma_b}(M(b))$, let $y \in (Cl_\sigma(G))(b)$ and let $V \in \sigma_b$ such that $y \in V$. Choose $S \in \sigma$ such that $S(b) = V$. Then we have $b_y \widetilde{\in} Cl_\sigma(M) \widetilde{\cap} S$, and hence $M \widetilde{\cap} S \neq 0_B$. Since $(Y, \sigma, B)$ is saturated, then $(M \widetilde{\cap} S)(b) = M(b) \cap S(b) = M(b) \cap V \neq \emptyset$. Therefore, $y \in Cl_{\sigma_b}(M(b))$.

(b) Since $1_B - K \in \sigma$, then by (a), $Cl_{\sigma_b}((1_B - K)(b)) = (Cl_\sigma(1_B - K))(b)$. And so,

$$Y - Cl_{\sigma_b}((1_B - K)(b)) = Y - (Cl_\sigma(1_B - K))(b)).$$

However, $Y - Cl_{\sigma_b}((1_B - K)(b)) = Y - Cl_{\sigma_b}(Y - K(b)) = Int_{\sigma_b}(K(b))$, and $Y - (Cl_\sigma(1_B - K))(b)) = (1_B - Cl_\sigma(1_B - K))(b) = (Int_\sigma(K))(b)$. Hence, $Int_{\sigma_b}(K(b)) = (Int_\sigma(K))(b)$.

(c) Since $Cl_\sigma(M) \in \sigma^c$, then by (b), $(Int_\sigma(Cl_\sigma(M)))(b) = Int_{\sigma_b}((Cl_\sigma(M))(b))$. Since $M \in \sigma$, then by (a), $(Cl_\sigma(M))(b) = Cl_{\sigma_b}(M(b))$. Thus,

$$\begin{aligned}(Int_\sigma(Cl_\sigma(M)))(b) &= Int_{\sigma_b}((Cl_\sigma(M))(b)) \\ &= Int_{\sigma_b}(Cl_{\sigma_b}(M(b))).\end{aligned}$$

□

**Theorem 13.** *Let $(Y, \sigma, B)$ be a saturated STS and let $M \in \sigma$. Then $M \in RO(Y, \sigma, B)$ if and only if $M(b) \in RO(Y, \sigma_b)$ for all $b \in B$.*

**Proof.** *Necessity.* Let $M \in RO(Y, \sigma, B)$ and let $b \in B$. Since $M \in RO(Y, \sigma, B)$, then $M = Int_\sigma(Cl_\sigma(M))$, and so $M(b) = (Int_\sigma(Cl_\sigma(M)))(b)$. However, by Theorem 12(c), $(Int_\sigma(Cl_\sigma(M)))$ $(b) = Int_{\sigma_b}(Cl_{\sigma_b}(M(b)))$. Therefore, $Int_{\sigma_b}(Cl_{\sigma_b}(M(b))) = M(b)$, and hence $M(b) \in RO(Y, \sigma_b)$.

*Sufficiency.* Suppose that $M(b) \in RO(Y, \sigma_b)$ for all $b \in B$. Then for every $b \in B$, $M(b) = Int_{\sigma_b}(Cl_{\sigma_b}(M(b)))$. However, by Theorem 12(c), $(Int_\sigma(Cl_\sigma(M)))(b) = Int_{\sigma_b}(Cl_{\sigma_b}(M(b)))$ for all $b \in B$. Therefore, $(Int_\sigma(Cl_\sigma(M)))(b) = M(b)$ for all $b \in B$, and hence $M = Int_\sigma(Cl_\sigma(M))$. Thus, $M \in RO(Y, \sigma, B)$. □

**Corollary 2.** *Let $(Y, \sigma, B)$ be saturated and soft anti-locally countable STS. Let $M \in \sigma$. Then $M \in R\omega O(Y, \sigma, B)$ if and only if $M(b) \in R\omega O(Y, \sigma_b)$ for all $b \in B$.*

**Proof.** This follows from Theorems 6 and 13. □

**Corollary 3.** *Let $(Y, \mu)$ be a TS and $B$ be any set of parameters. Let $Z \in \mathcal{P}(Y) - \{\emptyset\}$. Then $Z \in RO(Y, \mu)$ if and only if $C_Z \in RO(Y, C(\mu), B)$*

**Proof.** It is clear that $(Y, C(\mu), B)$ is saturated. So, the result follows from Theorem 13. □

**Corollary 4.** *Let $(Y, \mu)$ be an anti-locally countable TS and $B$ be any set of parameters. Let $Z \in \mathcal{P}(Y) - \{\emptyset\}$. Then $Z \in R\omega O(Y, \mu)$ if and only if $C_Z \in R\omega O(Y, C(\mu), B)$.*

**Proof.** It is clear that $(Y, C(\mu), B)$ is saturated and soft anti-locally countable. So, the result follows from Corollary 2. □

**Theorem 14.** *Let $\{(Y, \mu_b) : b \in B\}$ be a collection of TSs. Then $M \in RO(Y, \oplus_{b\in B}\mu_b, B)$ if and only if $M(b) \in RO(Y, \mu_b)$ for all $b \in B$.*

**Proof.** *Necessity*. Let $M \in RO(Y, \oplus_{b\in B}\mu_b, B)$ and let $b \in B$. Since $M \in RO(Y, \oplus_{b\in B}\mu_b, B)$, then $M = Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}\mu_b}(M))$ and so $M(b) = \left(Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}\mu_b}(M))\right)(b)$. However, by Lemma 4.9 of [7], $\left(Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}\mu_b}(M))\right)(b) = Int_{\mu_b}(Cl_{\mu_b}(M(b))$. Therefore, $M(b) \in RO(Y, \mu_b)$.

*Sufficiency*. Let $M(b) \in RO(Y, \mu_b)$ for all $b \in B$. Then for every $b \in B$, $M(b) = \left(Int_{\mu_b}(Cl_{\mu_b}(M(b)))\right)$. However, by Lemma 4.9 of [7], $Int_{\mu_b}(Cl_{\mu_b}(M(b))) = \left(Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}\mu_b}\right.$ $\left.(M))\right)(b)$ for all $b \in B$. Hence, $M \in RO(Y, \oplus_{b\in B}\mu_b, B)$. □

**Corollary 5.** *Let $(Y, \mu)$ be a TS and $B$ be any set of parameters. Let $M \in SS(Y, B)$. Then $M \in RO(Y, \tau(\mu), B)$ if and only if $M(b) \in RO(Y, \mu)$ for every $b \in B$.*

**Proof.** For each $b \in B$, put $\mu_b = \mu$. Then $\tau(\mu) = \oplus_{b\in B}\mu_b$ and the result follows from Theorem 14. □

**Theorem 15.** *Let $\{(Y, \mu_b) : b \in B\}$ be a collection of TSs. Then $M \in R\omega O(Y, \oplus_{b\in B}\mu_b, B)$ if and only if $M(b) \in R\omega O(Y, \mu_b)$ for all $b \in B$.*

**Proof.** *Necessity*. Let $M \in R\omega O(Y, \oplus_{b\in B}\mu_b, B)$ and let $b \in B$. Since $M \in R\omega O(Y, \oplus_{b\in B}\mu_b, B)$, then $M = Int_{\oplus_{b\in B}\mu_b}(Cl_{(\oplus_{b\in B}\mu_b)_\omega}(M))$. By Theorem 8 of [30], $(\oplus_{b\in B}\mu_b)_\omega = \oplus_{b\in B}(\mu_b)_\omega$ and so $M = Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}(\mu_b)_\omega}(M))$. Hence, $M(b) = \left(Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}(\mu_b)_\omega}(M))\right)(b)$. However, by Lemma 4.7 of [7], $\left(Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}(\mu_b)_\omega}(M))\right)(b) = Int_{\mu_b}(Cl_{(\mu_b)_\omega}(M(b))$. Therefore, $M(b) \in R\omega O(Y, \mu_b)$.

*Sufficiency*. Let $M(b) \in R\omega O(Y, \mu_b)$ for all $b \in B$. Then for every $b \in B$, $M(b) = \left(Int_{\mu_b}(Cl_{(\mu_b)_\omega}(M(b)))\right)$. However, by Lemma 4.7 of [7], $Int_{\mu_b}(Cl_{(\mu_b)_\omega}(M(b)) = \left(Int_{\oplus_{b\in B}\mu_b}(Cl_{\oplus_{b\in B}(\mu_b)_\omega}(M))\right)(b) = \left(Int_{\oplus_{b\in B}\mu_b}(Cl_{(\oplus_{b\in B}\mu_b)_\omega}(M))\right)(b)$ for all $b \in B$. Hence, $M \in R\omega O(Y, \oplus_{b\in B}\mu_b, B)$. □

**Corollary 6.** *Let $(Y, \mu)$ be a TS and $B$ be any set of parameters. Let $M \in SS(Y, B)$. Then $M \in R\omega O(Y, \tau(\mu), B)$ if and only if $M(b) \in R\omega O(Y, \mu)$ for every $b \in B$.*

**Proof.** For each $b \in B$, put $\mu_b = \mu$. Then $\tau(\mu) = \oplus_{b\in B}\mu_b$ and the result follows from Theorem 15. □

## 4. The Soft Topology of Soft $\delta_\omega$-Open Sets

In this section, we define the soft $\delta_\omega$-closure operator and use it to define soft $\delta_\omega$-open sets as a new class of soft open sets which form a soft topology. Moreover, we will study the correspondence between soft $\delta_\omega$-open in soft topological spaces and $\delta_\omega$-open in topological spaces.

**Definition 8.** *Let $(Y, \sigma, B)$ be an STS and let $K \in SS(Y, B)$. The soft $\delta_\omega$-closure of $K$ in $(Y, \sigma, B)$ is denoted by $Cl^\sigma_{\delta_\omega}(K)$ and defined as follows:*

*$b_y \widetilde{\in} Cl^\sigma_{\delta_\omega}(K)$ if and only if for each $S \in \sigma$ with $b_y \widetilde{\in} S$, we have $Int_\sigma(Cl_{\sigma_\omega}(S)) \widetilde{\cap} K \neq 0_B$.*

**Remark 1.** *Let $(Y, \sigma, B)$ be an STS and let $K \in SS(Y, B)$. Then $b_y \widetilde{\in} Cl^\sigma_{\delta_\omega}(K)$ if and only if for each $M \in R\omega O(Y, \sigma, B)$ with $b_y \widetilde{\in} M$, we have $M \widetilde{\cap} K \neq 0_B$.*

**Definition 9.** *Let $(Y, \sigma, B)$ be an STS and let $K \in SS(Y, B)$. Then $K$ is called*

*a. a soft $\delta_\omega$-closed set in $(Y, \sigma, B)$ if $K = Cl^\sigma_{\delta_\omega}(K)$.*
*b. a soft $\delta_\omega$-open set in $(Y, \sigma, B)$ if $1_B - K$ is a soft $\delta_\omega$-closed set in $(Y, \sigma, B)$.*

The family of all soft $\delta_\omega$-open sets in $(Y, \sigma, B)$ will be denoted by $\sigma_{\delta_\omega}$.

**Theorem 16.** *Let $(Y, \sigma, B)$ be an STS and let $M \in SS(Y, B)$. Then*

*a. $Cl_\sigma(M) \widetilde{\subseteq} Cl^\sigma_{\delta_\omega}(M) \widetilde{\subseteq} Cl^\sigma_\delta(M)$.*
*b. If $M$ is a soft $\delta$-closed set in $(Y, \sigma, B)$, then $M$ is a soft $\delta_\omega$-closed set in $(Y, \sigma, B)$.*
*c. If $M$ is a soft $\delta_\omega$-closed set in $(Y, \sigma, B)$, then $M$ is a soft closed set in $(Y, \sigma, B)$.*

**Proof.** Point (a) follows from the definitions and Theorem 3.

Points (b) and (c) follow from the definitions and part (a). □

**Theorem 17.** *Let $(Y, \sigma, B)$ be an STS and let $M, N \in SS(Y, B)$. Then*

*a. If $M \widetilde{\subseteq} N$, then $Cl^\sigma_{\delta_\omega}(M) \widetilde{\subseteq} Cl^\sigma_{\delta_\omega}(N)$.*
*b. $Cl^\sigma_{\delta_\omega}(M \widetilde{\cup} N) = Cl^\sigma_{\delta_\omega}(M) \widetilde{\cup} Cl^\sigma_{\delta_\omega}(N)$.*
*c. $Cl^\sigma_{\delta_\omega}(M) \in \sigma^c$.*
*d. If $M \in \sigma_\omega$, $Cl^\sigma_{\delta_\omega}(M) = Cl_\sigma(M)$.*
*e. If $M \in \sigma$, $Cl^\sigma_\delta(M) = Cl^\sigma_{\delta_\omega}(M) = Cl_\sigma(M)$.*

**Proof.** (a) Let $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(M)$ and let $S \in \sigma$ such that $b_y \tilde{\in} S$. Then $Int_\sigma(Cl_{\sigma_\omega}(S)) \tilde{\cap} M \neq 0_B$. Since $M \tilde{\subseteq} N$, then $Int_\sigma(Cl_{\sigma_\omega}(S)) \tilde{\cap} N \neq 0_B$. Thus, $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(N)$.

(b) By (a), $Cl^{\sigma}_{\delta_\omega}(M) \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N)$ and $Cl^{\sigma}_{\delta_\omega}(N) \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N)$. Thus, $Cl^{\sigma}_{\delta_\omega}(M) \tilde{\cup} Cl^{\sigma}_{\delta_\omega}(N) \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N)$. To show that $Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N) \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(M) \tilde{\cup} Cl^{\sigma}_{\delta_\omega}(N)$, let $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N) - Cl^{\sigma}_{\delta_\omega}(M)$. We are going to show that $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(N)$. Let $K \in R\omega O(Y, \sigma, B)$ such that $b_y \tilde{\in} K$. Since $b_y \tilde{\in} 1_B - Cl^{\sigma}_{\delta_\omega}(M)$, then there exists $L \in R\omega O(Y, \sigma, B)$ such that $b_y \tilde{\in} L$ and $L \tilde{\cap} M = 0_B$. By Theorem 8, $K \tilde{\cap} L \in R\omega O(Y, \sigma, B)$. Since $b_y \tilde{\in} K \tilde{\cap} L$ and $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N)$, then $(K \tilde{\cap} L) \tilde{\cap} (M \tilde{\cup} N) \neq 0_B$. However,

$$\begin{aligned} (K \tilde{\cap} L) \tilde{\cap} (M \tilde{\cup} N) &= (K \tilde{\cap} L \tilde{\cap} M) \tilde{\cup} (K \tilde{\cap} L \tilde{\cap} N) \\ &= 0_B \tilde{\cup} (K \tilde{\cap} L \tilde{\cap} N) \\ &\tilde{\subseteq} K \tilde{\cap} N \end{aligned}$$

Therefore, $K \tilde{\cap} N \neq 0_B$. Hence, $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(N)$.

(c) We will show that $1_B - Cl^{\sigma}_{\delta_\omega}(M) \in \sigma$. Let $b_y \in 1_B - Cl^{\sigma}_{\delta_\omega}(M)$. Then we find $S \in R\omega O(Y, \sigma, B)$ such that $b_y \tilde{\in} S$ but $S \tilde{\cap} M = 0_B$. Thus, $S \tilde{\cap} Cl^{\sigma}_{\delta_\omega}(M) = 0_B$. Hence, $1_B - Cl^{\sigma}_{\delta_\omega}(M) \in \sigma$.

(d) Suppose that $M \in \sigma_\omega$. By Theorem 16 (a), $Cl_\sigma(M) \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(M)$. To see that $Cl^{\sigma}_{\delta_\omega}(M) \tilde{\subseteq} Cl_\sigma(M)$, suppose to the contrary that there exists $b_y \tilde{\in} \left(Cl^{\sigma}_{\delta_\omega}(M)\right) \tilde{\cap} (1_B - Cl_\sigma(M))$. Since we have $b_y \tilde{\in} (1_B - Cl_\sigma(M)) \in \sigma$ and $b_y \tilde{\in} Cl^{\sigma}_{\delta_\omega}(M)$, then $Int_\sigma(Cl_{\sigma_\omega}(1_B - Cl_\sigma(M))) \tilde{\cap} M \neq 0_B$, and so $Cl_{\sigma_\omega}(1_B - Cl_\sigma(M)) \tilde{\cap} M \neq 0_B$. Choose $b_y \tilde{\in} Cl_{\sigma_\omega}(1_B - Cl_\sigma(M)) \tilde{\cap} M$. Since $M \in \sigma_\omega$, then $(1_B - Cl_\sigma(M)) \tilde{\cap} M \neq 0_B$ which is a contradiction.

(e) Suppose that $M \in \sigma$. By Theorem 16 (a), it is sufficient to show that $Cl^{\sigma}_{\delta}(M) \tilde{\subseteq} Cl_\sigma(M)$. Suppose to the contrary that there exists $b_y \tilde{\in} (Cl^{\sigma}_{\delta}(M)) \tilde{\cap} (1_B - Cl_\sigma(M))$. Since we have $b_y \tilde{\in} (1_B - Cl_\sigma(M)) \in \sigma$ and $b_y \tilde{\in} Cl^{\sigma}_{\delta}(M)$, then $Int_\sigma(Cl_\sigma(1_B - Cl_\sigma(M))) \tilde{\cap} M \neq 0_B$, and so $Cl_\sigma(1_B - Cl_\sigma(M)) \tilde{\cap} M \neq 0_B$. Choose $b_y \tilde{\in} Cl_\sigma(1_B - Cl_\sigma(M)) \tilde{\cap} M$. Since $M \in \sigma$, then $(1_B - Cl_\sigma(M)) \tilde{\cap} M \neq 0_B$ which is a contradiction. □

**Theorem 18.** *Let $(Y, \sigma, B)$ be an STS and let $\mathcal{A}$ be the family of all soft $\delta_\omega$-closed sets in $(Y, \sigma, B)$. Then*

*a. $0_B, 1_B \in \mathcal{A}$.*
*b. If $M, N \in \mathcal{A}$, then $M \tilde{\cup} N \in \mathcal{A}$.*
*c. If $\{M_\alpha : \alpha \in \Gamma\} \subseteq \mathcal{A}$, then $\underset{\alpha \in \Gamma}{\tilde{\cap}} M_\alpha \in \mathcal{A}$.*

**Proof.** a. Obvious.

b. Let $M, N \in \mathcal{A}$. Then $M = Cl^{\sigma}_{\delta_\omega}(M)$ and $N = Cl^{\sigma}_{\delta_\omega}(N)$. Thus, by Theorem 17 (b), $M \tilde{\cup} N = Cl^{\sigma}_{\delta_\omega}(M) \tilde{\cup} Cl^{\sigma}_{\delta_\omega}(N) = Cl^{\sigma}_{\delta_\omega}(M \tilde{\cup} N)$. Therefore, $M \tilde{\cup} N \in \mathcal{A}$.

c. Let $\{M_\alpha : \alpha \in \Gamma\} \subseteq \mathcal{A}$. Then for each $\alpha \in \Gamma$, $M_\alpha = Cl^{\sigma}_{\delta_\omega}(M_\alpha)$. It is clear that $\underset{\alpha \in \Gamma}{\tilde{\cap}} M_\alpha \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(\underset{\alpha \in \Gamma}{\tilde{\cap}} M_\alpha)$. On the other hand, by Theorem 17 (a), we have $Cl^{\sigma}_{\delta_\omega}(\underset{\alpha \in \Gamma}{\tilde{\cap}} M_\alpha) \tilde{\subseteq} Cl^{\sigma}_{\delta_\omega}(M_\beta) = M_\beta$ for all $\beta \in \Gamma$. Hence, $Cl^{\sigma}_{\delta_\omega}(\underset{\alpha \in \Gamma}{\tilde{\cap}} M_\alpha) \tilde{\subseteq} \underset{\alpha \in \Gamma}{\tilde{\cap}} M_\alpha$. □

**Theorem 19.** *For any STS $(Y, \sigma, B)$, $(Y, \sigma_{\delta_\omega}, B)$ is a STS.*

**Proof.** This follows directly from Theorem 18. □

**Theorem 20.** *Let $(Y, \sigma, B)$ be an STS and let $K \in SS(Y, B)$. Then the following are equivalent:*

*a. $K \in \sigma_{\delta_\omega}$.*
*b. For any $b_y \tilde{\in} K$, there exists $S \in \sigma$ such that $b_y \tilde{\in} Int_\sigma(Cl_{\sigma_\omega}(S)) \tilde{\subseteq} K$.*
*c. For any $b_y \tilde{\in} K$, there exists $M \in R\omega O(Y, \sigma, B)$ such that $b_y \tilde{\in} M \tilde{\subseteq} K$.*

**Proof.** (a) $\longrightarrow$ (b): Let $b_y \widetilde{\in} K$. Since by (a) $K \in \sigma_{\delta_\omega}$, then $Cl^{\sigma}_{\delta_\omega}(1_B - K) = 1_B - K$, and so $b_y \widetilde{\in} \left(1_B - Cl^{\sigma}_{\delta_\omega}(1_B - K)\right)$. Thus, there exists $S \in \sigma$ such that $b_y \widetilde{\in} S$ and $Int_\sigma(Cl_{\sigma_\omega}(S)) \widetilde{\cap} (1_B - K) = 0_B$. Hence, $b_y \widetilde{\in} Int_\sigma(Cl_{\sigma_\omega}(S)) \widetilde{\subseteq} K$.

(b) $\longrightarrow$ (c): Let $b_y \widetilde{\in} K$. Then by (b), there exists $S \in \sigma$ such that $b_y \widetilde{\in} Int_\sigma(Cl_{\sigma_\omega}(S)) \widetilde{\subseteq} K$. Put $M = Int_\sigma(Cl_{\sigma_\omega}(S))$. Then by Theorem 9, $M \in R\omega O(Y, \sigma, B)$, which ends the proof.

(c) $\longrightarrow$ (a) Suppose to the contrary that $K \notin \sigma_{\delta_\omega}$. Then $Cl^{\sigma}_{\delta_\omega}(1_B - K) \neq 1_B - K$, and so there exists $b_y \widetilde{\in} Cl^{\sigma}_{\delta_\omega}(1_B - K) - (1_B - K)$. Since $b_y \widetilde{\in} K$, then by (c), there exists $M \in R\omega O(Y, \sigma, B)$ such that $b_y \widetilde{\in} M \widetilde{\subseteq} K$, and thus $M \widetilde{\cap} (1_B - K) = 0_B$. Hence, $b_y \widetilde{\in} 1_B - Cl^{\sigma}_{\delta_\omega}(1_B - K)$ which is a contradiction. □

**Corollary 7.** *For any STS $(Y, \sigma, B)$, $R\omega O(Y, \sigma, B)$ is a soft base for $(Y, \sigma_{\delta_\omega}, B)$.*

**Theorem 21.** *For any STS $(Y, \sigma, B)$, $\sigma_\delta \subseteq \sigma_{\delta_\omega} \subseteq \sigma$.*

**Proof.** Since $RO(Y, \sigma, B)$ and $R\omega O(Y, \sigma, B)$ are soft bases for $(Y, \sigma_\delta, B)$ and $(Y, \sigma_{\delta_\omega}, B)$, respectively, and $RO(Y, \sigma, B) \subseteq R\omega O(Y, \sigma, B)$, then $\sigma_\delta \subseteq \sigma_{\delta_\omega}$. Moreover, by Theorem 3 and Corollary 7, we have $\sigma_{\delta_\omega} \subseteq \sigma$. □

**Theorem 22.** *For any soft locally countable STS $(Y, \sigma, B)$, $\sigma_{\delta_\omega} = \sigma$.*

**Proof.** This follows from Theorem 5 and Corollary 7. □

**Theorem 23.** *For any soft anti-locally countable STS $(Y, \sigma, B)$, $\sigma_\delta = \sigma_{\delta_\omega}$.*

**Proof.** This follows from Theorem 6, Corollary 7, and the fact that $RO(Y, \sigma, B)$ is a soft base for $(Y, \sigma_\delta, B)$. □

**Theorem 24.** *If $(Y, \sigma, B)$ is soft regular, then $\sigma_\delta = \sigma_{\delta_\omega} = \sigma$.*

**Proof.** According to Theorem 21, it is sufficient to show that $\sigma \subseteq \sigma_\delta$. Let $M \in \sigma$ and let $b_y \widetilde{\in} M$. Since $(Y, \sigma, B)$ is soft regular, then there exists $N \in \sigma$ such that $b_y \widetilde{\in} N \widetilde{\subseteq} Cl_\sigma(N) \widetilde{\subseteq} M$ and so $b_y \widetilde{\in} Int_\sigma(Cl_\sigma(N)) \widetilde{\subseteq} M$. However, $Int_\sigma(Cl_\sigma(N)) \in RO(Y, \sigma, B)$. This implies that $M \in \sigma_\delta$. □

**Theorem 25.** *If $(Y, \sigma, B)$ is soft $\omega$-regular, then $\sigma_{\delta_\omega} = \sigma$.*

**Proof.** According to Theorem 21, it is sufficient to show that $\sigma \subseteq \sigma_{\delta_\omega}$. Let $M \in \sigma$ and let $b_y \widetilde{\in} M$. Since $(Y, \sigma, B)$ is soft $\omega$-regular, then there exists $N \in \sigma$ such that $b_y \widetilde{\in} N \widetilde{\subseteq} Cl_{\sigma_\omega}(N) \widetilde{\subseteq} M$, and so $b_y \widetilde{\in} Int_\sigma(Cl_{\sigma_\omega}(N)) \widetilde{\subseteq} M$. This implies that $M \in \sigma_{\sigma_\omega}$. □

The assumption that $(Y, \sigma, B)$ is soft anti-locally countable in Theorem 23 is not superfluous, as the following example shows:

**Example 4.** *Let $Y$ be any non-empty set, $B$ be any set of parameters, and $b_y \in SP(Y, B)$. Let $\sigma = \{0_B\} \cup \{M \in SS(Y, B) : b_y \widetilde{\in} M\}$. Then $(Y, \sigma, B)$ is soft locally countable. So, by Theorem 22, $\sigma_{\delta_\omega} = \sigma$. Since $\sigma^c = \{1_B\} \cup \{K \in SS(Y, B) : b_y \widetilde{\in} 1_B - K\}$, then for every $M \in \sigma - \{0_B\}$, $Cl_\sigma(M) = 1_B$. This shows that $\sigma_\delta = \{0_B, 1_B\} \neq \sigma_{\delta_\omega}$.*

The assumption that $(Y, \sigma, B)$ is soft $\omega$-regular in Theorem 25 is not superfluous, as the following example shows:

**Example 5.** *Let $Y = \mathbb{R}$ and $B = \mathbb{Z}$. Let $\sigma = \{0_B, 1_B, C_{\mathbb{R}-\mathbb{Q}}\}$. Then $(Y, \sigma, B)$ is soft anti-locally countable. So, by Theorem 23, $\sigma_{\delta_\omega} = \sigma_\delta = \{0_B, 1_B\} \neq \sigma$.*

**Theorem 26.** *For any STS $(Y, \sigma, B)$, $(\sigma_\omega)_\delta = (\sigma_\omega)_{\delta_\omega}$.*

**Proof.** Since $RO(Y, \sigma_\omega, B)$ and $R\omega O(Y, \sigma_\omega, B)$ are soft bases for $(Y, (\sigma_\omega)_\delta, B)$ and $\left(Y, (\sigma_\omega)_{\delta_\omega},\right.$ $B)$, respectively, and by Theorem 7, $RO(Y, \sigma_\omega, B) = R\omega O(Y, \sigma_\omega, B)$, then $(\sigma_\omega)_\delta = (\sigma_\omega)_{\delta_\omega}$. □

**Remark 2.** *Let $(Y, \sigma, B)$ be an STS and let $K \in SS(Y, B)$. Then $Cl^\sigma_{\delta_\omega}(K) = Cl_{\sigma_{\delta_\omega}}(K)$.*

**Theorem 27.** *If $(Y, \sigma, B)$ is soft locally countable, then $\left(\sigma_{\delta_\omega}\right)_{\delta_\omega} = \sigma_{\delta_\omega}$.*

**Proof.** By Theorem 22, $\sigma_{\delta_\omega} = \sigma$ and thus, $\left(\sigma_{\delta_\omega}\right)_{\delta_\omega} = \sigma_{\delta_\omega}$. □

**Theorem 28.** *If $(Y, \sigma, B)$ is soft $\omega$-regular, then $\left(\sigma_{\delta_\omega}\right)_{\delta_\omega} = \sigma_{\delta_\omega}$.*

**Proof.** By Theorem 25, $\sigma_{\delta_\omega} = \sigma$ and thus, $\left(\sigma_{\delta_\omega}\right)_{\delta_\omega} = \sigma_{\delta_\omega}$. □

**Corollary 8.** *If $(Y, \sigma, B)$ is soft regular, then $\left(\sigma_{\delta_\omega}\right)_{\delta_\omega} = \sigma_{\delta_\omega}$.*

**Theorem 29.** *For any STS $(Y, \sigma, B)$, $\left(\sigma_\delta\right)_\delta = \sigma_\delta$.*

**Proof.** By Theorem 21, $\left(\sigma_\delta\right)_\delta \subseteq \sigma_\delta$. To show that $\sigma_\delta \subseteq \left(\sigma_\delta\right)_\delta$, let $M \in \sigma_\delta$ and let $b_y \widetilde{\in} M$. Then there exists $K \in \sigma$ such that $b_y \widetilde{\in} K \widetilde{\subseteq} Int_\sigma(Cl_\sigma(K)) \widetilde{\subseteq} M$. Put $S = Int_\sigma(Cl_\sigma(K))$. Then $S \in \sigma_\delta$ with $b_y \widetilde{\in} S \widetilde{\subseteq} Int_{\sigma_\delta}(Cl_{\sigma_\delta}(S))$. By Theorem 4.5 (e), $Cl_{\sigma_\delta}(M) = Cl^\sigma_\delta(M) = Cl_\sigma(M)$. Thus, by Theorem 21, $b_y \widetilde{\in} S \widetilde{\subseteq} Int_{\sigma_\delta}(Cl_{\sigma_\delta}(S)) \widetilde{\subseteq} Int_\sigma(Cl_\sigma(S)) \widetilde{\subseteq} M$. It follows that $M \in \left(\sigma_\delta\right)_\delta$. □

**Corollary 9.** *If $(Y, \sigma, B)$ is soft anti-locally countable, then $\left(\sigma_{\delta_\omega}\right)_{\delta_\omega} = \sigma_{\delta_\omega}$.*

**Proof.** It follows form Theorems 23 and 29. □

**Theorem 30.** *Let $(Y, \sigma, B)$ be a saturated STS. Then $(\sigma_\delta)_b = (\sigma_b)_\delta$ for all $b \in B$.*

**Proof.** Let $b \in B$. To show that $(\sigma_\delta)_b \subseteq (\sigma_b)_\delta$, let $U \in (\sigma_\delta)_b$ and let $y \in U$. Choose $M \in \sigma_\delta$ such that $U = M(b)$. Then $b_y \widetilde{\in} M$, and so there exists $S \in RO(Y, \sigma, B)$ such that $b_y \widetilde{\in} S \widetilde{\subseteq} M$. Thus, $y \in S(b) \subseteq M(b) = U$ and by Theorem 3.17, $S(b) \in RO(Y, \sigma_b)$. To show that $(\sigma_b)_\delta \subseteq (\sigma_\delta)_b$, let $U \in (\sigma_b)_\delta$ and let $y \in U$. Then there exists $V \in \sigma_b$ such that $y \in V \subseteq Int_{\sigma_b}(Cl_{\sigma_b}(V) \subseteq U$. Choose $M \in \sigma$ such that $M(b) = V$. Then we have $y \in M(b) \subseteq Int_{\sigma_b}(Cl_{\sigma_b}(M(b))) \subseteq U$. However, by Theorem 12 (c), $Int_{\sigma_b}(Cl_{\sigma_b}(M(b))) = (Int_\sigma(Cl_\sigma(M)))(b)$. Moreover, since $M \in \sigma$, then $Int_\sigma(Cl_\sigma(M)) \in \sigma_\delta$. It follows that $U \in (\sigma_\delta)_b$. □

**Corollary 10.** *Let $(Y, \sigma, B)$ be saturated and soft anti-locally countable STS. Then $(\sigma_{\delta_\omega})_b = (\sigma_b)_{\delta_\omega}$ for all $b \in B$.*

**Proof.** This follows from Theorems 23 and 29. □

**Corollary 11.** *Let $(Y, \mu)$ be a TS and $B$ be any set of parameters. Then $((C(\mu))_\delta)_b = \mu_\delta$ for all $b \in B$.*

**Proof.** It is clear that $(Y, C(\mu), B)$ is saturated. So, by Theorem 30, $((C(\mu))_\delta)_b = ((C(\mu))_b)_\delta$ for all $b \in B$. However, $(C(\mu))_b = \mu_\delta$ for all $b \in B$. This ends the proof. □

**Theorem 31.** *Let $\{(Y, \mu_b) : b \in B\}$ be a collection of TSs. Then $(\oplus_{b\in B}\mu_b)_\delta = \oplus_{b\in B}(\mu_b)_\delta$.*

**Proof.** To see that $(\oplus_{b\in B}\mu_b)_\delta \subseteq \oplus_{b\in B}(\mu_b)_\delta$, let $M \in (\oplus_{b\in B}\mu_b)_\delta$. Let $b \in B$. We will show that $M(b) \in (\mu_b)_\delta$. Let $y \in M(b)$. Then $b_y \widetilde{\in} M \in (\oplus_{b\in B}\mu_b)_\delta$. So, there exists $S \in RO(Y, \oplus_{b\in B}\mu_b, B)$ such that $b_y \widetilde{\in} S \widetilde{\subseteq} M$ and hence, $y \in S(b) \subseteq M(b)$. Now, by Theorem 14,

$S(b) \in RO(Y,\mu_b)$. It follows that $M(b) \in (\mu_b)_\delta$. To see that $\oplus_{b\in B}(\mu_\delta)_b \subseteq (\oplus_{b\in B}\mu_b)_\delta$, let $M \in \oplus_{b\in B}(\mu_b)_\delta$ and let $b_y \widetilde{\in} M$. Then $y \in M(b) \in (\mu_b)_\delta$. So, there exists $U \in RO(Y,\mu_b)$ such that $y \in U \subseteq M(b)$. Let $T = b_U$. Then $b_y \widetilde{\in} T \widetilde{\subseteq} M$. On the other hand, since $T(b) = U \in RO(Y,\mu_b)$ and $T(a) = \emptyset \in RO(Y,\mu_a)$ for all $a \in B - \{b\}$. Thus, by Theorem 14, $T \in RO(Y, \oplus_{b\in B}\mu_b, B)$. It follows that $M \in (\oplus_{b\in B}\mu_b)_\delta$. □

**Theorem 32.** *Let $\{(Y,\mu_b) : b \in B\}$ be a collection of TSs. Then $((\oplus_{b\in B}\mu_b)_\delta)_b = (\mu_b)_\delta$ for all $b \in B$.*

**Proof.** Let $b \in B$. To see that $((\oplus_{b\in B}\mu_b)_\delta)_b \subseteq (\mu_b)_\delta$, let $U \in ((\oplus_{b\in B}\mu_b)_\delta)_b$ and let $y \in U$. Choose $M \in (\oplus_{b\in B}\mu_b)_\delta$ such that $M(b) = U$. By Theorem 31, $M \in \oplus_{b\in B}(\mu_b)_\delta$ and so $M(b) = U \in (\mu_b)_\delta$. To see that $(\mu_b)_\delta \subseteq ((\oplus_{b\in B}\mu_b)_\delta)_b$, let $U \in (\mu_b)_\delta$. Then $b_U \in \oplus_{b\in B}(\mu_b)_\delta$. So, by Theorem 31, $b_U \in (\oplus_{b\in B}\mu_b)_\delta$. Hence, $(b_U)(b) = U \in ((\oplus_{b\in B}\mu_b)_\delta)_b$. □

**Corollary 12.** *Let $(Y,\mu)$ be a TS and $B$ be any set of parameters. Let $M \in SS(Y,B)$. Then $((\tau(\mu))_\delta)_b = \mu_\delta$ for all $b \in B$.*

**Proof.** For each $b \in B$, put $\mu_b = \mu$. Then $\tau(\mu) = \oplus_{b\in B}\mu_b$ and the result follows from Theorem 32. □

**Theorem 33.** *Let $\{(Y,\mu_b) : b \in B\}$ be a collection of TSs. Then $(\oplus_{b\in B}\mu_b)_{\delta_\omega} = \oplus_{b\in B}(\mu_b)_{\delta_\omega}$.*

**Proof.** To see that $(\oplus_{b\in B}\mu_b)_{\delta_\omega} \subseteq \oplus_{b\in B}(\mu_b)_{\delta_\omega}$, let $M \in (\oplus_{b\in B}\mu_b)_{\delta_\omega}$. Let $b \in B$. We will show that $M(b) \in (\mu_b)_{\delta_\omega}$. Let $y \in M(b)$. Then $b_y \widetilde{\in} M \in (\oplus_{b\in B}\mu_b)_{\delta_\omega}$. So, there exists $S \in R\omega O(Y, \oplus_{b\in B}\mu_b, B)$ such that $b_y \widetilde{\in} S \widetilde{\subseteq} M$ and so, $y \in S(b) \subseteq M(b)$. Now, by Theorem 15, $S(b) \in R\omega O(Y,\mu_b)$. It follows that $M(b) \in (\mu_b)_{\delta_\omega}$. To see that $\oplus_{b\in B}(\mu_{\delta_\omega})_b \subseteq (\oplus_{b\in B}\mu_b)_{\delta_\omega}$, let $M \in \oplus_{b\in B}(\mu_b)_{\delta_\omega}$ and let $b_y \widetilde{\in} M$. Then $y \in M(b) \in (\mu_b)_{\delta_\omega}$. So, there exists $U \in R\omega O(Y,\mu_b)$ such that $y \in U \subseteq M(b)$. Let $T = b_U$. Then $b_y \widetilde{\in} T \widetilde{\subseteq} M$. On the other hand, since $T(b) = U \in R\omega O(Y,\mu_b)$ and $T(a) = \emptyset \in R\omega O(Y,\mu_a)$ for all $a \in B - \{b\}$. Thus, by Theorem 15, $T \in R\omega O(Y, \oplus_{b\in B}\mu_b, B)$. It follows that $M \in (\oplus_{b\in B}\mu_b)_{\delta_\omega}$. □

**Theorem 34.** *Let $\{(Y,\mu_b) : b \in B\}$ be a collection of TSs. Then $\left((\oplus_{b\in B}\mu_b)_{\delta_\omega}\right)_b = (\mu_b)_{\delta_\omega}$ for all $b \in B$.*

**Proof.** Let $b \in B$. To see that $\left((\oplus_{b\in B}\mu_b)_{\delta_\omega}\right)_b \subseteq (\mu_b)_{\delta_\omega}$, let $U \in \left((\oplus_{b\in B}\mu_b)_{\delta_\omega}\right)_b$ and let $y \in U$. Choose $M \in (\oplus_{b\in B}\mu_b)_{\delta_\omega}$ such that $M(b) = U$. By Theorem 33, $M \in \oplus_{b\in B}(\mu_b)_{\delta_\omega}$ and so $M(b) = U \in (\mu_b)_{\delta_\omega}$. To see that $(\mu_b)_{\delta_\omega} \subseteq \left((\oplus_{b\in B}\mu_b)_{\delta_\omega}\right)_b$, let $U \in (\mu_b)_{\delta_\omega}$. Then $b_U \in \oplus_{b\in B}(\mu_b)_{\delta_\omega}$. So, by Theorem 33, $b_U \in (\oplus_{b\in B}\mu_b)_{\delta_\omega}$. Hence, $(b_U)(b) = U \in \left((\oplus_{b\in B}\mu_b)_{\delta_\omega}\right)_b$. □

**Corollary 13.** *Let $(Y,\mu)$ be a TS and $B$ be any set of parameters. Then $\left((\tau(\mu))_{\delta_\omega}\right)_b = \mu_{\delta_\omega}$ for all $b \in B$.*

**Proof.** For each $b \in B$, put $\mu_b = \mu$. Then $\tau(\mu) = \oplus_{b\in B}\mu_b$ and the result follows from Theorem 34. □

## 5. Conclusions

The growth of topology has been supported by the continuous supply of topological space classes, examples, properties, and relationships. As a result, expanding the structure of soft topological spaces in the same way is important.

The targets of this work are to scrutinize the behaviors of soft $R\omega$-open sets via soft topological spaces, to introduce the soft topology of soft $\delta_\omega$-open as a new soft topology, and to open the door to redefine and investigate some of the soft topological concepts such as soft compactness, soft correlation, soft class axioms, soft assignments, etc., via soft $R\omega$-open sets.

In this paper, soft $R\omega$-open sets as a strong form of soft open sets are introduced. We show that the family of soft $R\omega$-open sets forms a soft basis for some soft topology that lies between the soft topologies of soft regular-open sets and soft open sets. In addition, the soft $\delta_\omega$-closure operator as a new operator in soft topological spaces is defined. Via the soft $\delta_\omega$-closure operator, soft $\delta_\omega$-open sets as a strong form of open sets and a weaker form of soft $R\omega$-open sets are introduced. Moreover, the correspondence between soft $\delta_\omega$-open in soft topological spaces and $\delta_\omega$-open in topological spaces is studied.

In the upcoming work, we plan to: (1) Introduce some soft topological concepts using soft $R\omega$-open sets such as soft continuity and soft sepapration axioms; and (2) investigate the behavior of soft $\delta_\omega$-open sets under product soft topological spaces.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The author declares no conflict of interest.

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