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Editorial

Modern Problems of Mathematical Physics and Their Applications

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1. Introduction

There are many applications of mathematical physics in several fields of basic science and engineering. Thus, we have tried to provide the Special Issue "Modern Problems of Mathematical Physics and Their Applications" to cover the new advances of mathematical physics and its applications. In this Special Issue, we have focused on some important and challenging topics, such as integral equations, ill-posed problems, ordinary differential equations, partial differential equations, system of equations, fractional problems, linear and nonlinear problems, fuzzy problems, numerical methods, analytical methods, semi-analytical methods, convergence analysis, error analysis and mathematical models. In response to our invitation, we received 31 papers from more than 17 countries (Russia, Uzbekistan, China, USA, Kuwait, Bosnia and Herzegovina, Thailand, Pakistan, Turkey, Nigeria, Jordan, Romania, India, Iran, Argentina, Israel, Canada, etc.), of which 19 were published and 12 rejected.

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Citation: Juraev, D.A.; Noeiaghdam, S. Modern Problems of Mathematical Physics and Their Applications. *Axioms* **2022**, *11*, 45. https://doi.org/10.3390/axioms11020045

Received: 18 January 2022
Accepted: 21 January 2022
Published: 23 January 2022

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2. Brief Overview of the Contributions

Qaraad et al., in [1], have considered a class of quasilinear third-order differential equations with a delay argument. They have established some conditions of a certain third-order quasi-linear neutral differential equation as oscillatory or almost oscillatory. They have solved some examples to demonstrate the importance of the results.

Extending the SABO technique (Semi-Analytical method for Barrier Options), based on the collocation Boundary Element Method (BEM), to the pricing of Barrier Options with a payoff dependent on more than one asset has been discussed by Aimi and Guardasoni in [2]. The numerical results have been presented to show the efficiency and accuracy of the method in the case of a single asset.

With the rapid development of the Internet, the speed with which information can be updated and propagated has accelerated, resulting in wide variations in public opinion. Usually, after the occurrence of some newsworthy event, discussion topics are generated in networks that influence the formation of initial public opinion. After a period of propagation, some of these topics are further derived into new subtopics, which intertwine with the initial public opinion to form a multidimensional public opinion. In [3], Chen et al. were concerned with the formation process of multi-dimensional public opinion in the context of derived topics. Firstly, the initial public opinion variation mechanism was introduced to reveal the formation process of derived subtopics, then Brownian motion was used to determine the subtopic propagation parameters, and their propagation was studied based on complex network dynamics according to the principle of evolution. The formula of the basic reproductive number has been introduced to determine whether derived subtopics can form derived public opinion, thereby revealing the whole process of multi-dimensional public opinion formation. Secondly, through simulation experiments, the influences of various factors, such as the degree of information alienation, environmental forces, topic correlation coefficients, the amount of information contained in subtopics, and network topology on the formation of multi-dimensional public opinion have been studied by the authors. The simulation results showed that: (1) Environmental forces and the amount of information contained in subtopics are key factors affecting the formation of multi-dimensional public opinion. Among them, environmental forces have a greater impact on the number of subtopics, and the amount of information contained in subtopics determines whether the subtopic can be the key factor that forms the derived public opinion. (2) Only when the degree of information alienation reaches a certain level will derived subtopics emerge. At the same time, the degree of information alienation has a greater impact on the number of derived subtopics, but it has a small impact on the dimensions of the final public opinion. (3) The network topology does not have much impact on the number of derived subtopics but has a greater impact on the number of individuals participating in the discussion of subtopics. The multidimensional public opinion dimension formed by the network topology with a high aggregation coefficient and small average path length is higher.

In [4], the existence of mild solutions to a multi-term fractional integro-differential equation with random effects has been investigated by Diop and Wu. The results relied on stochastic analysis, Mönch's fixed point theorem combined with a random fixed point theorem with stochastic domain, and the measure of noncompactness and resolvent family theory. The authors have established the existence of random mild solutions under the condition that the nonlinear term is of Carathéodory type and satisfies some weakly compactness condition. By presenting a nontrivial example, they showed the obtained results.

Vasilyev and Eberlein [5] have studied a certain conjugation problem for a pair of elliptic pseudo-differential equations with homogeneous symbols inside and outside of a plane sector. They found that the solution is sought in corresponding Sobolev–Slobodetskii spaces. Using the wave factorization concept for elliptic symbols, they derived a general solution of the conjugation problem. Adding some complementary conditions, a system of linear integral equations has been obtained. For homogeneous symbols, they applied the Mellin transform to such a system to reduce it to a system of linear algebraic equations with respect to unknown functions.

In [6], Providas et al. attempted to find the solution of boundary value problems for ordinary differential equations with general boundary conditions. They have obtained the closed-form solutions in a symbolic form with the general n-th order differential operator, as well as the composition of linear operators. Furthermore, their method is based on the theory of the extensions of linear operators in Banach spaces.

Pham in [7] has presented a mathematical modeling of the virus-infected development in the body's immune system considering the multiple time-delay interactions between the immune cells and virus-infected cells with autoimmune disease. In the proposed model, he tried to determine the dynamic progression of virus-infected cell growth in the immune system. The patterns of how the virus-infected cells spread and the development of the body's immune cells with respect to time delays have been derived in the form of a system of delay partial differential equations. The model can be used to determine whether the virus-infected free state can be reached or not as time progresses. It has been used to predict the number of the body's immune cells at any given time. Several numerical examples have been discussed to illustrate the proposed model. The model provided a real understanding of the transmission dynamics and other significant factors of the virus-infected disease and the body's immune system subject to the time delay, including approaches to reduce the growth rate of virus-infected cells and the autoimmune disease and also enhance the immune effector cells.

The Ensemble Intermediate Coupled Model (EICM) is a model used for studying the El Nino-Southern Oscillation (ENSO) phenomenon in the Pacific Ocean, where anomalies in the Sea Surface Temperature (SST) are observed. In [8], Injan et al. aimed to implement Cressman to improve SST forecasts. The simulation considers two cases in this work: the control case and the Cressman initialized case. These cases are simulations using different inputs where the two inputs differ in terms of their resolution and data source. The Cressman method has been used to initialize the model with an analysis product based on satellite data and in situ data, such as ships, buoys, and Argo floats, with a resolution of 0.25×0.25 degrees. The results of this inclusion are the Cressman Initialized Ensemble Intermediate Coupled Model (CIEICM). Forecasting of the sea surface temperature anomalies was conducted using both the EICM and the CIEICM. The results showed that the calculation of the SST field from the CIEICM was more accurate than that of the EICM. The forecast using the CIEICM initialization with the higher-resolution satellite-based analysis at a 6-month lead time improved the root mean square deviation to 0.794 from 0.808 and the correlation coefficient to 0.630 from 0.611 compared the control model that was directly initialized with the low-resolution in situ analysis.

In [9], Sidi has discussed the secant method, which is a very effective numerical procedure for solving nonlinear equations $f(x) = 0$. In their recent work (A. Sidi, Generalization of the secant method for nonlinear equations. Appl. Math. E-Notes, 8:115–123, 2008), he presented a generalization of the secant method that used only one evaluation of $f(x)$ per iteration, and he provided a local convergence theory for it that concerned real roots. For each integer k, this method has generated a sequence $\{x_n\}$ of approximations to a real root of $f(x)$, where, for $n \geq k$, $x_{n+1} = x_n - f(x_n)/p'_{n,k}(x_n)$, $p_{n,k}(x)$ being the polynomial of degree k that interpolates $f(x)$ at $x_n, x_{n-1}, \ldots, x_{n-k}$, the order s_k of this method satisfies $1 < s_k < 2$. Clearly, when $k = 1$, this method reduces to the secant method with $s_1 = (1 + \sqrt{5})/2$. In addition, $s_1 < s_2 < s_3 < \ldots$, such that $\lim_{k\to\infty} s_k = 2$. The author has studied the application of this method to simple complex roots of a function $f(z)$. He showed that the local convergence theory developed for real roots can be extended almost as is to complex roots, provided suitable assumptions and justifications are made. He has illustrated the theory with two numerical examples.

"Odd" factor approximants of the special form are based on the idea that the critical index by itself should be optimized through the parameters of the power transform to be calculated from the minimal sensitivity (derivative) optimization condition. The critical index is a product of the algebraic self-similar renormalization that contributes to the expressions of control parameters typical to the algebraic self-similar renormalization and of the power transform that corrects them even further. The parameter of power transformation is, in a nutshell, the multiplier connecting the critical exponent and the correction-to-scaling exponent. In [10], Gluzman has studied the minimal model of critical phenomena based on expansions with only two coefficients and critical points. The optimization appears to bring quite accurate, uniquely defined results given by simple formulas. Many important cases of critical phenomena have been covered by the simple formula. For the longer series, the optimization condition possesses multiple solutions, and additional constraints have been applied. In particular, the author constrained the sought solution by requiring it to be the best in the prediction of the coefficients not employed in its construction. In principle, the error and measure of such a prediction have been optimized by itself, with respect to the parameter of the power transform. Methods of calculation based on optimized power-transformed factors have been applied, and results are presented for critical indices of several key models of conductivity and viscosity of random media, swelling of polymers, permeability in two-dimensional channels. The author has discussed several quantum mechanical problems as well.

In [11], Veeresha et al. have tried to analyze the nature and capture the corresponding consequences of the solution obtained for the Gardner–Ostrovsky equation with the help of the q-homotopy analysis transform technique. The fractional operator was used to illustrate its importance in generalizing the models associated with kernel singular. The authors have considered the fixed-point theorem and the Banach space to present the existence and uniqueness within the frame of the Caputo–Fabrizio fractional operator. Furthermore, for different fractional orders, the nature has been captured in plots. The realized consequences confirmed that the considered procedure is reliable and highly methodical for investigating the consequences related to the nonlinear models of both integer and fractional order.

The canonical gravitational partition function Z associated to the classical Boltzmann–Gibbs (BG) distribution $\frac{e^{-\beta H}}{Z}$ has been considered in [12] by Hameeda et al. It is popularly thought that it cannot be built up because the integral involved in constructing Z diverges at the origin. On the contrary, it was shown in (Physica A 497 (2018) 310), by appeal to sophisticated mathematics developed in the second half of the last century, that this is not so. Z has been computed by recourse to (A) the analytical extension treatments of Gradshteyn and Rizhik and Guelfand and Shilov, which permit tackling some divergent integrals, and (B) the dimensional regularization approach. Only one special instance was discussed in the above reference. In [12], the authors obtained the classical partition function for Newton's gravity in the four cases that immediately come to mind.

The Lagrange dynamics generated by a class of isoperimetric constrained controlled optimization problems involving second-order partial derivatives and boundary conditions have been investigated by Treanţă in [13]. The author has derived necessary optimality conditions for the considered class of variational control problems governed by path-independent curvilinear integral functionals. Moreover, the theoretical results are accompanied by an illustrative example. Furthermore, an algorithm has been proposed to emphasize the steps to be followed to solve a control problem.

With the rapid development of "We media" technology, netizens can freely express their opinions regarding enterprise products on a network platform. Consequently, online public opinion about enterprises has become a prominent issue. Negative comments posted by some netizens may trigger negative public opinion, which can have a significant impact on an enterprise's image. In [14], Chen et al. applied the perspective of helping enterprises deal with negative public opinion by combining the user portrait technology and a random forest algorithm to help enterprises identify high-risk users who have posted negative comments and thus may trigger negative public opinion. In this way, enterprises can monitor the public opinion of high-risk users to prevent negative public opinion events. Firstly, they crawled the information of users participating in discussions of product experience, and they constructed a portrait of enterprise public opinion users. Then, the characteristics of the portraits were quantified into indicators, such as the user's activity, the user's influence, and the user's emotional tendency, and then the indicators were sorted. According to the order of the indicators, the users were divided into high-risk, moderate-risk, and low-risk categories. Next, a supervised high-risk user identification model for this classification was established based on a random forest algorithm. In turn, the trained random forest identifier has been used to predict whether the authors of newly published public opinion information are high-risk users. Finally, a back propagation neural network algorithm was used to identify users and compared with the results of model recognition in this paper. The results showed that the average recognition accuracy of the back propagation neural network is only 72.33%, while the average recognition accuracy of the model constructed in this paper is as high as 98.49%, which verifies the feasibility and accuracy of the proposed random forest recognition method.

The aim of [15] was to explain the result concerning the Navier–Stokes problem (NSP) in $\mathbb{R}^3$ without boundaries to a broad audience. Ramm proved that the NSP is contradictory in the following sense: if one assumes that the initial data $v(x,0) \not\equiv 0$, $\nabla \cdot v(x,0) = 0$ and the solution to the NSP exists for all $t \geq 0$, then the author proved that the solution $v(x,t)$ to the NSP has the property $v(x,0) = 0$. This study showed that the NSP is not a correct description of the fluid mechanics problem and the NSP does not have a solution. In the exceptional case, when the data are equal to zero, the solution $v(x,t)$ to the NSP exists for all $t \geq 0$ and is equal to zero, $v(x,t) \equiv 0$. Thus, one of the millennium problems has been solved by the author.

Christodoulou et al. in [16] have derived a family of associated differential equations that share the same "degenerate" canonical form. These equations can be solved easily if the original equation is known to possess analytic solutions; otherwise, their properties and the properties of their solutions are de facto known as they are comparable to those already deduced for the fundamental equation. The authors analyzed several particular cases of new families related to some of the famous differential equations applied to physical problems, and the degenerate eigenstates of the radial Schrödinger equation for the hydrogen atom in N dimensions.

Traffic management is a significantly difficult and demanding task. It is necessary to know the main parameters of road networks in order to adequately meet traffic management requirements. In [17], Vrtagić et al. focused on an integrated fuzzy model for ranking road sections based on four inputs and four outputs. The goal was to determine the safety degree of the observed road sections by the methodology developed. The greatest contribution of the paper was reflected in the development of the improved fuzzy step-wise weight assessment ratio analysis (IMF SWARA) method and integration with the fuzzy measurement alternatives and ranking according to the compromise solution (fuzzy MARCOS) method. First, the data envelopment analysis (DEA) model was applied, showing that three road sections have a high traffic risk. After that, IMF SWARA was used to determine the values of the weight coefficients of the criteria, and the fuzzy MARCOS method was used for the final ranking of the sections. The obtained results were verified through a three-phase sensitivity analysis with an emphasis on forming 40 new scenarios in which input values were simulated. The stability of the model was proven in all phases of sensitivity analysis.

An explicit formula for the approximate solution of the Cauchy problem for the matrix factorizations of the Helmholtz equation in a bounded domain on the plane has been presented by Juraev and Noeiaghdam in [18]. The formula for an approximate solution has included the construction of a family of fundamental solutions for the Helmholtz operator on the plane. This family was parameterized by function $K(w)$, which depends on the space dimension. The authors have improved the results using the function $K(w)$.

In [19], Chashechkin has tried to formulate a list of principles that substantiates the choice of axioms and methods for studying nature. He showed that the axiomatics of fluid flows are based on conservation laws in the frames of engineering mathematics and technical physics. In the theory of fluid flows within the continuous medium model, a key role for the total energy has been distinguished. To describe a fluid flow, a system of fundamental equations has been selected and supplemented by the equations of the state for the Gibbs potential and the medium density. The system has been supplemented by the physically based initial and boundary conditions and analyzed, taking into account the compatibility condition. The complete solutions showed both the structure and dynamics of non-stationary flows. The results of compatible theoretical and experimental studies have been compared for the cases of potential and actual homogeneous and stratified fluid flow past an arbitrarily oriented plate.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: We are thankful the editors and reviewers of the journal *Axioms* for their help and support from our issue.

Conflicts of Interest: The authors declare no conflict of interest.

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