

Article

Set Theory, Dynamism, and the Event: Reinjecting Time into the Foundations of Mathematics

Said Mikki

Zhejiang University/University of Illinois at Urbana-Champaign (ZJU-UIUC) Institute, Zhejiang University, International Campus, Haining 314400, China; said.m.mikki@gmail.com

Abstract: This article concentrates on exploring the relevance of the postmodernist concept of the *event* to mathematical philosophy and the foundations of mathematics. In both the scientific and philosophical study of nature, and particularly event ontology, we find that space and dynamism are fundamental. However, whether based on set theory or category theory, modern mathematics faces conceptual and philosophical difficulties when the temporal is intentionally invoked as a key aspect of that intrinsic dynamism so characteristic of mathematical being, physical becoming, process, and thought. We present a multidisciplinary investigation targeting a diverse audience including mathematicians, scientists, and philosophers who are interested in exploring alternative modes of doing mathematics or using mathematics to approach nature. Our aim is to understand both the formal character and the philosophy of time as realized through a radical mode of thinking that goes beyond the spatial in mathematics. In particular, we suggest the need to transcend the purely geometrical view altogether in future foundational research in both mathematics and mathematical philosophy. We reexamine these issues at a fundamental and comprehensive level, where a detailed exposition and critique of both modern set theories and theories of space is outlined, with emphasis on how the philosophy of Idealism has been permeating much of old and new mathematics. Furthermore, toward the end of the article, we explore some possible constructive directions in mathematical ontology by providing new proposals on how to develop a fragment of mathematics for the description of dynamic events.

Keywords: foundations of mathematics; set theory; time; the event; dynamism; geometry; topological flow; postmodernism; mathematical philosophy; natural philosophy



Citation: Mikki, S. Set Theory, Dynamism, and the Event: Reinjecting Time into the Foundations of Mathematics. *Axioms* **2022**, *11*, 670. <https://doi.org/10.3390/axioms11120670>

Academic Editor: Hans J. Haubold

Received: 22 December 2021

Accepted: 11 October 2022

Published: 25 November 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The issue of the relation between time and mathematics is very old, possibly dating back to as early as the beginning of mathematics itself. In a nutshell, the problem we are concerned with here is how concepts such as time, temporality, dynamism, synchronization, and so on, can, at least partially, manifest themselves at the most elementary level of the foundations of mathematics as such. Ever since the invention (or codification) of the axiomatic method by Euclid, mathematical structures have been traditionally viewed as the domain of rigorous and exact thought, where propositions are generated through a deductive apparatus fed with initial principles called axioms [1,2]. Such a formalistic picture leaves almost nothing to temporal ideas and concepts: an axiom is an eternal truth [3,4]; a theorem is a product produced by mechanical proof machines [5]; mathematical theories are domains of permanent abstract structures [6–8]; and so on. However, within the restrictive worldview of such a rigid formalistic scaffolding, the only chance left for temporality to express itself is via the computational, Turing-like, machinic time of recursive calculations [9]: deductions, at least within the Hilbert program [1,10], can be seen as the unfolding of discrete step-wise calculations executed by the machinic order of axiomatic mathematical becoming [11]. Not much is left then for heavily temporalized concepts when such a mechanical succession of algebraic compositions is the sole generator

of change. Note that in the standard mathematical theory of dynamical systems, which is most frequently expressed as discrete dynamics, dynamism is generated through the repeated application of iterated maps. If $\phi : X \rightarrow X$ is a continuous map from a topological space X to itself, then computational, Turing-like dynamics may be generated by the n th iteration $\phi^n := \phi \phi \dots \phi$ (n times) [12]. This structure exhausts the philosophical concept of computational or machinic time, though, of course, there is a very rich mathematical theory of such deceptively simple dynamical systems [12–14].

In contrast to such overtly “static” pictures, which are favoured and encouraged by the ubiquitous drift of modern mathematics toward axiomatization [5], formalization [15,16], and algebraization [8,17], influential figures in the history of ideas, such as Aristotle [18], Leibniz [19], and Bergson [20], saw *dynamism* as an essential ingredient in both mathematics and nature. In fact, Bergson even considered that mechanical thought, expressed mathematically, is the antithesis of life per se since the fundamental essence of the living is disclosed as the sheer dynamism of becoming or coming-to-be [21]. Nevertheless, even as late as the last decade of the nineteenth century, when Bertrand Russell initiated his researches into the foundations of mathematics [22], there had been no consensus on the ultimate status of time in the foundations of mathematics. The most striking example illustrating this foundational impasse is the unsatisfactory state of the calculus. In spite of the fact that the differential calculus had been invested with dynamic and temporal concepts since its original formulation in the seventeenth century by Leibniz and Newton, e.g., the vanishing differentia, fluxes, fluxions, etc., the *rigorous* mathematical foundations of the field, developed in the nineteenth century, mainly by Cauchy [23] and Weierstrass [24], made no use of explicit temporalized concepts [25,26]. Indeed, the idea of the limit, while conceptually very dynamic, was expressed using “static” definitions utilizing inequalities with epsilon and delta [27].

While there are some notable exceptions, the general trend in the field of the foundations of mathematics, which is fundamentally shaped by Hilbert’s program [5], has been aligned with the axiomatic, formalized framework of mathematical logic [10], whereby one eschews temporality for the sake of a Platonic timelessness to be sought in the ultimate foundations of the mathematical field [28]. Mathematical physics has been also developed along such Hilbertian lines [29], where abstract mathematical disciplines such as algebraic groups [30], functional analytic operators [31], the theory of invariants [32], differential equations [33], complex function theory [34], and geometry [35], are all united into a single ambitious comprehensive program aiming at expressing the totality of nature’s structures in a purely mathematical language [17] using abstract symbolic representations of the Real governed and regulated by axiomatic variational principles [36]. That leaves still open for us a third direction of fundamental research less explored than the others: mathematical philosophy. Indeed, the first two directions, namely logical and formalized mathematics on one hand, and mathematical physics on the other, both, to some extent, have been dominated by the axiomatic—essentially non-dynamical—approach advocated by Hilbert and their numerous influential followers working throughout the twentieth century and beyond. However, mathematical philosophy, whose principal figures include Russell [27,37], Whitehead [38,39], Brouwer [40], Lautman [41], is a quite different field less dominated by the Hilbert’s school (thought not completely) than the other directions of foundational research. For that reason, in this article we suggest approaching the problem of the relation between mathematics and time through the prism of mathematical philosophy proper.

Mathematical philosophy is a branch of mathematics focused on developing mathematical concepts, structures, and methods that tend to be motivated by philosophical problems. The archetypal text representative of this movement is Russell’s short but hugely influential book [37]. While the subject has dramatically changed ever after Hilbert’s and Gödel’s entry into the field [4,5,10,42,43], its connection with fundamental philosophy appears to have weakened in recent decades, especially with the rise of new areas in the foundations of mathematics after the end of the Second World War such as the theory of computation [44], information theory [45], category theory [46], and constructive math-

ematics [47,48], which have contributed to eclipsing the old, purely philosophical flavor of the first school pioneered by the founding fathers of mathematical philosophy proper, namely figures like Frege [15,16], Husserl [49–51], and Russell [27,37].

In our opinion, one of the most neglected aspects of mathematical philosophy, especially when viewed as a branch of mathematics, not philosophy, is the relation between mathematics and *postmodernism*. Roughly speaking, postmodernism is a European philosophical movement that reached its peak in the 1960s and 1970s, mainly centered in Paris around these times, though it has its roots in the German pre-war philosophies of Nietzsche, Husserl, and Heidegger, see for example [52] and more references given throughout this article. We note that postmodernism brought a radical change to our outlook of the world, especially in connection with the philosophies of nature developed by authors like Serres [53–56], Simondon [57,58], Ruyer [59,60], Deleuze [61–64], and Guattari [65–67]. Indeed, those writers, and few others, had invented a series of remarkable and creative models of the Real, which take into account modern mathematics, biology, and physics, yet without reducing their new philosophical and foundational analysis of nature and mathematics to mere commentary on, and exposition of, the traditional approach of science and mainstream mathematics. In general, and especially starting from the 1920s, both theoretical physics (quantum physics and general relativity) and ontology (Bergson [20,21,68], Heidegger [69]) have revolutionized our understanding of time, introducing multiple radical changes within various theoretical and philosophical frames of references and points of contact with the concepts of temporality *qua* dynamism. However, modern mathematics, particularly in its emphasis on invariance, symmetry groups, and axiomatizations, appears to continue to evade a radical engagement with time at the very foundational level of the mathematical experience as such. However, while nontemporal thinking, attitudes, and ideologies have historically overdominated most fields of mainstream mathematical research, especially number theory, algebra, and geometry, but less so in general topology and analysis, an undercurrent, or a “substream” variety of research, has persisted, whereby theoretical concepts such as dynamism, change, temporality, become the prime subjects. In fact, temporal thinking in mathematics never completely died out. Unfortunately, the history of the “problem of time” has not received the same critical attention it received in other areas, such as physics, for example as in works like [70–75]. Nevertheless, we may mention Cantor’s work [76] as an exception that only proves the rule [77]. Temporality has also enjoyed the uncanny habit of suddenly reemerging when it is least expected. An example of the resurgence of the temporal in mathematical thinking is the rejection by Russell and Whitehead of the concept of “spacetime point” [39,78], a critique which informed their joint entry into the subject of event ontology [79,80], which will be discussed further below (cf. Sections 3 and 4).

One of the main objectives of this article is to acquaint the mathematical and physical communities with the impact of postmodernism on contemporary thought and its relation with the possibility of a revised foundation for mathematics more receptive to concepts of time. Indeed, postmodernism has been motivated and influenced by earlier theories of time due to Bergson [20], Heidegger [69], and Einstein [81]. We would like to reexamine the impact of the postmodernist critique of the foundations of mathematical physics on our current understanding of the foundations of mathematics, especially in regard to the latter’s connection with the natural sciences and philosophy [17,70,82]. It should be stressed from the beginning that postmodernist writers had sometimes tended to be critical of mainstream mathematics, yet while also engaging strongly with some of the latter’s very technical aspects, e.g., see the works of the distinguished mathematical philosopher Albert Lautman in this regard [41], and also Badiou’s work [83–85]. The same double-edged approach will be adopted in our formulation below. Indeed, in addition to the commitment to continue to work mainly from within the parameters of modern mathematics, we attempt to outline a critique of some of the conceptual and philosophical themes that have dominated the field in recent times. A major focus point in our presentation is the neglect of time and temporality in modern mathematics, and the need to reflect on how alternative forms of

doing mathematics, for example “postmodern mathematics”, could be envisioned from within the formal examination and solution of the problem of dynamism and temporality in set theory, geometry, category theory, and other areas. We highlight in particular the idea of event ontology, which goes back to Russell’s mathematical philosophy [78,86], as a promising pathway for a future “postmodern mathematics”.

The article is divided into three main parts. Section 2 examines the modern (axiomatic) approach to set theory but also highlights from the beginning some possible alternative modes of dynamically thinking the idea of the set. In Section 3, we consider issues related to spatialization in geometry and how the concept of transformation, which is shared by both modern and postmodern mathematics, could be utilized in order to rethink the foundations of mathematics in terms of the idea of the event. Event ontology proper is examined more specifically in Section 4 from the philosophical viewpoint, Section 4.3 projects the problem through a more technical mathematical perspective. Finally, we end up with a conclusion.

2. Modern and Postmodern Set Theories

2.1. Are Sets “Heaps of Things”?

The event is a dynamic set, though one cannot claim that we have fully comprehended that elusive and subtle concept, set-hood. A set can be anything; the concept of set-hood as such is non-philosophical: it is anthropological, even psychological, often contingent, or apparently dispensable. In contrast to the traditional attitude toward set theory, we approach the topic through the lens of *space theory*, that is, neither via logic nor grammar. This will serve several purposes, chief among them is downplaying the role of the Cartesian Subject [87–90] in the construction of the Set-Theoretic Universe. Careful analysis reveals how space is always at the heart of most of the standard operations of set theory. Indeed, a set cannot be defined as a “heap of things” since the concept of *thing* is not available yet. Instead, modern mathematics has always opted for the *logical* approach, best represented in the Anglo-American tradition by Frege [15,16], where the being of the set is *posited* from the very beginning on the basis of *intensional* logical propositions. In that way, instead of defining a set *by extension*, i.e., through a complete *listing* of “itemized objects” belonging to the set, the pan-logical approach creates a specialized formal structure, a “logical criterion” satisfied by all of those “legitimate” elements qualifying as genuine members of that particular set.

If we start with a set S , we must then specify a logical predicate $p(x)$ such that the entity denoted by x represents a generic possible variable item that may *or* may not “belong” to the set S . Therefore, according to the intensional view, we say that ‘ s belongs to S ’ if the predicate $p(s)$, obtained by substituting s into the general propositional function $p(x)$, is True. It is clear that the very concept of *belonging* as such is being *defined* here in terms of the logical predicate $p(x)$. There is then a logical *relation* R connecting s to S , say a form sRS through which the element s gets associated with S in a specific manner. In the modern notation of set theory, we write $s \in S$.

The previous deceptively straightforward analysis contains, in a nutshell, the most essential core of philosophical set theory. We do not bother (at this stage) about the purely *formalistic* operations involving manipulations of unions, intersections, complements, and so on (these will be discussed later). What is at stake at this very initial stage is the subtle manner in which the truly *ontological* content of set theory has been swept under the rug by Frege [15,16], Zermelo [91], Frankel [92], von Neumann [93], Hilbert [5,10], and the rest of the founders of the modern theory. What we have in mind here is the deliberate emphasis by the founders of modern mathematics on the use (and abuse) of the Parmenidean categories of being, which all amount to the machination of mathematical being according to the ontological idealistic formula Being is being-as-presence [94,95]. Becoming, according to this dominant view, is treated as less fundamental than being [96]. The All is all about *invariance*, that is, *permanence-in-and-through-variation* (a Kantian scheme and theme [97]), endurance, and so on. The symmetry group, for instance, becomes more important than

dynamic operations in space because the former is supposed to capture the “essence” of a geometrical being, while the latter is seen as ephemeral, transitory, capricious, and so on.

However, while such an intrinsically non-metaphysical, non-temporal approach might have been completely justified in the first half of the twentieth century, we would like to argue below that a fully-fledged integration of dynamic (temporalized) thinking with the metaphysical foundations of mathematics is a highly important priority urgently needed now, that is, more than one hundred years after Russell’s publication of *Principia Mathematica* [38,98,99] in the period 1911–1913, i.e., what has been considered the defining event of mathematical philosophy and the foundations of mathematics [1]. The reasons are many, some of them are historico-philosophical in nature and so will not be fully stressed in what follows. Others are more formal and technical, in the main relating to what we wish to highlight in our presentation as the “internal affair” of set theory, the theory of relations, topological flows, and so on, where it appears to us that a strong sense of urgency can be discerned in the general drift toward *dynamics*, which may be detected in both theoretical and mathematical physics. Theories such as dynamics sets [100], event ontologies [86], algebraic quantum field theories [101], topological dynamics [12,102], nonequilibrium dynamics [103], just to mention few examples, all require a more intensive engagement with the purely dynamical aspects of mathematical being [71–73,104].

Indeed, there is more to a set than being “a heap of things” held together via rigorous and exact control statement like the intensional predicate $p(x)$. A set can *never* be exhausted by logical, linguistic, or semiotic formulations. Whether intension or extension, it is simply not possible to move from part-elements to whole-sets just by magically summoning the force of some *decision* in which a metaphysically presupposed all-knowing Cartesian-Kantian Subject would *posit* the set-hood of the whole by merely invoking the computational power of purely logical statements (the process of *enunciation* as the ontological foundation, see [62,66]). In other words, modern set theory, a branch of Idealism *par excellence*, continues the age-old myth in which *language* is assigned a higher priority than “natural processes”. Set theory understands its content through the eyes of a universal rational Mind capable of delimiting the world according to purely combinatorial operations (the computation of truth values of logical functions). If modernism is to triumph over ancient natural philosophy (pre-Platonic philosophy, China, India), the new Aristotelian order (the organon of language, logic, Euclidean deduction) must be allowed to rein in so it may master and contain the explosive *open* nature of sets. Indeed, it is precisely this ontological *openness* – inherent in every process of forming wholes – what is at stake when attempting to dissect the concept of set-hood.

Recall that a set is not well-defined until some *logical* factor enters into the picture. A collection of “objects lying out there” does not qualify as a set until some *intentionally* selected intensive or extensive processes are imposed on the collection [27,38,105,106]. This “selective” logical process is enacted by the presupposed knowing Subject of modern logic and grammar [66]. Conventionally speaking, we may readily appreciate how relative to every process involving words and signs one may pick out a correlated process of “semiotic transfer”, which is implicitly presupposed and imposed by a Universal Signifier, whose main function is to plan and calculate with the aim of *dominating* nature. Knowledge, the framework of logic and language, is that science of absolute ontological control and discipline at the disposal of a despotic signifier that can be manifested in various forms: Man, Mind, Axioms, Universal Reason, Symmetry Group, and others. More than even Royal Science [62], Set Theory might be viewed as the culmination of the long history of Idealism’s devastation of the environment and the world inside which the human lives and thrive [55].

In our opinion, the deeper reality behind the concept of set is that it is *not a logical concept at all*. While of course formalism is indispensable in ontology, every time you bring formal considerations into thinking about sets the very structure of set-hood gets entangled with language and knowing, hence the inevitable emergence of some sort of Universal Subject, Transcendental Ego, or Signifier, to whom the entire process of *forming a*

set will be referred. However, sets need not be viewed as “collections of things”, especially given the undisputed fact that the status of *things* is already highly questionable from the ontological viewpoint. A possible alternative? Replace sets by zones of “eventuation”. Instead of talking about elements belonging to sets, we prefer to think in terms of flows and becomings condensing and consolidating themselves into a globalized quasi-whole, a sort of metastable “super-event”, or a large-scale event-structure composed of series of events, eventual strata *par excellence* [107]. In fact, we even believe that such an ontological model is closer to the spirit of Cantor’s and Russell’s than Frege or Hilbert, where in fact the latter two had been Idealist formalists in and through.

2.2. On Extension and Intension in Set Theory

In order to determine the “internal” structure of a set, it is usually enough to “list” all of its elements. For example, one way to describe a set S is to say that “ s_1 is in S , s_2 is in S , s_3 is in S , and so on”. More formally, we write

$$S = \{s_1, s_2, s_3, \dots\}. \quad (1)$$

However, it is this last innocently looking “so on” that will cause problems. Indeed, how do we know that *all* elements s are to be “included” in S ? Can we count them all? Furthermore, if so, why should the fundamental definition of a set depend so shamefully on the “properties of the observer”, in this case, the supposed capacity of a Cartesian *subject* to *list* all elements by enunciating extension via one sweeping declarative statement like the one prescribed above? If there is a “continuum” of elements s , say those possessing the form s_x , where x is a continuous variable serving as an index, then it follows that no one, not even a full explicit human subject (or classical computer) can perform the impossible feat of “listing” all elements s_x . It is for this reason that in the nineteenth century mathematicians had rapidly moved toward an intension-based approach to the logic of set theory, preferring it over the more intuitive definition by extension. In the former approach, a continuum of elements indexed by x can be ascribed to a set S using a predicate $p(x)$ in the following manner:

Definition 1. (Set-theoretic intensionality). Consider a set S and an object s . We say that S contains s intensionally if the following is satisfied:

$$s_x \in S \longleftrightarrow p(s_x) \text{ is True}. \quad (2)$$

where $p(\cdot)$ is a logical schema or propositional function. Sometimes $p(\cdot)$ is called the set-theoretic membership fundamental predicate.

Remark 1. The intensional statement (2) is to be understood as holding for “all” values of the index x in an index set I (formally, we should append ‘ $\forall x \in I$ ’ to the intensional statement above).

However, what is the sense of “all” in a statement like the above? How is the quantification over the totality of “all” elements to be best understood here? Are we implicitly presupposing that the universal quantifier $\forall$ is indispensable for the most primordial definition of set-hood? At a first sight, the answer may appear to be unequivocal “sure it is.” However, this would open pandora box, many troubles will come out. Even more worrying is this: the logical process of intension as described by Definition 1 itself is fundamentally problematic. First, it seems we must operate on an index set I , but *before* constructing a definition of the new set S . In other words, the method is *recursive* (in terms of sets), meaning that in order to define a new set, we have to first accept the presence or existence of a previous set, here the indexing set I , out of which the new set S may be intensionally constructed (with the help of the propositional function-predicate $p(s)$.) A related concept in the foundations of mathematics is that of impredicative definitions, which haunted mathematicians in the early twentieth century such as Russell and Poincare [106,108]. The second

major difficulty concerns the enormous ambiguity involved with any process of building a rigorous definition of propositional functions that is *philosophically* acceptable [27]. It is well known that no such fully satisfactory definition is formally possible [109], which implies that in axiomatic set theory the thorny issue of the predicate function remains an open gap in the foundations of mathematics that may never be closed. Overall, it appears to us then that intention, as practiced in set theory and logic, can be best captured as more or less a form of “thinking” or “logical viewing” more appropriate for *human* or computerized agents than anything else. For modern mathematics, with its essentially Idealist tilt toward Kant and Plato, the above difficulty with defining prepositional functions or general predicates has not been considered a major problem, but to us here it presents a serious weakness haunting the entire modernist mathematical movement, a problem that needs to be addressed right from the very beginning.

2.3. The Great Reversal of Set Theory

Many writers may prefer to believe that the “essence” of set theory is reducible to that fundamental relation of “belongingness,” best symbolized by the ‘in-operator’ $\in$. Indeed, such formal relation draws a strict distinction between the set and its elements. (We avoid for now dealing with the infamous “very large sets” that “belong to themselves”, like those classes obeying relations of the form $S \in S$, an especially dangerous sub-class of the larger class of self-reflective collections or inconsistent multiplicities [27,37,38].) A view like this is inherently biased since it only sheds light on the *elemental*-compositional character of sets, overemphasizing then the initial intuitive understanding of a set as a kind of “heap of things” envisioned, gathered, and enunciated by a universal Subject or transcendental Ego. (See Husserl’s work on the foundations of mathematics and its relation to Idealism or transcendental phenomenology [49–51]. Furthermore, see Section 2.1 for additional discussion from the viewpoint of this article.) As will be repeatedly shown throughout this article, such exaggerated focus on the heap theory of set-hood is a trademark of the Idealist position of philosophy and the foundations of mathematics. Observe that such an emphasis on the role played by a transcendental Subject or Ego-consciousness is already manifest in *both* the extensional *and* the intensional views (Cf. Section 2.2). Indeed, the propositional form $s \in S$ appears in the two standard modes of defining sets. In extension, we merely list all elements s satisfying $s \in S$; in intension, the list is replaced by the set predicate, a propositional function of the form $p(s)$ or $p(s_x)$, but still with the whole implicit package of the ontology of being-in/here/there, which is entailed by the form $s \in S$, already serving as the ultimate constitutive principle lurking behind the fundamental ontogenetic operation of creating a set. This is because in set theory it is still the *human mind* that is at stake. Everything in modernism is supposed to be serviceable to Reason, Mind, Nous, Rational Thought, Order, Law, and so on [63,94]. Mind literally “sees” sets through the *mind’s eyes*, which are cosmological generalizations of Foucault’s concept of the Gaze gleaned from the social sciences [110]. Within such purview, sets are construed as emerging wholes or unified patterns coming into being *after* their elements or composing parts have already been (ontologically) established in a previous existential stage [49,50]. Take in some elements, group them together, search for a principle of set formation (extension, intension, or anything else), and *then* you get your set. Classical reason sees in this the ultimate justification of set-hood: hypostatizing the in-operator in expressions like $s \in S$, with a de facto acceptance of a *directionality* by means of which some “action” takes place *from* an initial source s toward a terminus S . So if a set S is accepted (constructed), then the form $s \in S$ can be mentally visualized as literally a “sticking” of some already-existing object-element s into the “bigger bag” S ; the relation $\in$ now “feels like a movement”, usually captured by arrows, so one may claim, in an admittedly ontologically vague fashion, that underlying $s \in S$ there is something hidden in there which looks like a directive process of the form $s \rightarrow S$. Let us then express such a process by the following form:

$$s \xrightarrow{\in} S, \quad (3)$$

which will be taken as the traditional onto-dynamic content of the logical form $s \in S$. In general, the assignment (3) contains more information than $s \in S$, for the former includes the latter but also tells us that s must be treated as an *already existing object* that is being *attached to, moved into, sourced into, pushed toward* the “whole” S , giving rise to the *set* itself, denoted by S , through the classical principle of part-whole. (The general structure of the part-whole ontology will be taken up again in Section 2.8. Furthermore, see the second volume of Husserl’s *Logical Investigations* [50] for more information on how the formal apparatus of part-whole may be deployed in order to construct *logical spaces* in a way strikingly reminiscent of the later concept of Hausdorff’s *topological space* [111]. The connection between logic and topology is fundamental for event ontology but cannot be explored in details here. For some previous attempts in this direction, see [112].) We would like now to *reverse* this traditional ontological structure of traditional set theory, i.e., the movement (3), by turning the emphasis from elements to sets. That is, by reversing the direction of the arrow in (3), we propose the structure

$$s \xleftarrow{\in} S,$$

where it would be more convenient now to introduce the converse operator $\ni$, which serves the logical proposition $S \ni s$, meaning

$$S \xrightarrow{\ni} s. \quad (4)$$

So what does (4) mean, and how it compares with (3)? You may think they are formally identical. Strictly speaking, that seems right; the statements $s \in S$ and $S \ni s$ have the same “truth values”; they are “computationally equivalent” to each other. So since modern logic and set theory are thought to be ultimately grounded in computational logic and combinatorialism [10], this conclusion is unavoidable. From our own viewpoint, however, there is nothing further from truth. The two forms (3) and (4) are completely different; they represent diametrically opposite worldviews and cannot be taken as instances of one and the same philosophical set-theoretic Universe. When we write $s \in S$, it is understood that s is *already given*. The *element* s is right here, presented to me as a datum of experience after which we can engage in the process of attaching this pre-given element to an open form, the set S to be defined by this very process as such. *Subsequently*, the set S will come into being. It is this *unavoidably subjective* process of attaching the *already-given* s to the *soon-to-be-determined* set S that defines the set-hood of sets. The idea behind constructive definitions in modern mathematics is precisely such ontogenesis of set-hood via processes of direct attachments. By adjoining the *pre-given* to that which *will* be given, the *Idea* of sets emerges into the ontological scene. Therefore, in modern mathematics and analytical philosophy, the creation of S is an enactment of ontogenesis in and through a universal Subject: the logical Ego or the Transcendental Kantian Subject who is in charge of completing the process of enjoining elements to their “total container”, i.e., the set with which they are bonded by way of the purely formalistic relation *being-in* $\in$ [49,50,113,114].

Now, against this fundamental formal ontological operator being-in $\in$, we contrast the operator of giving-out $\ni$, which is not merely a formal “inverse” of the relation $\in$, but rather a fundamentally new and irreducible happening: the *process of production* as such, though still not in the sense of the extremely generic and broad concept of *genesis* already familiar in nature philosophies (for example, the natural Stoic ontologies of Schelling [115], Bergson [21], Whitehead [79], Simondon [57,58], Deleuze [63,64]). The particular process captured by the operator $\ni$ is in fact a form of *condensation of Becoming*, a “slowing down” of pre-given *fluxes*—not things—culminating in the production of a new fresh object. The writing of the form $S \ni s$ implies that an already-given flux S has just *produced* the element s . The point s is *generated* by its enveloping space S . This is intended as a reversal of set theory that should be one of the main concerns of mathematical philosophy: Sets are not formed by sending given elements into one container or bag called *set*. You do not get a set by “heaping up stuff” into one mental closed space created in the mind’s landscape

(“mindscape”?) of the Universal Subject and then baptize the just delimited space as *set*. Instead, *sets are initial open dynamic processes always in flux. Moreover, some of these “flux-like sets” may eventuate by producing new “elements” out of their own internal immanent dynamics of becoming other.* Within the general parameters of a viable mathematical philosophy serving the ontology of nature, set theory should be redefined then as the generic, minimally skeletal coarse-grained logical framework of the *event-as-a-dynamic-happening*, a concept we have already encountered before in the philosophy of nature [19,39,78,80,115]. The new set theory will be based on the giving-out of elements captured by the formal expression $S \ni s$. Belonging-in, the relation of being-in $\in$, or the entire apparatus of the part-whole calculus, container-elements, and so on, are to be relegated to the background since they represent the less “primordial” aspect of the set-theoretic universe. The reason is that this approach requires an ontological Subject (cf. Section 2.1), hence making the former not fundamental enough for the purposes of the philosophy of nature. Yes, strange as this may appear, at first sight, sets are *not* defined by their elements. On the contrary, these elements that Classical Thought has characterized as belonging to the set, hence allegedly constituting its true existential character, are in fact nothing but occasional *products* formed by condensations of set-like fluxes of becoming. We still need to develop a more careful theory of what is meant here by those ‘set-like flows’, but the main insight to be recorded now boils down to this: A set (according to *postmodern* mathematics) is an inherently *open* system that is *perpetually changing*. We should never forget that Classical Thought likes to express sets as fixed static unified wholes because Classical Thought *is* Platonism [63,95], and modern mathematics, logic, and analytical philosophy are essentially a harping on the very same classical theme [62].

2.4. A New Set Theory?

The interesting question now is whether what we called above “The Great Reversal” may be further advanced in order to construct a genuine new set theory, a postmodern discourse replacing modern axiomatic set theory and its earlier Cantorian forerunner. Can we substitute the “giving-out” relation $\ni$ for the classical being-in operator $\in$? This is a very difficult question, whose full answer is beyond the scope of this article. However, from a mathematical point of view, it is not impossible to imagine achieving precisely such a goal. For example, one may view the now popular category theory [46] as an attempt (with partial success) to replace set theory by something else [116]. However, both set and category theories trace their origins back to one and the same underlying philosophical substratum, that of Classical Thought, here exemplified by intension (set theory) and extension (category theory) [84]. It is possible to offer a general critique of category theory and its philosophy, where it is suggested that such a new approach to the foundations of mathematics is not as radical as some writers would like us to believe and that categorical thinking is deeply rooted in traditional Greek and Platonic modes of research and speculations. This, however, cannot be dealt with in full here, but see Section 3.4 for some additional remarks on category theory drawn from within the larger framework of geometrism. For the time being, let us examine more closely the new proposed view of sets enacted through the giving-out operator $\ni$.

The first thing to note is that the traditional being-in relation $\in$ and the new one $\ni$ should not be treated as inverse of each other, but rather as *dual* operations. That is, $\ni$ can be understood as the dual of $\in$, and vice versa. A first philosophical insight is the one summarized by writing

$$\ni = \in^{\text{dual}}, \quad (5)$$

where the operation of taking the dual is yet to be made more precise through the context. The advantage of such formulation is that it allows us to draw on the formal calculus of set theory while building something surpassing its traditional discourse. However, in order to *effectively* move out of the de facto territory of modern mathematics, it is important to avoid trading the dual operation $\ni$ by a streamlined generalization based on the well-known duality theory of topological vector spaces [117] and category theory [46], with roots tracing

back to special *algebraic* structure highly prized by mainstream mathematical thinking [116]. Consequently, in the remaining parts of this section we try to formulate a conceptual and philosophical approach to the giving-out operator $\ni$, rather than attempting to reuse parts of the formal algebraic-topological duality theory of modern mathematics such as that presented in texts like [46,116].

Let us start by considering a situation where a given set produces several elements according to the directed formula (interpreted through (4)):

$$S \ni s_1, s_2, \dots, \quad (6)$$

where the series of generated elements is infinite but need not be explicitly ordered. Since there is no order relation externally imposed by the generative set S , the system (6) can be viewed as an operation occurring “in parallel.” We then propose the following view:

Definition 2. (The abstract schema of parallelism). Consider objects $s_1, s_2, \dots$, etc, and a set S . A parallel schema of generation associated with this configuration is written as

$$S \left\{ \begin{array}{l} \rightarrow s_1 \\ \rightarrow s_2 \\ \vdots \\ \rightarrow s_n \\ \vdots \end{array} \right. \quad (7)$$

Each of the “parallel arrows” above constitutes an individual event, the happening of the giving-out of one element of S at a time. On the other hand, we say that parent set S , which has created all of these produced elements $s_1, s_2, \dots$, etc., acts like a base or total ground form supporting the existential status of those elemental being exemplified by $s_1, s_2, \dots$, and so on.

The parallel schema in Definition 2 should be contrasted to the purely *serial* ontological mode of presentation entailed by conventional formulas such as (6). Indeed, the latter abstract scheme fits with both the intensional and extensional views. In what follows we explore new possible modes of thinking set-hood based on generative dynamism and intrinsic set-theoretic temporality. We propose that the *sense* of the set S , its “proper set-hood”, is secured by the generative action (7) leading to the production of elements. Elements are *beings*. The generative set is *Being*. We do not get the set, the “big thing”, by amassing “smaller” parts and then imposing some sort of metaphysical “unity” on them, as in Rational Order, Religions, Axiomatic Science. Instead, the Great Reversal of set theory aspires to direct the entire problem along the other, opposite direction where the set always *overflows* its composition. It is true that eventually I can speak of beings like s_1 or s_n as particular or generic elements of the set S , respectively, but this is really beside the point now. The most characteristic trait that we would like to highlight at this point is that in modernist mathematics and Idealism sets are formally constructed by *finite combinatorial operations* enacted in and through a formal machine-like axiomatic apparatus such as the systems of Peano, Zermelo-Frankel (ZF) [83,92,118], von Neumann [93], and many others [1,119]. On the other hand, the real content of a mathematical being (provided something like sets must be brought up into fundamental ontology) is that a set is already an irreducible whole perpetually evading analysis into final parts. The operation $\ni$ records one aspect of the set S , the latter’s capacity to delimit zones of indetermination latent in the set itself and then channel them into an outward-looking concrete form, an elemental being like s_n . It is only *afterward* that we can claim that s_n “belongs” to S , writing $s_n \in S$, but then merely as a matter of *retrospectively* describing what has already happened: the fact of never being able to capture in truth the real force of the Ontological *as such*.

The statement ‘a set overflows its elements’ requires further elaboration. In a nutshell, it explains why there is more to a given set than the collection of elements it produced.

For example, if we say that $S \ni s_1, s_2$, then it may end up the case that S is capable of producing *many more* elements than just s_1 and s_2 . In Idealism, Being is the *determinate*, the *actual* [120,121]. On the other hand, abstract materialism finds in *primary matter* the ontological Open horizon of actualization (Bakhtin [122], Heidegger [123,124], Uexküll [125], Russell [126]), where Being can never be exhausted by mere enumeration or listing of all that exist or have been. Consequently, instead of thinking the set S as a “bag of stuff” s_1 and s_2 , the situation captured by the formal statement $s_1, s_2 \in S$, we reverse the construction and interpret the generic process entailed by $S \ni s_1, s_2$ as an *indication* of some happening, that in which the Undetermined or Open horizon named S has given rise to (or condensed into) concrete elemental beings s_1 and s_2 , while *more still could come out from S* , that is, additional concretizations still affiliated with the very same mother set S . Sets, therefore, may be viewed neither as collections nor aggregates, but rather like *abstract generative principles* existing outside logic and grammar, truly immanent to Nature and incorporating within their territories the seeds of that great crystallization of space and time underlying ontologies of becoming and metamorphoses.

However, how can we *formally* capture this ‘abstract generative principle’ at the roots of the ontological structure of being-a-set? We may try to express the productive operation of the giving-out relation $\ni$ as a processual power of *maps*; for instance, as in the following form:

$$\ni: S \rightarrow s, \quad (8)$$

where the arrow is to be interpreted here as indicative of a function mapping S into s . Note, however, that the codomain is not the “set of possible outcomes $s_1, s_2, \dots$ ”, but is merely *one* of these, say a generic element s . Therefore, the form (8) is not in fact a function in the traditional sense of set theory because the codomain fails to be a set. Moreover, we do not also want to postulate that the domain S is a traditional set since this will undermine our main goal, which is the introduction of postmodern concepts of sets not founded on the intuition of collections and aggregation.

Let us now replace (8) by the following more comprehensive system:

$$\ni (s_1) : S \rightarrow s_1, \quad \ni (s_2) : S \rightarrow s_2, \quad \dots, \quad \ni (s_n) : S \rightarrow s_n, \quad \dots, \quad (9)$$

where instead of one “giving-out” operation $\ni$, we have multiple relation-forms $\ni (s_n)$ enumerated by $n = 1, 2, \dots$, each representing one generative process leading to the production of the corresponding element s_n by the same original set S . In the transition from (8) to (9), we have eliminated the need to interpret each individual relation-form (8) as a *function* in the traditional set-theoretic fashion; but also we have created a more complex and richer content for set theory because the alternative (9) now represents a *nexus* of relation-forms, indexed by the possible outcomes $s_n, n = 1, 2, \dots$, rather than being given functionally after completing the totalities of those traditional sets often appearing as domain and codomain in modern set theory. In other words, the nexus of relation-forms (9) bypasses the classical system of sets, where the latter turns out to be nothing but an incarnation of *geometric-arithmetic multiplicity*, an instantiation of the ontological concept of the *many*. Each one of those relation-forms, say $\ni (s_n)$, is not a function mapping elements of the set S onto elements of another set s_n , but is the *name* of the onto-transformation that *generates*, not map, S into s_n .

Therefore, we suggest that the key mechanism behind the constitution of set-hood needs *not* be fundamentally linked to the concept of the multiple. *Contrary to classical thought, Idealism, and modern (mainstream) mathematics, a set is neither the register of multitude, nor the locus of the many-in-the-one*. Relation-forms like $\ni (s_n), n = 1, 2, \dots$, do not exhaust S precisely because in reality they do not enumerate, count, list, or collectivize a group of “given things”. The elements s_n are not given, but *produced*; that is, the set S generating them *antedates its own elements*, a situation impossible to imagine in Idealism or modern set theory. Indeed, the modernist discourse, which is sufficiently characteristic of this dogmatic philosophical and conceptual orientation of thought, has always been obsessed with closed

systems, complete forms, axiomatic closure, the finished, etc. The discourse of idealism and axiomatic post-Cantorian, post-Russellian, set theory is antagonistic to postmodernist mathematical philosophy, the latter being the position defended in this article. Note that it is not our aim here to introduce $\ni (s_n)$ in order to serve as a proxy for a hidden *function* mapping the set $\{s_1, s_2, \dots\}$ onto the relation set

$$\{\ni (s_1), \ni (s_2), \ni (s_3), \dots\}, \quad (10)$$

for in that case nothing has been gained. Our interest is devoted instead to the *individuality* of each relation-form $\ni (s_n)$, the fact that it stands alone as a *singular factum* or *datum* of experience, a move toward making sets closer to *events* than static completed wholes. A set S is associated with some relation-form $\ni (s_n)$; but *more* forms may be added, expanding then the ontological reach of the set's existential status in the world. Let us remember that S is not the intersection of all inverses of maps of the form $\ni (s_n)$. The relation $\ni$ is a *transformation*, a genetic process. On the other hand, the being-in operator $\in$, is a *syntactical* operation unthinkable without that Universal Subject enacting the corresponding *logical judgment* perpetually at work while buried deep into the innermost depths of Nature's noematic side of the mathematical experience.

It is nearly inevitable that one may fall onto the typical mistake of proclaiming the "new set S " as nothing but that captured by the "set" of generative relations $\ni (s_n)$, $n = 1, 2, \dots$, but in such case we immediately find ourselves trapped in a vicious logical circle since set-hood would have then been already invoked in an impredicative definition of the set itself. This is why we believe the second (detailed) functional form of the giving-out operations (9) is a potentially fundamental change of orientation, a shift of focus that modern mathematics has not yet been able to fully master or even comprehend. The idea is to first note that the arrows $\rightarrow$ represent *neither* set-theoretical functions *nor* category-theoretic morphisms. Those arrows are direct semiotic *representations* of an underlying ontological reality that can never be seized by language or signs: the *pure relation-form of becoming-other in transformation* that perpetually evades representation. However, what if such an underlying, deeper onto-layer of the Real is non-mathematical, un-mathematizable, non-formalizable [11]? A non-axiomatic beginning surpassing all principles of beginning and termination, a rather strange hybrid, an "anti-algorithmic Geist" more in harmony with Heidegger's Second Beginning and Turning [76] than Descartes's first philosophy [87,88] or Hegel's [127] concept of Nature? 'Everything is possible', a hyperbolic proposition meaning that Nature's innermost core could be both mathematical but un-mathematizable [128,129]. The key is to realize that there is *not* only one "kind of mathematics", largely inherited from the Greeks and now practiced everywhere, namely modern mathematics; for it is also imaginable that a non-orthodox *postmodern* mathematics is still waiting to be invented in the future.

2.5. A Philosophical Interlude: Death and Birth in the Set-Theoretic Universe

It seems that we are stuck at a difficult position where the set is constructed only a posteriori, that is, after completing the giving-out operation of its elements as described above. However, even those special elements that have been *already given-out* are not the only objects capable of existing. In the end, a set in *postmodern* mathematics is like the *event*, an inherently *open* system that is never finished. In fact, post-set-theoretic sets are nothing but the pure formal concept underlying events in the philosophy of nature. Although the set S appears to have "expired" after the termination of the act of "giving-out" certain elements s_n , $n = 1, 2, \dots$, the truth is that there always remains the possibility that something new will come up, an erupting chance or an aleatory reshuffling of internal strata that may lead to a return to either the previous onto-generative mode that has given out the elements s_n , $n = 1, \dots$; or that the set S may *implode* into a shrinking state of being less prominent than before, losing elements it already produced. Becoming is both explosive and implosive, progress and decay at the same time. Mathematics captures this through the device we call the constructive/de-constructive definition of the set, the set is determined only as an

“afterthought” consequent on a priori genetic processes of either growth or decay. Birth and death are at the heart of postmodern set theory. Sets are no longer mere collections composed of exactly determined elements with individual existential states controlled by rigorous logical propositions. *A set is a dynamic being.* A set is very formal. It is more generic than the event, but not less profound, for there are “more” sets in Life than events. Life-in-itself is constituted by the act of forming other relations out of previously given ones. A set is *all* those generative relations (9) describing the productions of its elements. Consequently, you can never formally close off the gushing forth of that everlasting stream of elements s_n antedating the naming of a set S in postmodern mathematics. Sets, then, are the *ontological* marker of the most striking affirmative expression of Life’s *surplus value*.

2.6. The Operational Edifice of Classical Thought: A Philosophical Critique of Modern Set Theory

Conventional set theory attempts to proceed from logic to constructive operations systemically deployed in order to create an edifice, a “larger” system composed of “smaller” or “less complex parts”. Obviously, at this point we are unable to spell out the exact details of how such “largeness” or “smallness” can be determined without falling back into Idealism with its obsession with measures and evaluation metrics. Set theory, however, especially in its purest original Cantorian form, has been to a great extent a proposal to establish exactly that difficult position of perpetually *evading* any form of *metric* thinking whatsoever, all while continuing to reflect on the nature of the large and small, the part and the whole, and so on. This modern theory was constructed by actually defining concrete *set-theoretic operations* allowing one to *compose*, and hence *construct*, more complex systems starting from already given subsystems. These include, most importantly, the classical operations of *union* and *intersection*. Others like *difference* and *complement* can be easily defined but they do not figure as prominently as the first two. To a large extent, *combinatorialism*, the ontology founded on finite combinatorial operators, is currently the prevailing underlying philosophy of modern (Idealist) mathematics. Set unions and intersections are exemplary combinatorial operations as we will see below.

Recall how a set is defined in Idealism (modern set theory): A set S is the being-united in mind (or by a Subject) of a multitude of elements $s_n, n = 1, 2, \dots$, such that a logical predicate, the *set predicate* $P(s)$, is true [15,27,76]. That is, we write

$$S = \{s | P(s) \text{ is True}\}. \quad (11)$$

Now let us imagine that we are given *two* sets instead of one, say S_1 and S_2 , with corresponding set predicates $P_1(s)$ and $P_2(s)$. How can you combine the two sets in order to produce a new one? The answer is that there is an infinite number of possible ways to do that, depending on how advanced is the formal *logical* apparatus available for doing calculus on predicate spaces. However, at least in an initial approximation, sets impose themselves as *spatial* beings, that is, not as logical constructs. Whatever the importance of logic in set theory, the purely formalistic logical approach to the subject was introduced only later, *after* securing the main intuitions of set theory on the basis of spatial considerations alone. In this geometric approach to sets, people (including Cantor [76] though not Russell [27]) did think of sets as blocks of spatial parts (regions, domains, neighborhoods) that can be mentally visualized, even when not readily available to immediate sense perception.

Fix your imagination on the two sets S_1 and S_2 . Bring them “close” to each other. How close? It does not matter now. Just follow *your* thought, *your* imagination. It is all in the head, literally. Platonizing away subjective Idealism by replacing thoughts with so-called Objective Ideas is in order. Now, you only need to look into the problem from the viewpoint of the Cartesian Subject. What do you see? Two sets “hanging up in idealized abstract space”. Bring them near to each other. They come close, they *touch*. They *overlap*. As a matter of fact, they *unite*. The merging of the two into one. The two become one. The Many a unity. Change? Transformation? Not really. It is only the *appearance* of change, for nothing, absolutely nothing, has been altered when it comes to the individual characters of the concerned sets. We say that after touching each other the two sets united into one

set, say S . We write this as $S = S_1 + S_2$, or in the modern notation $S = S_1 \cup S_2$. However, S_1 remains S_1 , S_2 remains S_2 after this union operation. To see how this situation may be escalated to an extreme level, consider that the two sets do *not* need to even “touch” each other. Indeed, imagine S_1 and S_2 held apart. You still can *think* the act of combining the two (mentally, just visualize it to yourself). The new name, the set $S_1 \cup S_2$, is merely the *representation* of this imagined, perfectly *visualizable* process of the combining of the two into one set S .

Set unions then are essentially logical, not physical. Theoreticians already know this [1]. They historically defined the operation $\cup$ as follows [38,119]:

$$S_1 \cup S_2 := \{s \mid P_1(s) \text{ OR } P_2(s) \text{ is True}\}, \quad (12)$$

from which you may readily see that it is the *combined proposition* ‘ $P_1(s) \text{ OR } P_2(s)$ ’ what plays the role of the set predicate of $S_1 \cup S_2$. The operator OR is a logical operator. *Logic is the foundation of the operation of uniting two sets*. For this reason, the union of two sets is not a process, but a logical (combinatorial) operation after which the two sets that have just entered into this $\cup$ -scheme retain their individual identity. In logic, predicates like $P_1(s)$ and $P_2(s)$ “stick around”. (No predicate left behind). Thus, there are now *three sets*: S_1 , S_2 , and $S = S_1 \cup S_2$. This is in direct contrast to dialectic, where in the latter the thesis and antithesis are pronounced “dead” after their joint sublimation into the synthetic unity of the two [127].

Now what is the precise meaning of “touching”? How do you know that two sets brought “close” to each other actually “overlap”? Again, the technical solution rests on the use of logic, which will be deployed in order to render what was originally a vague *spatial* picture a very precise *computational mechanism*. The definition is very obvious. If we have two sets S_1 and S_2 , then their *intersection* set $S = S_1 \cap S_2$ is defined by the same form (12), with the direct replacement of the logical OR operator by the AND operator. That is, we have

$$S_1 \cap S_2 := \{s \mid P_1(s) \text{ AND } P_2(s) \text{ is True}\}, \quad (13)$$

where again, the predicate calculus is deployed in order to define new sets from old ones. As we can see, underlying the set-theoretic construction there is always a *parallel logical construction*. Set union is founded on logically combining predicates via the binary OR operation in order to form a new predicate defining the union set; set overlap or intersection is likewise defined such that a new proposition is formed by the AND operation. Those new, more complex propositional functions are then always invoked to justify the existence of newly constructed sets.

Without logical calculus, it would not be possible to define set operations. However, set operations are not logical operations. Logic is about the laws of thinking, thinking happening in the Subject’s mind, or in Mind (whatever that is), while sets are supposed to be formal ontological categories of the Real. What is missing here? Why should logic and reality be so tightly correlated in such a manner? Logical positivism [130,131], the early Wittgenstein [132], and orthodox analytical philosophy [133] believed there is some deep organic connection between the world of things and the world of thoughts. Words and things are presumed co-ordinated. Spinoza and their stratification of the Real (The Absolute [3]). Various forms, multiple theories, precarious ontologies, all amounting to the profound illusion of the Global Isomorphism between expression and content. Even Deleuze and Guattari fell into the trap of *pre-supposing* the isomorphism without demonstrating even its so-called universality in spite of the fact that the ontological structure of expression/content in *Capitalism and Schizophrenia* [61,62,65] is certainly not Idealistic and remains very sophisticated. The truth is that logic and semiology are both inadequate for dealing with the project of a total ontology of the Real. Neither words nor things are fundamental ontological blocks. These two concepts are “fabricated stuff” [11,134]; like propaganda slogans and ideological affairs, they are merely noematic constructions [113,114] produced by existing mechanisms of subjectification already at work in nature [66]. The only difference

is that the corresponding process that created the Grand Isomorphism (between words and things) is a collective (unconscious) process of nature, for example Guattari's abstract desiring-production machines [65,67].

Set-theoretic "operations" like union and intersection are the ultimate archetypical representatives of combinatorialism, the ideological counterpart to Idealism's perception-based ontological agenda. To *combine* one being with another you claim that beings are to *be composed* with each other through the execution of set-theoretic intersection and union operations. Several sets can then be *combined* into one whole (union), or new "smaller" beings can be generated through the removing of the nonoverlapping regions (intersection). In other words, union and intersection are the fundamental *spatio-logical* operators of *thought* deployed for its grand quest to construct a total ontology of the world. Now, in modernism (Idealism), being is to be a set [83]. A set is a whole unified by thought [76,127]. Thought is mediated by the enactment of the fundamental set predicate (Definition 1). Following Heidegger's ontological difference, we distinguish between beings and Being [69,135], so we say that beings are *included* in Being via the operator of set-belonging $\in$. Unity (of set) is logical unity. Being is Unity-in-Multiplicity. Classical Thought believes that Being is exhausted by such spatio-logical approach to being-as-set, that is, generation through combination, unity through logical unification, and so on.

However, now the question is this: how do you know that several elements belong to a set? Because they satisfy the set predicate. However, where is this predicate? In the Mind, Ego, the Absolute, all different instantiations of the same ontological structure: the Transcendental Subject [113,114]. This subject is the fundamental metaphysical agent presupposed by modern mathematics and philosophy, by idealism, by crude materialism, by empiricism [83,136]. It is the founding figure behind the direct approach to reality through language, logic, geometry, and arithmetic. The predicate can become impersonal (Hegel's Logic [127], see also Heidegger's critique [137]), but in truth, it remains fully modeled after the human subject introduced by Descartes, with roots going back to Plato, Plotinus, and Augustine [28,95,138]. This is why we believe it is very dangerous to take set operations like unions and intersections very seriously. They fail to probe deeper into the being of the set since their entire execution hinges only on mathematico-logical considerations, merely the computation of truth values of various propositional functions. For example, sets remain sets after union. There is nothing ontologically new in taking the operation of the intersection. Unions and intersections are inherently passive operations of thought. Both fail to bring into the world something that never existed before. This is why modern mathematics, in its complete reliance on *spatial intuition*, risks getting fatally trapped within a narrow framework of thinking the mathematical that is neither ontologically productive nor synthetically adequate.

The combining of two sets S_1 and S_2 into one set S as defined above is purely *algebraic*. Rules of "proper conduct" obeyed by union and intersection operations can be readily drawn out and strictly enforced. Nature's "behavior" is then supposed to be capable of being brought under close observation, and measures have been taken to ensure that every happening in nature comes under the regulation of strict law. However, is such joining up of two sets into one a true *event*? Is this a genuine *happening* of the Real? Does an *algebrized* set theory provide an access to the inner depth of *dynamic* being at the root of every *substantial* being? Well, if we define 'substance' as the counting-as-one-of-the-many, the bringing up of unity-in-multiplicity-through-cognition, then certainly the rich diversity and complexity of Beings-in-the-world *can* be exhausted in and through set-theoretic ontological modelizations, for here Cognition joins logic in the quest to conquer Nature. However, Being is not a heap of beings. Being is not an abstract generalization of set-hood where beings are members of a set coextensive with Being-in-itself. Being is not determined by logical concepts. Being is non-dialectical in and through, refusing hence to enter into rivalry with thought as long as the latter models itself after cognition.

Let us now briefly outline a view against axiomatization. Clearly, the well-known—though rarely openly acknowledged—facts concerning the metaphysical shallowness of

axiomatic set theory can be traced back to this fundamental ontological blunder: excessive reliance on logic in which ontology was replaced by computations, metaphysics traded for “mathemes” or “rules of conduct” regularizing operations and dictating allowable combinations. The ongoing mathematization of philosophy is in fact not mathematization at all; it is a ruthless *subordination* of the inherently open questioning endeavor of authentic philosophical thinking to the fashion of closed and completed axiomatic exact systematization. Idealism, especially in its most vulgar reincarnations found in Badiou’s “System of the world” [83,85], is the refusal to admit deep and profound meditative reflection on being and worldhood. On the other hand, the philosophy of nature continues to resist giving the upper hand to perception and introspection, favoring instead the more penetrating anti-personalistic thinking style of abstract art, non-axiomatic mathematical exploration, and poetic reflection (Taoism [139], Heidegger [124], and the Stoics [140]). However, mathematics itself remains the proper ontological stage on which the drama of the field of the non-personal experience of Nature unfolds, a space of creative production as fundamental as poetry and abstract painting. It is a world on its own that possesses a strong independent inner will even while—as in postmodern mathematics—free of the fetters of exact axiomatic rules of affairs. This is why we diverge from Heidegger’s overall rejection of modern (Cartesian) mathematics [95], the latter a relatively recent field of human knowledge whose mother matrix he (rightly though) equated with mathematical physics [94]. Heidegger’s critique has forced us then to tilt toward the alternative “hybrid” approach of Leibniz’s philosophy of nature [19,141], especially in its confrontation with the labyrinth of dynamism in space, which brings in the vexing problem of time in mathematical being [142]. The only thing that matters in real mathematics is that you remain *rigorous* and *abstract*. Being *exact*, however, is *not* part of the essence of the Mathematical [143]. The modernist Euclideanization of mathematical being, for example as in Royal Science [55,62], has been a major blunder we are still paying its heavy price up to the present moment. However, there *are* alternatives to modern mainstream mathematics, though extremely rare (aside from Exodus, Leibniz, Cantor, and Russell, how many more thinkers do you know who can serve as good representatives of *postmodern* mathematics?) The problem then is neither to reject mathematics altogether (Heidegger) nor to subordinate philosophy to it (Kant [120,144], Weyl [17], Lacan [145], Gödel [4,42,43], Badiou [83]). Moreover, patchy attempts to hybridize Euclidean mathematics and ontology (Hegel [127,146], Whitehead [147], Lautmann [41]) will never suffice as long as the philosopher is not actively engaged in changing the very concept of the mathematical as such. Indeed, as far as modern mathematical structures are still being exported into the philosopher’s machine and reappropriated for their own agenda, previous idealistic archetypes will keep resurfacing again in the very new philosophical system being constructed for the simple (but subtle) reason that idealism has always been lurking behind mainstream mathematics itself since its inception by Plato and Euclid until today. Therefore, “being inspired by mathematics” (à la Lacan, analytical philosophy, etc) does not imply that something revolutionary will be reached at the end of a bold mathematics-inspired “new” philosophy, for it is precisely *that* so-called “inspiring discourse” itself, Euclidean mathematics, what has been plagued by Idealism, logicism, anthropomorphism, semiotism, and absolutism.

2.7. Being and Set-hood

2.7.1. Being a Set

We are trapped in the *idea* of the set. The problem: given a “heap of things”, unite its elements into a set. However, what is a thing? the generic name of an element belonging to the set you just formed? It is either no set at all, or *is* a set. Axiomatic set theory assumes that all things are sets. A mathematical being is a set. The being of an object is its set-hood. Again: A set is a heap of things. So you *gather* various things. Multiple beings coming into *one*. Is a set the *name* of the One? If so, then mathematics becomes the science behind *henology*. However, mathematics is not henology; it is ontology. Can we have multiple ones, just in case someone may interject that there are already many sets in the world?

So a set is not *the* One's name, but could it be the name of *a* One? Well, what does that mean, anyway, to speak of "multiple Ones"? The problem again: Should set-hood in mathematical being be posited as the starting point of ontology, or, instead, the latter's mere end product? It appears to us that set-hood in modern set theory is the *termination of ontology*, and that is so for reasons that have to do with the undeniable absence of even a minimally sufficient *ontological* depth in axiomatic set theory. You cannot regulate being by declaring axioms, for instance by merely pronouncing that one set exists, say the *void set* $\emptyset$, only to proceed after that to build on the basis of this declared void-being all other sets using the now familiar sophisticated constructive recursive set-theoretic apparatuses taught in standard texts [83,92,119]. What is missing in modern set theory, also and the various proposed mathematical ontologies revolving around it, is a deep and *independent* investigation into the metaphysical core of the concept of being-a-set. We must even change the emphasis to write the last expression as being-*a*-set. The structure of this ontological formula is very curious and has not been taken into account so far, except in Russell's early work. A set is not *the* set, which means every set is both *one* being and *the* unity underlying its so-called constituents. So there is the gathering-together of the many into one [69] that brings the discrete or demarcated "stuff" called things into a common abstract framework, *set-hood*, capable of giving the entire cluster of things we started with a form of unity that did not exist before. However, it never existed in the Mind of the observer, right? What is the basis of unity in set theory? The logical predicate, the set predicate $P(s)$, cf. Definition 1. However, that makes the gathering-together into set a *logical* concept grounded (exclusively) in the presence of a Subject enacting the logical *judgment* through which every propositional function can be enunciated [114]. However, we are trying to see if this is really necessary. Do we actually need a transcendental Subject like that in order to describe the unity of a set? If you are looking for an *assertion* about sets, then the answer is yes [113]. However, if ontology is to become a *descriptive* science, then there is a need to determine unity without recourse to traditional formal logic [148], and it is precisely here where we find the *axioms* of modern mathematics playing their destructive part. They just *presupposed* a solution to the dilemma of the set's (onto-)unity, one of the most difficult and challenging problems of formal ontology, without in fact conducting any serious inquiry into the matter. However, this is not all. The immediate next problem is that the unity of the set presupposed in set theory does not exhaust the Real because *many other sets still need to be accounted for*.

How can logical unity be sufficient for pushing the world forward through successive developmental and evolutionary stages of progressive complexification and diversification, all without relying, at every step, on that not-so-sincere *axiomatic* intervention by a wise, all-knowing Universal Subject who "flexes its muscles and fixes things" by merely erecting one principle here or posing an axiom there? It seems that in set theory there is an unavoidable tension already at work between the multiplicity of readily completed sets on one hand, and the multiplicity inherent in the individual set's elements togetherness-within-one-set on the other hand. A set was brought into unity by gathering-into-one the various elements it is supposed to encompass, while *other fully-unified sets must also be around*. How can there be one being and the One Being at the same time? Why is it that we approach the unity of one set through the logical predicate, while the diversity of the world into multiple sets must be left for subsequent context-dependent constructions? Is it the case that being and beings are fundamentally different in set theory? Russell's theory of types [27,37,109] is probably the most important ontology we have now that is based on recognizing the importance of explicitly positing a fundamental distinction (Heidegger's ontological difference?) between being and beings, between a set of elements and a set of sets. However, we are not there yet. We do not know if set-hood is Being. We do not know if sets are beings. Nothing clear or decisive regarding this matter can be found in the formal content of axiomatic set theory, nor Gödel's theorems, nor Russell's system of *Principia Mathematica*.

2.7.2. Being-*a*-Set

Formally speaking, in axiomatic set theory a set is taken as an undefined entity. Modern mathematics believes it is a matter of being “ontologically profound” not to engage in defining the most primary terms. Yes, every definition is circular [108]. From Aristotle to Hilbert and Frege, no real progress in the problem of the formal definition has been attained. We need a different strategy. Axiomatic set theory offered one, the *formalist* approach famously espoused by the Hilbert Court and its two crown princes, Weyl and von Neumann. There, a set is never fully defined. Instead, you *start* with some *pre-given* set, most conveniently the void set, then build new sets out of the old ones using set-theoretic operations like union and intersection. The exact procedures are not difficult to follow but they are both clever and philosophically not very interesting.

We will use the formula ‘being-*a*-set’ to describe the situation encountered by every attempt to either ontologize modern set theory or to build a new formal ontology directly based on modern (mainstream) mathematics. The idea here is to emphasize how objects and entities in mathematics are ultimately understood as either some sets or forms defined in terms of pre-given sets, an observation that remains true even after the advent of category theory.

Let us take up again the problem of unity. Consider a group of multiple things, like those composing a heap of stones or objects. First problem: How do you know that the objects or stones are in fact *things*? What is the thing here? Is the thing-ing of a pre-given object something consequent or subsequent to being part of a set? Again, in order to rescue set theory, the Universal Subject is invoked. It is presupposed that things have very concrete meaning since they are mental constructs created by a *cognitive* faculty advanced by the Universal Subject as a response to the reception (and internal processing) of various sense data, like those passing through the circuits of touch, smell, vision [94]. Set theory should then be at best approached as a theory of mental constructions, a field of nature belonging to the *mindscape*, the noosphere [149], instead of material nature’s landscape. Mindscape is that parallel Platonic cosmos where only fully-formed Ideas live [79,138,149,150]. It is the Perfect Sphere of Absolute or Eternal Existence. By way of this orthodox (or heretodox, depending on your view) Platonic perspective, an intrinsic and authentic superiority of mathematical being acquires a special ontological value because now an object can be founded on those cognitive processes at work in humans or intelligent automata, which are in turn still somehow “naturally attuned” to the objective Platonic realm of pure Ideas. Since Ideas are as real as sense data, the concept of objects and things, as mediated by the human or Cartesian Subject, becomes then ontologically legitimate or at least highly defensible. So in Idealism, set theory indeed presents a very natural formal ontology or a basis for constructing new ontologies of the world. By proclaiming being to be a set, the emerging ontological structure, the one denoted by being-*a*-set, is precisely the marker of set-hood as the joining up of things into one unity. It does not matter that in some formulations the only things you start with are the “singular object $\emptyset$ ”, the so-called void set; even the void set is supposed to be *a thing*; it is not *the* thing, but one possible thing that is also *singular* in the sense of being extraordinary. Now, by being *a* thing, *other* things can be constructed out of mere thing-hood, such as the set of *one* void $\{\emptyset\}$ marking the onto-form of one-ness, the *two*-ness form $\{\emptyset, \{\emptyset\}\}$, and so on.

However, in this manner being is fully determined by its characterization as one possible set among others. First, there is the generic and abstract concept of simply being a *set*. Next comes the more concrete determination of being *a* set, that is, a *specific something*. (These ideas were fully articulated first by Fichte [121], and then passed over to his famous pupils Schelling and Hegel.) What concerns us here is the rich and double ontological sense of the structure being-*a*-set, which betrays a complexity of hidden significations that does not often pop up in standard accounts of set theory, not even the philosophical work of Badiou’s *Being and Event* [83]. More and more we find ourselves drawn toward the situation where a set in modern mathematics can signify a doubled, deeper hidden character by

being (1) first and foremost the name of being-unity on one hand; and (2) the marker of a concrete actualization of some latent potential on another hand.

A fundamental inadequacy of modern set theory can be captured by the observation that it does not permit the treatment of *a* set as an *event*. Indeed, the *concrete* character of being-*a*-set does not receive proper foundational attention in the mainstream mathematical account of the subject, the reason, we believe, being that modern mathematics is itself founded on transcendental idealism, where a Subject must always be presupposed (Descartes, Kant, Husserl, Weyl) in order to effectuate invariance, logical judgment, and computations of symmetry groups [17,108,151]. This all-one “view from the top”, instantaneously sweeping then absorbing the entire horizon of the world, is the crux of the standard Western metaphysical dogma further ratified by mathematics’ embrace of what was dubbed *mindscapes*, which is an inherently Platonic-Idealist ontologization of the Real [138], an unwarranted grounding of modern set theory based on exporting Set-hood to the realm of *concepts*, then refashioning the latter after logical unity, then erecting the idol of self-certainty through a return to the self via regulated change and controlled variability.

However, can we consider *a* set *the* form of an event? By no means. Sets are less event-like than being generic empty names of some logical possibilities. It is true that the abstract name of a collection, the set *S*, is still *something*, and therefore a form of a *happening*, but here there is nothing more than what first meets the eyes of the beholder: a linguistic, semiotic *name*, hence one signifier among others.

Sets do not signify. They are *the* signified *par excellence*.

Since in mainstream mathematics the ultimate constructive definition of a set rests on a logical evaluation of some propositional function, set-hood is inherently “static” and lacks that vibrant dynamism characteristic of *becoming*. One feature of this shortcoming is the inability of sets to *refer* to extrinsic being. Sets may only *represent* or *stand for* something else, usually by the sheer force of a metaphysical, physical, or logical argument, but they would never constitute the “thing-in-itself”: the residual remainder left after clearing away all sense impressions and logical constructions; the Heideggerian abyssal hole-in-being crafted by techniques of becoming-other; the marker of perpetual shift and displacement along the infinite chain of beings, eternally evading getting pinned down onto the idealist table of constant essence, permanent presence, beautiful soul, and so on.

2.7.3. Thing or Object?

Is being-*a*-set the formula of a thing or an object? However, do we know the difference between the two concepts? We tended to deploy the two terms somehow interchangeably, though in general there is an inclination toward assigning a stronger ontological sense to things than objects.

Things are beings [69]. Beings are actualized Being [63]. Actualized being is a being-present state of being something. A thing is already *something*. If a thing is considered *anything* (a kind of “no-matter-what”), then we would risk falling back on the discredited Hegelian categories of abstract representation.

We may think of a thing as that which is in the process of “thing-ing” [94]. To thing is to *express* (but never to assert) the thing-hood of the thing, which is related (but not reducible) to being’s coming to be (in the world). Only concepts and representations *assert* (usually via propositions and judgments) [152], while processes in immanent ontologies of nature [79,153] only *express* the interior happening of that which is in the act of becoming-other [154].

On the other hand, an object is always, and without exception, that which comes to be in and through a subject [136]. Objects are never isolated or self-sufficient causes; they must be incorporated into an active ulterior subject that is itself more of a thing on its own, but never objective (there is no “objective subject” [155].) So objects can be only *thought* through a thinking subject; objects can be *viewed* from the vantage point of a *transcendental* structure [49]; the objective is “justified” by the organic [146]; and so on. Let us look into the ontological process at a more formal level:

- (i) *Entities* are things concertized by non-subjective processes already at work in nature.
- (ii) A thing antedates an object. Things are formal roles (empty “containers” of onto-sense) partially fulfilled by objects and entities.
- (iii) Being leads to beings, beings become things, things metamorphesize into objects and entities.
- (iv) Being-*a*-set is the ontological structure combining *both* entities and objects.

Let us see how. First, we note that according to modern mathematics and its Idealist background ontology, sets are initiated by a *subject* that is forced to deal with entities: physically-existing stuff like chairs or hadrons; or cognitive entities such as ideas, images, sense-impressions. All these *are* entities, whether being stones or dreams it does not matter, for they were fabricated by *generic* non-subjective (non-intentional) processes already in natures (for example whether high-energy or brain processes, both are non-subjective in the generic sense). However, the *act* of bringing into unity all such plethora of entities under the name-banner of the set *S* is precisely the infamous task rigourously carried out by the transcendental *subject* (through the logical machinery of set predicates). Hence, in the structure of being-*a*-set we are dealing with a singular form of being hybridizing *two ontologically distinct categories of potential being*, the category of objects and the category of entities.

However, does merely acknowledging that being-*a*-set integrates both objective and entitative concepts within one and the same framework allows us to start from the fact ‘being a thing ontologically antedates being an object or entity’, then move on to deduce that ‘sets are things’? Not so fast. A thing is a being stripped of virtuality. Being is the original gushing-forth of becoming in the one, the joining up of mystery and the cosmic sharing of all that is; in one word: The Virtual. When Being loses virtuality, it does not become “dead matter”, it becomes a *thing*. Thing-hood is the deathly state of being, the first precursor to beings in the world. Only when Being loses virtuality does something come into being out of the infinite virtual field of primary matter. The transition from virtuality to the event in our understanding of set-hood in mathematics and ontology will be taken up again in Section 4.1.

2.8. The Doctrine of Part and the Whole: From Organic Being to Power Sets

We would like now to examine more carefully the doctrine of the whole and part, especially the often vexing manner in which holism has inadvertently inserted itself into the biological in general, the organism in particular. There is something uncanny regarding this philosophical obsession with totalities, the complete, the whole, and the various ontological modes exposed throughout the “Journey toward the Organic”. To a large extent, a focus on the biological has characterized mathematical thinking in the first half of the last century, especially the works of Bergson [20,21,68] and Whitehead [79,156]. Husserl, who was originally a mathematician, approached mathematical philosophy also from a biological perspective, especially Brentano’s empirical psychology [49,50]. No comprehensive reexamination of the foundations of modern mathematics is complete then without some reflections on the doctrine of holism and how it relates to the philosophy of organism.

Is the Whole the Complete? Is the Organism a Completed Totality? Can we claim, with numerous others, that the Organic is yet another name of the Whole? Probably one way forward is to summon the Mathematical into the philosophical scene, asking whether Sets-as-Totalities can play the role of the Organic Whole. So the concept of set-hood is injected back into the picture, meaning we must question the very idea of the set by subjecting it to a rigorous and meticulous examination taking into consideration concepts of whole, totality, the local, parts, and so on. No comprehensive analysis at this level is possible here. We only give brief remarks and suggestions connected with the main objective of this article, which is reviving the concepts of time and temporality in contemporary mathematical thinking.

For the set, the *static* object studied by mainstream mathematicians, is a fully-developed totality of elements. (Recall the concept of *consistent* totalities so important in the early

years of set theory, that is, late nineteenth, late twentieth centuries [1]). A given set S is a *totality*. However, it is a totality of *things*. The “things” out of which a totality called set is formed are nothing but the set’s *elements*. So for a set S to constitute a totality, it must be viewed as being comprised of elements. Again, common sense proclaims sets to be essentially “heaps of things” (cf. Section 2.1). Now, let us envision those “heaps” called sets as each representing a “total whole” in the sense of a completed object standing on its own, whether presentable to common sense and intuition or not is not the main issue now. All that matters here is that some logical or ontological *unity* is enjoined with some collections, in effect turning the latter into sets.

So let us start with one such totality, the set S . Because it originated from a collective mode of co-existence distributed through the totality of all S -elements, we are motivated to imagine the creation of a *new* set, the power set $\mathcal{P}(S)$, which is the set of all subsets of S . Each element of this new set $\mathcal{P}(S)$ is a subset of S . That is, if s , which is some generic element of S , is in a set u , then u is a subset of S , which, in turn, makes u an element of $\mathcal{P}(S)$, the power set of S [27,92]. In other words,

$$s \in u \implies u \subset S. \quad (14)$$

In the extensional view of set theory, the relation ‘part’, represented by the set-theoretic symbol $\subset$, is intuitively defined in terms the set’s elements, so we say that, by definition, $u \subset S$ if and only if it holds that for every given generic object s , the proposition ‘ $s \in u$ ’ implies ‘ $s \in S$ ’. That is, conceptually speaking, we start with a purely generic givenness (any given object s), then adding a membership condition ($s \in u$) we attain a *transfer of set-theoretic membership* by logically deducing the completely distinct proposition ‘ $s \in S$ ’. From the ontological perspective, this transfer operation moves us from a local setting ($s \in u$) to—at least a relatively—global level ($s \in S$).

Furthermore, from the viewpoint of this article, we have two characterizations here. First, in the language of “static sets,” i.e., the conventional system of axiomatic set theory critically reexamined above, the previous definition maybe formulated as follows:

$$(s \in u \rightarrow s \in S) \rightarrow u \subset S. \quad (15)$$

For *dynamic* sets, the corresponding rendering of this definition reads like

$$(u \ni s \rightarrow S \ni s) \rightarrow u \subset S. \quad (16)$$

Therefore, in the dynamic setting a set u is said to be part of S if and only of the following happens: whenever the set u gives out an element s , the set S also gives out the same element s . However, now the question is whether (15) and (16) are equivalent. This is hardly important. The two definitions might appear at first sight as *logico-computationally* equivalent, but their meanings are quite different. The static version (15) is interpreted as follows: If the mind of the mathematician is engaged with the process of examining all elements of u , it will find that every element $s \in u$ is also in S , which means that set-part-hood relations symbolized by $\subset$ are effectively *discoveries* made by logical calculations. On the other hand, the dynamic version (16) suggests that there exists some form of *coordination* between various processes of onto-production: The two initially independent productive operations $u \ni s$ and $S \ni s$ always occur together. A transcendental vantage point, a universal all-knowing mind, computing and distributing truth values, is no longer needed in the dynamic version (16). In order to conclude that u is part of S , all that one may need is the purely coordinative or consistency conditions mandating that two productive giving-out dynamic set theoretic processes are entitled to take place together: if one of them is the case, the other will be, and if one of them does not obtain, the other will not too. This is now a story taking place in the theatre of ontogenesis, not the static being of axiomatic set theory.

We can make now the following observations on the philosophical form of the dynamic version of the part-whole relation (16):

- (i) Dynamic set-part-hood, then, is a matter of *coordination* or *orchestration* of what may otherwise appear at first sight as several unrelated ontological processes of genetic production. These are precisely the parallel giving-out subprocesses (Definition 2) by which different sets give rise to (or produce) their elements.
- (ii) Note further how the concept of part-hood is *global*, at least at the level of the sets considered: it is *not* enough that *some* elements produced by u turn out to be also produced by S ; instead, it is absolutely essential that *all* elements ever produced by u happen to be also generated by S .
- (iii) However, not vice versa: it does *not* matter that some elements produced by S are never found to be produced by u . A sort of *directionality* is then always found to be present in part-relations; the relation $\subset$ is *asymmetric* [27].
- (iv) We may infer then that the relations $\in$, $\ni$ and $\subset$ have something in common: the three are asymmetric. However, the relation $\subset$, which is already more complex than $\in$ and $\ni$, differs from the latter two in being global in character: the manner in which $\ni$ enters into the composition of $\subset$ necessarily makes the latter an operator of *totality* (in logic this is called *quantification over entire set* [38]), where for carrying out part-relations processes like $\subset$, the *full* or *total* body of a given set is traversed through inherently *local* relations like $\in$ and $\ni$.

Regarding the philosophical concept of power sets, power sets, or the bare *belief* in the existence of such sets, constitute one of the boldest leaps of imagination ever undertaken by the human mind. Mathematicians have always believed in the ability to form a set of parts out of a pre-given whole, but it was only with Cantor [76] that this systematic, almost “automatic” mental process, was institutionalized. Indeed, one usually assumes that if I am given a “block of something”, then it is a straightforward matter to *divide* this something into “smaller” parts. The parts are supposed now to be somehow *separated* or *differentiated* from that mother block. The latter is the ‘parent’, while the former are ‘children’, ‘offspring’, and so on. However, is it really as obvious as just prescribed? In fact, nothing is less *intuitively* obvious. As usual, fundamental confusion has crept into the mathematical picture. Let us see how. First, the mind typically operates with a given a priori intuition of position space that is both inherited and acquired (Piaget [157,158]). The prototype of the process of forming the power set $\mathcal{P}(S)$ out of S , that is, the operation

$$S \rightarrow \mathcal{P}(S), \quad (17)$$

is *line division*: given a geometric line L , one imagines a subdivision of this line into a set of generally unequal *segments*. When those segments are joined in together they would give back the original line. This process is often treated as an exemplary and unproblematic generalization to be ultimately based on common experience. However, the process of *segmentation* is very limited: you cannot divide a line L unless you have *already* secured a solid grasp of the *whole* object L . A whole is decomposed into parts, and the segmentation is nothing but the pure, crude expression of this process when executed from within the view of a *knowing subject*. What a *subject* can *not* foresee, however, is the inherent element of *surprise* latent in every operation leading to the formation of a power set. Indeed, Cantor’s celebrated *diagonal element argument*, a landmark in the history of thought, reflects this awkward moment of being “caught by surprise” in the most direct manner: Starting from a countable set S for instance, one ends up with “more” elements in the associated power set $\mathcal{P}(S)$ than what you have “seen” already present in the original mother collection S itself.

2.9. Objects and Elements

Up to now, we have been freely using vague expressions such as ‘things in sets’, even referring to them as ‘objects’ or ‘elements’. At this stage, it is preferable to draw out some important distinctions. In the axiomatic set theory of Zermelo and Fraenkel (ZF), a new set

is generated out of an old one through a logical predicate $p(x)$. That is, if we are given a set S , one can generate a new subset $u \subset S$ out of S by introducing a “property” enjoyed by all members of u . The “property” in question is captured by exactly the logical predicate $p(x)$, where x is a logical variable ranging over propositions. The Axiom of Separation, the principle of comprehension as it appears within **ZF**, is then given the following form: If an object a possesses the property captured by the logical propositions obtained by substituting a for x , i.e., the statement $p(a)$, then the object a becomes an *element* of u . Therefore, there is a fundamental distinction between objects and elements in modern set theory. However, it is important at this stage not to mix the *philosophical* concept of objecthood (that which is comprehensible only in reference to Cartesian–Kantian–Hegelian Subject) with the technical (overtly generic) scope the term possesses in set theory. In the latter case, objects are “things not yet baptized as members of a new subset u separated (demarcated) out of an already existing mother set S ”. As such, the concept of objects, when used to describe the relation between an item and other sets, is not very significant. Mainstream mathematics is really concerned with *elements*, rather than objects, the reason being that intuitively most working mathematicians believe that sets constitute the ultimate foundational layer of their discipline.

It is interesting in this context to recall the so-called Urelements, or sets without elements. Can there be a set that has no elements? We have seen that *objects*, when it is not decided whether they belong to a given set or not, i.e., while *not* yet treated as *elements*, would behave like a “pure stuff” that is *undetermined* from the set-theoretical viewpoint. However, some mathematicians did consider *urelements*, which are complete sets that can become elements of other “larger” sets, without the former containing any element at all [1,83]. That is, an urelement u can be a *member* (not part) of another set V , or $u \in V$, though no elements whatsoever have been found to compose u itself. This is very interesting since it paves the way for the concept of *dynamic sets* introduced earlier, those wholes-without-parts that themselves are going to give-out (or produce) elements. The urelement or urset can be thought of then as a kind of dynamic whole, like the event, irreducible to “smaller” components, yet capable of either producing “future members” or entering into various membership and/or part-whole relations with other sets.

3. Geometry, Space, and Events

In this section, we leave the topic of set-theoretic foundations and move toward a closer examination of generic spatial and geometrical concepts naturally embedded into some of the abstract structures already encountered in the critical reexamination of set theory outlined above. Here, we concentrate on the philosophical ideas of the geometric object, figure, events, and transformations as understood within mathematical philosophy, mathematics, and ontology. Thus, we would like to emphasize the close connection between dynamics, dynamism, topology, and space. The intention is to prepare for the following Section 4, especially Section 4.3, which deal more directly with event ontology as often presented in mathematical philosophy and hence contain more detailed and specific technical treatment of one particular theory, that of dynamic events, which may be viewed as a special subclass of the more abstract and general concept of dynamic sets that we sketched out in Section 2.

3.1. The General Concept of Dynamic Space

We start by examining what a *dynamic space* is, especially in relation to the traditional concept of space as practiced within mainstream mathematics. In a certain sense, our terminology tends to suggest that the traditional space of classical Greek geometry and modern mathematical physics is somehow “static”. Strangely, this is both true and untrue. It is true because the concept of space dynamics is a major theme whose inclusion in a positive contribution to the philosophy of nature is not only desirable, but in fact inevitable. Nevertheless, it is also untrue because dubbing classical space “static” suggests that problems of time and motion have been historically ignored. However, the systems

of Descartes [87–89], Galileo [159,160], Newton [161] and above all, Leibniz [19], had all attempted to uncover a dynamic concept of reality, yet this concept is most often found buried into what are essentially *typical* phenomena, mainly observables collected in perceptual space. Kant would later consolidate the modern formulation by postulating the transcendental aesthetic as comprising *both* space and time [120]. According to the early and middle Heidegger’s reading of Kant [94,97,162], the Kantian move was a revolutionary step in Western philosophy because it is *time* what eventually transpired to have been de-hypostatized – in certain enigmatic passages in the Kantian First Critique [120] – by positing temporality as constituting the ultimate foundational ground or mainspring of being [95]. If we move from Kant to Edmund Husserl (like Kant himself, Husserl was an Idealist, but a very special one: a professional mathematician by training), we encounter those “atoms of space” now conceived as little dynamic wholes. In our own interpretation, let us view them as topological part-wholes enlivened by intentionality [49,50]. Those “atoms” are also to be understood locally as “parts” of the totality of all events. To start with, while each “part” is an event, the overtly spatial language utilized by Russell [78,80,163] and Whitehead [39,79,164,165] will not be followed here. Indeed, events are *not* merely *partial* sets differentiated from a mother class. The interaction between two events is determined by something new, *the consistency condition* in the overlap region [107]. That is, two events may overlap in a set-theoretic sense, but this is not how they come into mutual relation with each other as differentiated from within the global context of the totality of all events. In order to admit a new non-spatial apparatus into the philosophy of nature, an ontological discourse not reducible to the classical part-whole relation, the definition of the event as a spatially privileged set must be given up [86]. The key concept needed for a true understanding of events in nature is that which integrates time directly into the inner system of space [20,21,68,166]. Instead of spatializing time, as was famously done in Einstein–Weyl’s work [17,35,81] or the general influential program of geometric physics [36,167], we retain in temporality the latter’s fundamental, irreducible quality as a *flow, passage, becoming* [54,56,72,73,79,166,168].

As a summary of the new concept of dynamic space, we propose that the fundamental structure of reality is determined neither by the space of the Greeks (Plato’s *receptacle* [169,170]), nor the modern mathematical concept (Descartes [90], Galileo [159,160], Newton [161]), nor non-Euclidean geometries [171], but rather by something more akin to older but less known formulations propounded by Leibniz in their late philosophy, those circling around concepts of *events, primary matter, the Continuum*, and *dynamic monads* [19,172]. We take into account that ‘points’ in space do not represent the proper point of departure for our journey toward a better grasp of the fundamental constitution of thinghood. Rather, and instead of geometric points, we seek to present an alternative ontological *principle of generation* based on events and dynamic atoms now seen to constitute the ultimate system underlying reality [39,78,86]. In fact, it is *this* generative principle what, in a *later* stage, will eventually produce the ‘points’ of classical geometry [141]. *At the first fundamental ontological level, there are strictly speaking no points in space.* Instead, a set of interacting events exists, each defined as a flow [107]. The *outcome* of this collective interaction is that highly abstract space comprised of multidimensional ordered series (Russell’s definition [27,80]) of points. In other words, classical space in mainstream mathematics and mathematical physics is a *derivative* concept, not a primary one [39,80,173]. Interestingly, recent researches into the foundations of quantum field theory and quantum gravity have also reached such conclusions about the primacy of an underlying dynamical “subclassical” or “subquantum” (the jargon is a bit tricky) levels of reality [173–175].

Within modern mathematics, especially in the aftermath of Russell’s 1901–1903 *Principles of Mathematics* [27], a formalization of the concept of space was achieved for the first time when the Euclidean metric structure was *not* taken explicitly into considerations. Fréchet space (we refer to their general-topological, not the metric space, defined as systems of abstract neighborhoods [176]) is an immediate fine tuning of the concepts created by Cantor in his early topological researches on limit sets [76]. Currently, it is sufficiently known in

the mathematical community that the hidden concept implied by the term ‘space’ does not need to correspond to the direct perceptual space of the observational world [108,167,177]. This was originally required in order to satisfy the needs of analysis since the urgent demand to work with sets of functions treated as a whole was stimulated by problems coming directly from mathematical physics (see Hilbert’s groundbreaking invention of the concept of function space in his theory of integral equations). However, Fréchet topological space immediately degenerated to the Fréchet *metric* space, where the latter, though very abstract, is still closely modeled after the Euclidean metric world of perception. In the main, this is due to mainstream mathematics’ apathy toward the general spirit of Cantorian thinking, and the lack of serious engagement with Russell’s mathematical philosophy. Regardless to that sad generic trend, what we would like to bring into focus at this stage is the following: the general concept of abstract space accepted by the international mathematical community is in fact very close to the one Russell proposed in the *Principles of Mathematics* [27]: A set of points with additional structure. For example, a metric space is a set of points with an additional distance structure satisfying certain axioms [111]. Departure from Russellian philosophy, however, came later, with the mathematical community’s notorious refusal to abandon its traditional overemphasis on arithmetic and Euclidean geometry that has been the distinctive mark of the majority of mathematicians, in particular, Hilbert [36,178] and Poincaré [151]. Consequently, more or less, the metric system of Fréchet space has come to resemble a kind of strange “perceptual residue,” or a residual effect, like the surplus value of code [61,65], always hankering back to a ubiquitous background forming the basis of every presentable mainstream modern mathematical space concept. The original thrust of the *free* and *wild* open dynamism of the early Russellian space, understood as a *multi-dimensional ordered series that can always be dynamically reconfigured* [27,80,86], has been unfortunately downplayed and marginalized by the postmodernist thinkers Deleuze and Guattari’s complaints against Russell’s (and Chomsky’s) adherence to “Royal Science’s” mathematicians (the Hilbert school) [62,66]. In spite of that, we are now beginning to appreciate (again) the originality and vitality of Russell’s mathematical ontology and the philosophy of nature, especially his early work [27].

3.2. From Geometrical Space to Ontospace: A High-Level Overview

Geometry is not the theory of points (that would be general topology instead [176,179]). The Geometrical is about the Figure. However, in order to grasp the essence of geometry as science, then you must learn how to *personally* connect with the Figure. The Figure is the quintessential feature of the Object. Each of them, Object and Figure, has been made possible *exclusively* on the basis of perception and introspection. Both the Objective and the Figurative are products of Idealism. They are quite “natural”, falling then into the open expanse of *common sense*. Finally, classical art, a servant enlisted under the beautiful soul, human Reason and Sensibility, is unthinkable without that elemental necessity acquired and required by Geometrical Being. However, what is the Figure? A line or a plane or a sphere are much more than mere collections of points (though the concept of a ‘set of points’ is very rich and will occupy us later). They are *totalities* or *holistic ensembles* but in a very unique and specialized sense: What makes a line, for instance, a whole or a total entity is *essentially* a “principle” of cohesion imposed by the *geometer’s* mind. In other words, something emerges out of the depths of the beautiful soul of the observer-scientist in order to *posit* the Figure. Reason tyrannically imposes its will. *The Figure in geometry is always posited*, and this is the key philosophical issue here. It remains an open question whether there are really lines or triangles in Nature, but there are indeed such objects in the mind’s inner space of imagination and contemplation, what we call mindscapes. Furthermore, this open-inner space is never a collective mind or part of the unconscious. Euclidean mathematics, the true foundations of modern mainstream mathematics, is a product of *conscious* thinking, something that would be altogether impossible if the Greek Ego had not reached that advanced developmental stage it acquired sometime between the fifth and third centuries before the Common Era.

A line *appears* to move, but it never really moves. A triangle is seen rotating, but it never rotates. A sphere moves and rotates, but spheres are always static and fixed. Permanent. Geometric Being is Static Being. The science of geometry is the science of silence, rest, and tranquility. The Eternal. Hence both religion and the State love and support geometers [55,56,62]. Capitalism has been financing their activities. Furthermore, the world in the large treated them with respect. Why so? Because mathematicians have always been docile, passive, obedient, and submissive. They did exactly what they were told to do: Measure, calculate, estimate, predict. Most importantly: *Tame the Infinite*. Nothing terrifies organized religion or the State or Capitalism more than the inherently uncontrollable expansion of Nature's Infinite Being, the inner, outward-looking explosive force of creation and progressive development. This is why the State needed Euclid more than Euclid needed the patron King. To a large extent, axiomatic mathematics was created by the State for political reasons, even if mathematics may occasional push back against those who brought it into existence to start with [55]. State mathematicians' main job was to assassinate the Infinite and replace it with Figure [62].

Now, for dealing with the infinite or the continuum, we find ourselves trapped into a doublebind, facing the binary ontological bifurcation of the point-being traversing two potential branches:

- (i) *There are no points*: As a matter of fact, geometry is *not* a theory of points. Therefore, the modern concept of the Figure is not set-theoretic. This may appear strange at first sight, but the progressive development of such proposition consumed the good part of more than twenty five centuries of intense work. Klein's [180] and Lie's [5] re-axiomatized the same subject. Eventually Klein and Lie lost to that new mathematical Idealism of the twentieth century built on the foundations of Hilbert and Poincaré [29,108]. Nowadays, Klein and Lie are remembered mainly for the least philosophical and radical part of their itineraries, that overlapping the ultra-modernistic obsession with algebrization and axiomatization: the theory of invariants [32,36].
- (ii) *There are points*: Georg Cantor, on the other hand, is the Father of the Theory of Points [77]. What does that mean? Certainly not that no one before Cantor had ever theorized about points. It only says that Cantor constructed the first and most *abstract* discourse on points [76]. His points, and it is precisely this what underlies our adjectival quantifier 'abstract', are *non-geometrical*, in other words, there is a theory founded on neither spatial intuition nor the latter's intimate connection with vision, visualization, seeing, perceiving. Cantor's points are so abstract to the degree one begins to suspect that—like Aristotle, Avicenna, and Leibniz before them—he was in fact doing *ontology*, rather than being engaged with an official piece of professional mathematics. In fact, this is exactly the case. The Cantorian theory of points is much more profound than what academic historians would later baptize as *naive set theory*. Cantor's is a post-Leibnizian ontology of objects, not as fundamental as Heidegger's and Russell's (because it still presupposes a theoretical attitude toward *objects*), but at least it was certainly *post-modernist*, not modernist. To a large extent, the **ZF** axiomatic set theory of the early twentieth century [83,92,118] is better described as a *setback* than a presumed advance over Cantor's so-called "naive set theory", the latter term itself is nothing but a caricature of the *ontological* Theory of Points of the early years of 1870s and 1880s.

But in reality there has been a viable path out of this dilemma, a trajectory chosen by Bertrand Russell more than a century ago that now is nearly forgotten: Russell Space. Indeed, following in the footsteps of Cantor's Theory of Points, Russell invented (or discovered) the "next big thing", the concept of Spaces of Points, which is essentially a *structured set of points* [27]. This is *superspace*, which means it is mainly the nexus of order relations obtained between points what makes the collection a space [86]. Although Cantor was the first to introduce the concept of ordinal relations, it was Russell who, as a matter of fact, had *explicitly* created the idea of *generalized space* from the *ordinal* viewpoint [80]. 'Superspace', a term we will not further use or formalize here (for some background,

see [181]), is to a large extent an attempt to purge away the last residues of the geometrical and spatial which Cantor, in his early theory of points, was unable to fully suppress or eliminate from the traditional system of points of plane-set topologies and their natural higher-order generalizations. So how did Russell accomplish this arduous task? No direct conclusions can be made now, one must first go through various actual detailed technical constructions in order to see in precise terms what is meant by ‘generalized space’ in contrast to ‘geometric space’. For the time being, let us see into it that some initial insight into the Russellian concept can be recorded at this entry level of our exposition. A more detailed analysis and exposition should await future research.

In effect, Russell had destroyed the necessity of the long-held conviction that in order to find geometry one must deploy a subjective or ego-based *perspective*. Projective geometry was one towering intellectual achievement of nineteenth-century mathematics [182], but it had also gradually lead to the steady growth of Idealism. This had happened in spite of Cantor and Weierstrass efforts to shift the current away from spatial visualization toward more abstract modes of thinking closer to the spirits of *analysis*, that is, to bring us closer to the tradition of Leibniz infinitesimals and Cantor’s set-theoretic topology. A theory of space is not merely a collection of facts about observable phenomena that can be always checked by direct comparison and measurement [182]. The latter *is* indeed *geometry*, the science created by the Egyptians and the Mesopotamians and formalized by the Greeks; but in reality, the idea of *space as such* is very recent, dating to Descartes, Leibniz, and Newton, though it had already existed as a side curiosity in the works of Aristotle, Avicenna, and Averroes [180]. In our view, Space is not a locus of geometric possibility. Instead, Space vs. space is the *onto*-logical concept to be emphasized against the *onto-logical*. Geometry and logic are closely related. Geometry is a formalized science. However, Space, the non-geometrical system of virtuality, is more fundamental than the central Subject, Axiom, Mind, Ego, State, and so on. With Russell, you are in the company of a nexus of relations defined on sets of “objects” or “points” [78], but the key to their enormous innovation in this regard is the *disregarding of objects* by demonstrating that points can be constructed by means of the inner logic of relations [80]. In other words, *relations* defined on “points”, giving rise to the structure of space, *are* also themselves the productive source of objecthood or point-hood as such [39]. A point is a derived structure, an inferred object, a derivative, while relations or morphisms are the mainspring of being per se, the founding constitutive act giving rise to the creative emergence of beings and entities [107]. Space is the matrix of creation, not a construct produced by a seeing intellect or mind. It is the forming-presence of objecthood, outside human or mechanical minds [163], where the world literally “worlds” (Heidegger) [69], and Life outflows bio-matter in the path toward a non-universalized “unification” with the “totalities of all that can exist”. Again, Russell did not say this exactly, but their work provided pointers into the direction of seeing Space as more of a horizon of creation than being created (by mind). Shortly after, both Jakob von Uexküll (1920) [125] and Heidegger (1927) [69] would develop independent rediscoveries of the same basic (formal) Russellian insight.

It now all boils down to this: Do *not* define Space in terms of grand final objectives such as metric invariance, geometrico-topological relations, perspectives, and so on; but, instead, do so in lieu of these unique and singular enabling capacities allowing us to *produce* being out of sheer virtuality (primary matter). *The fundamental problem of the ontology of space is to understand production, not invariance*. The theory of Space-folds will displace spatial manifolds (Riemannian-Poincaré manifolds), substituting for them an inherently infinitely and open system tailored to evade geometrism and its associated finitism and combinatorialism. A metastable state of being-open: *that is* “big” Space, while “little” space collapses to the adorable compact, closed, and triangulable Riemann-Poincaré manifold so dear to modern mathematicians. To Space-as-Ontospace belongs an inner structure external to all those genres of measures or conceptual/theoretical determinations still counting as an order or form; the very idea of such “radical formalism” (ontospace) is a revolt against axiomatics, logics, and grammar. How come an order is free of axioms? How can you even

describe such a monster? The answer (for now) is that you cannot, but also you do not need to. Classical thought had sought to dominate nature by *describing* her, by inserting the possible into the observable via the feedback circuit of representation. This method, *inscription by description* is indeed the signature of representational thought, and we are done with it, thanks to the works of figures like Artaud, Nietzsche, Heidegger, Jung, and Russell. On the other hand, there is more to ontology than critique. If philosophy, from Kant to Adorno and Deleuze, has been dominated by critical theory, then the new twenty-first century should be more interested in *positive constructive work* than the mere negative critique that had dominated the previous epoch. Even positive critique does not count if it is still a critique. Ontology joins art [183], and mathematics is the medium employed by the artist in order to create ontology [184]. Science, physics and biology, are pointers to new research directions or suppliers of ideas and data, images and experiences, but the real active building is that pursued by the ontologist-artist [185]. This brings us to a direct confrontation with the French ontologies of space. If Space (in contrast to space) is not geometrical, then should it be treated as Deleuze and Guattari's *smooth space* [62] or Serres' hydrodynamic (Epicurean) space [56]? Neither, we believe. We are afraid both Deleuze and Serres have not gone far enough. None of them was successful in creating a viable ontology of space. The trouble we find in their approaches is that they were just too close to science, formalism and idealism, even while critical of the latter. Why did Deleuze fail in this regard? Because, contrary to Guattari [67,186], his method of writing and thinking put the Deleuzian outside the true field of ontology, which is the actual *technical construction of fully new worlds of experience*. On the other hand, this is *precisely* what Russell and Heidegger *did* succeed in doing: Heidegger created *Being and Time* [69], Russell's *The Principles* [27] and *Principia* [38,98,99]. Each work presented a singularity in terms of the history of idea, a remarkable and genuine production of the new, that is, it was neither a mere collage of past thoughts nor a reassembling of already prefabricated parts. It is strange to notice how the ultimate radical of French philosophy, Guattari, the non-academic, the "college dropout" [187], surpassed many others in philosophical and conceptual matters. We are still waiting for a chance encounter to emerge in the future, a window opening into technical ontological thinking proper where one may begin to read Guattari's last texts [67], learn how to admire them in the same way we do now with Leibniz's or Avicenna's. However, regardless of this, it remains important to observe the extremely productive role played by "doing mathematics" in the literal sense of the expression. Heidegger [188], Uexküll [125], and Russell [22] were, in at least a philosophical sense, first-rate mathematicians even if "officially" this is not supposed to be true. To them belongs something uncanny: the power of singularizing the *unneeded*, the *redundant*, the *dispensable*. This is why they were (predictably) rejected by their professional communities. Nowadays the academy celebrates Gödel but not Russell; Husserl and Hegel but not Heidegger; Darwin but not Uexküll, and so on. What unites the three is a deep, profound commitment to Space, not the spatial. For them, Space is the locus of creation, the *topos of being* elevated into Being per se, as if place is replaced by a strange topology of extra-spatial nonpresence: you are present in nonspace, or nonpresent in Space. No contradiction, logic and grammar suspended. The goal is to create the present out of nothing. The nothing becomes primary matter, its sheer virtuality, *cantaus* of creative production and making-present; the future, in a nutshell; to project the current onto the upcoming, an implementation of the Real in and through the virtual.

We note that the concept of ontospace discussed above has strong historical roots in French twentieth-century philosophy. It should be recalled that from the perspective of this article, the key motivation behind moving from space to ontospace is to allow for dynamism, and hence intrinsic temporality, to penetrate into the fundamental fabric of mathematical being, with the hope that such temporalized mathematical systems may displace the long-held emphasis on geometrical thinking and the centrality of the visualizable Figure. This subject will be taken up again in Section 4.3 from a more formal viewpoint, namely that of the event structure of set theory.

3.3. The Structure of Space According to Classical Thought: A Critique

In modern mathematics, Lie and Klein had proposed and worked on the concept of *space structures*, the implementation of the idea of structure going through the machinery of group theory [180,189]. Space linked into *invariance* [32], the Kantian Sublime [17]. Group theory is more general than Lie theory [190,191]. Lie theory is not a special case of the geometry of invariants [192]. We believe the former is a richer domain of philosophical investigation than the one officially exhausted by Lie's and Cartan's computational virtuosity [193] (the so-called "problem of classification"). Why so? Because Lie theory is the first (and probably the last) great science of the local/global. Even the calculus of Newton and Leibniz did not go far enough in envisaging this profound, inherently open and dynamic relation between the micro and macro, the limited and the unlimited. Against the common current of professional mathematics, we do not view space as the seat of geometrical objects. Instead, space is like Heidegger's Being, a violent flux of infinite fluctuating "essence" so agitated in its internal explosive impulses and metamorphoses to the degree "essence" transforms into *nonessence*, paving the way for *nonspace*. Such space like nonspace is anti-geometrical in and through, which implies, among many other things, that it has nothing to do with the ontology of invariants so fundamental for modern mathematicians. There is literally nothing "preserved" in ontospace. The latter is an *open* system, while geometrical space is closed, compact, triangularizable, and possesses algebraic invariants. Ontospace is the foundation of the Real, while geometric space is the matrix of cognition and idealistic thought. Geometry is associated with Plato [182], the State [55], while ontospace is of the future [86], the non-present, the nomadic outsider [62], an endeavour carried out by minds like those of Heidegger, Russell, and the Later Leibniz.

In Classical Thought, the fundamental structure of space is the repetition of the Same. In modern mathematics, the Same is the Invariant [17]. The Invariant is the return-to-itself of the changeable, the different, the transformed. Groups are bundles of transformations [189]. Objects are changed by *transformation groups* [180]. However, if the net effect of this change is reproducing the *same* object we started with, then the transformation group will be elevated to a very special category, a *symmetry* of the object, one of the most revered sacred elements in the idealist world [17]. Symmetry is a special group operation that *preserves* the initial character of the object transformed [194]. In other words, if the object is $\mathcal{O}$ and the object obtained by applying the transformation group g is $\mathcal{O}' = g\mathcal{O}$, then we say that g is a symmetry of the object $\mathcal{O}$ precisely when $\mathcal{O} = \mathcal{O}'$; that is, when

$$g\mathcal{O} = \mathcal{O}. \quad (18)$$

What does that mean? The special g satisfying the relation above acquires then the singular role played by *operators of identity*, those ontological markers of self-hood celebrated by Idealism [84]. The transformation element g enacts the identitarian being-for-itself of the initial object $\mathcal{O}$, which now becomes really nothing but an empty formal mark on the paper. In other words, within this Kantian-Hegelian moment of vision, the very being of the object is exhausted in precisely this coming back-into-the-self-in-the-other when the initial self $\mathcal{O}$ is *re-discovered*, rehabilitated as nothing but the other $\mathcal{O}'$.

What about space as such? According to this philosophy, space is *essentially* the totality of all such symmetries [171,180,182]. The set of all symmetry operations constitutes a *concrete* transformation group, the *symmetry group* G_s of the space under consideration. Furthermore, as was shown by Lie and Klein, the space itself inside which all symmetrical objects live can be effectively produced by a quotient operation involving subgroups of the symmetry group G_s [195]. Therefore, you do not need to go beyond a given space's symmetry group in order to know what that space is. Not only the very space itself can be generated by a quotient operation involving a symmetry group and one of its subgroups, but the very *structure* of space under inquiry is *exhausted* by the symmetry group itself [193].

This colossal reductive oversimplification of the problem of space, however, comes with a heavy price: we now no longer find in space anything except the implicit structure

of cognition and perception that has been already put into the concept of geometric invariance through the very process of constructing the operation of transformation group. Indeed, this last operation is fundamentally perceptual in nature. As Helmholtz [167,196], Poincaré [108], Einstein [81], and Reichenbach [197] have all unconsciously demonstrated in their own respective works, the very idea of space as such is unworkable without the implicit assumption of an “absolute eye”, a sort of universal observer who is more than the sum of all individual contingent (local) observers. This Universal Subject is played by the role of the cosmological metric in Einstein’s gravity [198], or the invariant symmetry group of the Weyl-Cartan type [17,30,35,193]. The modern formulation of mathematical physics is then an inherently *static* ontology of space. It has nothing to do with *ontospace*. For us, the Ontological is not only to be set against the Ontic, but the former is also the enemy of all sorts of “ontologies of X” such as “the ontology of realism”, “the ontology of space”, “the ontology of Marx”, and so on. Idealism’s predilection for grand formal generalizations, accompanied by the false pretension of possessing a profound metaphysical depth, is well known. However, we cannot locate any *philosophical* substance in those statements made by modern “philosophically-oriented mathematicians” (all of them tend to be Kantian and Platonist), regardless to how great they are in the computations of invariants and the construction of isomorphisms. At least from the conceptual viewpoint, they seem to be more interested in retaining the *status quo* of space-as-geometry against the more ontological formalization introduced by thinkers like Aristotle, Leibniz, and Russell.

Let us see, for instance, how the previous formal statement regarding geometric invariance will work out philosophically in the quest to ground Idealist space (Euclidean, homogeneous, uniform, and compact). The object $\mathcal{O}$ is the Figure. The Figure is an abstraction founded on the mechanisms of sense, perception, and introspection (cognition ontologized). The Object $\mathcal{O}$ is the Geometric Figure. Big Object against little object. The formalization of this transition is effected via the symbolic marker $\mathcal{O}$. On the other hand, the Figure enjoys a higher level of sensuality than the Object since the former necessarily contains within its inner structure the entire framework of onto-cognition, which is at the core of Kantianism, Hegelianism, Husserl, Merleau-Ponty, Sartre, Badiou, etc, i.e., most of the key figures of Idealism. The determination of an object is possible via sense perception, after which sensual data will be purified, formalized, and essentialized by means of cognitive mental abstractions like conceptual logic and categorical thinking. So in order to effectuate this objective determination of the Given, you need to undergo a process involving an encounter with the *Subject*, an event out of which the Object will emerge. However, the Figure is not to be equated with the Object. The Object, however, remains much more formal and abstract than the object, a statement that remains true even while we continue to link the Figure with the Geometrical, whereas the latter, at least in the public imagination, is still associated with mathematics. In fact, the Figure is unthinkable without its initial sensual context of givenness, while Object-hood is a late product involving the dialectic of subject/object and the rest of the familiar Idealist grand narratives and myths.

By intentionally conflating Objects with Figures, though only in the sense of a first approximation, Geometry has erected a new Idol of the Tribe, that of the Eye, the Seeing Big Brother, the Gaze, the Focal Referent, the Signifier, the Universal Viewpoint, and the suchlike. The only method permitted to an object in order to acquire being is by going through invariance-in-change, the precise process through which an initial given, the object $\mathcal{O}$, eventually comes back to itself via the equality $\mathcal{O} = g\mathcal{O}$. Afterwards, and only then, mathematicians can rigorously define the Figure as that *formal* invariance *as such*. Consequently, *geometrized* Figures initially manifest themselves as purely empty abstractions; they are exhaustively formalistic and lack content. However, the original Figure, that initiated through a complex process of sensual cognition of givenness, will *always remain alive and kicking in the background of the latter’s formalized essence*. Even in its most extreme abstractions, as found, for example, in category theory and algebraic topology, one continues to find the Figure of modern mathematics as basically a complex system ultimately grounded in a Cartesian Subject, the absolute agent enacting geometric and

algebraic operations and distributing essence. Now, this orthodox ontological structure of the return-to-itself, found at the heart of the process of objective determination in geometry, culminating in the erection of the Figure, is *never truly dynamic* precisely because of the presence of an ultimate *Ego-Pole* at the center of the Mind-Nature system. In order to disclose *processual* essence, you must completely eliminate the ontological structure of subjectivity, that is, you have to abolish Idealism once and for all. In contrast to the (ontologically) dynamic, the (ontologically) static characterizes the full process of invariance-in-change, which is a foundational cornerstone of modern geometrized mathematics [17]. The return of the same, short-circuited via the equality $\mathcal{O} = g\mathcal{O}$, represents Thought, though it is never the Thought of the Real as such, which can never be dominated by the apparatuses of representation and coding. You literally see the *same* formal object $\mathcal{O}$ after the completion of the false ontological loop entailed by $\mathcal{O} = g\mathcal{O}$, and hence the deceptive illusion of self-certitude characteristic of the writings of Idealists like Kant, Hegel, Husserl, and Weyl. However, nothing is less apodictic than the formal appearance of decision or announcement that what I say is “logically beyond dispute”. For, indeed, language and grammar are the very same trap that has ensnared Classical Thought since Parmenides and Plato, the paralogism of the return to the self and the superiority of the Familiar, the latter here taken as Cognitive Thinking, the process of thinking modeled after ego-consciousness, the ontological matrix of the Cartesian Subject.

3.4. A Critique of Category Theory

No discussion of space in modern mathematics is complete without invoking category theory [199]. Even though this characteristically twentieth-century mathematical discipline is often connected with abstract algebra and algebraic topology [46,116], in recent years it has become increasingly clear that most of the fundamental intuition behind abstract categories find its origin in ideas of space and geometric figures [158,195]. Here, we only examine the subject philosophically at the high level of concepts of objects, space, and dynamism. Our goal is to suggest that, to the contrary of some commentator’s views [84], category theory is still deeply entangled with Idealism, in fact as much as set theory. (For example, Badiou’s defense of orthodoxy and Idealism [83,84] is not surprising. His “ontology” is essentially a commentary on modern mainstream mathematics. Everything he says about being can be already found in Lacan [145], Lautman [41], Deleuze [63,64], and some mainstream mathematicians such as [199,200]).

Consider the following propositions: The ultimate ontological model of Life is vectorial being, or the thrusting forward of the unidirectional, forward-looking, and progressive [21,27]. How did we get there? The great founders of the formal ontology of vectorial being are Russell [27,78], Heidegger [148], Uexküll [125]. However, category theory failed to grasp Russell’s ontology of relation. Categories make becoming nothing but an algebraic arrow obeying certain strict rules of compositions, which are now supposed to become continuous with a new version of formal ontology proper [84,85]. In contrast to this idealist position, event ontology is against *both* geometrization and algebrization. While we agree that category theory is very different from set theory, this should not lure you into the trap of thinking that the former presents a true advance over the latter. Categories are still deeply embedded into set-theoretic thinking [201], and the notorious debate on whether one can fully purge sets from the categorical is still an open and active field of inquiry.

An “arrow” ‘ $\rightarrow$ ’ represents becoming, but is not becoming in-itself. Morphisms are arrows. An arrow is an object obeying certain rules [46]. In category theory, rules play into the role axioms have occupied in mathematics since the latter’s codification by Euclid [195]. However, what is the difference between *object* and *rule*? We are required to establish the conceptual distinction between the two *before* embarking on the long career path of category theory. However,—at least in set theory—it has been already realized that the very concept of being a mathematical *object* was questionable [84,200]. Nevertheless, the inner force of such problematization of objecthood would lose much of its power when, right from the start, morphisms are *declared*, ontologically speaking, to be “already there,”

enjoying a high degree of existence. An arrow is a *geometrical representation* of relational becoming. As such, it fails to touch the deeper ontological issues. Formalization often obscures what is important if pursued very seriously. (Leibniz critique of Spinoza was right on this point [19]).

The categorical construction is the one inspired and based on projective geometry and algebraic geometry, that is, mathematical fields so dear to Idealism and its closely related mainstream mathematics. The very basic fabric of the categorical centers on possessing the power to *view*, *see*, and *project* distant objects through the prism of another *privileged* object [157,202]. Furthermore, “verification processes” of this “seeing operation” are to be enacted via the algorithmic building up of “commutative diagrams”, which are essentially combinatorial finitary calculations effectuated in accordance with rigorous and precise algebraic rules of engagement already set long in advance [158]. *Organization* is the system of connectivity linking all morphisms together [158,203]. In Classical Thought, morphisms replace events, so it is precisely how different arrows relate to each other that really matters. How did category theory accomplish this explication of organization? By building universal *functors*, while harping on the theme of isomorphism [116,195]. Both functors and natural isomorphism are operations best viewed from the vantage point of a global Cartesian Subject. In order to say that an object O_1 is isomorphic to another object O_2 , usually one express this in writing as

$$O_1 \simeq O_2.$$

“Optics” matters here. It appears as if there is indeed a preferred “viewing angle” through which one may see the world, the superior elevated platform on top of which one can administer the relational nexus enjoining O_1 and O_2 . That implies the unification of the two objects into one unit, the higher abstract level engendered by the existence of the isomorphism $\simeq$ itself. So the new “object”, say O , is the one encompassing both O_1 and O_2 through the isomorphism $O_1 \simeq O_2$. However, there is something very distinctive about the way in which category theory produces new objects like O . This production of the new comes only through a *dominating transcendental*, effectively *hierarchical*, operating on and combining lower-order elements in order to produce higher-order representations and structures. It is *not* a genuine process of emergent evolutionary change, the reason being that true emergence requires *pure immanence*, something that category theory in itself is incapable of providing.

The grand failure of modern mathematics is reincarnated in category theory. It can be best captured through the conspicuous absence of time in mathematical being. Arrows are representations of timeless structuration of the Real, hence giving rise to merely one approach — among many possible others — deployed by Idealism in the Platonist quest to eternalize time. While responding to the same problem, Euclid, Plotinus [204], and Proclus [170,205] may be considered exemplary pre-Idealist figures where each had attempted a very different approach, yet while all fitting into the very same universal abstract structure: Plato’s theory of structure-as-eternity. (In fact, it is possible to trace the origins of modern mathematics back to the special manner by which the Euclidean program had sidelined the progress made by Plotinus and Proclus.) Coming to the present contemporary moment, we notice how the algebraic arrow-composition picture (morphisms) advocated by category theory has become the ultimate statement of the death of time in modern mathematics. Indeed, Modernism is the spatialization and geometrization of the Real on the expense of pure movements and change. The morphism becomes a mere direction or directed link in position space. Arrows are bad models of relational becoming and intentional comportment. Consequently, the “arrow-ing” of becoming is the Idealist’s murder of time.

3.5. Affirming Events and Dynamism: Against Structuralism

Overall, structuralism, and even including its various French “postmodern” versions developed in the 1960-2000 period, has maintained a consistent position of endorsing the tendency—in our opinion faulty—toward associating transformations with something like

a “higher authority” overseeing the return-to-self’s moment of invariance. Historically speaking, the nineteenth-century theory of algebraic invariants, projective geometry, and algebraic geometry/topology had all travelled down to the end of this fundamentally misguided and philosophically unfruitful direction of re-enforcing the same trend that originally encouraged mathematicians to overemphasize the role played by the observer or the subject in the constructive processes of mathematical activity. This tradition of mathematics and science, with its hidden, nearly invisible Idealism, has continued to influence French thinkers up to the present. Therefore, we do not believe that postmodernism, especially the “militant philosophies of the 1950s–1970s” [206], has not been able to shake off the Cartesian heritage that so crippled French thinking in general, and those in Paris in particular. All what writers like Lacan [145], Thom [207,208], and Badiou [83,84] did was merely *reinstating* mainstream mathematics in new jargon, that of psychoanalysis, algebraic topology, and refurbished Idealism.

Let us take a closer look at the formal essence of structuralism. The preferred viewpoint of analyzing mathematical systems is the categorical perspective, i.e., the process of seeing being as sheer directivity and vecoriality (“arrow-ness”), the going from here into there. An example can be given as follows:

$$a \leftarrow X \rightarrow b$$

in which X is the “vantage point” of the system comprised of a and b put together through a relation R , i.e., aRb , which reads a and b are related to each other via R :

$$aRb$$

Consequently, we may think of X as playing the role of the “transcendental Subject” underlying the apparently immanent relational structure aRb . Indeed, in mainstream mathematics, the important structures are precisely those viewed through isomorphisms expressing deeper or hidden inner symmetries. *Invariant* forms are what emerge into the scene when one preferred node, here X , succeeds in establishing or “verifying” that a and b are connected to each other through the bidirectional isomorphism R .

The erection of a privileged metaphysical viewing point, the categorical vantage point X , is only a semblance of scientificity, not an effective genuine advance as such. The mathematician installs himself as God, Subject; the Universal Signifier regulating and dictating various modes of interrelatedness and interactions among beings and things; the essence of the (mainstream) mathematical always revolves around that which *remains, invariant, identical*. For instance, the “truth” of a geometrical object is its set of algebraic invariants. Why? We are told the answer is that only through such self-identical quantities can you justify talking about a *permanent* character possessed by the object under consideration. It is as if mathematicians are so terrified of the abstractness of their discipline to the degree they have zealously endeavored to backup the abstract by the good, old fashioned stable stuff of straightforward identity. So regardless to how wild things may go, we are told that the only important aspect of the topological manifold, just to give a high-profile example, is that it has this particular homology group or that homotopy type, which can be efficiently encoded by a set of simple algebraic systems, the latter in themselves being direct generalizations from arithmetic to abstract algebra [209]. Hence modernism celebrates the arithmetization of space through the algebrization of the geometrical, the taming of the wild in calculus by the finite closed algebraic form, and so on.

4. Event Ontology: Concept and Mathematical Structures

4.1. Sets and Events

By losing virtuality, things become available to a *world* (cf. Section 2.7.3). Here, by ‘availability’ we understand a specific technical requirement in ontology pertaining to the need to capture, as formally as possibly, the concept of *composability*, that fundamental nexus of ingression, embedding, inclusion, in-formation at the heart of emergence and

diversification so characteristic of Nature [19,79,141,142,210]. Beings then can be accepted into a world only after transforming into things. A thing is either an entity or object, depending on whether or not it is being “viewed” (comprehended, manipulated, redirected) by an *active* other (subject). Consequently, things present themselves as “beings available in a world” ready to enter into all types of coupling and interactions. However, will this nexus of mutual ingression reveal itself in the form of spatio-logical coupling? This is where we locate a major shortcoming of modern mathematics: its incapacity to work with *dynamic* structuration and processes. If a set is to be viewed as a total static whole, then sets can *not* couple with each other—at least not in any ontologically interesting manner—except via “pure” formal operations such as set unions and intersections, which would lead to merely *topological*—hence purely *spatio-geometric*—image of thought (cf. Section 2). So we are now forced to deal with the all-important question of whether a set is an event or not. If it is, then set theory and event ontology become either one and the same field of inquiry, or at least very closely related to each other and since set theory is the ultimate product of Idealism, then our entire project would face great difficulty. However, sets are *not* events. This is not a decision. Nor a choice nor a wish. It is a fact. Sets are static wholes regulated by logical forms of thinking founded on the *act* of executing judgments (Husserl [114]). Only the act of *performing* the logical evaluation of the set predicate is dynamic, not the set itself. The set *S* is *produced* by logical judgment. According to mainstream modern mathematics, sets do not exist out there in nature. Instead, set-hood is a happening in the mind of the thinker, an object held or controlled or comprehended by a transcendental subject. Whatever corner in the great temple of modern mathematics you single out for admiration, surely you will find Platonism lurking underneath. However, the ‘act of performing a judgment’ belongs to regional or secondary domains of nature, that of the actualized—so-called “empirical”—world of sense, introspection, and perception: individual psychology [202,211]. No implication can be found for our main project, which is the laying down of a fundamental ontology of nature. So sets remain static after all, even if the act of producing them (through the performative processuality of logical judgment) is dynamic in and through.

On the other hand, underground mathematics, the postmodern moment of mathematical being, treats all of its objects as *processes*, dynamic entities that are perpetually pushing forward toward something never attained. This style of thinking does not respect logicism or Idealism. Underground mathematics flees rigid determinations and enforced fixations and territorialization through replacing sets with events. Events are true dynamic atoms, themselves quasi-wholes [107]. Since sets are characterized by a fundamental membership relation $\in$ serving as a razor-sharp operator deciding whether any pre-given thing can be included or excluded from that set, then being-*a*-set (Section 2) cannot represent or stand for an event, not to mention that a set is inherently never an event “in-itself”. This is because an event is a singular happening that is always on the move from one state to another [64,79]. If an event is a set, such set must be correlated with a changing or fluctuating membership relation $\in$ indicating that one is never sure which pre-given things are included or not in *S*. Note that this is not a “fuzzy set” in the sense of soft computing [212], for there is no indetermination caused by lack of “knowledge”, a stand that is again possible only in Idealism. An essential core of the event concept relates to its perpetual changeability, being always in the mode of continuous variability and alteration. Indeed, the innermost underlying structure of the Real is pure change. Such a state of affairs is not the result of some contingent fact like that of an observer or thinker who happens to be studying nature without having sufficient knowledge enabling them to somehow “eternalize away” all variability in order to end up with a static Parmenidean-Platonic Idea. Instead, let us agree that being an event is the most primordial manner or way of being. Events are not epistemological constructions; they are purely ontological givens.

4.2. What is Event Ontology?

A detailed exposition of event ontology is outside the scope of this article. There are several different ontologies based on the idea of the event, which is fundamentally a postmodernist concept. Examples include Mach [213], who called events “sensations”; James [214,215]; Russell [78,80,163]; Whitehead [39,79]; Auyang [173]. For a recent view, see [86,107]. There are also several other versions not mentioned here. In what follows, we only provide some high-level remarks on the subject, emphasizing the conceptual relevance to the main topic of this article, which is dynamism and temporality in mathematics.

Let us begin with an intentionally fragmented presentation. Following Leibniz, but replacing his monads with events, we declare that *the world is constructed out of events*. The unfolding of this extraordinary rich proposition may go into several stages:

- (i) Everything is a process. The event is a process.
- (ii) The event is a block of becoming (Whitehead [39,79]), an arrested subdomain of Bergson’s *Élan vital* [21], a region of Bergson’s intensive field of *duration* [20].
- (iii) The event is a dynamic process of an arrested topological flow [107].
- (iv) It is not true that sometimes there are things, and sometimes events. No, there are events and only events.
- (v) The world’s events enter into nexuses of interactions. Interaction is what constitutes composition.
- (vi) Composition is the secret of being.
- (vii) Being and Becoming are the same.
- (viii) Becoming is becoming-other.
- (ix) Transformation. Change. Metamorphoses. The vicissitudes of appearing: Being remains.
- (x) Events are blocks of becoming (Leibniz [19], Schelling [115], Bergson [20,21,68], Nietzsche [216]), frozen snapshots of the Real.
- (xi) The Real is the dynamic. The dynamic as perpetual otherness.

Event ontology rejects the Whiteheadian spatialization [21,39] of Bergson’s *duration*. Russell also made the mistake of bringing in Weyl, Einstein, and Eddington [80]. On the other hand, the postmodern concept of the event must involve an intrinsic element of dynamism, but captured through a local topological group or semigroup of transformations that exhausts what an event is [107,183]. Moreover, position space as such must be derivable from the dynamical topological flow itself [86]. What would be then the ultimate scope of an ontology of nature based on such an onto-mathematization of the Real? To be pursued only after the rejection of the Einstein–Minkowsky–Weyl’s four-dimensional spacetime manifold [17,35,81,217,218] since one of the main objectives of event ontology is to generate space, geometry, and algebraic structures out of the sheer power of an underlying sub-classical, sub-quantum eventual system of the world [80,86,107,173,174]. Events are not four-dimensional blocks of spacetime. Events *are* temporality [20,21,68,79].

To *connect* is to *compose*. But how? This is the relational theory of consistent multiplicities [62,65–67,107,210], which we may develop in the following manner:

- (i) Composition is *harmony*. Harmony is *anti*-symmetry.
- (ii) Harmony is variance, *not* in-variance: against Identity.
- (iii) To harmonize is not the bringing of the different into unity. Harmony is about understanding, mutual intercourse leading to active building and constructive behavior.
- (iv) Harmony shall not be calculated on the basis of preset transcendental rules. Harmony is pure immanence.
- (v) That which belongs to time-in-itself is the event. Yet, Space, that is, *onto*-space, is *produced* by interacting events.
- (vi) The generative principle lying at the heart of the onto-production of Space and spaces is *harmony*.

Guattari had invented a technical term for the construal of harmony as an ontological principle of coming into being: *conditions of consistency* [65,66]. In a certain sense then,

one may argue that he is the High Priest of event ontology, at least in modern times. However, one may argue that the roots of this idea can be traced back to natural philosophy, Leibniz [19,141,142] and Schelling [115] being the best known precursors. Deleuze had absorbed the idea and integrated it into his own system of the world [63,64]. But let us dial back two hundred years of intellectual evolution. In a rather short time, Hegel had managed to annul a major progress that philosophy had attained by the time of Fichte [121] and Schelling [115], chiefly their re-introduction of dynamism into ontology, a theme that can be traced back to Aristotle [18] and Leibniz [19]. What was Hegel's ultimate sin? He transposed Fichte's rediscovery of dynamism into a purely circular onto-theology of Identity as return-to-itself, return-to-the-Self. With Hegel, Being is the Circle, the Sphere. For Fichte and Schelling, Being is becoming-other, and becoming-other is nothing but being a vector. Hegel's sin: reinstating back into ontology what he criticized in the famous Preface to *Phenomenology of Geist* [152]: the Platonic eternal return of the self to the same. The mathematics of Idealism: circles, spheres, symmetry groups. Mathematical Idealism, from Plato to Weyl, has been the theory of the self-identical as essence, e.g., groups, invariant measures, homology, spaces of constant curvature, Conserved quantities, and so on. Since Plato, Being is defined as Invariance, i.e., that dynamical process which, eventually and ultimately, would return to its own "permanent essence". So, in an admittedly circular fashion, 'essence' is impredicately construed as "that which circles back toward itself by reconnecting and merging with its own initial essence". The term 'essence' is mentioned twice. Dialectic embraces this circularity. Heidegger, at least Early Heidegger, tried to revisit the same term through the concept of the 'hermeneutical circle' [69,219]. (We believe the Later Heidegger had moved beyond this [124]. See also Lautman's essay on essence in modern mathematics [41] and Deleuze's commentary [63]). In modern mathematics and Idealism: Essence is Invariance. There is no better way to define it than the modern mathematical approach through symmetry groups [17].

Movement is not to be understood as some kind of "transfer" happening "in" space. Instead, pure relational becoming, the *sense* of the dynamic *asymmetric* relation $\rightarrow$, is that which encodes the intentional comportment of one being toward another. It is only within this framework that we can put together a formal ontology of the *created* world, where things, entities, beings, are already there, or have been actualized, produced. On the other hand, the more primordial movement from Being to beings, the *horizontal* clearing or enactment of beings out of the virtual field of Being, is the essence of *disclosure*, *Unconcealment*, *unfolding*, the most fundamental *processes* in nature expressing the coming to be of the *event* as such. Consequently, the *event* is the marker of the latter, more ontologically fundamental relational type, while the formal relations obtaining between *already actualized* events belong to the former type, the one we have in mind in mathematics and is symbolized by arrows or morphisms. In general, we would like to adhere to the following three "slogans" or maxims:

- (i) Replace *morphisms* by *processes*.
- (ii) The *event* is a process.
- (iii) Make no exceptions to the above.

As strategic points of actions, categorical morphisms do not exhaust or saturate the intrinsic dynamism of the Cantorian–Russellian–Hausdorffian set-theoretic universe [27,76,111].

4.3. A Fragment of Mathematics for Event Ontology

4.3.1. Preliminary Considerations

Let us imagine that the membership relation $\in$ is itself time-varying. Assuming further that time can be captured by a real variable $t \in \mathbb{R}$, then we may replace $\in$ by a "time-varying" set-theoretic membership relation as follows:

$$\in \rightarrow \in_t . \quad (19)$$

Formally, we may introduce the following definition:

Definition 3. Let $\mathcal{E}$ be the set of all membership relations $\in$. A temporalization of set-theoretic membership, denoted by $\in_t$, is a map of the form

$$\in_t: \mathbb{R} \rightarrow \mathcal{E}, \quad (20)$$

where $t \in \mathbb{R}$ is the time index of the present state of the temporalized membership relation. The image of $t \in \mathbb{R}$ under the map $\in_t$ is written as $\in_t$.

Remark 2. Probably the most fundamental definition of the membership relation $\in$ is that given by Russell and Whitehead [38], but see also [15,16,27,37]. We note that if a suitable topology is introduced on the set of all set-theoretic membership relations $\mathcal{E}$, then one can further require that the temporalization map in Definition 3 is continuous.

An element x may belong to a set S at time t , and we write

$$x \in_t S \quad (21)$$

in order to express the following happening:

$$\text{The event } x \text{ being a member of the set } S \text{ happening at the precise moment } t. \quad (22)$$

The hope then is that $\in_t$ can be interpreted as a kind of “dynamic membership operator” capable of replacing the static set predicate $P(x)$ deployed before by set theory in the quest to define being- a -set. (cf. Section 2, especially Section 2.2).

The strategy outlined above would make sense only if we can foretell a priori which of those elements x will be in S for all times $t \in \mathbb{R}$. Otherwise, we may not be able to even refer to the set S in statements like ‘ $x \in_t S$ ’. If the element x belonging to the set S at time t_2 , $t_2 > t_1$, is not known to exist in S at earlier moments t_1 , then in general we can *not* write down the name of the set S . In other words, the very idea of constructing a specific set like S is that there exists a predicate $P(x)$ such that the computation of its truth value allows the determination of *all* elements belonging to S , yet, and this is the subtle point, only while this very determination per se does not involve time. Technically speaking, the dynamic determination should be “eternal” (on the concept of eternal truth in mathematics and philosophy, see the views of Spinoza [3], Leibniz [19], and Whitehead [79]). Strangely, there exists a fundamentally irreducible *nonlocality* in the classical idea of sethood: in order to know a set by listing its elements via the membership relation, whether temporalized or not, the totality of all elements must be known in advance or in an absolute or eternal mode of ontological knowing. Therefore, even with temporalizations such as (19), the original dynamic picture of the Real [168], especially mathematical being, has not been fully disclosed yet by the formalization associated with (21) and (22).

Apparently, then, one may conclude that the proposed time-dependent membership relation-function $\in_t$ does not really introduce a fundamental change after all. We somehow smuggled back into the proposed dynamization of the inclusion operator $\in$ the very same static concept of the set S heavily criticized in Section 2. Yet, one of our main objectives in this article has been demonstrating the fundamental need for genuinely dynamic concepts of time to be directly integrated into the foundations of mathematics. The event ontology outlined in Section 4 may supply at least a partial dynamization of mathematical objects since the event as such is a self-contained whole naturally capable of accommodating nonlocality. Let us see if event ontology may fare better in the quest for a reintegration of time with set-theoretic systems.

4.3.2. Mathematics of the Event: First Construction

It seems that our set S should be replaced by a dynamic whole, a “slice of life” or *block of becoming* [79]. In place of the set S , we operate with *set plus transformation* [107]. What does that mean? First, we note that the statement $x \in_t S$ approximately says the following:

instead of dealing with a *fixed* item x that will belong to S at time t , one can also proclaim that a *varying* item x_t is related to the (now unavoidably static) set S at the moment t by the same old classic membership relation $\in$. Therefore, we write

$$x_t \in S, \quad (23)$$

but keep the classical concept of set-hood, which is here realized by our continuing to use the name S . So at any given moment of time, a set can be described by those *potential* elements having the form x_t , that is, a function of time. The “real” set is not the one you “measure” or “observe” at any specific time instant, but the *dynamic whole* obtained by combining all “ t -slice-sets” together. In order to codify this proposal, we introduce the following formalization of the above idea.

Definition 4. The set S is said to be temporalized when it is replaced by the augmented structure $\langle S, \phi_t \rangle$, where ϕ_t is the transformation

$$\phi_t : \mathbb{R} \times S \rightarrow S, \quad (24)$$

with

$$x_t := \phi_t(x), \quad (25)$$

defined as the image by the map ϕ_t at the time moment $t \in \mathbb{R}$.

The key conceptual idea in Definition 4 is that x_t in (25) should be interpreted as one of elements belonging to the “dynamic set” at the time instant t . Here, ϕ_t is a time-transformation (t -transform) of the set S . It *acts* on S in order to produce another set the totality of which is still S . Consequently, in dynamic set theory, there is no longer a clear-cut logical membership relation in the sense of the predicate calculus of propositional functions (Definition 1). Each action ϕ_t is governed or controlled by the real parameter $t \in \mathbb{R}$, here serving as a *representation* of time, not time *per se*. Therefore, ϕ_t is in fact a *family* of set-transformations, denoted by $\mathcal{T}$, which is an infinite set with the cardinality of the continuum. A generic member ϕ_t of the family $\mathcal{T}$ is indexed by the real time variable $t \in \mathbb{R}$.

A t -transform ϕ_t assigns one and only one element $x_{0t} \in S$ to every given (arbitrary) element $x_0 \in S$ via the formula

$$x_{0t} = \phi_t x_0. \quad (26)$$

The mathematical orthography of this expression (and others) is carefully chosen such that it may reflect the *dynamic* operator-like nature of ϕ_t . Indeed, it is clear from the context that x_0 plays the role of some “initial point” or “starting element”, say at moment t_0 , which is acted upon by the operator ϕ_t in order to *produce* a new “point” or “element” x_{0t} at time t . We always read the action sequence from right to left, so in a one-dimensional ordered time series $t > t_0$, we can see that x_0 indeed comes “first” then followed by ϕ_t , a machine or operator-like device, eventually “spitting out” x_{0t} .

Somehow we managed to put our fingers on a first impression of time in mathematical thinking. The expression (26) contains—in a very compact form—the germ of the familiar concept of time, the time of physics and the time of simple consciousnesses: a directed marching forward, progressing in sequential linear order, where there is always a precise meaning of past, present, and future. More rigourously, the past is t_0 , the present is t , and the future is any state at $t + \epsilon$, where $t_0, t, \epsilon \in \mathbb{R}$, and $\epsilon > 0$. So x_{0t} expresses the fact that an initial state, x_0 , already a member of S , was “shifted” to another state or element, the one we now call x_{0t} .

4.3.3. The Idea of t -Slices and Set Blocks of the Past

In the transition from static being to dynamic being, a decisive change in perspective is effectuated through attaching time to the very essence of being-a-set. Temporality is the most crucial factor in the ontology of set-hood: time is what holds a set into itself as a whole.

It should be noted, though, that such a fully dynamic philosophy should not be conflated with the historically more influential position based on the traditional Platonic eternal being of the Same, which has served as the metaphysical ground of Identity. Nor should the dynamism of event ontology be conflated with the Hegelian–Kantian false process of the Return to the Self, which many authors, for instance Marcuse [220] and Badiou [83], mistake for a genuine dynamism. Sets themselves *are* dynamic wholes, with the important modification that it is more accurate to describe them as *quasi*-whole than completed wholes. Note that the operator prefix *quasi* indicates the Bakhtinian unfinished character of dynamic sets, their frustratingly persistent refusal to be pinned down or delimited in advance [122]. Formally, we would like to express this richer structure that sets acquire in postmodern mathematics using more technical means, say mathematical bodies of thought true to the open and dynamic objects under consideration. To do this, we try to analyze the internal structure of dynamic sets by isolating a concrete and specific concept of set *history*, the substructure we call *t*-slice. Informally and intuitively, we say that a *t*-slice is the marker of the past’s impressions remaining on the surface of the present after the onset of the event. More formally, We wish to find a proper mode expressing the fact that a subset of S enjoys the following distinguishing property: that a special sub-collection of its elements happens to belong to one, and only one, historical trajectory or orbit of the set-flow map ϕ_t , resulting in something that may look like the set-event’s “community,” “home nation,” or “a private pathway of occasions.” (Compare also with Whitehead’s ontology [79]).

However, does the dynamic set “change with time”? In a certain way, the answer is yes, at least if we remember that a generic element can be written as x_t so to express the dynamic membership relation we introduced above. Hence, the most direct proposal would be something like the following:

Definition 5. *In the spirit of Definitions 3 and 4, assume that all elements of a given subset of S can be expressed in the form x_t , where t is a fixed number and x is an element of S . We then say that such a subset is a time-slice or historical capsule. If we denote by S_t one such slice-set, we may then write*

$$S_t := \{x_t \mid x \in S, t = c\}, \quad (27)$$

where $c \in \mathbb{R}$ is some constant.

What Definition 5 says is that all elements of S_t share the property that they can be written as a *t*-transformation of some “common germ”, a special initial element x in S . Every element in S_t is the result of an evolution of past elements in S , but what all such evolutionary outcomes have in common is not that they have started from a specific subset of S , but that they all have evolved exactly through a time interval with length t . The subset $S_t \subset S$ can be thought of then as a “block of the past of S ” observed at the present time moment.

Definition 6. *In terms of the proposed event’s mathematical language, we formally define a ϕ_t -induced *t*-slice of the event-set $\langle S, \phi_t \rangle$ as*

$$S(\phi_t) = \{x \mid \exists s \in S, x = \phi_t(s), t = c\}, \quad (28)$$

where $c \in \mathbb{R}$ is some constant.

Remark 3. *In what follows, we do not distinguish $S(\phi_t)$ (Definition 6) and the more generic concept S_t (Definition 5) as long as a specific temporal set-event transformation ϕ_t is fixed by the context.*

Note that since the time transformation ϕ_t is still very generic, one cannot know for sure that those slices are nonoverlapping, so in general we must always admit the more generic mode

$$\bigcap_{t \in \mathbb{R}} S_t \neq \emptyset. \quad (29)$$

However, it is clear that

$$\bigcup_{t \in \mathbb{R}} S_t \subseteq S. \quad (30)$$

In other words, (30) says that the net effect of all possible dynamical transformations is to reconstitute the original complete universe of the mother set S . On the other hand, the generic state (29) expresses the expectation that in nature dynamic evolution mixes up all previous and future historical levels of representations. The past catches up with the present and soon will engulf the future.

Definition 7. (Base sets and spaces). The set S will be called the base set or base space of the event $\langle S, \phi_t \rangle$.

Definition 8. (Relative t -slices). If we have a subset of the base $A \subset S$, then a t -slice relative to this subset is

$$S_t(A) = \{x \mid \exists s \in A, x = \phi_t(s), t = c\}, \quad (31)$$

where $c \in \mathbb{R}$ is some constant. Here, the inducing transformation ϕ_t is still understood to be defined on the entire base set S containing A .

The meaning of $S_t(A)$ is as follows. It signifies a sub-block of the t -history of S , a kind of “sub-history” or “sub-narrative” crafted from within the jargon of t -slices. The following representation of a t -slice in terms of its relative t -slices is immediate:

Theorem 1. The total t -history S_t is the sum of all of its sub-histories:

$$S_t = \bigcup_{A \in \mathcal{P}(S)} S_t(A), \quad (32)$$

where $\mathcal{P}(S)$ is the power set of S .

It is interesting to further note that since our transformation ϕ_t is still extremely general, we cannot guarantee that the evolution of two nonoverlapping subsets of the base set of a dynamic set are themselves nonoverlapping. Indeed, if one has

$$A_1 \subset S, A_2 \subset S, A_1 \cap A_2 = \emptyset, \quad (33)$$

then in general

$$S_t(A_1) \cap S_t(A_2) \neq \emptyset. \quad (34)$$

For example, two distinct regions of the base set may evolve into the same “final” outcome if ϕ_t happens not to be injective (one-to-one). This is how two “different beings” merge into “one being”, or in more familiar terms, when “two things become one”. In the classical set-theoretic universe, the static ontology of sets does not allow this to happen as distinct sets remain distinct and new sets are formed only through the standard operations of intersection, union, complement, and so on. The only way two different sets can become one is through *logic*, that is, Classical Thought invoking its universal power of nominal representation by re-naming the two sets by giving them *identical* fundamental set predicates. In the theory of dynamic sets, on the other hand, sets change through time, and no fixed identity is possible in general, only *quasi*-identities held over finite periods of time. Furthermore, it may happen after such a finite period that the evolution operator ϕ_t will

project two mutually exclusive regions into one domain, resulting in two different beings becoming one.

We end with a remark on the “size” of the universe S . Clearly, the image of any element $s \in S$ under ϕ_t is still in S , which means that somehow we are presupposing that a “large enough” base set S (cf. Definition 7) has been chosen right from the outset in order to include all potential, possible, or virtual “future” elements x_t generated by the transformation ϕ_t . This is a subtle point: the base set is in itself not ontologically significant; its role, at least for now, is to merely serve as a reference or background space supporting the definition of the dynamic transformation ϕ_t , while only the latter remains] what really lays at the heart of the idea of the event as dynamic set-hood.

4.3.4. Events and Sets

Dynamic sets constitute the initial mathematical system of events. Indeed, we now no longer think of S as the main object of study. The newly introduced explicitly dynamic structure is the one controlled by the manner in which various t -slices relate to each other. Knowledge of this can be made available through the detailed structure of the transformation ϕ_t itself. The overall *dynamic* content of S boils down to precisely the ordered pair $\langle S, \phi_t \rangle$, which is very different from the original (mother) set S when the latter is treated as a separate object in itself. Note how various dynamic structurings of the same universe S are possible. Indeed, the essence of dynamism here is nothing but the *inter-relational mode* in which the variety of ϕ -induced t -slice coexist together while enduring the shared experience of one fundamental “tension”. We are no longer mainly interested in merely “listing” the elements of S (via the standard inclusion/membership relation $\in$), but would rather accept the infinity of all possible stratifications of the same “set” S entailed by admitting the transformation ϕ_t as a fundamental constitutive element of what could be characterized as a dynamic whole-set. In fact, this dynamization of sets is the first step needed in order to grasp the technical formal-ontological concept of the *event*.

We now provide a formal definition of the event.

Definition 9. (*The event*). A set-event, or simply the event in short, is the dynamic set-structure captured by the ordered pair $\langle S, \phi_t \rangle$, where the temporal map ϕ_t is as given in Definition 4.

A set-event is a set-theoretic approach to defining the original event. In the sense of Definition 9, the set event is nothing but the temporalized set in Definition 3. However, it should be noted that from philosophico-physical viewpoint such a definition does not exhaust the very concept of the event. Indeed, Russell himself in *The Analysis of Matter* [80], Part III, appeared not very enthralled by the prospects of a purely set-theoretic approach to the foundations of nature. His highly original concept of what we called Russell space [86] seems to be an attempt to make events more fundamental than set-hood as such. It is, however, not very clear whether or not he fully succeeded in doing that in texts such as [80,163], nor did Whitehead fare better in [79].

4.4. The Internal Structure of Dynamism: First Forays into Propagating Being

Consider the internal structure of Dynamism as a first foray into propagating Being. The base set S remains the same after performing the transformation ϕ_t . In other words, we have

$$\text{domain}(\phi_t) = \text{codomain}(\phi_t) = S. \quad (35)$$

The “state of the membership relation in S ” is changing, implying dynamism since the “cause” of such change as such is known, namely the transformation ϕ_t . In order to grasp the deeper significance of this new view, we need to examine more carefully the internal structure of S but from the viewpoint of the transformation ϕ_t itself. This will require an evaluation of how ϕ -induced slices of the base set “propagate” under ϕ_t . If we start from an “earlier” t -slice, say S_{t_1} , applying the transformation ϕ_t on the entire slice, then a new

time-history or historical stratum will be generated. We capture this process formally using the following definition:

Definition 10. (Generated time-histories and t -slices). Let S'_{t_2} denotes the new set obtained by transforming every element in S_{t_1} under ϕ_{t_2} . The generated time-slice relative to S_t is defined by:

$$S'_{t_2} := \phi_{t_2} S_{t_1} = \{x \mid \exists s \in S_{t_1}, x = \phi_{t_2} s\}.$$

Here, we assume $t_2 > t_1$.

That is, every element in S'_{t_2} must be obtained by propagating some germ element $s \in S_{t_1}$ in the earlier slice. Those germs are not necessarily identical, adding more complexity to the process of historical evolution. It is not difficult then to see that the following proposition holds:

Proposition 1. For any transformation ϕ_t , the following holds

$$\phi_{t_2} S_{t_1} \subset S_{t_2}$$

for any $t_1, t_2 \in \mathbb{R}, t_2 < t_1$.

Proof. Recall that a t -slice was defined as that subset of S such that every element of it is a ϕ_t -image of some “earlier” element in S . Since the common feature of all elements of $\phi_{t_2} S_{t_1}$ is precisely the fact that each of its members is an ϕ_{t_2} -image of an element $s \in S_{t_1} \subset S$, it follows that indeed all elements in S'_{t_2} satisfy the condition of S_{t_2} , but not necessarily exhausting all such possible conditions, so we only conclude that $S'_{t_2} = \phi_{t_2} S_{t_1} \subset S_{t_2}$. $\square$

Remark 4. We may call sets like S'_{t_2} partial t -slices. The transformation ϕ_t seems to “propagate” a t -slice into a partial t -slice.

Further mathematical constructions conducted along lines similar to those drawn in this section may also be developed and elaborated. However, we stop at this point and move to our overall conclusion of this article. More detailed and comprehensive mathematical formulations of event ontology will be undertaken in the future.

5. Conclusions

We presented an extensive critique emphasising the need to introduce more dynamic and temporal concepts into the foundations of mathematics. Our approach was historico-philosophical and the framework is that of mathematical philosophy, especially within the tradition of Bertrand Russell and Albert Lautman. A main theme in our undertaking was uncovering the long-held but often ignored link between modern (i.e., mainstream) mathematics and the philosophy of Idealism, the latter a venerable — though rarely explicit — intellectual orientation that goes back to Plato, Plotinus, and Proclus. On the other hand, postmodernism, a distinctly twentieth-century critique of modernism, has attempted to surpass Idealism and advocate an approach to nature, science, and life that replaces Idealism’s transcendence with materialism’s immanence. We suggested that a new form of mathematics, which was dubbed postmodern mathematics, is needed in order to connect mathematical research with contemporary cultural and philosophical development on one hand, and, on the other, to introduce new ideas to mathematical theories themselves, such as temporality, intrinsic dynamism, and the ontology of the event. Aspects of time, temporality, dynamism, and the event were reviewed in light of our brief reexamination of some of the conceptual foundations of set theory in modern mathematics (Section 2), criticizing in particular the latter’s emphasis on spatialization, the connection between Idealism and Geometrism in both philosophy and modernism, and how the latter had filtered through into modern mathematics (Section 3). Events were seen

then as dynamic sets coupled with primordial transformations introducing their own local time parameterizations (Sections 2.4 and 4.3). Moreover, in light of the earlier critiques of set theory (Section 2) and geometry (Section 3), we attempted to contrast the dynamic concept entailed by the event to the static picture of axiomatic modern mathematics often connected with the philosophy of Idealism. A condensed fragment of event ontology presented in Section 4.3 aimed at introducing a simple model, worked out using methods and ideas borrowed from mathematical philosophy, in order to demonstrate how time and temporality may be injected into the concept of the set. Overall, the main conclusion of this article is that we need to incorporate elements from various postmodernist theories in order to motivate expanding mathematics and its applications by integrating the latter with a candidate future “postmodern mathematics”, where concepts of time, temporality, and dynamism are projected to play the central role. It is the hope that this work can help bridge the gap between professional mathematicians on the technical side, and artists and philosophers on the other side of postmodernism. In addition, philosophers of mathematics and mathematical philosophers, who tend to operate from within the Analytical tradition of philosophy, may benefit from several approaches discussed here that are usually presented mainly within the opposing Continental camp. The multidisciplinary approach of this article is based on the author’s conviction that it will be fruitful for future investigations and dialogues to reduce or even bridge the two big gaps: the first one is that separating mathematical research and mathematical philosophy; the second one is that between Analytical and Continental philosophies.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Guinness, I. *The Search for Mathematical Roots, 1870–1940 : Logics, Set Theories and the Foundations of Mathematics From Cantor through Russell to Gödel*; Princeton University Press: Princeton, NJ, USA, 2000.
2. Ferreirós, J. *Labyrinth of Thought: A History of Set Theory and Its Role in Modern Mathematics*, 2nd ed.; Springer: Basel, Switzerland, 2007.
3. De Spinoza, B. *Ethics*; Penguin Books: London, UK, 1996.
4. Gödel, K. *Collected Works, Volume III: Unpublished Essays and Lectures*; Oxford University Press: Oxford, UK, 2001.
5. Hilbert, D. *David Hilbert’s Foundations of Arithmetic and Logic: 1917–1933*; Springer: Berlin, Germany, 2013.
6. Bourbaki, N. The architecture of mathematics. *Am. Math. Mon.* **1950**, *57*, 221–232. [[CrossRef](#)]
7. Bourbaki, N. *Theory of Sets*, 1st ed.; Springer: Berlin, Germany, 2004.
8. Corry, L. *Modern Algebra and the Rise of Mathematical Structures*; Birkhauser: Basel, Switzerland, 2004.
9. Turing, A. *The Essential Turing: Seminal Writings in Computing, Logic, Philosophy, Artificial Intelligence, and Artificial Life plus The Secrets of Enigma*, Copeland, B.J., Ed.; Clarendon Press: Oxford, UK, 2004.
10. Hilbert, D.; Ackermann, W. *Principles of Mathematical Logic*; AMS Chelsea: Providence, RI, USA, 1999.
11. Hofstadter, D. *Gödel, Escher, Bach: An Eternal Golden Braid*; Basic Books: New York, NY, USA, 1999.
12. Vries, J. *Topological Dynamical Systems: An Introduction to the Dynamics of Continuous Mappings*; De Gruyter: Berlin, Germany, 2014.
13. Robinson, R. *Dynamical Systems: Stability, Symbolic Dynamics, and Chaos*; CRC Press: Boca Raton, FL, USA, 1999.
14. Sell, G. *Topological Dynamics and Ordinary Differential Equations*; Van Nostrand-Reinhold: London, UK, 1971.
15. Frege, G. *The Foundations of Arithmetic: A Logical-Mathematical Investigation into the Concept of Number (1884)*; Pearson Education: New York, NY, USA, 2007.
16. Frege, G. *Basic Laws of Arithmetic: Derived Using Concept-Script (1893, 1903)*; Oxford University Press: Oxford, UK, 2016.
17. Weyl, H. *Philosophy of Mathematics and Natural Science*; Princeton University Press: Princeton, NJ, USA, 2009.
18. Aristotle. *Physics*; Oxford University Press: Oxford, UK, 2008.
19. Leibniz, G. *Philosophical Papers and Letters*; D. Reidel Publishing Company: Dordrecht, The Netherlands, 1976.
20. Bergson, H. *Time and Free Will: An Essay on the Immediate Data of Consciousness*; Harpays: New York, NY, USA, 1960.
21. Bergson, H. *Creative Evolution*; University Press of America: Lanham, MD, USA, 1984.
22. Monk, R. *Bertrand Russell: The Spirit of Solitude (1872–1920)*; Vintage: London, UK, 1997.
23. Belhoste, B. *Augustin-Louis Cauchy: A Biography*; Springer: New York, NY, USA, 1991.
24. Kleiner, I. *Excursions in the History of Mathematics*; Springer Science & Business Media: New York, NY, USA, 2012; Volume 178.

25. Gray, J. *The Real and the Complex: A History of Analysis in the 19th Century*, 1st ed.; Springer Undergraduate Mathematics Series; Springer: Cham, Switzerland, 2015.
26. Hairer, E.; Wanner, G. *Analysis by Its History*, 1st ed.; Series Undergraduate Texts in Mathematics; Springer: New York, NY, USA, 2008.
27. Russell, B. *The Principles of Mathematics*; W.W. Norton: New York, NY, USA, 1996.
28. Gray, J. *Plato's Ghost: The Modernist Transformation of Mathematics*; Princeton University Press: Princeton, NJ, USA, 2008.
29. Hilbert, D. *David Hilbert's Lectures on the Foundations of Physics, 1915–1927: Relativity, Quantum Theory and Epistemology*; Springer: Dordrecht, The Netherlands, 2009.
30. Weyl, H. *The theory of Groups and Quantum Mechanics*; Martino Publishing: Mansfield Centre, CT, USA, 2014.
31. Neumann, J. *Mathematical Foundations of Quantum Mechanics*; Princeton University Press: Princeton, NJ, USA, 2018.
32. Weyl, H. *The Classical Groups: Their Invariants and Representations*; Princeton University Press: Princeton, NJ, USA, 1946.
33. Showalter, R.E. *Hilbert Space Methods in Partial Differential Equations*; Dover Publications: Mineola, NY, USA, 2010.
34. Weyl, H. *The Concept of A Riemann Surface*; Dover Publications: Mineola, NY, USA, 2009.
35. Weyl, H. *Space, Time, Matter*; Dover Publications: Mineola, NY, USA, 1952.
36. Gray, J. *The Symbolic Universe: Geometry and Physics 1890–1930*; Oxford University Press: Oxford, UK, 1999.
37. Russell, B. *Introduction to Mathematical Philosophy*; Barnes & Noble: New York, NY, USA, 2005.
38. Whitehead, A.; Russell, B. *Principia Mathematica: Volume I*; Rough Draft Printing: San Bernardino, CA, USA, 2011.
39. Whitehead, A. *The Concept of Nature: The Turner Lectures Delivered in Trinity College, November 1919*; Cambridge University Press: Cambridge, UK, 2015.
40. Dalen, D. L.E.J. Brouwer: *Topologist, Intuitionist, Philosopher*; Springer: London, UK, 2013.
41. Lautman, A. *Mathematics, Ideas, and the Physical Real*; Continuum: New York, NY, USA, 2010.
42. Gödel, K. *Collected Works, Volume I: Publications 1929–1936*; Oxford University Press: Oxford, UK, 2001.
43. Gödel, K. *Collected Works, Volume II: Publications 1938–1974*; Oxford University Press: Oxford, UK, 2001.
44. Boolos, G.S.; Burgess, J.P.; Jeffrey, R.C. *Computability and Logic*, 5th ed.; Cambridge University Press: Cambridge, UK, 2007.
45. Gallager, R.G. *Information Theory and Reliable Communication*; John Wiley & Sons: Nashville, TN, USA, 1968.
46. Mac Lane, S. *Categories for the Working Mathematician*; Springer: New York, NY, USA, 1998.
47. Kushner, B.A. The constructive mathematics of A. A. Markov. *Am. Math. Mon.* **2006**, *113*, 559. [[CrossRef](#)]
48. Edwards, H.M. *Essays in Constructive Mathematics*, 2005th ed.; Springer: New York, NY, USA, 2004.
49. Husserl, E. *Logical Investigations: Volume 1*; Routledge, Taylor & Francis Group: London, UK, 2001.
50. Husserl, E. *Logical Investigations: Volume 2*; Routledge, Taylor & Francis Group: London, UK, 2001.
51. Husserl, E. *Philosophy of Arithmetic: Psychological and Logical Investigations (with Supplementary Texts from 1887 to 1901)*; Kluwer Academic Publishers: Dordrecht, The Netherlands, 2003.
52. Deleuze, G.; Guattari, F. *What is Philosophy?*; Columbia University Press: New York, NY, USA, 1994.
53. Serres, M. *Conversations on Science, Culture, and Time*; University of Michigan Press: Ann Arbor, MI, USA, 1995.
54. Serres, M. *Genesis*; University of Michigan Press: Ann Arbor, MI, USA, 1995.
55. Serres, M. *Geometry: The Third Book of Foundations*; Bloomsbury Academic: London, UK; New York, NY, USA, 2017.
56. Serres, M. *The Birth of Physics*; Rowman & Littlefield International: London, UK, 2018.
57. Simondon, G. *Individuation in Light of Notions of Form and Information, Part I*; University of Minnesota Press: Minneapolis, MN, USA, 2020.
58. Simondon, G. *Individuation in Light of Notions of Form and Information, Part II*; University of Minnesota Press: Minneapolis, MN, USA, 2020.
59. Ruyer, R. *Neofinalism*; University of Minnesota Press: Minneapolis, MN, USA, 2016.
60. Ruyer, R. *The Genesis of Living Forms*; Rowman & Littlefield Publishers: London, UK, 2020.
61. Deleuze, G.; Guattari, F. *Anti-Oedipus: Capitalism and Schizophrenia*; University of Minnesota Press: Minneapolis, MN, USA, 1983.
62. Deleuze, G.; Guattari, F. *A Thousand Plateaus: Capitalism and Schizophrenia*; University of Minnesota Press: Minneapolis, MN, USA, 1987.
63. Deleuze, G. *Difference and Repetition*; Columbia University Press: New York, NY, USA, 1994.
64. Deleuze, G. *Logic of Sense*; Bloomsbury Academic: London, UK, 2015.
65. Guattari, F. *The Anti-Oedipus Papers*; Semiotext(e) Distributed by MIT Press: Cambridge, MA, USA, 2006.
66. Guattari, F. *The Machinic Unconscious: Essays in Schizoanalysis*; Semiotext(e) Distributed by the MIT Press: Cambridge, MA, USA, 2011.
67. Guattari, F. *Schizoanalytic Cartographies*; Bloomsbury Academic: London, UK, 2013.
68. Bergson, H. *Matter and Memory*; Zone Books: New York, NY, USA, 1988.
69. Heidegger, M. *Being and Time*; HarperPerennial/Modern Thought: New York, NY, USA, 2008.
70. Anderson, E. *The Problem of Time: Quantum Mechanics Versus General Relativity*; Springer: Cham, Switzerland, 2017.
71. Prigogine, I. Time, structure, and fluctuations. *Science* **1978**, *201*, 777–785. [[CrossRef](#)] [[PubMed](#)]
72. Prigogine, I. *From Being to Becoming: Time and Complexity in the Physical Sciences*; W.H. Freeman: San Francisco, CA, USA, 1980.
73. Prigogine, I.; Stengers, I. *Order Out of Chaos: Man's New Dialogue with Nature*; Bantam Books: Toronto ON, Canada; New York, NY, USA, 1984.

74. Mikki, S. On the direction of time: From Reichenbach to Prigogine and Penrose. *Philosophies* **2021**, *6*, 79. [[CrossRef](#)]
75. Davies, P.C.W. *The Physics of Time Asymmetry*; University of California Press: Berkeley, CA, USA, 1977.
76. Cantor, G. *Contributions to the Founding of the Theory of Transfinite Numbers*; Dover Publications: New York, NY, USA, 1955.
77. Dauben, J. *Georg Cantor: His Mathematics and Philosophy of the Infinite*; Princeton University Press: Princeton, NJ, USA, 1990.
78. Russell, B. *Our Knowledge of the External World: As A Field for Scientific Method in Philosophy*; Routledge: London, UK, 2009.
79. Whitehead, A. *Process and Reality: An Essay in Cosmology*; Free Press: New York, NY, USA, 1978.
80. Russell, B. *The Analysis of Matter*; Spokesman: Nottingham, UK, 2007.
81. Einstein, A. *Ideas and Opinions*; Crown Trade Paperbacks: New York, NY, USA, 1995.
82. Barbour, J. *The End of Time: The Next Revolution in Physics*; Oxford University Press: Oxford, UK, 2000.
83. Badiou, A. *Being and Event*; Bloomsbury Academic: London, UK, 2013.
84. Badiou, A. *Mathematics of the Transcendental*; Bloomsbury Academic: London, UK, 2014.
85. Badiou, A. *Logics of Worlds*; Bloomsbury Academic: London, UK, 2018.
86. Mikki, S. On Russell's 1927 Book *The Analysis of Matter*. *Philosophies* **2021**, *6*, 40. [[CrossRef](#)]
87. Descartes, R. *The Philosophical Writings of Descartes: Volume 1*; Cambridge University Press: Cambridge, UK, 1985.
88. Descartes, R. *The Philosophical Writings of Descartes: Volume 2*; Cambridge University Press: Cambridge, UK, 1985.
89. Descartes, R. *The Philosophical Writings of Descartes: Volume 3 (The Correspondence)*; Cambridge University Press: Cambridge, UK, 1993.
90. Descartes, R. *Discourse on Method, Optics, Geometry, and Meteorology*; Hackett Publishing: Indianapolis, IN, USA, 2001.
91. Heijenoort, J. (Ed.) *From Frege to Godel: A Source Book in Mathematical Logic (1879–1931)*; Harvard University Press: Cambridge, UK, 1967.
92. Fraenkel, A. *Abstract Set Theory*; North-Holland Publishing Company: Amsterdam, The Netherlands; American Elsevier Publishing Company: New York, NY, USA, 1976.
93. Neumann, J. *Collected Works, Vol. 1: Logic, Theory of Sets and Quantum Mechanics*; Pergamon Press: Oxford, UK, 1961.
94. Heidegger, M. *What is a Thing?* H. Regnery Company: Chicago, MI, USA, 1968.
95. Heidegger, M. *The Basic Problems of Phenomenology*; Indiana University Press: Bloomington, Indiana, 1988.
96. Popper, K. *The World of Parmenides: Essays on the Presocratic Enlightenment*; Routledge: London, UK, 1998.
97. Heidegger, M. *Kant and the Problem of Metaphysics*; Indiana University Press: Bloomington, Indiana, 1962.
98. Whitehead, A.; Russell, B. *Principia Mathematica: Volume II*; Rough Draft Printing: San Bernardino, CA, USA, 2011.
99. Whitehead, A.; Russell, B. *Principia Mathematica: Volume III*; Rough Draft Printing: San Bernardino, CA, USA, 2011.
100. Oriti, D. (Ed.) *Approaches to Quantum Gravity: Toward a New Understanding of Space, Time and Matter*; Cambridge University Press: Cambridge, UK, 2009.
101. Haag, R. *Local Quantum Physics: Fields, Particles, Algebras*; Springer-Verlag: Berlin/Heidelberg, Germany, 1992.
102. Alongi, J.; Nelson, G.S. *Recurrence and Topology*; American Mathematical Society: Providence, RI, USA, 2007.
103. Prigogine, I. *Non-Equilibrium Statistical Mechanics*; Dover Publications: Mineola, NY, USA, 2017.
104. Prigogine, I. *The End of Certainty: Time, Chaos, and the New Laws of Nature*; Free Press: New York, NY, USA, 1997.
105. Russell, B. *The Philosophy of Logical Atomism*; Open Court: LaSalle, IL, USA, 1985.
106. Russell, B. *Collected Writings, Volume III: Toward the Principles of Mathematics: 1900–1902*; Routledge: London, UK, 1993.
107. Mikki, S. The event ontology of nature. *Philosophies* **2021**, *6*, 88. [[CrossRef](#)]
108. Poincaré, H. *The Value of Science: Essential Writings of Henri Poincaré*; Modern Library: New York, NY, USA, 2001.
109. Russell, B. *My Philosophical Development*; Spokesman: Nottingham, UK, 2007.
110. Foucault, M. *Discipline and Punish: the Birth of the Prison*; Vintage Books: New York, NY, USA, 1995.
111. Hausdorff, F. *Set Theory*; American Mathematical Society: Providence, RI, USA, 2005.
112. Epperson, M. *Foundations of Relational Realism: A Topological Approach to Quantum Mechanics and the Philosophy of Nature*; Lexington Books: Lanham, MD, USA, 2013.
113. Husserl, E. *Formal and Transcendental Logic*; Martinus Nijhoff: The Hague, The Netherlands, 1969.
114. Husserl, E. *Experience and Judgment: Investigations in a Genealogy of Logic*; Routledge and K. Paul: London, UK, 1973.
115. Schelling, F. *First Outline of a System of the Philosophy of Nature*; State University of New York Press: Albany, NY, USA, 2004.
116. Mac Lane, S. *Mathematics: Form and Function*; Springer: New York, NY, USA, 1986.
117. Grothendieck, A. *Topological Vector Spaces*; Gordon and Breach: New York, NY, USA, 1973.
118. Moore, G. *Zermelo's Axiom of Choice: Its Origins, Development, & Influence*; Dover Publications, Inc: Mineola, NY, USA, 2013.
119. Potter, M. *Set Theory and Its Philosophy: A Critical Introduction*; Oxford University Press: Oxford, UK, 2004.
120. Kant, I. *Critique of Pure Reason*; Dover Publications: Mineola, NY, USA, 2003.
121. Fichte, J. *Foundations of Transcendental Philosophy (Wissenschaftslehre) Nova Methodo (1796/99)*; Cornell University Press: Ithaca, NY, USA, 1992.
122. Bakhtin, M.M. *The Dialogic Imagination: Four Essays*; University of Texas Press: Austin, TX, USA, 1981.
123. Heidegger, M. *On the Way to Language*; Harper & Row: San Francisco, MA, USA, 1982.
124. Heidegger, M. *Contributions to Philosophy: Of the Event*; Indiana University Press: Bloomington, IN, USA, 2012.
125. Uexküll, J. *Theoretische Biologie*; J. Springer: Berlin, Germany, 1928.
126. Russell, B. *The Analysis of Matter*; Martino Fine Books: Mansfield Centre, CT, USA, 2014.

127. Hegel, G. *The Science of Logic*; Cambridge University Press: Cambridge, UK, 2010.
128. Penrose, R. *The Emperor's New Mind: Concerning Computers, Minds and the Laws of Physics*; Oxford University Press: Oxford, UK, 2016.
129. Penrose, R. *Shadows of the Mind: A Search for the Missing Science of Consciousness*; Oxford University Press: Oxford, UK, 1994.
130. Carnap, R. *The Logical Structure of the World and Pseudoproblems in Philosophy*; Open Court: Chicago/La Salle, IL, USA, 2003.
131. Carnap, R. *The Logical Syntax of Language*; Open Court: Chicago, IL, USA, 2002.
132. Wittgenstein, L. *Tractatus Logico-Philosophicus*; Routledge: London, UK, 2001.
133. Quine, W.V. *Word and Object*; Martino Fine Books: Mansfield Centre, CT, USA, 2013.
134. Wittgenstein, L. *Philosophical Investigations: the English Text of the Third Edition*; Prentice Hall: New York, NY, USA, 1958.
135. Heidegger, M. *Identity and Difference*; Harper Torchbooks: New York, NY, USA, 1969.
136. Husserl, E. *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*; M. Nijhoff Distributors: The Hague Boston Hingham, MA, USA, 1980.
137. Heidegger, M. *Hegel*; Indiana University Press: Bloomington, IN, USA, 2015.
138. Penrose, R. *The Road to Reality: A Complete Guide to the Laws of the Universe*; Vintage Books: New York, NY, USA, 2007.
139. Michael, T. *In the Shadows of the Dao*; SUNY series in Chinese Philosophy and Culture; State University of New York Press: Albany, NY, USA, 2016.
140. Cicero, M. *The Nature of the Gods*; Oxford University Press: Oxford, UK, 2008.
141. Leibniz, G. *The Leibniz-Arnould Correspondence: with Selections from the Correspondence with Ernst, Landgrave of Hessen-Rheinfels*; Yale University Press: New Haven, CT, USA, 2016.
142. Leibniz, G. *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672–1686*; Yale University Press: New Haven, CT, USA, 2001.
143. Husserl, E. *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy*; Northwestern University Press; Evanston, IL, USA, 1970.
144. Kant, I. *Metaphysical Foundations of Natural Science*; Cambridge University Press: Cambridge, UK, 2004.
145. Lacan, J. *Ecrits: The First Complete Edition in English*; Norton: New York, NY, USA, 2006.
146. Hegel, G. *Hegel's Philosophy of Nature: Being Part Two of the Encyclopaedia of the Philosophical Sciences (1830)7*; Clarendon Press: Oxford, UK, 2004.
147. Whitehead, A. *The Principle of Relativity*; Cosimo Classics: New York, NY, USA, 2007.
148. Heidegger, M. *Logic: The Question of Truth*; Indiana University Press: Bloomington, Indiana, 2016.
149. Bateson, G. *Steps to An Ecology of Mind*; University of Chicago Press: Chicago, IL, USA, 2000.
150. Bateson, G. *Mind and Nature: A Necessary Unity*; Hampton Press: Cresskill, NJ, USA, 2002.
151. Gray, J. *Henri Poincaré: A Scientific Biography*; Princeton University Press: Princeton, NJ, USA, 2013.
152. Hegel, G. *The Phenomenology of Mind*; Dover Publications: Mineola, NY, USA, 2003.
153. Deleuze, G. *Pure Immanence: Essays on a Life*; Zone Books Distributed by the MIT Press: Cambridge, MA, USA, 2001.
154. Deleuze, G. *Expressionism in Philosophy: Spinoza*; Zone Books Distributed by the MIT Press: Cambridge, MA, USA, 1990.
155. Husserl, E. *Cartesian Meditations: An Introduction to Phenomenology*; Springer-Science + Business Media, B.V.: Dordrecht, The Netherlands, 1973.
156. Whitehead, A. *Symbolism: Its Meaning and Effect*; Fordham University Press: New York, NY, USA, 1985.
157. Piaget, J. *The Child's Conception of Geometry*; Basic Books, Inc.: New York, NY, USA, 1960.
158. Piaget, J. *Morphisms and Categories: Comparing and Transforming*; L. Erlbaum Associates: Hillsdale, NJ, USA, 1992.
159. Galilei, G. *Dialogue Concerning the Two Chief World Systems: Ptolemaic and Copernican*; Modern Library: New York, NY, USA, 2001.
160. Galilei, G. *Dialogues Concerning Two New Sciences*; William Andrew Publishing: Norwich, NY, USA, 2001.
161. Newton, I. *Isaac Newton: Philosophical Writings*; Cambridge University Press: Cambridge, NY, USA, 2014.
162. Heidegger, M. *The Question Concerning the Thing: on Kant's Doctrine of the Transcendental Principles*; Rowman & Littlefield International Ltd: London, UK; Lanham, MD, USA, 2018.
163. Russell, B. *Human Knowledge: Its Scope and Value*; Taylor & Francis: Abingdon, UK, 2016.
164. Whitehead, A. *Science and the Modern World*; Free Press: New York, NY, USA, 1967.
165. Whitehead, A. *An Enquiry Concerning the Principles of Natural Knowledge*; Cosimo, NY, USA, 2007.
166. Bergson, H. *Duration and Simultaneity: Bergson and the Einsteinian Universe*; Clinamen Press: Manchester, UK, 1999.
167. Pesic, P. (Ed.) *Beyond Geometry: Classic Papers from Riemann to Einstein*; Dover Publications: Mineola, NY, USA, 2007.
168. Zubiri, X. *Dynamic Structure of Reality*; University of Illinois Press: Urbana, IL, USA, 2003.
169. Plato. *Timaeus and Critias*; Oxford University Press: Oxford, UK, 2008.
170. Proclus. *The Commentaries of Proclus on the Timaeus of Plato: in Five Books, Containing a Treasury of Pythagoric and Platonic Physiology*; Translated by Thomas Taylor, CreateSpace: Scotts Valley, CS, USA, 2012.
171. Gray, J. *Worlds Out of Nothing: A Course in the History of Geometry in the 19th Century*; Springer: London, UK, 2011.
172. Lawrenz, J. *Leibniz: The Nature of Reality and the Reality of Nature: A Study of Leibniz's Double-Aspect Ontology and the Labyrinth of the Continuum*; Cambridge Scholars: Newcastle upon Tyne, UK, 2010.
173. Auyang, S. *How is Quantum Field Theory Possible?* Oxford University Press: Oxford, UK, 1995.
174. Wheeler, J. *Geons, Black Holes, and Quantum Foam: A Life in Physics*; Norton: New York, NY, USA, 2000.

175. Rovelli, C. *Reality is Not What It Seems: The Journey to Quantum Gravity*; Riverhead Books: New York, NY, USA, 2018.
176. Sierpinski, W. *General Topology*; Dover Publications, Inc: Mineola, NY, USA, 2020.
177. Jammer, M. *Concepts of Space*, 3rd ed.; Dover Publications: Mineola, NY, USA, 2003.
178. Corry, L. *David Hilbert and the Axiomatization of Physics (1898–1918): From Grundlagen der Geometrie to Grundlagen der Physik*; Kluwer: Dordrecht, The Netherlands, 2004.
179. Kelley, J. *General Topology*; Dover Publications, Inc: Mineola, NY, USA, 2017.
180. Klein, F. *Elementary Mathematics from An Advanced Standpoint: Geometry*; Dover Publications: Mineola, NY, USA, 2004.
181. Mikki, S. On the topological structure of nonlocal continuum field theories. *Foundations* **2022**, *2*, 20. [[CrossRef](#)]
182. Russell, B. *An Essay on the Foundations of Geometry*, Series Routledge Classics; Taylor & Francis: London, UK, 2022.
183. Mikki, S. Aesthetic theory and the philosophy of nature. *Philosophies* **2021**, *6*, 56. [[CrossRef](#)]
184. Mikki, S. Homo philosophicus: Reflections on the nature and function of philosophical thought. *Philosophies*, **2021**, *6*, 77. [[CrossRef](#)]
185. Guattari, F. *Chaosophy: Texts and Interviews 1972–1977*; Semiotext(e): Los Angeles, CA, USA, 2009.
186. Guattari, F. *Lines of Flight: for Another World of Possibilities*; Bloomsbury Academic: London, UK, 2016.
187. Dosse, F. *Gilles Deleuze & Félix Guattari: Intersecting Lives*; Columbia University Press: New York, NY, USA, 2010.
188. Safranski, R. *Martin Heidegger: Between Good and Evil*; Harvard University Press: Cambridge, MA, USA, 1998.
189. Lie, S. *Theory of Transformation Groups I*; Springer: Berlin/Heidelberg, Germany, 2015.
190. Montgomery, D.; Zippin, L. *Topological Transformation Groups*; Dover Publications, Inc: Mineola, NY, USA, 2018.
191. Pontryagin, L.S. *Topological Groups*; Gordon and Breach Science Publishers: New York, NY, USA, 1986.
192. Godement, R. *Introduction to the Theory of Lie Groups*; Springer: Cham, Switzerland, 2017.
193. Sharpe, R.W. *Differential Geometry: Cartan's Generalization of Klein's Erlangen Program*; Springer: New York, NY, USA, 1997.
194. Thyssen, P.; Ceulemans, A. *Shattered Symmetry: Group Theory from the Eightfold Way to the Periodic Table*; Oxford University Press: New York, NY, USA, 2017.
195. Kromer, R. *Tool and Object: A History and Philosophy of Category Theory*, 2007th ed.; Series Science Networks, Historical Studies; Birkhauser Verlag AG: Basel, Switzerland, 2007.
196. Helmholtz, H. *Science and Culture: Popular and Philosophical Essays*; University of Chicago Press: Chicago, IL, USA, 1995.
197. Reichenbach, H. *The Philosophy of Space & Time*; Dover Publications: Mineola, NY, USA, 1958.
198. Wald, R. *General Relativity*; University of Chicago Press: Chicago, IL, USA, 1984.
199. Lawvere, F.W.; Schanuel, S.H. *Conceptual Mathematics: A First Introduction to Categories*, 2nd ed.; Cambridge University Press: Cambridge, UK, 2009.
200. Adamek, J.; Herrlich, H.; Strecker, G.E. Mathematics. In *Abstract and Concrete Categories*; Series Dover Books on Mathematics; Dover Publications: Mineola, NY, USA, 2009.
201. Lawvere, F.W.; Rosebrugh, R. *Sets for Mathematics*; Cambridge University Press: Cambridge, UK, 2003.
202. Piaget, J. *Origin of Intelligence in the Child*; Routledge: London, UK, 1997.
203. Piaget, J. *Biology and Knowledge: An Essay on the Relations between Organic Regulations and Cognitive Processes*; University of Chicago Press: Chicago, IL, USA, 1971.
204. Plotinus. *The Enneads*; Cambridge University Press: Cambridge, UK, 2018.
205. Proclus. *A Commentary on the First Book of Euclid's Elements*; Princeton University Press: Princeton, NJ, USA, 1970.
206. Cusset, F. *French Theory: How Foucault, Derrida, Deleuze, & Co. Transformed the Intellectual Life of the United States*; University of Minnesota Press: Minneapolis, MN, USA, 2008.
207. Thom, R. *Mathematical Models of Morphogenesis*; Ellis Horwood Halsted Press: Chichester, NY, USA, 1983.
208. Thom, R. *Structural Stability and Morphogenesis: An Outline of a General Theory of Models*; Addison-Wesley Publishing: Reading, MA, USA, 1989.
209. Poincare, H. *Papers on Topology: Analysis Situs and Its Five Supplements*; American Mathematical Society London Mathematical Society: Providence, RI, USA, 2010.
210. Deleuze, G. *The Fold: Leibniz and the Baroque*; University of Minnesota Press: Minneapolis, MN, USA, 1993.
211. Piaget, J. *The Psychology of Intelligence*; Series Routledge Classics; Routledge: London, UK, 2001.
212. Russell, S.; Norvig, P. *Artificial Intelligence*, 4th ed.; Pearson: Upper Saddle River, NJ, USA, 2020.
213. Mach, E. *The Analysis of Sensations, and the Relation of the Physical to the Psychical*; Dover Publications: Mineola, NY, USA, 1959.
214. James, W. *Writings: 1902–1910*; Literary Classics of the United States Distributed to the trade in the U.S. and Canada by Viking: New York, NY, USA, 1987.
215. James, W. *Essays In Radical Empiricism*; Wilder Publications: Saint Paul, MN, USA, 2018.
216. Nietzsche, F. *The Will to Power: Selections from the Notebooks of the 1880s*; Series Penguin classics; Penguin Books: London, UK, 2017.
217. Lorentz, H.A.; Einstein, A.; Minkowski, H.; Weyl, H.; Sommerfeld, A. *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity*; Dover Publications: Mineola, NY, USA, 1952.
218. Weyl, H. *Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics*; Princeton University Press: Princeton, NJ, USA, 2009.
219. Heidegger, M. *Ontology: The Hermeneutics of Facticity*; Indiana University Press: Bloomington, Indiana, 1999.
220. Marcuse, H. *Hegel's Ontology and the Theory of Historicity*; MIT Press: Cambridge, MA, USA, 1987.