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Article

A Reliable Technique for Solving Fractional Partial Differential Equation

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Abstract: The development of numeric-analytic solutions and the construction of fractional-order mathematical models for practical issues are of the greatest importance in a variety of applied mathematics, physics, and engineering problems. The Laplace residual-power-series method (LRPSM), a new and dependable technique for resolving fractional partial differential equations, is introduced in this study. The residual-power-series method (RPSM), a well-known technique, and the Laplace transform (LT) are elegantly combined in the suggested technique. This innovative approach computes the fractional derivative in the Caputo sense. The proposed method for handling fractional partial differential equations is provided in detail, along with its implementation. The novel approach yields a series solution to fractional partial differential equations. To validate the simplicity, effectiveness, and viability of the suggested technique, the provided model is tested and simulated. A numerical and graphical description of the effects of the fractional order γ on approximating the solutions is provided. Comparative results show that the suggested method approximates more precisely than current methods such as the natural homotopy perturbation method. The study showed that the aforementioned method is straightforward, trustworthy, and suitable for analysing non-linear engineering and physical issues.

Keywords: fractional partial differential equations; Laplace transform; residual power series; Caputo operator

MSC: 26A33; 60H15; 35R11; 34A25

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1. Introduction

It has been noted that fractional-order α derivatives, often in the range between 0 and 1, are a helpful tool for describing a variety of events [1]. To obtain the desired order of a fractional derivative, the Caputo fractional derivative computes an ordinary derivative first, then a fractional integral. In reverse order, the Riemann–Liouville fractional derivative is calculated. As a result, although the Riemann–Liouville fractional derivative permits initial conditions in terms of fractional integrals and their derivatives, and the Caputo fractional derivative only enables the involvement of conventional initial and boundary conditions. These two operators coincide under the homogeneous initial condition assumption. Anyone who has studied fundamental calculus is familiar with the differentiation operator $D = \frac{d}{d\varsigma}$. Furthermore, provided that n is a positive integer, the nth derivative of u, denoted by $D^{\gamma}u(\varsigma) = \frac{d^n u(\varsigma)}{d\varsigma^n}$, is well defined for suitable functions u. L'Hôpital asked Leibnitz in 1695 what significance could be assigned to $D^n u(\varsigma)$ if n were a fraction. However, it was not until 1884 that the theory of generalized operators reached a stage in its evolution that was appropriate for the modern mathematician to use as a starting point. By that time, the theory had been expanded to include operators D^v, where m could be real or complex, rational or irrational, and positive or negative [2]. Although derivatives of arbitrary order were discussed by Leibniz, Euler, Laplace, Lacroix, and Fourier, Niels Henrik Abel used fractional operations for the first time in 1823. In order to solve the tautochrone paradox, Abel used fractional calculus [2]. Liouville is to be credited with making what is likely the first sincere attempt to define a fractional derivative logically. He wrote nine articles on the topic between 1832 and 1837, with the latest one in the field appearing in 1855. In recent years, it has been discovered that the fractional calculus is very effective in describing a wide variety of physical phenomena, including damping laws and diffusion processes [2–5]. Kilbas and Trujillo [1], Caputo [6], Debanth [7], Jafari and Seifi [8], Kemple and Beyer [9], Oldham and Spanier [10], Momani and Shawagfeh [11], and others provide some fundamental works on various elements of the fractional calculus [12–16].

Over the past forty years, fundamental research and advancements on fractional derivatives and differential equations have been made. Traditional differential equations with non-local and genetic significance in material characteristics are generalized as fractional-order differential equations. Fractional partial differential equations are increasingly used in the creation of non-linear models and the analysis of dynamical systems. The theory of fractional-order calculus has been connected to real-world projects and used to examine and investigate a variety of phenomena, such as chaos theory [17], financial models [18], a noisy environment [19], optics [20], and others [21–24]. The characteristics of non-linear issues that occur in nature are largely described by the solutions of fractional differential equations. Since it is challenging to find an accurate solution for fractional differential equations representing non-linear phenomena, many analytical and numerical techniques are employed [25–28].

In recent years, scholars and researchers have paid close attention to both the numerical and analytical solutions of PDE systems. For resolving fractional FPDEs such as this, numerous numerical and analytical algorithms have been used, including the first integral method [29], the Elzaki transform decomposition method [30,31], the double Laplace transform method [32], the homotopy perturbation transform method [33,34], the conformable fractional Laplace transform method [35], the Yang transform decomposition method [36,37], the generalized two-dimensional differential transform method [38], the Fourier transform [39], He's variational iteration method [40], the fractional complex differential transformation method [41], and the fractional variational iteration method [42].

The power-series method (PSM), which results in a closed-form solution of known functions, is well proven as an efficient method for solving linear ordinary-partial differential equations. In the case of nonlinear problems, it is impossible to obtain a closed-form solution, and finding out the series coefficients is a highly challenging task. A modified version of the PSM that treats the coefficients as transformed functions that follow a set of rules and are determined in recurrence relations is introduced to address the aforementioned limitations of the standard PSM. The differential transform method is the name of this improvement (DTM). Different kinds of integro-differential equations and linear-nonlinear equations have both been solved using it. Another advancement is the establishment of the residual-power-series method (RPSM) through the differentiation of the nth ordered coefficient of the PSM's nth partial sum of the PSM (n-1)-times.

It was necessary to increase the use of the power-series method to deal with fractional difficulties during the modification of the ordinary derivative to a fractional derivative because it is more general. Many significant models that arise in various branches of science and engineering are constructed and solved analytically using the fractional DTM and fractional RPS methodologies. By including the Laplace transform (LT) into the RPSM's technique, we hope to improve its accuracy in this work. This RPSM promotion is known as the Laplace residual-power-series method (LRPSM). Solving the FPDEs introduces the building of this innovative approach. Accuracy to the necessary level has been attained. The suggested technique has a very easy and uncomplicated process. The findings indicate that, in comparison to other analytical procedures, the current method has the appropriate accuracy.

The framework of the study is detailed as follows. First, we use key FC theory ideas and findings in Section 2. Additionally, several original findings that provide the basis for the innovative technique in Section 2 are provided. The solutions to time-fractional PDEs are then determined in Section 3 using the LRPSM. Some of the problems in Section 4 are solved using LRPSM. A brief conclusion ends Section 6.

2. Preliminaries

Here, we provide some definitions in terms of Caputo and Riemann–Liouville, along with the Laplace transform theorem .

Definition 1. *The fractional derivative in terms of Caputo is stated as [6,43]*

$$ {}^{C}D_{\eta}^{\gamma}u(\varsigma,\eta) = J_{\eta}^{m-\gamma}u^{m}(\varsigma,\eta), \;\; m-1<\gamma\leq m, \;\; \eta>0, \tag{1}$$

where $m \in N$ and J_{η}^{γ} represents a fractional integral in terms of Riemann–Liouville (RL) as

$$ J_{\eta}^{\gamma}u(\varsigma,\eta) = \frac{1}{\Gamma(\gamma)}\int_{0}^{\eta}(\eta-t)^{\gamma-1}u(\varsigma,t)dt. \tag{2}$$

Definition 2. *The LT is stated as [43]*

$$ u(\varsigma,\mu) = \boldsymbol{L}_{\eta}[u(\varsigma,\eta)] = \int_{0}^{\infty} e^{-\mu\eta}u(\varsigma,\eta)d\eta, \;\; \mu>\gamma, \tag{3}$$

with inverse LT as

$$ u(\varsigma,\eta) = \boldsymbol{L}_{\eta}^{-1}[u(\varsigma,\mu)] = \int_{l-i\infty}^{l+i\infty} e^{\mu\eta}u(\varsigma,\mu)d\mu, \;\; l=Re(\mu)>l_{0}. \tag{4}$$

Lemma 1. *Suppose $u(\varsigma,\eta)$ is a piecewise continuous function with $U(\varsigma,\mu) = \boldsymbol{L}_{\eta}[u(\varsigma,\eta)]$, so*

1. $\boldsymbol{L}_{\eta}[J_{\eta}^{\gamma}u(\varsigma,\eta)] = \frac{U(\varsigma,\mu)}{\mu^{\gamma}}, \;\; \gamma>0.$
2. $\boldsymbol{L}_{\eta}[D_{\eta}^{\gamma}u(\varsigma,\eta)] = \mu^{\gamma}U(\varsigma,\mu) - \sum_{k=0}^{m-1}\mu^{\gamma-k-1}u^{k}(\varsigma,0), \;\; m-1<\gamma\leq m.$
3. $\boldsymbol{L}_{\eta}[D_{\eta}^{n\gamma}u(\varsigma,\eta)] = \mu^{n\gamma}U(\varsigma,\mu) - \sum_{k=0}^{n-1}\mu^{(n-k)\gamma-1}D_{\eta}^{k\gamma}u(\varsigma,0), \;\; 0<\gamma\leq 1.$

Proof. For proof, see [44]. □

Theorem 1. *Let us assume that $u(\varsigma,\eta)$ is a continuous piecewise on $I\times[0,\infty)$ and that ϑ is the order of the exponential function. Take the function $U(\varsigma,\mu) = \boldsymbol{L}_{\eta}[u(\varsigma,\eta)]$ with fractional expansion as*

$$ U(\varsigma,\mu) = \sum_{n=0}^{\infty}\frac{f_{n}(\varsigma)}{\mu^{1+n\gamma}}, 0<\gamma\leq 1, \varsigma\in I, \mu>\vartheta. \tag{5}$$

So, $f_{n}(\varsigma) = D_{\eta}^{n\gamma}u(\varsigma,0)$.

Proof. For proof, see [43]. □

Remark 1. *On taking the inverse LT of Equation (5) as provided in [43]:*

$$ u(\varsigma,\eta) = \sum_{i=0}^{\infty}\frac{D_{\eta}^{\gamma}u(\varsigma,0)}{\Gamma(1+i\gamma)}\eta^{i(\vartheta)}, \;\; 0<\vartheta\leq 1, \;\; \eta\geq 0. \tag{6}$$

This corresponds to the fractional Taylor's formula described in [45].

The convergence of the FPS in Theorem (1) is explained and proven by the following theorem.

3. General Methodology of LRPSM

$$D_{\eta}^{\gamma}u(\varsigma,\eta) = cD_{\varsigma}^{2}u(\varsigma,\eta) + au(\varsigma,\eta) - bu^{4}(\varsigma,\eta), \;\; 1<\gamma\leq 2, \tag{7}$$

with initial source

$$u(\varsigma,0) = f_0(\varsigma), \quad u_{\eta}(\varsigma,0) = g_0(\varsigma). \tag{8}$$

By employing LT to (7),

$$\mathbf{L}\left[D_{\mu}^{\gamma}u(\varsigma,\eta)\right] = c\mathbf{L}\left[D_{\varsigma}^{2}u(\varsigma,\eta)\right] + a\mathbf{L}^{2}[u(\varsigma,\eta)] - b\mathbf{L}\left[u^{4}(\varsigma,\eta)\right]. \tag{9}$$

As from the fact that $\mathcal{C}\left[D_{1}^{a}w(\varsigma,\eta)\right] = \mu^{a}\mathbf{L}[w(\varsigma,\eta)] - \mu^{a-1}u(\varsigma,0) - \mu^{a-2}u'(\varsigma,0)$ and by utilizing the initial condition (8), we have

$$U(\varsigma,\mu) = \frac{f_0(\varsigma)+g_0(\varsigma)}{\mu} + \frac{c}{\mu^{a}}D_{\mu}^{2}U(\varsigma,\mu) + \frac{a}{\mu^{a}}U(\varsigma,\mu) - \frac{b}{\mu^{a}}L^{2}\left[\left(\mathcal{C}^{-1}[U(\varsigma,\mu)]\right]^{a}\right], \tag{10}$$

with $U(\varsigma,\mu) = \mathbf{L}[w(\varsigma,\eta)]$.

We may express the transformed function $U(\varsigma,\mu)$ in the following manner:

$$U(\varsigma,\mu) = \sum_{n=0}^{\infty}\frac{f_{\mu}(\varsigma)}{\mu^{n\gamma+1}}. \tag{11}$$

The kth-truncated series of (11) can be expressed as

$$U_k(\varsigma,\mu) = \sum_{n=0}^{k}\frac{f_{\mu}(\varsigma)}{\mu^{n\gamma+1}} = \frac{f_o(\varsigma)+g_0(\varsigma)}{\mu} + \sum_{n=1}^{k}\frac{f_k(\varsigma)}{\mu^{n\gamma+1}}. \tag{12}$$

As provided in [46], from the definition of the Laplace residual function

$$\begin{aligned}\mathbf{L}Res_k(\varsigma,\mu) =& U_k(\varsigma,\mu) - \frac{f_0(\varsigma)+g_0(\varsigma)}{\mu} - \frac{c}{\mu^{\gamma}}D_{\mu}^{2}U_k(\varsigma,\mu) - \frac{a}{\mu^{\gamma}}U_k(\varsigma,\mu) \\ &+ \frac{b}{\mu^{\gamma}}\mathbf{L}\left[\left(\mathbf{L}^{-1}[U_k(\varsigma,\mu)]\right]^{q}\right].\end{aligned} \tag{13}$$

We provide several features that emerge in the common residual power series approach [46]:
$\mathbf{L}\,\mathrm{Res}(\varsigma,\mu) = 0$ and $\lim_{k\to\infty}\mathbf{L}\,\mathrm{Res}\,\mu_k(\varsigma,\mu) = \mathbf{L}\,\mathrm{Res}(\varsigma,\mu)$ for each $\mu > 0$.
$\lim_{\mu\to\infty}\mu\mathbf{L}\,\mathrm{Res}(\varsigma,\mu) = 0 \Rightarrow \lim_{\mu\to\infty}\mu\mathbf{L}\,\mathrm{Res}(\varsigma,\mu) = 0$.
$\lim_{\mu\to\infty}\mu^{k\gamma+1}\mathbf{L}\,\mathrm{Res}(\varsigma,\mu) = \lim_{\mu\to\infty}\mu^{k\gamma+1}\mathbf{L}\,\mathrm{Res}_k(\varsigma,\mu) = 0, 0<\gamma\leq 1, k = 1,2,3,\ldots$ We will now solve the system below recursively in order to define the coefficient functions $f_n(\varsigma)$.

$$\lim_{\mu\to\infty}\left(\mu^{ka+1}\mathbf{L}Res_k(\varsigma,\mu)\right) = 0, \;\; 0<\gamma\leq 1, \;\; k = 1,2,3,\ldots$$

The next step is to take the inverse LT of $U_k(\varsigma,\mu)$ to obtain the kth approximation $u_k(\varsigma,\eta)$.

4. Numerical Examples

Here, we solve three problems to show the accuracy of the proposed method.

4.1. Problem

Assume the fractional partial differential equation of the following form:

$$D_{\eta}^{\gamma}u - u_{\varsigma\varsigma} - 2u_{\ell\ell} = 0, \quad 1<\gamma\leq 2, \tag{14}$$

with the initial condition

$$u(\varsigma,\ell,0)=\sin(\varsigma)\sin(\ell),\quad u_\eta(\varsigma,\ell,0)=0. \tag{15}$$

By employing LT to Equation (1) and by using Equation (9), we obtain

$$U(\varsigma,\ell,\mu)-\frac{\sin(\varsigma)\sin(\ell)}{\mu}+\frac{1}{\mu^\gamma}\mathbf{L}_\eta\left[\mathbf{L}_\eta^{-1}[U_{\varsigma\varsigma}]+\mathbf{L}_\eta^{-1}[U_{\ell\ell}]\right]=0. \tag{16}$$

The kth-truncated series can be expressed as

$$U(\varsigma,\mu)=\frac{\sin(\varsigma)\sin(\ell)}{\mu}+\sum_{n=1}^{k}\frac{f_n(\varsigma,\ell,\mu)}{\mu^{n\gamma+1}},\ \ k=1,2,3,4\cdots \tag{17}$$

so the kth-LRFs are

$$\mathbf{L}_t Res_{u,k}(\varsigma,\ell,\mu)=U_k(\varsigma,\ell,\mu)-\frac{\sin(\varsigma)\sin(\ell)}{\mu}+\frac{1}{\mu^\gamma}\mathbf{L}_\eta\left[\mathbf{L}_\eta^{-1}[U_{\varsigma\varsigma,k}]+\mathbf{L}_\eta^{-1}[U_{\ell\ell,,k}]\right]. \tag{18}$$

The kth-truncated series Equation (17) will be substituted into the kth-truncated residual function Equation (18) to yield $f_k(\varsigma,\ell,\mu)$. The resulting equation, $\mu^{k\gamma+1}$, will then be multiplied, and the relation $\lim_{\mu\to\infty}(\mu^{k\gamma+1}\mathbf{L}_t Res_{u,k}(\varsigma,\ell,\mu))=0$, $k=1,2,3,\cdots$. Several terms are as

$$\begin{aligned}
f_1(\varsigma,\ell,\mu)&=-(4)\sin(\varsigma)\sin(\ell),\\
f_2(\varsigma,\ell,\mu)&=(4)^2\sin(\varsigma)\sin(\ell),\\
f_3(\varsigma,\ell,\mu)&=-(4)^3\sin(\varsigma)\sin(\ell),\\
f_4(\varsigma,\ell,\mu)&=(4)^4\sin(\varsigma)\sin(\ell),
\end{aligned} \tag{19}$$

and so on.

We may now obtain by altering the values of $f_k(\varsigma,\mu)$, $k=1,2,3,\cdots$, in Equation (17).

$$U(\varsigma,\ell,\mu)=\frac{\sin(\varsigma)\sin(\ell)}{\mu}+\frac{-(4)\sin(\varsigma)\sin(\ell)}{\mu^{\gamma+1}}+\frac{(4)^2\sin(\varsigma)\sin(\ell)}{\mu^{2\gamma+1}}+\frac{-(4)^3\sin(\varsigma)\sin(\ell)}{\mu^{3\gamma+1}}+\frac{(4)^4\sin(\varsigma)\sin(\ell)}{\mu^{4\gamma+1}}+\cdots. \tag{20}$$

By employing inverse LT, we obtain

$$u(\varsigma,\eta)=\sin(\varsigma)\sin(\ell)-4\sin(\varsigma)\sin(\ell)\frac{\eta^\gamma}{\Gamma(\gamma+1)}+(4)^2\sin(\varsigma)\sin(\ell)\frac{\eta^{2\gamma}}{\Gamma(2\gamma+1)}-(4)^3\sin(\varsigma)\sin(\ell)\frac{\eta^{3\gamma}}{\Gamma(3\gamma+1)}+(4)^4\sin(\varsigma)\sin(\ell)\frac{\eta^{4\gamma}}{\Gamma(4\gamma+1)}+\cdots.$$

$$u(\varsigma,\ell,\eta)=\sin(\varsigma)\sin(\ell)\left(1-\frac{4\eta^\gamma}{\Gamma(\gamma+1)}+\frac{(4\eta^\gamma)^2}{\Gamma(2\gamma+1)}-\frac{(4\eta^\gamma)^3}{\Gamma(3\gamma+1)}+\frac{(4\eta^\gamma)^4}{\Gamma(4\gamma+1)}+\cdots\right). \tag{21}$$

On putting $\gamma=1$, we have

$$\begin{aligned}
u(\varsigma,\ell,\eta)&=\sin(\varsigma)\sin(\ell)\left(1-\frac{4\eta}{1!}+\frac{(4\eta)^2}{2!}-\frac{(4\eta)^3}{3!}+\frac{(4\eta)^4}{4!}+\cdots\right),\\
u(\varsigma,\ell,\eta)&=\sin(\varsigma)\sin(\ell)e^{-4\eta}.
\end{aligned} \tag{22}$$

4.2. Problem

Assume the fractional partial differential equation of the following form:

$$D_\eta^\gamma u-6u_\varsigma u+u_{\varsigma\varsigma\varsigma}=0,\quad 0<\gamma\le 1, \tag{23}$$

with the initial condition

$$u(\varsigma,0)=\frac{1}{6}(\varsigma-1). \tag{24}$$

By employing LT to Equation (23) and by using Equation (24), we obtain

$$U(\varsigma,\mu)-\frac{\frac{1}{6}(\varsigma-1)}{\mu}-\frac{1}{\mu^{\gamma}}\mathbf{L}_{\eta}\left[6\mathbf{L}_{\eta}^{-1}[U_{\varsigma}]\mathbf{L}_{\eta}^{-1}[U]-\mathbf{L}_{\eta}^{-1}[U_{\varsigma\varsigma\varsigma}]\right]=0. \tag{25}$$

The kth-truncated series can be expressed as

$$U(\varsigma,\mu)=\frac{\frac{1}{6}(\varsigma-1)}{\mu}+\sum_{n=1}^{k}\frac{f_n(\varsigma,\mu)}{\mu^{n\gamma+1}},\ \ k=1,2,3,4\cdots \tag{26}$$

so the kth-LRFs are

$$\mathbf{L}_{\eta}Res_{u,k}(\varsigma,\mu)=U_k(\varsigma,\mu)-\frac{\frac{1}{6}(\varsigma-1)}{\mu}+\frac{1}{\mu^{\gamma}}\mathbf{L}_{\eta}\left[6\mathbf{L}_{\eta}^{-1}[U_{\varsigma,k}]\mathbf{L}_{\eta}^{-1}[U_k]-\mathbf{L}_{\eta}^{-1}[U_{\varsigma\varsigma\varsigma,k}]\right]. \tag{27}$$

The kth-truncated series Equation (26) will be substituted into the kth-truncated residual function Equation (27) to yield $f_k(\varsigma,\ell,\mu)$. The resulting equation, $\mu^{k\gamma+1}$ will then be multiplied, and the relation $\lim_{\mu\to\infty}(\mu^{k\gamma+1}\mathbf{L}_tRes_{u,k}(\varsigma,\ell,\mu))=0$, $k=1,2,3,\cdots$. Several terms are as

$$\begin{aligned}f_1(\varsigma,\mu)=&\frac{(\varsigma-1)}{6},\\ f_2(\varsigma,\mu)=&\frac{(\varsigma-1)}{6},\\ f_3(\varsigma,\mu)=&\frac{(\varsigma-1)}{6},\end{aligned} \tag{28}$$

and so on.

We may now obtain by altering the values of $f_k(\varsigma,\mu)$, $k=1,2,3,\cdots$, in Equation (26).

$$U(\varsigma,\mu)=\frac{\frac{1}{6}(\varsigma-1)}{\mu}+\frac{\frac{(\varsigma-1)}{6}}{\mu^{\gamma+1}}+\frac{\frac{(\varsigma-1)}{6}}{\mu^{2\gamma+1}}+\frac{\frac{(\varsigma-1)}{6}}{\mu^{3\gamma+1}}+\cdots. \tag{29}$$

By employing inverse LT, we obtain

$$u(\varsigma,\eta)=\frac{1}{6}(\varsigma-1)+\frac{(\varsigma-1)}{6}\frac{\eta^{\gamma}}{\Gamma(\gamma+1)}+\frac{(\varsigma-1)}{6}\frac{\eta^{2\gamma}}{\Gamma(2\gamma+1)}+\frac{(\varsigma-1)}{6}\frac{\eta^{3\gamma}}{\Gamma(3\gamma+1)}+\cdots.$$

On putting $\gamma=1$, we have

$$u(\varsigma,\eta)=\frac{1}{6}\frac{\varsigma-1}{1-\eta}. \tag{30}$$

4.3. Problem

Assume the fractional partial differential equation of the following form:

$$D_{\eta}^{\gamma}u-u^3u_{\varsigma}u=0,\ \ 0<\gamma\le 1, \tag{31}$$

with the initial condition

$$u(\varsigma,0)=\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{\frac{2}{3}}. \tag{32}$$

By employing LT in Equation (31) and by using Equation (32), we obtain

$$U(\varsigma,\mu)-\frac{\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{\frac{2}{3}}}{\mu}+\frac{1}{\mu^{\gamma}}\mathbf{L}_{\eta}\left[\mathbf{L}_{\eta}^{-1}[U^{3}]\mathbf{L}_{\eta}^{-1}[U_{\varsigma}]\mathbf{L}_{\eta}^{-1}[U]\right]=0. \tag{33}$$

The kth-truncated series can be expressed as

$$U(\varsigma,\mu)=\frac{\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{\frac{2}{3}}}{\mu}+\sum_{n=1}^{k}\frac{f_{n}(\varsigma,\mu)}{\mu^{n\gamma+1}},\ \ k=1,2,3,4\cdots \tag{34}$$

so the kth-LRFs are

$$\mathbf{L}_{\eta}Res_{u,k}(\varsigma,\mu)=U_{k}(\varsigma,\mu)-\frac{\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{\frac{2}{3}}}{\mu}+\frac{1}{\mu^{\gamma}}\mathbf{L}_{\eta}\left[\mathbf{L}_{\eta}^{-1}[U_{k}^{3}]\mathbf{L}_{\eta}^{-1}[U_{\varsigma,k}]\mathbf{L}_{\eta}^{-1}[U_{k}]\right]. \tag{35}$$

The kth-truncated series Equation (34) will be substituted into the kth-truncated residual function Equation (35) to yield $f_k(\varsigma,\ell,\mu)$. The resulting equation, $\mu^{k\gamma+1}$, will then be multiplied, and the relation $\lim_{\mu\to\infty}(\mu^{k\gamma+1}\mathbf{L}_t Res_{u,k}(\varsigma,\ell,\mu))=0$, $k=1,2,3,\cdots$. Several terms are as

$$\begin{aligned}
f_{1}(\varsigma,\mu)=&-b^{\frac{2}{3}}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{1}{3}},\\
f_{2}(\varsigma,\mu)=&-\frac{b^{3}}{2}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{4}{3}},\\
f_{3}(\varsigma,\mu)=&b^{\frac{9}{2}}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{7}{3}}\left(\frac{15}{2}\frac{\Gamma(2\gamma+1)}{2(\Gamma(\gamma+1))^{2}}-16\right),
\end{aligned} \tag{36}$$

and so on.

We may now obtain by altering the values of $f_k(\varsigma,\mu)$, $k=1,2,3,\cdots$, in Equation (34).

$$U(\varsigma,\mu)=\frac{\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{\frac{2}{3}}}{\mu}+\frac{-b^{\frac{2}{3}}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{1}{3}}}{\mu^{\gamma+1}}+\frac{-\frac{b^{3}}{2}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{4}{3}}}{\mu^{2\gamma+1}}+\frac{b^{\frac{9}{2}}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{7}{3}}\left(\frac{15}{2}\frac{\Gamma(2\gamma+1)}{2(\Gamma(\gamma+1))^{2}}-16\right)}{\mu^{3\gamma+1}}+\cdots. \tag{37}$$

By employing inverse LT, we obtain

$$\begin{aligned}
u(\varsigma,\eta)=&\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{\frac{2}{3}}-b^{\frac{2}{3}}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{1}{3}}\frac{\eta^{\gamma}}{\Gamma(\gamma+1)}-\frac{b^{3}}{2}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{4}{3}}\frac{\eta^{2\gamma}}{\Gamma(2\gamma+1)}+\\
&b^{\frac{9}{2}}\left(a-\frac{3\sqrt{b}}{2}\varsigma\right)^{-\frac{7}{3}}\left(\frac{15}{2}\frac{\Gamma(2\gamma+1)}{2(\Gamma(\gamma+1))^{2}}-16\right)\frac{\eta^{3\gamma}}{\Gamma(3\gamma+1)}+\cdots.
\end{aligned}$$

On putting $\gamma=1$, we have

$$u(\varsigma,\eta)=\left(a-\frac{3\sqrt{b}}{2}(\varsigma+b\eta)\right)^{\frac{2}{3}}. \tag{38}$$

5. Results and Discussion

The numerical analysis between exact and approximative solutions, as shown in Tables 1–3, has been investigated in detail and with more precision in this study. The correctness and simplicity of the suggested method are demonstrated by computing the numerical values at different fractional orders. Tables 1–3 display the numerical comparison of the accurate and approximative solutions, demonstrating that the series solution soon converges to a small value. As a result, adding more terms for an approximate solution increases the accuracy of the analytical result. Figure 1 shows how the accurate and suggested approaches behave, as well as the characteristics of the approximative solution. For a better understanding of the problem's characteristics, we also provide the suggested method solution at various fractional orders in Figures 2 and 3. Figure 4 calculates the solution to problem 2 using the suggested and actual method. Figure 5 displays the graphical representations for $\gamma = 0.8, 0.6$. The behaviour of problem 2 in 2D and 3D for different fractional orders is shown in Figure 6. Similarly, Figure 7 presents the actual and suggested methods' solutions at $\gamma = 1$, whereas Figures 8 and 9 present the proposed approach solution at different fractional orders. Based on the tables and graphs, we came to the conclusion that the proposed technique solution was in good agreement with the precise solution.

Table 1. Comparison of the accurate and suggested technique solution at different values of γ for problem 1.

η	ς	$\gamma = 1.4$	$\gamma = 1.6$	$\gamma = 1.8$	$\gamma = 2(approx)$	$\gamma = 2(NHPM)$ [47]	$\gamma = 2(exact)$
0.01	0.2	0.0914144	0.0915022	0.0915102	0.0915124	0.0915123	0.0915124
	0.4	0.1792532	0.1793622	0.1793743	0.1793766	0.1793765	0.1793766
	0.6	0.2600133	0.2600643	0.2600832	0.2600895	0.2600894	0.2600895
	0.8	0.3303134	0.3304152	0.3304302	0.3304335	0.3304334	0.3304335
	1	0.3875321	0.3875931	0.3876011	0.3876042	0.3876041	0.3876042
0.02	0.2	0.0878231	0.0879102	0.0879211	0.0879242	0.0879241	0.0879242
	0.4	0.1722330	0.1723331	0.1723412	0.1723431	0.1723430	0.1723431
	0.6	0.2497621	0.2498821	0.2498901	0.2498913	0.2498911	0.2498913
	0.8	0.3173292	0.3174609	0.3174721	0.3174770	0.3174769	0.3174770
	1	0.3723141	0.3723920	0.3724021	0.3724060	0.3724060	0.3724060
0.03	0.2	0.0843204	0.0844602	0.0844720	0.0844766	0.0844765	0.0844766
	0.4	0.1654231	0.1655713	0.1655831	0.1655854	0.1655853	0.1655854
	0.6	0.2400015	0.2400820	0.2400912	0.2400929	0.2400928	0.2400929
	0.8	0.3050004	0.3050121	0.3050242	0.3050286	0.3050285	0.3050286
	1	0.3577116	0.3577930	0.3578010	0.3578038	0.3578037	0.3578038
0.04	0.2	0.0810192	0.0811513	0.0811614	0.0811642	0.0811641	0.0811642
	0.4	0.1590002	0.1590105	0.1590903	0.1590927	0.1590926	0.1590927
	0.6	0.2305280	0.2306696	0.2306754	0.2306787	0.2306786	0.2306787
	0.8	0.2930001	0.2930530	0.2930632	0.2930682	0.2930681	0.2930682
	1	0.3436561	0.3437631	0.3437709	0.3437741	0.3437740	0.3437741
0.05	0.2	0.0778561	0.0779702	0.0779800	0.0779817	0.0779816	0.0779817
	0.4	0.1527245	0.1528461	0.1528502	0.1528546	0.1528545	0.1528546
	0.6	0.2215890	0.2216241	0.2216312	0.2216337	0.2216336	0.2216337
	0.8	0.2814600	0.2815653	0.2815731	0.2815769	0.2815768	0.2815769
	1	0.3301343	0.3302863	0.3302916	0.3302945	0.3302944	0.3302945

Table 2. Comparison of the accurate and suggested technique solution at different values of γ for problem 2.

η	ς	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1(approx)$	$\gamma = 1(NHPM)$ [47]	$\gamma = 1(exact)$
	0.2	−0.1334995	−0.1334877	−0.1334768	−0.1334668	−0.1334669	−0.1334668
	0.4	−0.1001246	−0.1001158	−0.1001076	−0.1001001	−0.1001002	−0.1001001
0.01	0.6	−0.0667497	−0.0667438	−0.0667384	−0.0667334	−0.0667335	−0.0667334
	0.8	−0.0333748	−0.0333719	−0.0333692	−0.0333667	−0.0333668	−0.0333667
	1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
	0.2	−0.1336590	−0.1336381	−0.1336185	−0.1336005	−0.1336006	−0.1336005
	0.4	−0.1002443	−0.1002286	−0.1002139	−0.1002004	−0.1002005	−0.1002004
0.02	0.6	−0.0668295	−0.0668190	−0.0668092	−0.0668002	−0.0668001	−0.0668002
	0.8	−0.0334147	−0.0334095	−0.0334046	−0.0334001	−0.0334002	−0.0334001
	1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
	0.2	−0.1338163	−0.1337871	−0.1337597	−0.1337345	−0.1337346	−0.1337345
	0.4	−0.1003622	−0.1003403	−0.1003197	−0.1003009	−0.1003009	−0.1003009
0.03	0.6	−0.0669081	−0.0668935	−0.0668798	−0.0668672	−0.0668673	−0.0668672
	0.8	−0.0334540	−0.0334467	−0.0334399	−0.0334336	−0.0334337	−0.0334336
	1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
	0.2	−0.1339721	−0.1339352	−0.1339005	−0.1338688	−0.1338689	−0.1338688
	0.4	−0.1004791	−0.1004514	−0.1004253	−0.1004016	−0.1004017	−0.1004016
0.04	0.6	−0.0669860	−0.0669676	−0.0669502	−0.0669344	−0.0669345	−0.0669344
	0.8	−0.0334930	−0.0334838	−0.0334751	−0.0334672	−0.0334673	−0.0334672
	1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
	0.2	−0.1341270	−0.1340828	−0.1340411	−0.1340033	−0.1340034	−0.1340033
	0.4	−0.1005952	−0.1005621	−0.1005308	−0.1005025	−0.1005026	−0.1005025
0.05	0.6	−0.0670635	−0.0670414	−0.0670205	−0.0670016	−0.0670017	−0.0670016
	0.8	−0.0335317	−0.0335207	−0.0335102	−0.0335008	−0.0335009	−0.0335008
	1	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

Table 3. Comparison of the accurate and suggested technique solution at different values of γ for problem 3.

η	ς	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$	$\gamma = 1(approx)$	$\gamma = 1(NHPM)$ [47]	$\gamma = 1(exact)$
	0.2	2.3919191	2.3920191	2.3921191	2.3922195	2.3922194	2.3922195
	0.4	2.2607964	2.2608964	2.2609964	2.2610967	2.2610966	2.2610967
0.01	0.6	2.1257538	2.1258538	2.1259538	2.1260541	2.1260540	2.1260541
	0.8	1.9862757	1.9863757	1.9864757	1.9865760	1.9865759	1.9865760
	1	1.8417147	1.8418147	1.8419147	1.8420150	1.8420150	1.8420150
	0.2	2.3919181	2.3920181	2.3921181	2.3922188	2.3922187	2.3922188
	0.4	2.2607954	2.2608954	2.2609954	2.2610961	2.2610960	2.2610961
0.02	0.6	2.1257528	2.1258528	2.1259528	2.1260534	2.1260533	2.1260534
	0.8	1.9862747	1.9863747	1.9864747	1.9865753	1.9865752	1.9865753
	1	1.8417137	1.8418137	1.8419137	1.8420142	1.8420141	1.8420142
	0.2	2.3919171	2.3920171	2.3921171	2.3922182	2.3922181	2.3922182
	0.4	2.2607944	2.2608944	2.2609944	2.2610954	2.2610953	2.2610954
0.03	0.6	2.1257518	2.1258518	2.1259518	2.1260527	2.1260526	2.1260527
	0.8	1.9862737	1.9863737	1.9864737	1.9865746	1.9865745	1.9865746
	1	1.8417127	1.8418127	1.8419127	1.8420135	1.8420134	1.8420135
	0.2	2.3919161	2.3920161	2.3921161	2.3922175	2.3922174	2.3922175
	0.4	2.2607934	2.2608934	2.2609934	2.2610947	2.2610946	2.2610947
0.04	0.6	2.1257508	2.1258508	2.1259508	2.1260520	2.1260519	2.1260520
	0.8	1.9862727	1.9863727	1.9864727	1.9865739	1.9865738	1.9865739
	1	1.8417117	1.8418117	1.8419117	1.8420128	1.8420127	1.8420128
	0.2	2.3919151	2.3920151	2.3921151	2.3922169	2.3922168	2.3922169
	0.4	2.2607924	2.2608924	2.2609924	2.2610941	2.2610940	2.2610941
0.05	0.6	2.1257498	2.1258498	2.1259498	2.1260514	2.1260513	2.1260514
	0.8	1.9862717	1.9863717	1.9864717	1.9865732	1.9865731	1.9865732
	1	1.8417107	1.8418107	1.8419107	1.8420120	1.8420120	1.8420120

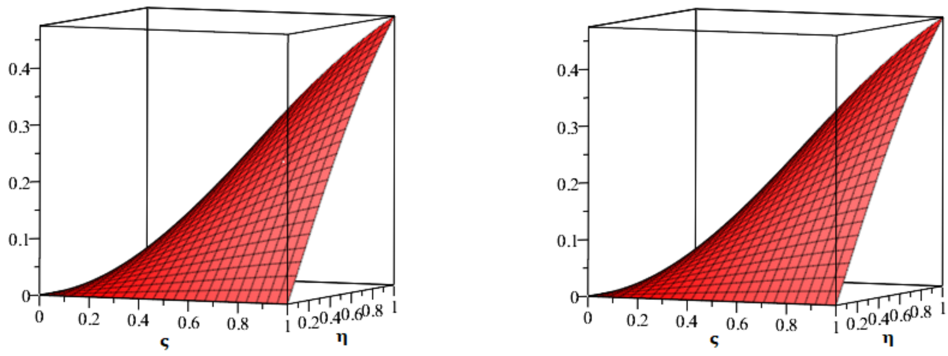

Figure 1. The proposed method solution and accurate solution at $\gamma = 2$.

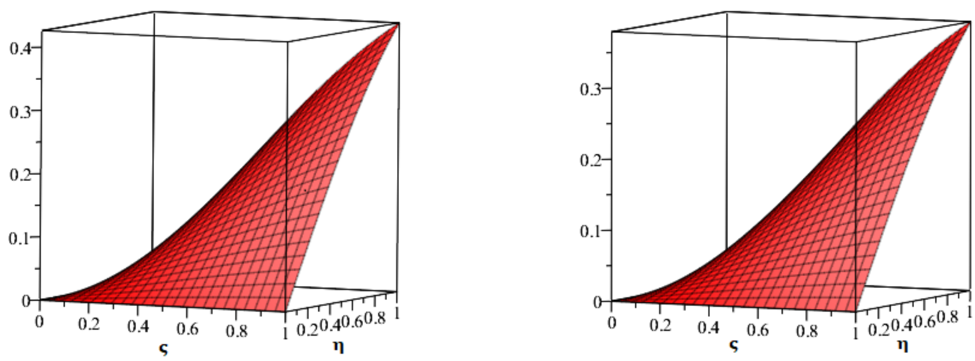

Figure 2. The proposed method solution at $\gamma = 1.8, 1.6$ for example 1.

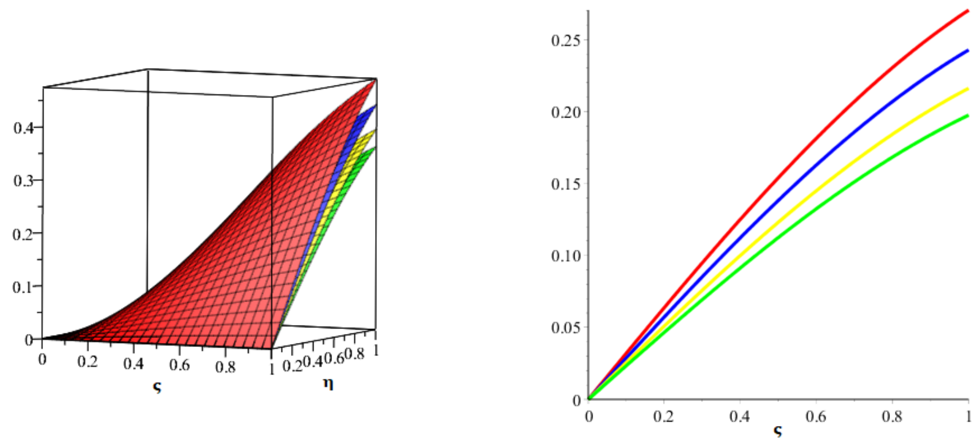

Figure 3. The proposed method solution for problem 1 at different values of γ.

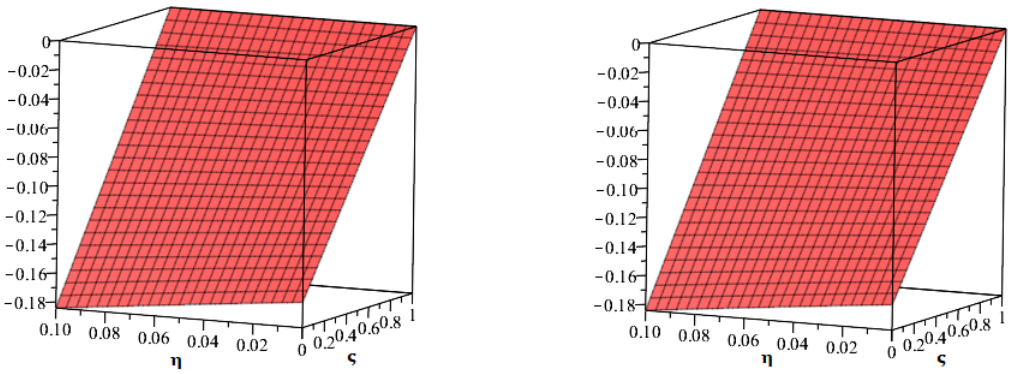

Figure 4. The proposed method solution and accurate solution at $\gamma = 1$.

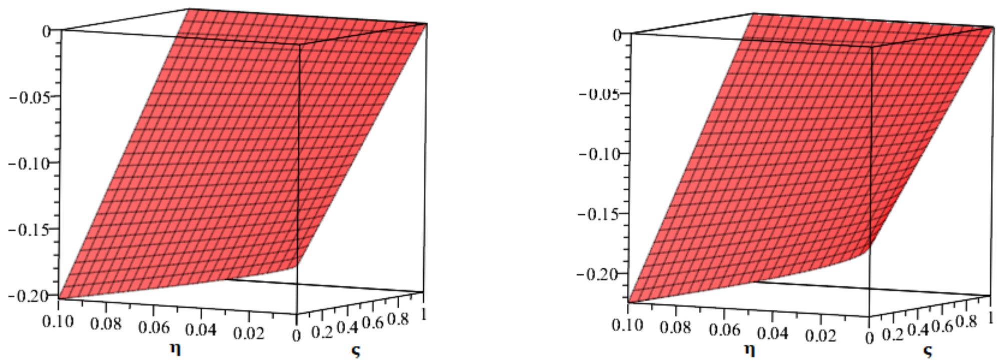

Figure 5. The proposed method solution at $\gamma = 0.8, 0.6$ for example 2.

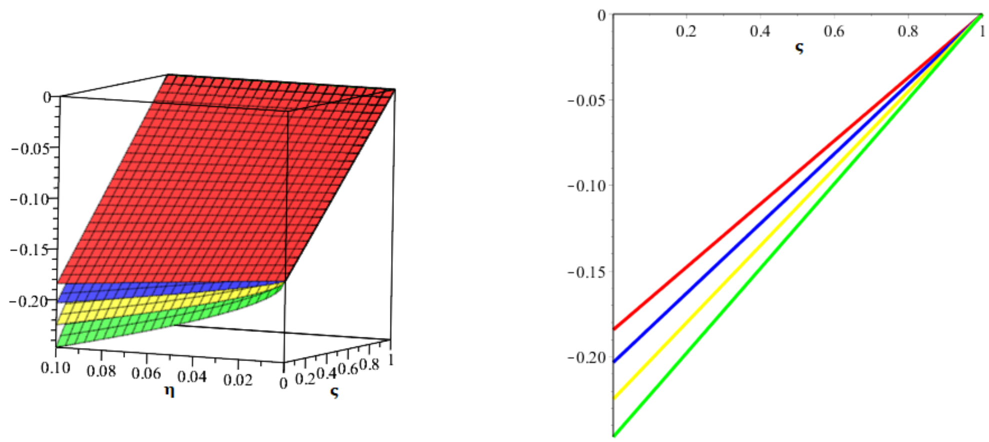

Figure 6. The proposed method solution at different values of γ for example 2.

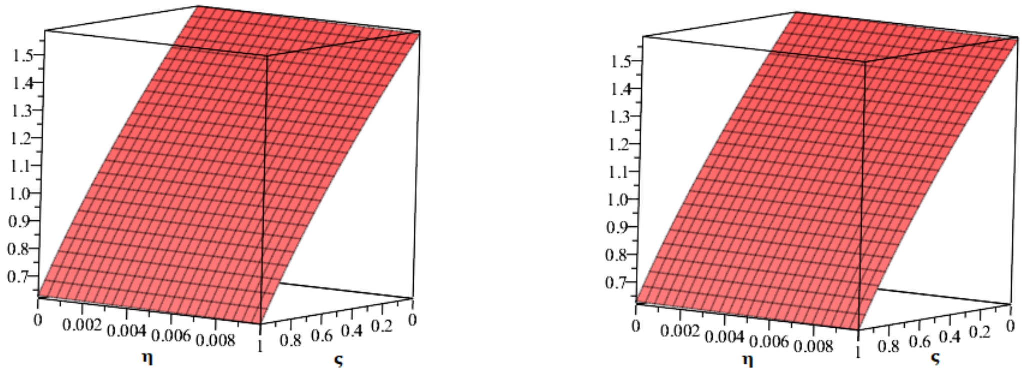

Figure 7. The proposed method solution and accurate solution at $\gamma = 1$.

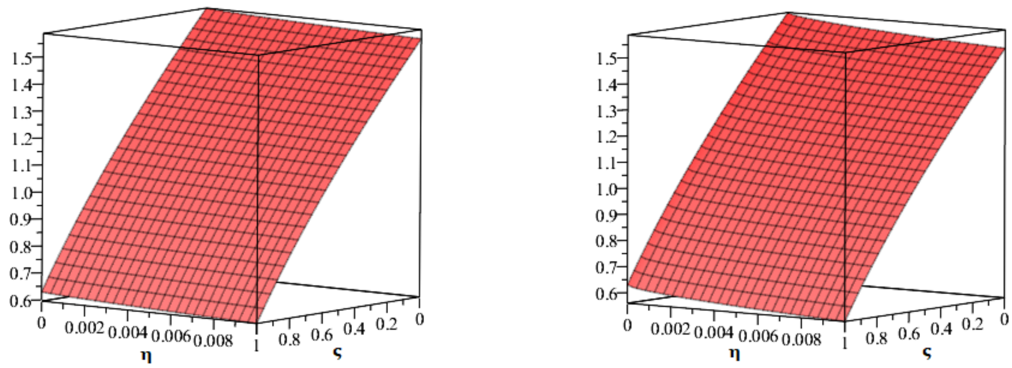

Figure 8. The proposed method solution at $\gamma = 0.8, 0.6$ for example 3.

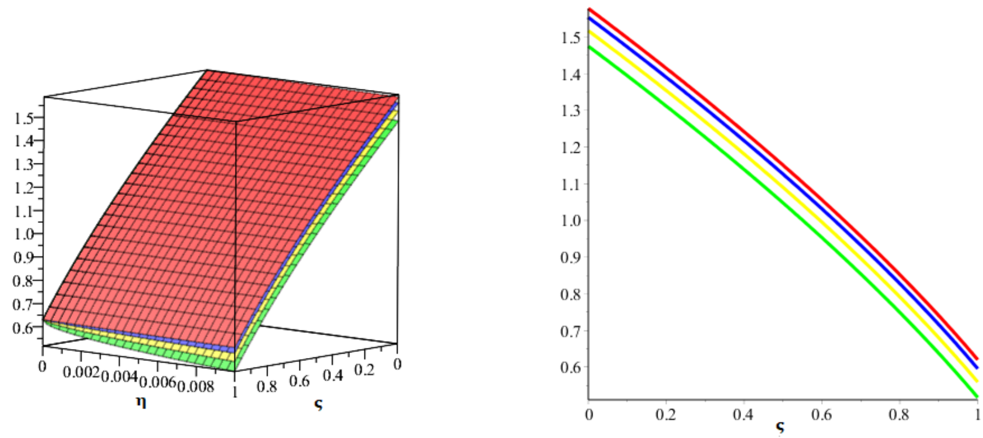

Figure 9. The proposed method solution at different values of γ for example 3.

6. Conclusions

In this study, the Laplace residual-power-series method (LRPSM), a powerful new technique for solving fractional partial differential equations, is developed by successfully combining the residual-power-series method (RPSM) and the Laplace transform. The new technique provides a series solution with elegant computational terms that quickly converges to an exact or approximate solution. The fractional derivative is handled in the Caputo sense in this novel analytical technique. With the help of the new analytical technique, fractional partial differential equations are successfully solved precisely. The obtained results via our technique are compatible with the results obtained by the natural homotopy perturbation method. In order to understand the behavior of the provided problems, solutions at different fractional orders are taken and are shown with the help of graphs and tables, which confirm that we get closer to the exact solution as the order of γ goes from fractional-order towards the integercorder. It is clearly proven that the new reliable approach is both straightforward and highly accurate. The Laplace residual-power-series method is a powerful and reliable technique for handling fractional partial differential equations that are both linear and nonlinear.

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