Correction

Correction: Bates et al. Vector Fields and Differential Forms on the Orbit Space of a Proper Action. *Axioms* 2021, 10, 118

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There was an error in the original article [1]. In the proof of lemma 5, Equation (19) is incorrect. A correction has been made to the proof of Lemma 5 in Section 2 Vector fields on M/G to the fifth, sixth and seventh paragraphs.

The corrected fifth paragraph is:

The module $\mathfrak{X}(B)^H$ of H invariant smooth vector fields on B is finitely generated by polynomial vector fields, see [16], and we denote a generating set by $\{X_j\}_{j=1}^N$. Hence, every H invariant smooth vector field X_B on B is of the form $X_B = \sum_{j=1}^N h_j X_j$ for some $h_1, \dots, h_n \in C^\infty(B)$. Similarly, every K invariant smooth vector field on $B_{(K)}$ can be written as $\sum_{j=1}^N f_j X_j$, where $f_j \in C^\infty(B_{(K)})$. Since $B_{(K)}$ is open and dense in B , a generic $f \in C^\infty(B_{(K)})$ need not extend to a smooth function on B . Therefore a generic vector field on $B_{(K)}$ need not extend to a smooth vector field on B . On the other hand, the H invariant vector field $X_{B_{(K)}}$ on $B_{(K)}$, is obtained above from a smooth bounded vector field Y on B/H . Therefore

$$X_{B_{(K)}}(\mathbf{x}) = \sum_{j=1}^N k_{j|B_{(K)}}(\mathbf{x}) X_j(\mathbf{x}), \quad (19)$$

for each $\mathbf{x} \in B_{(K)} \subseteq B \subseteq \mathbb{R}^n$, where every $k_j \in C^\infty(\mathbb{R}^n)$, and $k_{j|B_{(K)}}$ is the restriction of k_j to $B_{(K)}$.

Since $B_{(K)}$ is open and dense in B , we may define

$$X(\mathbf{x}) = \begin{cases} X_{B_{(K)}}(\mathbf{x}) & \text{if } \mathbf{x} \in B_{(K)} \\ \sum_{i=1}^N \lim_{k \rightarrow \infty} k_j(\mathbf{x}_k) X_j(\mathbf{x}_k), & \text{where } \{\mathbf{x}_k\} \subseteq B_{(K)} \text{ and } \mathbf{x} = \lim_{k \rightarrow \infty} \mathbf{x}_k \in B \setminus B_{(K)}, \end{cases} \quad (20)$$

provided that $\lim_{k \rightarrow \infty} k_j(\mathbf{x}_k)$ exists and is unique. Since the vector fields $X_j(\mathbf{x}_k)$ are smooth on B ,

$$\lim_{k \rightarrow \infty} k_j(\mathbf{x}_k) X_j(\mathbf{x}_k) = \left(\lim_{k \rightarrow \infty} k_j(\mathbf{x}_k) \right) X_j \left(\lim_{k \rightarrow \infty} \mathbf{x}_k \right) \text{ for every } j = 1, \dots, N.$$

The corrected sixth paragraph is:

Moreover, since $B_{(K)}$ is open and dense in B , it is open and dense in $\bar{B}$, the closure of B . In Equation (19), each function $k_{j|B_{(K)}}$ is the restriction to $B_{(K)} \subseteq B \subseteq \mathbb{R}^n$ of a smooth function k_j on $\mathbb{R}^n$. Moreover, the choice of polynomial basis $\{X_j\}_{j=1}^N$ ensures that the right-hand side of Equation (19) extends to the closure $\bar{B}$ of B . Hence all the the limits $\lim_{k \rightarrow \infty} k_j(\mathbf{x}_k)$ in Equation (20) exist, and $X(\mathbf{x})$ is defined for all $\mathbf{x} \in B$.

The corrected seventh paragraph is:

We need to show that this definition of X on B depends only on $X_{B_{(K)}}$. Since each k_j is continuous on B and its first partial derivatives are bounded on $\bar{B}$, it follows that k_j are uniformly continuous on $\bar{B}$. In particular, if $c : [0, 1] \rightarrow \bar{B}$ is a smooth curve, such that $c([0, 1)) \subseteq B_{(K)}$ and $c(1) \in B$, then

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$$k_{j|B}(c(1)) = k_{j|B}(c(0)) + \int_0^1 \frac{\partial k_{j|B}}{\partial t}(c(t))dt = k_{j|B(K)}(c(0)) + \int_0^1 \frac{\partial k_{j|B(K)}}{\partial t}(c(t))dt.$$

Thus, the values of k_j on B are uniquely determined by $k_{j|B(K)}$. Repeating this argument for all the first-order partial derivatives of k_j , we deduce that the first-order partial derivatives of k_j on B are uniquely determined by $k_{j|B(K)}$ and its first partial derivatives. Continuing this process for every partial derivative of every order shows that the restriction of k_j to B is uniquely determined by $k_{j|B(K)}$.

The authors apologize for any inconvenience caused and state that the scientific conclusions are unaffected. The original article has been updated.

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