axioms

MDPI

Article

Evolutionary Relation of Mathematical Physics Equations Evolutionary Relation as Foundation of Field Theory Interpretation of the Einstein Equation

Ludmila Petrova

Department of Computational Mathematics and Cybernetics, Moscow State University, 119991 Moscow, Russia; ptr@cs.msu.ru

Abstract: It was shown that from the mathematical physics equations that are composed of the conservation laws equations for energy, momentum, angular momentum, and mass and describe material media such as thermodynamical, gas-dynamical, cosmic, and others, it follows the evolutionary relation that possesses the properties of field theory equations. The evolutionary relation, which is based on conservation laws, unites the field theory equations, reveals their internal connection, and discloses the properties, which are common for all equations of field theory. The correspondence between the equations of field theory and evolutionary relation physics indicates that the equations of field theory are related to the equations of mathematical physics. This can reveal the fundamentals of field theory. These results are obtained using skew-symmetric differential forms describing conservation laws on which the equations of mathematical physics and equations of field theory are based. In this paper, the Einstein equation will be investigated by application of skew-symmetric differential forms.

Keywords: skew-symmetric differential forms; mathematical physics equations; field theory equations; conservation laws; Einstein equation

check for updates

Citation: Petrova, L. Evolutionary Relation of Mathematical Physics Equations Evolutionary Relation as Foundation of Field Theory Interpretation of the Einstein Equation. *Axioms* **2021**, *10*, 46. https://doi.org/10.3390/axioms10020046

Academic Editor: Palle E. T. Jorgensen

Received: 18 February 2021
Accepted: 24 March 2021
Published: 29 March 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Copyright: © 2021 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

In the late 19th to early 20th century, there were many observed phenomena which could not be described by the differential equations of mathematical physics (Anry Poincare named this situation "the crisis of physics"). This led to the fact that new equations were obtained in many branches of physics. The field theory equations (such as those of Einstein, Maxwell, Schroedinger, Dirac, and Heisenberg) are such equations. These equations are based on the properties of invariance or covariance that are necessary for a description of measurable physical structures and observed phenomena.

It turns out that the mathematical physics equations describing material media, such as thermodynamic, gas-dynamic, cosmic media, systems of charged particles, and so on, possess the properties of invariance and covariance, on which the field theory equations are based.

It follows from the properties of conservation laws, which have a following feature.

The conservation laws for physical fields (described by the field theory equations) and for material media (described by the equations of mathematical physics) are different conservation laws. The conservation laws for physical fields are laws that state the presence of conservative physical quantities or objects (structures). However, the conservation laws for material media are laws for energy, linear momentum, angular momentum, and mass. However, there exists a connection between the conservation laws for material media and those for physical fields, which indicates the connection between the field theory equations and the mathematical physics equations.

This is described by evolutionary relation followed from the conservation law equations for material media.

In Section 2, it is shown that conservation laws for physical fields are described by closed exterior forms, which are conservative quantities and invariants. Closed exterior forms are solutions to field theory equations.

Section 3 examines the equations of mathematical physics that describe material media, such as thermodynamic, gas-dynamic, space, and so on. It is shown that from the equations of conservation laws the evolutionary relation is obtained, which turns out to be non-identical, due to the properties of conservation laws. Such a non-identical relation possesses the properties of field theory equations. It is shown that closed exterior forms that are solutions of the equations of field theory are obtained from the evolutionary relation.

Section 4 demonstrates a correspondence between evolutionary relation and field theory equations, that indicates the connection of the equations of field theory with the equations of mathematical physics and reveals the foundations of field theory.

In Section 5, Einstein's equation will be investigated by application of skew-symmetric differential forms.

Note that invariant and covariance properties of mathematical physics equations are hidden ones. They do not directly follow from mathematical physics equations, but are realized discretely (in the presence of degrees of freedom) when describing evolutionary processes. Such properties of mathematical physics equations are described by skew-symmetric differential forms that correspond to the equations of conservation laws. In addition to exterior skew-symmetric forms which are based on integrable manifolds and structures, skew-symmetric forms defined on non-integrable manifolds were used. Such skew-symmetric forms (obtained by the author from differential equations) possess evolutionary properties that enable describing evolutionary processes and the processes of emergence of various structures.

2. The Closed Exterior Forms: Conservation Laws for Physical Fields

The conservation laws for physical fields are conservation laws that state the presence of conservative physical quantities or objects (structures). Such conservation laws, which can be named exact ones, are described by closed exterior skew-symmetric forms [1] (Noether is an example).

The exterior skew-symmetric form is called a closed one if its differential equals to zero:

$$d\theta^p = 0 \tag{1}$$

From condition (1), one can see that the closed form is a conservative quantity. This means that such a form can correspond to the conservation law for physical fields, i.e., a conservative physical quantity. (Such a physical quantity is preserved under non-degenerate transformations.)

If the form is closed only on a pseudostructure, i.e., the form is a closed inexact one, the closure condition can be written as

$$d_\pi \theta^p = 0 \tag{2}$$

In this case, the pseudostructure π obeys the condition

$$d_\pi {}^*\theta^p = 0 \tag{3}$$

Here, ${}^*\theta^p$ is the dual form.

From conditions (2) and (3) one can see that the dual form (pseudostructure which is a metric form of manifold) and closed inexact form (conservative quantity since its differential equals to zero) describe a conservative object that can also correspond to conservation law for physical fields.

The closed inexact exterior form and respective dual form made up a differential-geometric structure that describes a physical structure, namely, a pseudostructure with conservative physical quantity. It can be seen that this physical structure is an object on

which the conservation law for physical fields is preserved. Such physical structures form physical fields.

Note that closed inexact exterior or dual forms are solutions of the field-theory equations. Moreover, there exists the following correspondence:

- Closed exterior forms of zero degree correspond to quantum mechanics.
- The Hamilton formalism is based on the properties of closed exterior and dual forms of first degree.
- The properties of closed exterior and dual forms of second degree underlie the equations of electromagnetic field.
- The closure conditions of exterior and dual forms of third degree form the basis of equations for gravitational field.

It can be seen that field theory equations are connected with closed exterior forms of a certain degree. This enables one to introduce a classification of physical fields by degrees of closed exterior forms.

Such a classification shows that there exists an internal connection between field theory equations that describe physical fields of various types. It is evident that the degree of closed exterior forms is a parameter that integrates field theories into unified field theory.

Such features of closed exterior forms are associated with the fact that closed exterior forms are invariants and can describe measurable physical structures. The invariance of closed inexact exterior forms is related to the covariance of the corresponding dual forms.

As a closed form is a differential (the total one, if the form is exact, or the interior one, if the form is inexact) [1–3], it is obvious that a closed form will turn out to be invariant under all transformations that preserve the differential. [The non-degenerate transformations in mathematics and mathematical physics, such as the unitary, tangent, canonical, gradient, etc., are examples of such transformations that preserve the differential.]

It is obvious that conservative physical quantities and conservative physical structures are physical quantities and physical structures that are conserved in non-degenerate transformations.

It turns out that closed inexact exterior forms, possessing invariant and covariant properties on which the field theory equations are based, are realized from the evolutionary relation obtained from the equations of mathematical physics. As will be shown below, evolutionary relation is associated with the equations of conservation laws for material media, of which the equations of mathematical physics are composed.

Here, it is necessary to emphasize once again that the conservation laws for physical fields and the laws for material media are different laws. The conservation laws for physical fields are conservation laws that state the presence of conservative quantities or objects (structures). However, the conservation laws for material media are the conservation laws for energy, linear momentum, angular momentum, and mass.

3. Analysis of Conservation Law Equations for Material Media

3.1. Evolutionary Relation

It is known that the equations of mathematical physics for thermodynamical, gasdynamical, cosmic, and other material media consist of the equations of conservation laws for energy, linear momentum, angular momentum, and mass [2,3]. It turns out that the equations of conservation laws for material media are not consistent, which indicates the non-commutativity of the conservation laws. Properties of equations of mathematical physics corresponding to equations of field theory are connected namely with inconsistency of equations of conservation laws for material media.

The peculiarity of this study is that the conservation laws equations are transformed into equations expressed in terms of *state functionals* [4].

It turns out that the functionals such as wave function, entropy, the action functional, the Pointing's vector, the Einstein tensor, and so on (which appears to be a field-theory functionals) are functionals that specify the state of material media [4]. (As the physical quantities such as energy, pressure, and density relates each to a single material medium, a

connection between them should exist. The state functionals describe such a connection on which the state of the material medium depends.)

One more feature of present study is *using two frames of reference* [5].

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material medium), and the second is an accompanying one (this frame of reference is connected with the manifold built by the trajectories of the material medium elements). (The Euler and Lagrange frames of reference are examples of such frames.)

[The analysis of the consistency of the equations of conservation laws and the study of the hidden properties of the equations of mathematical physics are carried out in detail in [5]. This article repeats what is necessary to substantiate the connection of the equations of field theory with the equations of mathematical physics.]

Let us investigate consistency of the equation of energy and the equation for linear momentum. In the accompanying frame of reference that is connected with the manifold made up by the trajectories of material medium elements, the equation of energy and the equation for linear momentum may be written in the form

$$\frac{\partial \psi}{\partial \xi^1} = A_1 \tag{4}$$

$$\frac{\partial \psi}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, \ldots \tag{5}$$

Here, ψ is the state functional, A_1 and A_ν are the quantities that depend on specific features of material medium and accordingly on energy and force actions onto medium, and ξ^1 and ξ^ν are the coordinates along the trajectory and in the direction normal to the trajectory.

Equations (4) and (5) can be convoluted into the relation

$$d\psi = \omega \tag{6}$$

where $\omega = A_\mu \, d\xi^\mu$ is the skew-symmetric differential form of the first degree.

In the general case (for energy, linear momentum, angular momentum, and mass), this relation will be the form

$$d\psi = \omega^p \tag{7}$$

where ω^p is the skew-symmetric form of degree p takes the values $p = 0, 1, 2, 3$.

The relations obtained and the skew-symmetric forms ω^p are evolutionary ones, as conservation laws equations are evolutionary ones.

[Concrete forms of relations (6) and (7) and their properties were considered in paper [1]. In the case of Euler and Navier–Stokes equations, the functional ψ is the entropy s. A concrete form of relation (7) for $p = 2$ was considered for electromagnetic field. In this case the functional ψ is Pointing's vector. The relation for Einstein's tensor is obtained by integrating the evolutionary relation for $p = 3$ (about this it will be told below).]

3.2. The Non-Identity of Evolutionary Relation: Non-Commutativity of Conservation Laws

The evolutionary relation has a peculiarity. This relation turns out to be non-identical and self-changing one.

The relation can be an identical one if it contains only invariant measurable terms.

The evolutionary relation prove to be nonidentical since the skew-symmetric evolutionary form, which includes in the right-hand side of evolutionary relation, is unclosed form, and, therefore, it is not an invariant.

The skew-symmetric evolutionary form is not a closed form because its differential is not equal to zero. This relates to the fact that such evolutionary skew-symmetric forms (as opposed to exterior form which are based on is an integrable manifold) are defined on an accompanying manifold, which is a deforming non-integrable manifold (see in [1], Sections 2 and 3). The differential of such a form contains a differential of the manifold metric form, which specifies a manifold deformation and thus is nonzero.

[Let us consider this by the example of the form $\omega = A_\mu d\xi^\mu$ in evolutionary relation (6).

The differential $d\omega$ of the form ω can be represented as $d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$, where $K_{\alpha\beta}$ is the commutator of evolutionary form which can be written in the form (see in [1], Section 3):

$$K_{\alpha\beta} = \left(\frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta}) A_\sigma$$

Here, $\Gamma^\sigma_{\beta\alpha}$ is connectednesses of an accompanying manifold. The commutator $K_{\alpha\beta}$ is nonzero as the coefficients A_μ not potential, and connectednesses of an accompanying manifold, which is a deforming non-integrable manifold, is not symmetric (see in [6], Part four. Mathematical additions). Thus, it turns out that the commutator of the form ω, and its differential does not equal zero.]

The non-identical evolutionary relation turns out to be self-changing one, that is, a change of one term leads to a change of another one and so on.

The non-identity of evolutionary relation point to the fact that the conservation law equations appear to be inconsistent. This means that the conservation laws turn out to be non-commutative.

It turns out that the evolutionary relation has unique properties.

The evolutionary relation not only reveals the non-commutativity of conservation laws for material media (namely, conservation laws for energy, linear momentum, angular momentum, and mass). It describes the role of conservation laws in evolutionary processes that are accompanied by the emergence of physical structures that form physical fields. This is due to the transition from non-commutative conservation laws for material media to conservation laws for physical fields. In the next subsection, it will be shown that from the evolutionary relation obtained from the equations of *conservation laws for material media*, follow closed inexact exterior forms that describe *the conservation laws for physical fields*.

3.3. Physical Properties of the Evolutionary Relation. Realization of Closed Inexact Exterior Forms. The Emergence of Physical Structures

Material media are exposed to various actions such as energetic, force, molecular, and others exterior non-potential influences. These actions cannot (due to non-commutativity) directly convert into the physical quantities (such as energy, pressure, and density) of material medium itself as they are non-potential and do not correspond to the nature of material medium. They are accumulated in material media as unmeasurable (unobservable) physical quantity. Such an unmeasurable physical quantity, which is described by the commutator of evolutionary form, acts in the material medium as internal force. This means that material medium is in non-equilibrium (under the action of internal forces) state.

This is described by an evolutionary relation.

The evolutionary relation contains a differential $d\psi$ of functional ψ that specifies a state of material medium. Availability of the differential of functional means that there exists a state function, and this indicates equilibrium state of material medium.

However, as evolutionary relation turns out to be nonidentical, from evolutionary relation it is impossible to obtain the differential of functional. This means that a material medium is in non-equilibrium state.

However, as it follows from evolutionary relation, the material medium can go into a local equilibrium state.

Under degenerate transformation (that does not preserve the differential) from the evolutionary form (whose differential is not zero), the closed inexact exterior form (whose differential is zero) and corresponding closed dual form are realized.

The degenerate transformations can take place under additional conditions, which are due to degrees of freedom. The vanishing of such functional expressions as determinants, Jacobians, Poisson's brackets, and others corresponds to these additional conditions. Additional conditions can be realized in the process of changing the nonequilibrium state of the material medium. This is described by the self-change of the evolutionary relation. Note

that the nonequilibrium state of the material medium may change under the influence of internal forces, but at the same time it will remain non-equilibrium.

The realization of the closed inexact exterior forms and relevant dual forms has unique physical meaning.

The closed inexact exterior form and the corresponding dual form (as shown in Section 2) correspond to the exact conservation laws, which are the conservation laws for physical fields. In addition, the closed inexact exterior form (which is a conservative quantity as its differential equals to zero) and the corresponding dual form (the pseudostructure on which inexact exterior form is defined) describe the physical structure on which the exact conservation law is satisfied.

It can be seen that the realization of closed inexact exterior forms (which occurs discretely) is associated with the fulfillment of the conservation law for physical fields and describes the emergence of physical structures. Such physical structures, on which the exact conservation law is satisfied, form physical fields.

As closed exterior forms that correspond to conservation laws for physical fields are realized from evolutionary forms that correspond to conservation laws for material media, this indicates a connection between conservation laws for physical fields and conservation laws for material media. It can be seen that such a connection, which is realized discretely, leads to the emergence of physical structures on which physical fields are formed.

The realization of dual form (pseudostructure) and closed inexact (on a pseudostructure) exterior form means that on an pseudostructure π from nonidentical evolutionary relation $d\psi = \omega^p$ it follows the identical relation $d\psi_\pi = \omega^p_\pi$, as the closed form ω^p_π is a differential. From identical relation one can define the differential of the state functional, and this points to a presence of the state function and the transition of material medium from non-equilibrium state into locally equilibrium one (only on pseudostructure).

The transition of the material medium into locally equilibrium state means that unmeasurable physical quantity, which acts as internal force, converts into a measured (unobservable) physical quantity of material medium. This reveals in emergence of some observed formations in material medium, such as waves, vortices, fluctuations, turbulent pulsations, and so on. However, as the process of emergence of observable formations occurs only locally, only a part of unobservable and unmeasurable quantities converts into observable formations and measurable quantities, and a remainder part of unobservable and unmeasurable physical quantities, which are described by the evolutionary form commutator, is kept in material medium. Such remainder part of unobservable and unmeasurable physical physical quantities form the dark energy and dark matter.

Thus, it can be seen that the transition of material medium into locally equilibrium state is accompanied by emergence of measurable physical structures (that can form physical fields) and appearance of observable formations.

Note that physical structures and observed formations are not identical objects. An example would be the occurrence of a wave. The wave is an observable formation, whereas the wave front is a physical structure. Moreover, at the same time the occurrence of the observed formation and physical structure are a manifestation of the same phenomenon. Light is an example of manifestation of such a duality, namely, as a wave and as a massless particle (photon).

[As it was shown in [5], from evolutionary relation it follows that the mathematical physics equations possess double solutions, namely, the solution on initial coordinate space and the solutions on integrable structures. The first one, which depends on the evolutionary form commutator, describes a nonequilibrium state of material medium, and the second solution, which is a discrete function, corresponds to the locally equilibrium state of material medium and describes physical structures. The degenerate transformation describes a transition from initial coordinate space to integrable structures. Such properties of the mathematical physics equations describe processes of emerging physical structures and observable formations, as well as a forming physical fields [7]].

As noted in the introduction, field theory equations are based on the properties of invariance or covariance that are necessary for a description of measurable physical structures and observed phenomena.

The closed inexact exterior forms and relevant dual forms, derived from the evolutionary form included in the evolutionary relation, possess invariance and covariance and describe measurable physical structures.

It can be seen that the properties of invariance and covariance inherent in the equations of field theory follow from the evolutionary relation obtained from the equations of mathematical physics.

This indicates the correspondence of the evolutionary relation to the equations of field theory.

It should be emphasized that the evolutionary relation also describes *the process of the emergence of physical structures and observed formations (observed phenomena.)*

4. Correspondence between the Evolutionary Relation and the Field Theory Equations. Foundation of General Field Theory

The evolutionary relation has the properties of the equations of field theory, which indicates a correspondence between the evolutionary relation and the equations of field theory.

The evolutionary relation is a relation for functionals such as the action functional, entropy, wave function, Lagrangian, Einstein's tensor, Poynting's vector, and others. The field theory equations are equations for the same functionals. The evolutionary relation reveals the physical meaning of the functionals. It was shown that the functionals of the equations of mathematical physics, and, accordingly, the functionals of the equations of field theory, are functionals describing the state of material media. That is, they are the state functionals of material media.

From the evolutionary relation are realized closed inexact exterior forms, which are solutions to field theory equations.

The field theory equations have the form of relations as identity relations, obtained from the evolutionary relation, written in terms of skew-symmetric forms or their tensor or differential analogs. Thus,

- the Einstein equation is a relation in skew-symmetric forms (or a tensor relation),
- the Maxwell equations have the form of tensor relations, and
- the Schredinger's equations have the form of relations expressed in terms of derivatives and their analogs.

[The field theory equations are not differential equations. They have the form of relation, on the right side of which there is a potential. The solutions of such equations, in contrast to differential equations, are differentials, not functions.]

The evolutionary relation unites the field theory equations, reveals their internal connection, and discloses the properties, which are common for all equations of field theory.

The correspondence between the evolutionary relation obtained from the equations of mathematical physics and the equations of field theory points out to a connection of the field-theory equations with the equations of mathematical physics. Such a connection, which is based on the properties of conservation laws, can reveal the fundamentals of field theory.

However, the evolutionary relation, as shown in the work, not only corresponds to the equations of field theory, it also describes the mechanism of evolutionary processes, such as the emergence of measurable physical structures.

From the evolutionary relation, it follows that closed inexact exterior forms that correspond to solutions of the field theory equations and describe physical structures are realized discretely. This indicates physical fields are quantum.

Such processes cannot be described within the framework of field theory equations. This is illustrated by the example of the Einstein equation.

5. Interpretation of the Einstein Equation

The Einstein equation [8] can be written in the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu},$$

where $R_{\mu\nu}$ is the Ricci tensor obtained from the tensor of space-time curvature $R_{\mu\nu\delta\rho}$, R is the scalar curvature, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the energy-momentum tensor, and G is the gravity constant.

Furthermore, the Einstein equation can be written in the form

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

where $G_{\mu\nu} = R_{\mu\nu} - 1/2 g_{\mu\nu} R$ is the Einstein tensor.

5.1. The Correspondence between the Einstein Equation and the Evolutionary Relation for the Cosmic Medium

As is well known, the specific feature of the Einstein equation is the fact that it contains only covariant tensors [8]. The covariant divergence of the Einstein tensor equals zero $G^{\mu}_{\nu;\mu} = 0$, that is, the Einstein tensor as well as the energy-momentum tensor are covariant tensors, i.e., closed exterior forms.

That is, the Einstein equation is an identity relation.

Here, it should be emphasized that when deriving the Einstein equation it was assumed that from the space-time curvature tensor $R_{\mu\nu\delta\rho}$ follow the the Riemann–Chistophel tensor (the curvature tensor of pseudo-Riemann manifold), the Ricci tensor $R_{\mu\nu}$, and the scalar curvature R, characterizing the pseodo-Riemann manifold and the Riemann space-time. That is, the Einstein equation describes only the Riemann space-time.

As it was shown, the evolutionary relation is not an identical relation. However, identical relations can be obtained from the evolutionary relation and this fact points to the correspondence between the Einstein equation and the evolutionary relation.

From the conservation laws equations for cosmic material medium one can obtain the evolutionary relation $d\psi = \omega^3$, where ψ is the functional specifying the state of cosmic medium and ω^3 is the evolutionary form of degree $p = 3$.

The evolutionary relation for the cosmic material medium is obtained from the equations of conservation laws of energy, linear momentum, angular momentum, and mass when studying a consistency of conservation laws equations. The equations of conservation laws were used when deriving the Einstein equation in [8,9]. However, it was not taken into account that the equations of conservation laws may be inconsistent. From the equations of the laws of conservation of energy and momentum, the covariant energy-momentum tensor was obtained that appears to be possible only under additional conditions.)

The evolutionary relation is defined on accompanying manifold that is a deforming manifold. (This is manifold that corresponds to cosmic medium.) Such a manifold cannot be a metric one. Its curvature ($R_{\mu\nu\delta\rho}$) is not a covariant tensor. This indicates that the evolutionary form ω^3 is not a closed form and the evolutionary relation appears to be nonidentical one. (Note that the curvature tensor ($R_{\mu\nu\delta\rho}$) used when deriving the Einstein equation cannot be a covariant tensor. The covariant tensor is the Riemann–Christoffel G^{i}_{jkl} tensor (obtained from the curvature tensor), which characterizes the pseudo-Riemann manifold.)

However, as it follows from the properties of evolutionary relation, under realization of any degree of freedom of material medium (to which the degenerate transformation corresponds) from the evolutionary form ω^3 the closed inexact exterior forms third degree are realized on pseudostructures. Under subsequent degenerate transformations, the closed inexact exterior forms of lowest degree can be realized and the identity relation corresponding to the Einstein equation can be obtained.

As it is known, when deriving the Einstein equation it was assumed that the following conditions have to be satisfied:

- the Bianchi identity is fulfilled,
- the connectedness coefficients are symmetric ones (the connectedness coefficients are the Christoffel symbols), and
- and there exists a transformation under which the connectedness coefficient becomes zero.

Above conditions are those of realization of the degenerate transformations for the nonidentical evolutionary relation and transition to the identical relations. (If the Bianchi identity is satisfied, then from the tensor curvature $R_{\mu\nu\delta\rho}$, the Riemann–Christoffel tensor G^i_{jkl}, which is a characteristic of the pseudo-Riemann manifold, can be obtained.)

The following should be noted. As it follows from the properties of evolutionary relation, the conservation laws equations for energy and momentum turn out to be inconsistent (i.e., the conservation laws for energy and momentum do not commutate). Moreover, this means that the energy and momentum cannot be described by the energy-momentum tensor. However, as it follows from the evolutionary relation, the conservation laws for energy and momentum can become commutative (locally) ones under a realization of any degrees of freedom, and thus be described by the energy-momentum tensor. In this case, under degenerate transformation from the evolutionary relation it is possible to obtain an identical relation that can correspond to the Einstein equation.

5.2. Differences between Evolutionary Relation and the Einstein Equation

As one can see, there exist differences between evolutionary relation and the Einstein equation.

In the Einstein equation, identical transitions from original curvature tensor $R_{\mu\nu\delta\rho}$ to Einstein's tensorare supposed (that is necessary for deriving the relevant Einstein equation). In particular, the covariant tensor Ricci is derived from the curvature tensor.

Moreover, from the evolutionary relation it follows that the closed inexact exterior forms, which are the covariant tensors, can be obtained only under degenerate transformation (under realization of any degrees of freedom of material media). This points to emergence of physical structures, namely, a pseudostructure (dual skew-symmetric forms) with conserved physical quantities (the closed inexact exterior forms). In other words, the transformations from initial tensor curvature to covariant tensors of lower order are not identical as under the derivation of the Einstein equation, but they are realized discretely and describe the emergence of physical structures; that is, the Einstein equation, which contains only covariant tensors, as opposed to evolutionary relation, cannot describe the evolutionary processes the emergence of physical structures and pseudomanifolds.

6. Conclusions

It is shown that from the equations of mathematical physics describing material media, an evolutionary relation is obtained, which corresponds to the equations of field theory describing physical fields.

The evolutionary relation is a relation for functionals such as action functional, entropy, wave function, Einstein tensor, Poynting vector, and others, which are functionals of field theory equations.

It is shown that from the evolutionary relation follow closed inexact exterior forms, which are solutions of the field theory equations.

The evolutionary relation unites the field theory equations, reveals their internal connection and discloses the properties, which are common for all equations of field theory.

The correspondence between the evolutionary relation and the field-theory equations points to a connection of the field-theory equations with the equations of mathematical physics. Such a connection, which is based (as shown) on the properties of conservation laws, can reveal the fundamentals of field theory.

However, the evolutionary relation, as shown in the work, not only corresponds to the equations of field theory, but also describes evolutionary processes such as the emergence of physical structures. Such processes cannot be described within the framework of field theory equations.

This is illustrated by the example of the Einstein equation.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Petrova, L.I. Exterior and Evolutionary Skew-Symmetric Differential Forms and Their Role in Mathematical Physics. 2003. Available online: http://arxiv.org/pdf/math-ph/0310050v1.pdf (accessed on 22 March 2021).
2. Clark, J.F.; Machesney, M. *The Dynamics of Real Gases*; Butterworths: London, UK, 1964.
3. Tolman, R.C. *Relativity, Thermodynamics, and Cosmology*; Clarendon Press: Oxford, UK, 1969.
4. Petrova, L.I. Physical meaning and a duality of concepts of wave function, action functional, entropy, the Pointing vector, the Einstein tensor. *J. Math. Res.* **2012**, *4*, 78–88. [CrossRef]
5. Petrova, L. Discrete Quantum Transitions, Duality: Emergence of Physical Structures and Occurrence of Observed Formations (Hidden Properties of Mathematical Physics Equations). *J. Appl. Math. Phys.* **2020**, *8*, 1911–1921. [CrossRef]
6. Tonnelat, M.A. *Les Principles de la Theorie Electromagnetique et la Relativite*; Masson: Paris, France, 1959.
7. Petrova, L.I. Formatting Physical Fields and Pseudometric Manifolds. The Dark Matter. *J. Phys. Conf. Ser.* **2020**, *1557*, 012032. [CrossRef]
8. Einstein, A. *The Meaning of Relativity*; Princeton University Press: Princeton, NJ, USA, 1953.
9. Weinberg, S. *Gravitation and Cosmology. Principles and Applications of the General Theory of Relativity*; Wiley & Sons, Inc.: Hoboken, NJ, USA, 1972.