

Supporting information

Table S1. Three-variable CCD used for the Falcon UF tests and experimental results, i.e. yield, WO_3 (R_W), K_2O (R_K), P_2O_5 (R_P), and Fe_2O_3 (R_{Fe}) recoveries, and D_{eff} .

Tests #	x_1 (ω)	x_2 (‰)	x_3 (Q)	Yield	R_W	R_K	R_P	R_{Fe}	D_{eff}
Units	Hz	wt. %	$\text{kg}\cdot\text{min}^{-1}$	%	%	%	%	%	%
1	30 (−1)	2 (−1)	1 (−1)	49.5	76.5	40.5	52.6	48.5	60.2
2	70 (+1)	2 (−1)	1 (−1)	28.9	60.2	24.9	28.5	28.4	75.8
3	70 (+1)	10 (+1)	1 (−1)	56.9	81.7	50.2	58.8	56.2	47.4
4	30 (−1)	10 (+1)	1 (−1)	63.2	82.4	57.4	64.1	61.2	59.7
5	50 (0)	6 (0)	2 (0)	62.3	85.0	34.0	43.5	39.3	61.1
6	30 (−1)	2 (−1)	3 (+1)	45.0	83.9	32.7	46.2	41.1	70.7
7	70 (+1)	2 (−1)	3 (+1)	29.8	78.1	23.7	33.4	29.0	72.5
8	30 (−1)	10 (+1)	3 (+1)	42.1	88.2	52.3	69.7	59.8	62.3
9	70 (+1)	10 (+1)	3 (+1)	40.5	75.9	38.5	47.6	44.1	58.9
10	50 (0)	6 (0)	3 (+1)	40.4	87.9	33.6	49.0	42.2	64.4
11	50 (0)	10 (+1)	2 (0)	55.1	92.1	49.9	59.9	57.1	59.5
12	30 (−1)	6 (0)	2 (0)	49.6	88.9	45.6	60.7	52.9	67.6
13	50 (0)	6 (0)	1 (−1)	48.5	89.2	45.0	51.6	50.8	57.3
14	70 (+1)	6 (0)	2 (0)	38.8	84.9	35.0	44.1	40.2	63.8
15	50 (0)	6 (0)	2 (0)	42.2	88.2	38.3	47.2	45.2	62.6
16	50 (0)	2 (−1)	2 (0)	28.7	82.7	25.8	31.5	30.7	74.5

Table S2. Analysis of variance (ANOVA) for the WO_3 , K_2O , P_2O_5 , and Fe_2O_3 recoveries and the desliming efficiency for the Falcon UF presenting the degrees of freedom (DF), sum of squares (SS), F-ratios (F) and p-values (P) for the first and the final runs. Critical F-values for 0.05 significance level are $F_{(10,5)} = 4.74$, $F_{(3,12)} = 3.49$ for the R_W , R_K , R_{Fe} models, $F_{(2,13)} = 3.81$ for the R_P model, and $F_{(1,14)} = 4.60$ for the D_{eff} model.

Run Source of Variation	First				Final				First				Final			
	DF	R_W			DF	R_W			DF	R_K			DF	R_K		
		SS	F	P		SS	F	P		SS	F	P		SS	F	P
Model	10	808.64	5.41	0.0382	3	512.52	5.53	0.0128	10	1523.84	21.01	0.0018	3	1469.56	64.91	<0.0001
x_1 (ω)	1	152.33	10.19	0.0242	1	152.33	4.93	0.0464	1	315.96	43.56	0.0012	1	315.96	41.87	<0.0001
x_2 ($\%_s$)	1	151.01	10.10	0.0246	1	151.01	4.89	0.0472	1	1015.66	140.02	<0.0001	1	1015.66	134.59	<0.0001
x_3 (Q)	1	57.41	3.84	0.1073	-	-	-	-	1	137.94	19.02	0.0073	1	137.94	18.28	0.0011
$x_1 \times x_2$ ($\omega \times \%_s$)	1	10.60	0.71	0.4381	-	-	-	-	1	1.62	0.22	0.6564	-	-	-	-
$x_1 \times x_3$ ($\omega \times Q$)	1	0.14	0.01	0.9272	-	-	-	-	1	0.00	0.00	0.9920	-	-	-	-
$x_2 \times x_3$ ($\%_s \times Q$)	1	80.58	5.39	0.0679	-	-	-	-	1	7.68	1.06	0.3506	-	-	-	-
$x_1 \times x_2 \times x_3$ ($\omega \times \%_s \times Q$)	1	61.99	4.15	0.0973	-	-	-	-	1	21.78	3.00	0.1437	-	-	-	-
x_1^2 ($\omega \times \omega$)	1	50.85	3.40	0.1244	1	209.18	6.77	0.0232	1	10.94	1.51	0.2741	-	-	-	-
x_2^2 ($\%_s \times \%_s$)	1	39.62	2.65	0.1644	-	-	-	-	1	0.46	0.06	0.8110	-	-	-	-
x_3^2 (Q $\times$ Q)	1	19.46	1.30	0.3056	-	-	-	-	1	2.86	0.39	0.5576	-	-	-	-
Residuals	5	74.74			12	370.86			5	36.27			12	90.55		
Total	15	883.38			15	883.38			15	1560.11			15	1560.11		

Run		First			Final			
Source of Variation	DF	SS	R _F	P	DF	SS	R _F	P
Model	10	2014.82	19.42	0.0022	2	1820.67	48.10	<0.0001
x_1 (ω)	1	653.19	62.96	0.0005	1	653.19	34.51	<0.0001
x_2 (ω_s)	1	1167.48	112.53	0.0001	1	1167.48	61.69	<0.0001
x_3 (Q)	1	9.43	0.91	0.3842	-	-	-	-
$x_1 \times x_2$ ($\omega \times \omega_s$)	1	11.45	1.10	0.3416	-	-	-	-
$x_1 \times x_3$ ($\omega \times Q$)	1	3.93	0.38	0.5650	-	-	-	-
$x_2 \times x_3$ ($\omega_s \times Q$)	1	2.13	0.21	0.6693	-	-	-	-
$x_1 \times x_2 \times x_3$ ($\omega \times \omega_s \times Q$)	1	99.48	9.59	0.0270	-	-	-	-
x_1^2 ($\omega \times \omega$)	1	40.52	3.91	0.1051	-	-	-	-
x_2^2 ($\omega_s \times \omega_s$)	1	20.44	1.97	0.2194	-	-	-	-
x_3^2 (Q $\times$ Q)	1	8.50	0.82	0.4069	-	-	-	-
Residuals	5	51.88			13	246.03		
Total	15	2066.70			15	2066.70		

Run	First				Final				First				Final			
Source of Variation	DF	R_{Fe}			DF	R_{Fe}			DF	D_{eff}			DF	D_{eff}		
		SS	F	P		SS	F	P		SS	F	P		SS	F	P
Model	10	1616.52	13.84	0.0048	3	1527.77	41.53	<0.0001	10	768.78	16.79	0.0031	1	434.81	17.06	0.0010
x_1 (ω)	1	432.70	37.04	0.0017	1	432.70	35.28	<0.0001	1	0.40	0.09	0.7784	-	-	-	-
x_2 ($\%s$)	1	1011.84	86.62	0.0002	1	1011.84	82.51	<0.0001	1	434.81	94.95	0.0002	1	434.81	17.06	0.0010
x_3 (Q)	1	83.23	7.13	0.0444	1	83.23	6.79	0.0230	1	80.54	17.59	0.0085	-	-	-	-
$x_1 \times x_2$ ($\omega \times \%s$)	1	16.85	1.44	0.2835	-	-	-	-	1	137.20	29.96	0.0028	-	-	-	-
$x_1 \times x_3$ ($\omega \times Q$)	1	0.97	0.08	0.7845	-	-	-	-	1	3.06	0.67	0.4507	-	-	-	-
$x_2 \times x_3$ ($\%s \times Q$)	1	5.56	0.48	0.5209	-	-	-	-	1	5.60	1.22	0.3194	-	-	-	-
$x_1 \times x_2 \times x_3$ ($\omega \times \%s \times Q$)	1	43.95	3.76	0.1101	-	-	-	-	1	64.58	14.10	0.0132	-	-	-	-
x_1^2 ($\omega \times \omega$)	1	7.15	0.61	0.4695	-	-	-	-	1	3.78	0.83	0.4054	-	-	-	-
x_2^2 ($\%s \times \%s$)	1	2.79	0.24	0.6458	-	-	-	-	1	16.51	3.60	0.1161	-	-	-	-
x_3^2 (Q $\times$ Q)	1	6.85	0.59	0.4785	-	-	-	-	1	35.86	7.83	0.0381	-	-	-	-
Residuals	5	58.41			12	147.16			5	22.90			14	356.87		
Total	15	1674.93			15	1674.93			15	791.68			15	791.68		

Table S3. Four-variable CCD used for the Falcon SB tests and experimental results, i.e., WO_3 (R_W), K_2O (R_K), P_2O_5 (R_P), and Fe_2O_3 (R_{Fe}) recoveries, and D_{eff} .

Tests #	x_1 (ω)	x_2 (% $_{\text{os}}$)	x_3 (Q)	x_4 (f)	Yield	R_W	R_K	R_P	R_{Fe}	D_{eff}
Units	Hz	wt. %	$\text{kg}\cdot\text{min}^{-1}$	PSI	%	%	%	%	%	%
1	70 (+1)	2 (-1)	3 (+1)	3 (+1)	20.3	86.0	14.0	22.9	19.0	93.1
2	30 (-1)	2 (-1)	1 (-1)	3 (+1)	12.1	53.9	5.5	16.3	11.9	95.8
3	30 (-1)	2 (-1)	3 (+1)	1 (-1)	21.8	64.6	14.9	25.1	21.3	94.2
4	70 (+1)	2 (-1)	3 (+1)	1 (-1)	27.0	79.5	18.7	34.3	25.5	87.9
5	30 (-1)	2 (-1)	1 (-1)	1 (-1)	20.4	81.9	13.5	23.4	20.1	94.4
6	70 (+1)	2 (-1)	1 (-1)	1 (-1)	32.7	67.0	27.8	34.9	31.0	80.2
7	30 (-1)	10 (+1)	1 (-1)	3 (+1)	11.7	50.6	7.4	14.5	11.0	97.4
8	50 (0)	6 (0)	2 (0)	2 (0)	17.7	72.3	14.0	22.4	16.9	95.1
9	70 (+1)	10 (+1)	1 (-1)	1 (-1)	27.5	75.4	21.5	29.4	25.5	88.0
10	70 (+1)	10 (+1)	3 (+1)	1 (-1)	29.4	76.5	23.5	33.7	28.0	89.1
11	70 (+1)	10 (+1)	1 (-1)	3 (+1)	20.8	73.9	15.0	24.4	19.8	93.2
12	30 (-1)	10 (+1)	3 (+1)	3 (+1)	15.6	49.7	9.8	20.6	15.0	96.6
13	70 (+1)	2 (-1)	1 (-1)	3 (+1)	25.6	84.0	17.2	29.2	24.0	88.6
14	30 (-1)	2 (-1)	3 (+1)	3 (+1)	7.9	62.7	4.7	11.6	7.2	98.4
15	30 (-1)	10 (+1)	1 (-1)	1 (-1)	18.3	70.2	11.8	22.6	17.1	96.4
16	30 (-1)	10 (+1)	3 (+1)	1 (-1)	33.7	61.7	24.6	35.1	25.1	89.6
17	70 (+1)	10 (+1)	3 (+1)	3 (+1)	24.5	62.9	19.5	27.5	22.8	91.8
18	50 (0)	6 (0)	3 (+1)	2 (0)	16.5	74.4	12.8	20.5	19.8	97.4
19	50 (0)	6 (0)	1 (-1)	2 (0)	16.3	76.8	14.1	22.2	18.0	96.8
20	50 (0)	6 (0)	2 (0)	3 (+1)	15.2	71.5	11.3	20.2	19.3	97.9
21	70 (+1)	6 (0)	2 (0)	2 (0)	19.4	85.7	14.8	25.4	23.1	95.7
22	50 (0)	6 (0)	2 (0)	1 (-1)	18.3	81.9	13.3	27.2	19.6	96.5
23	50 (0)	2 (-1)	2 (0)	2 (0)	16.1	78.2	10.9	22.0	21.1	98.0
24	30 (-1)	6 (0)	2 (0)	2 (0)	17.2	77.5	11.1	22.9	21.3	97.8
25	50 (0)	10 (+1)	2 (0)	2 (0)	18.9	78.4	15.3	27.7	23.1	97.6
26	50 (0)	6 (0)	2 (0)	2 (0)	16.8	75.5	12.4	23.3	18.4	98.0

Table S1. Analysis of variance (ANOVA) for the WO₃, K₂O, P₂O₅, and Fe₂O₃ recoveries and the desliming efficiency for the Falcon SB presenting the degrees of freedom (DF), sum of squares (SS), F-ratios (F) and p-values (P). Critical F-values for 0.05 significance level are $F_{(19,6)} = 3.88$, $F_{(5,20)} = 2.71$, $F_{(2,23)} = 3.42$, and $F_{(3,22)} = 3.05$.

Run		First				Final				First				Final			
Source of Variation	DF	R _w			DF	R _w			DF	R _k			DF	R _k			
		SS	F	P		SS	F	P		SS	F	P		SS	F	P	
Model	19	2483.74	6.45	0.0144	5	1820.73	364.15	0.0001	19	728.16	7.00	0.0116	2	496.42	21.58	<0.0001	
x_1 (ω)	1	775.26	38.28	0.0008	1	775.26	19.76	0.0002	1	262.13	47.89	0.0005	1	262.13	22.79	<0.0001	
x_2 (% _s)	1	189.61	9.36	0.0222	1	189.61	4.83	0.0398	1	25.28	4.62	0.0752	-	-	-	-	
x_3 (Q)	1	13.64	0.67	0.4432	-	-	-	-	1	4.28	0.78	0.4105	-	-	-	-	
x_4 (f)	1	223.45	11.03	0.0160	1	223.45	5.70	0.0270	1	234.29	42.80	0.0006	1	234.29	20.37	0.0002	
$x_1 \times x_2$ ($\omega \times$ % _s)	1	0.59	0.03	0.8702	-	-	-	-	1	10.94	2.00	0.2072	-	-	-	-	
$x_1 \times x_3$ ($\omega \times$ Q)	1	31.89	1.58	0.2562	-	-	-	-	1	29.13	5.32	0.0605	-	-	-	-	
$x_1 \times x_4$ ($\omega \times$ f)	1	305.29	15.07	0.0081	1	305.29	7.78	0.0113	1	8.31	1.52	0.2640	-	-	-	-	
$x_2 \times x_3$ (% _s $\times$ Q)	1	40.29	1.99	0.2081	-	-	-	-	1	70.18	12.82	0.0116	-	-	-	-	
$x_2 \times x_4$ (% _s $\times$ f)	1	101.46	5.01	0.0665	-	-	-	-	1	0.94	0.17	0.6936	-	-	-	-	
$x_3 \times x_4$ (Q $\times$ f)	1	7.44	0.37	0.5667	-	-	-	-	1	1.19	0.22	0.6570	-	-	-	-	
$x_1 \times x_2 \times x_3$ ($\omega \times$ % _s $\times$ Q)	1	34.43	1.70	0.2401	-	-	-	-	1	1.19	0.22	0.6570	-	-	-	-	
$x_1 \times x_2 \times x_4$ ($\omega \times$ % _s $\times$ f)	1	84.50	4.17	0.0871	-	-	-	-	1	2.11	0.39	0.5576	-	-	-	-	
$x_1 \times x_3 \times x_4$ ($\omega \times$ Q $\times$ f)	1	198.04	9.78	0.0204	-	-	-	-	1	27.17	4.96	0.0675	-	-	-	-	
$x_2 \times x_3 \times x_4$ (% _s $\times$ Q $\times$ f)	1	25.68	1.27	0.3032	-	-	-	-	1	8.43	1.54	0.2611	-	-	-	-	
$x_1 \times x_2 \times x_3 \times x_4$ ($\omega \times$ % _s $\times$ Q $\times$ f)	1	17.74	0.88	0.3854	-	-	-	-	1	1.47	0.27	0.6228	-	-	-	-	
x_1^2 ($\omega \times \omega$)	1	3.00	0.15	0.7135	-	-	-	-	1	1.67	0.31	0.6010	-	-	-	-	
x_2^2 (% _s $\times$ % _s)	1	12.42	0.61	0.4634	-	-	-	-	1	2.20	0.40	0.5495	-	-	-	-	
x_3^2 (Q $\times$ Q)	1	62.17	3.07	0.1303	-	-	-	-	1	4.14	0.76	0.4178	-	-	-	-	
x_4^2 (f $\times$ f)	1	37.41	1.85	0.2230	1	327.12	8.34	0.0091	1	0.05	0.01	0.9311	-	-	-	-	
Residuals	6	121.52			20	784.54			6	32.85			23	264.59			
Total	25	2605.26			25	2605.26			25	761.00			25	761.00			

Run	Final				Final				Final					
Source of variation	DF	DF		R _p	DF	DF		R _{Fe}	DF	DF		R _{Fe}		
		SS	F			SS	F			SS	F			
Model	19	852.88	8.23	2	611.45	25.65	<0.0001	619.84	3.19	0.0780	2	485.35	28.50	<0.0001
x_1 (ω)	1	269.28	49.40	1	269.28	22.59	<0.0001	262.82	25.70	0.0023	1	262.82	30.86	<0.0001
x_2 (% _s)	1	13.78	2.53	-	-	-	-	2.38	0.23	0.6464	-	-	-	-
x_3 (Q)	1	11.33	2.08	-	-	-	-	1.69	0.17	0.6983	-	-	-	-
x_4 (f)	1	342.17	62.77	1	324.17	28.71	<0.0001	222.53	21.76	0.0035	1	222.54	26.13	<0.0001
$x_1 \times x_2$ ($\omega \times$ % _s)	1	31.58	5.79	-	-	-	-	7.52	0.74	0.4241	-	-	-	-
$x_1 \times x_3$ ($\omega \times$ Q)	1	13.95	2.56	-	-	-	-	11.54	1.13	0.3290	-	-	-	-

$x_1 \times x_4 (\omega \times f)$	1	13.88	2.55	-	-	-	-	12.48	1.22	0.3117	-	-	-	-
$x_2 \times x_3 (\%_{os} \times Q)$	1	80.55	14.78	-	-	-	-	61.58	6.02	0.0495	-	-	-	-
$x_2 \times x_4 (\%_{os} \times f)$	1	0.99	0.18	-	-	-	-	4.74	0.46	0.5214	-	-	-	-
$x_3 \times x_4 (Q \times f)$	1	24.60	4.51	-	-	-	-	4.87	0.48	0.5158	-	-	-	-
$x_1 \times x_2 \times x_3 (\omega \times \%_{os} \times Q)$	1	3.31	0.61	-	-	-	-	0.02	0.00	0.9695	-	-	-	-
$x_1 \times x_2 \times x_4 (\omega \times \%_{os} \times f)$	1	3.69	0.68	-	-	-	-	0.73	0.07	0.7987	-	-	-	-
$x_1 \times x_3 \times x_4 (\omega \times Q \times f)$	1	2.36	0.43	-	-	-	-	7.20	0.70	0.4337	-	-	-	-
$x_2 \times x_3 \times x_4 (\%_{os} \times Q \times f)$	1	1.22	0.22	-	-	-	-	0.28	0.03	0.8744	-	-	-	-
$x_1 \times x_2 \times x_3 \times x_4 (\omega \times \%_{os} \times Q \times f)$	1	1.23	0.23	-	-	-	-	0.20	0.02	0.8921	-	-	-	-
$x_1^2 (\omega \times \omega)$	1	3.96	0.73	-	-	-	-	7.10	0.69	0.4367	-	-	-	-
$x_2^2 (\%_{os} \times \%_{os})$	1	9.92	1.82	-	-	-	-	6.47	0.63	0.4566	-	-	-	-
$x_3^2 (Q \times Q)$	1	6.21	1.14	-	-	-	-	6.68	0.65	0.4498	-	-	-	-
$x_4^2 (f \times f)$	1	1.78	0.33	-	-	-	-	2.99	0.29	0.6083	-	-	-	-
Residuals	6	32.71		23	274.14			61.37			23	195.86		
Total	25	885.59		25	885.59			681.21			25	681.21		

Run Source of variation	First				Final			
	DF		D_{eff}		DF		D_{eff}	
		SS	F	P		SS	F	P
Model	19	466.28	5.87	0.0183	3	340.08	16.49	<0.0001
x_1 (ω)	1	155.17	37.14	0.0009	1	155.17	22.57	<0.0001
x_2 ($\%_{\text{os}}$)	1	4.76	1.14	0.3267	-	-	-	-
x_3 (Q)	1	2.96	0.71	0.4322	-	-	-	-
x_4 (f)	1	73.21	17.52	0.0058	1	73.21	10.66	0.0036
$x_1 \times x_2$ ($\omega \times \%_{\text{os}}$)	1	13.84	3.31	0.1186	-	-	-	-
$x_1 \times x_3$ ($\omega \times \text{Q}$)	1	18.02	4.31	0.0831	-	-	-	-
$x_1 \times x_4$ ($\omega \times \text{f}$)	1	4.04	0.97	0.3634	-	-	-	-
$x_2 \times x_3$ ($\%_{\text{os}} \times \text{Q}$)	1	31.58	7.56	0.0333	-	-	-	-
$x_2 \times x_4$ ($\%_{\text{os}} \times \text{f}$)	1	0.71	0.17	0.6937	-	-	-	-
$x_3 \times x_4$ (Q $\times$ f)	1	0.62	0.15	0.7125	-	-	-	-
$x_1 \times x_2 \times x_3$ ($\omega \times \%_{\text{os}} \times \text{Q}$)	1	0.37	0.09	0.7772	-	-	-	-
$x_1 \times x_2 \times x_4$ ($\omega \times \%_{\text{os}} \times \text{f}$)	1	4.08	0.98	0.3612	-	-	-	-
$x_1 \times x_3 \times x_4$ ($\omega \times \text{Q} \times \text{f}$)	1	12.92	3.09	0.1291	-	-	-	-
$x_2 \times x_3 \times x_4$ ($\%_{\text{os}} \times \text{Q} \times \text{f}$)	1	1.02	0.24	0.6388	-	-	-	-
$x_1 \times x_2 \times x_3 \times x_4$ ($\omega \times \%_{\text{os}} \times \text{Q} \times \text{f}$)	1	0.37	0.09	0.7772	-	-	-	-
x_1^2 ($\omega \times \omega$)	1	9.92	2.37	0.1743	1	111.70	16.25	0.0006
x_2^2 ($\%_{\text{os}} \times \%_{\text{os}}$)	1	2.00	0.48	0.5152	-	-	-	-
x_3^2 (Q $\times$ Q)	1	6.42	1.54	0.2615	-	-	-	-
x_4^2 (f $\times$ f)	1	5.63	1.35	0.2897	-	-	-	-
Residuals	6	25.07			22	151.27		
Total	25	491.35			25	491.35		