

Article

Reducing Bypass in Hydrocyclones: Part I—Preliminary Testing and Assessments

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Abstract

Hydrocyclones are widely applied devices in mineral processing due to their simple design, high capacity and low operational costs. Some of the main applications are classification in closed grinding circuits and desliming, as well as dewatering. However, hydrocyclones have an inherent inefficiency known as the fine particles bypass to the underflow stream, often associated with entrainment by water flow. Several approaches have been proposed to mitigate fine particle bypass, such as optimizing hydrocyclone design, adjusting apex and vortex finder diameters, water injection systems and improved inlet design. The objective of the present work was to assess hydrocyclone performance on different apex and vortex diameter combinations, seeking the reduction in fine particles bypass to underflow on the Paragominas bauxite processing industrial desliming circuit. Two different bauxite samples were used in a hydrocyclone classification test work, carried out on a specially built pilot plant. Six different combinations of apex and vortex were evaluated in a 254 mm diameter hydrocyclone, covering a range of apex-to-vortex diameters from 0.38 to 0.57. The results indicate operating conditions that significantly reduce fine particles bypass to underflow, increasing classification efficiency with minor effects in overflow selected size distribution parameter— d_{95} . Accordingly, smaller apex-to-vortex ratios result in overall better performances, reducing fine particles bypass to underflow from 33% to 7%, as well as reducing the partition curve slope from 0.52 to 0.21 for one of the tested samples. Significant benefits are also obtained in terms of reducing the contents of reactive silica in the underflow of the optimized desliming hydrocyclone.

Keywords: hydrocyclone; partition curve; fine particles bypass; pilot tests

Academic Editor: Weiran Zuo

Received: 16 February 2026

Revised: 27 March 2026

Accepted: 29 March 2026

Published: 31 March 2026

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1. Introduction

Hydrocyclones are widely applied devices in mineral processing due to their simple design, high capacity, relatively low operational costs, and absence of moving parts [1]. Their operation is based on the generation of centrifugal forces when slurry is laterally fed into the cylindrical section, creating a swirling flow that promotes particle classification according to size and density [2,3]. Coarse and dense particles' underflow are directed to the orifice located in the lower part of the conical section, apex, while finer and lighter ones are carried with water overflow, to the vortex finder, located in the cylindric section [4].

In industrial mineral processing plants, hydrocyclones are essential for closed grinding circuits, desliming and thickening [5–7].

In an ideal size separation process, particles larger than a specific size would report entirely to the coarse product, while finer particles would flow to the fine product. In practice, however, part of the fine material is bypassed to the coarse stream, and some coarse particles end up in the fine stream. The partition curve represents the weight fraction of feed reporting to either product stream [1]. Figure 1a illustrates an actual partition curve derived from measured data (dotted line) and a corrected partition curve (continuous line). A corrected curve is derived from the actual curve by removing the effect of bypassing material due to water split to underflow. This water bypass corresponds to the value on the actual partition curve at a zero-particle size. Three main parameters characterize the experimental partition curve: the cut point (d_{50}), which is the size where 50% of particles go to the underflow; the bypass (or short circuit), which represents the proportion of fines reporting to the underflow, corresponding to the intercept at the smallest particle size; and the sharpness of cut, indicating the slope of the central region, defined after d_{25} , d_{50} and d_{75} , shown in Figure 1b [8].

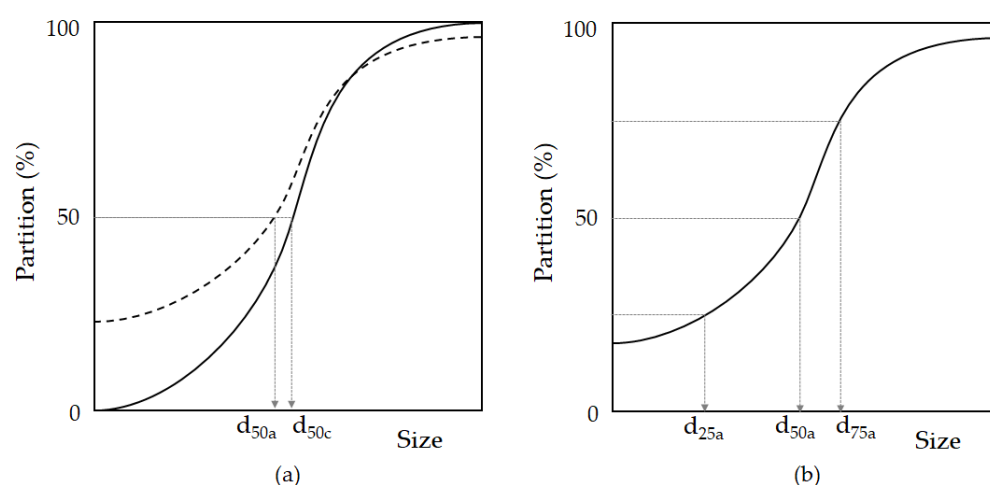


Figure 1. Partition curves: (a) experimental—dashed and corrected—solid; (b) highlighting d_{25} , d_{50} and d_{75} (adapted from [8]).

In bauxite beneficiation, hydrocyclones are used to remove clay minerals and improve the product quality [9]. However, hydrocyclones have an inherent inefficiency known as the fine particles bypass to the underflow stream, often associated with entrainment by water flow [10–12]. The theoretical framework to explain the bypass phenomenon was first assessed by Kelsall [13], whose work remains a cornerstone in understanding fine particle misplacement in hydrocyclones. This effect leads to excessive recirculation loads in grinding circuits, overgrinding, slime generation, and ultimately to losses in valuable mineral recovery [14,15]. In bauxite processing, misplaced fines reporting to the underflow decrease the beneficiation process product quality, leading to higher operational costs and residue generation in downstream processing [9,16].

Several approaches have been proposed to mitigate fine particle bypass. Geometrical modifications, such as optimizing cyclone dimensions [17] and adjusting apex, vortex finder diameters [18–20] and the use of semi-inverted cyclones [21], have shown significant effects on classification efficiency. Additionally, water injection systems have been investigated to displace entrained fines from the underflow back to the overflow, reducing misplacement [22,23]. Computational fluid dynamics (CFD) and experimental studies further support that careful control of operating conditions, including feed pressure and inlet design, can be a solution to enhance performance [24–26]. This study aimed to test and evaluate the effect of different apex and vortex finder diameter ratios on the classification efficiency in a hydrocyclone. Such an aspect is particularly important as it modulates not

only mass recovery, but also the product quality in the Paragominas industrial bauxite processing plant, operated by Norsk Hydro, in Pará state, within the Amazon region in Brazil. A total of six different apex and vortex finder diameter combinations were tested in a specially built pilot plant using two bauxite slurry samples with distinct particle size distributions.

2. Materials and Methods

2.1. Paragominas Industrial Plant

Figure 2 shows a simplified flow sheet of the two parallel processing lines installed at the Paragominas industrial plant. Despite the complexity, each line essentially comprises two grinding stages and a desliming circuit for processing 17 Mt per year bauxite, having as the main difference between each other ball mill classification, with hydrocyclones in one line and high-frequency screens in the other. The grinding circuit processes crushed ore to a final top size of 500 μm , the latter figure to comply with the specifications for transporting the bauxite pulp through a 244 km pipeline directly to Alunorte, an alumina refinery located in Barcarena, near the Pará state capital, Belém, in Brazil.

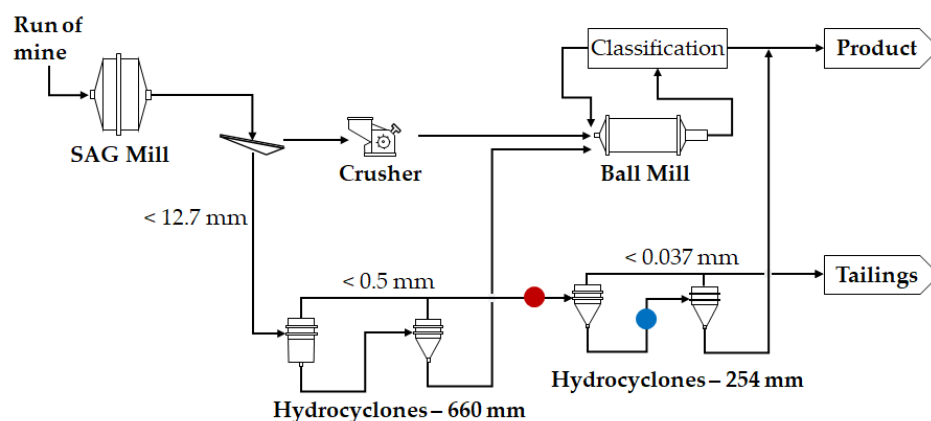


Figure 2. Simplified Paragominas industrial plant flow sheet. Streams submitted to testing highlighted in red—Sample 1 and blue—Sample 2.

The desliming circuit is an essential part of the processing plant, as it separates the relatively fine particles which concentrates the majority of the reactive silica, the latter a deleterious material for processing the bauxite at the alumina refinery. Primary and a secondary hydrocycloning stages are included in the desliming circuit, respectively, with 48 and 20 units of 254 mm (10") diameter in each processing line. Figure 3 shows the primary hydrocyclone stage installed in one of the two processing lines.



Figure 3. Industrial desliming stage.

2.2. Pilot Plant

A dedicated pilot plant was specially built and installed adjacent to the Paragominas industrial plant for carrying out hydrocyclone testing. The test rig comprising a single 254 mm diameter (10") hydrocyclone, with 10° cone angle and 1445 mm body length, including apex and vortex finder in various diameters, a 3 m³ stirred tank, and variable speed slurry pump, as well as slurry sampling devices.

Typical hydrocyclone operating feed flowrate was in the 85–150 m³/h range, at 200–314 kPa operating pressure.

The testing procedure started with filling the stirred slurry tank with a batch of bauxite slurry directly derived from the desliming stage feed in the industrial plant, as marked in the flowsheet shown in Figure 2—red circle for the finer stream (Sample 1) and blue circle for the coarser stream (Sample 2). The slurry was subsequently pumped to the hydrocyclone feed, while respective underflow and overflow were recirculated back to the feed tank, therefore forming a closed circuit, as shown in Figure 4. Valves installed in the pump discharge pipeline allow directing the slurry to cyclone feed, or the recirculation of the feed material to a sampling device and then back to the slurry tank. Manual vezin sampling devices installed on the hydrocyclone feed, underflow and overflow were used to obtain slurry samples, later used for solid concentration, size distribution and chemical composition determinations.

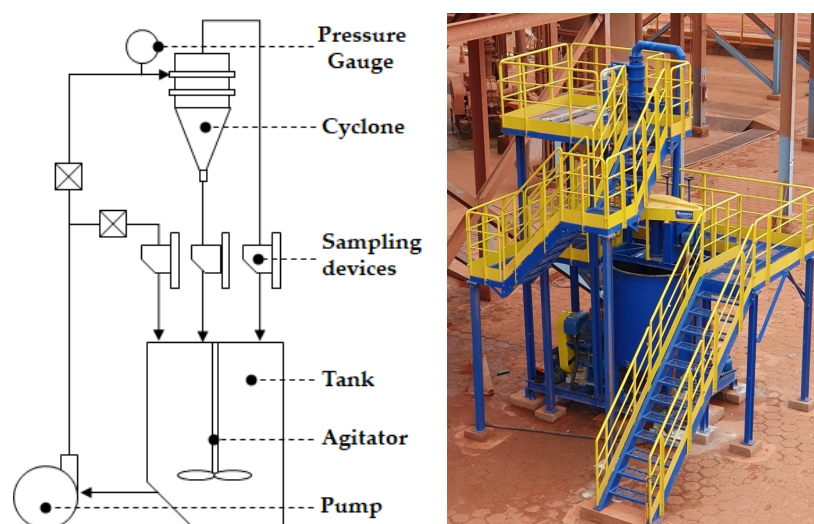


Figure 4. Pilot plant used on the present work. Schematic representation on the left side and photo of the actual installation on the right side (Source: Author's personal archive).

2.3. Material and Characterization Methods

Two testing campaigns were carried out to assess hydrocyclone performance according to different combinations of vortex and apex orifices. Each campaign was carried out with slurry directly gravitated from a sampling point located in the feed of the desliming stage of the Paragominas industrial plant. The respective slurry density was thus maintained, 1.07 g/cm³ and 1.09 g/cm³ for Samples 1 and 2, respectively.

The adopted nomenclature was Sample 1 for the first slurry batch, while Sample 2 corresponded to the second slurry batch. It is here emphasized that the former was obtained from the desliming first stage feed, i.e., red circle in Figure 2 flow sheet, while the latter was obtained from the desliming second stage feed, i.e., blue circle in Figure 2 flow sheet.

For each testing condition, the planned apex and vortex were assembled in the hydrocyclone, feed material was recirculated to the slurry tank to allow cyclone feed sampling,

and then, the recirculation valve was closed and the cyclone feed valve opened, directing slurry to the cyclone feed. A frequency inverter was used to fine tune the pump speed and keep 294 kPa pressure on the cyclone inlet in the different test conditions. Cyclone underflow and overflow samples were taken simultaneously, 2 min after setting inlet pressure. For the three cyclone streams, the sample acquisition procedure was to take eight increments with the vezin sampling devices, with a total duration of approximately 10 s, directing the analytic samples to 15 L polyethylene containers.

Samples obtained from the hydrocyclone feed, underflow and overflow during the testing campaigns were prepared according to the following procedures:

- Drying in a laboratory stove at 105 °C to obtain the slurry solids concentration by weight. Dry samples were quartered to generate aliquots for chemical and particle size analysis;
- Wet screening for obtaining the particle size distribution analysis, ranging from 600 µm to 37 µm. The fraction finer than 37 µm was used in laser size analysis, the latter carried out in a Malvern Mastersizer 3000 equipment;
- Determination of Al₂O₃, Fe₂O₃, SiO₂ and TiO₂ grades by X-ray Fluorescence—XRF. Available alumina and reactive silica grades determined after bauxite digestion at 150 °C in PARR bombs, followed by titration with CDTA (cyclohexanediaminetetraacetic acid) and Atomic Absorption Spectrometer analysis.

2.4. Experimental Testwork and Performance Indices

The testing conditions are listed in Table 1.

Table 1. Apex and vortex diameters used on each test.

Sample	1	2	1	2	1	2	1	2	1	2	1	2
Test	1	7	2	8	3	9	4	10	5	11	6	12
Vortex Diameter (mm)	102		102		102		89		89		89	
Apex Diameter (mm)	51		44		38		51		44		38	
Apex/Vortex Ratio	0.50		0.43		0.38		0.57		0.49		0.43	

The six conditions presented in Table 1 were separately tested for Sample 1 and Sample 2, therefore summing 12 complete classification tests. According to the adopted nomenclature, Tests 1 to 6 refer to Sample 1, while Tests 7 to 12 refer to Sample 2.

Raw data including slurry density, particle size distributions and hydrocyclone feed flowrate were mass balanced using JKSImMet V. 6.3. The results obtained are listed in Appendix A.

Mass balanced data were used to fit Nageswararao hydrocyclone model [27] and generate fitted partition curves, used to evaluate classification performance from the different test conditions. The error sum, as shown in Equation (1), was used to evaluate model fitting.

$$Error\ sum = \sum_{i=1}^m 100 \times (M_i - P_i)^2 \quad (1)$$

where

M_i = Partition to underflow calculated after balanced data associated with each particle size

P_i = Partition to underflow calculated using fitted model associated with each particle size

m = Number of data points

The following indicators were used to assess the classification performance on the different testing conditions:

- Bypass of ultrafines (%): Partition of particles smaller than 2 μm to the hydrocyclone underflow;
- Water split to underflow (%): Proportion of water reporting to the hydrocyclone underflow;
- Solids split to underflow (%): Partition of solids to the hydrocyclone underflow;
- Classification efficiency at 10 μm (%): The efficiency on removing particles finer than 10 μm to overflow, expressed by the ratio of respective solids flowrate finer than 10 μm on hydrocyclone overflow and feed, as shown in Equation (2).

$$Eff_{10\mu\text{m}} = \frac{O \times o_{10}}{f_{10}} \quad (2)$$

where

O = Solids split to cyclone overflow

o_{10} = Percent passing at 10 μm in overflow stream

f_{10} = Percent passing at 10 μm in feed stream

- Probable error (E_p): This value describes the “sharpness” of the separation. It is derived from the slope of the curve between the 25% and 75% partition points, as shown in Equation (3).

$$E_p = \frac{d_{75} - d_{25}}{2} \quad (3)$$

where

d_{75} = Particle size for which 75% of the feed particles reports to underflow in corrected partition curve

d_{25} = Particle size for which 25% of the feed particles reports to underflow in corrected partition curve

A lower E_p value indicates a more efficient, sharper separation.

- Imperfection (I) [28]: This measure normalizes the probable error by the cut-point, as shown in Equation (4).

$$I = \frac{E_p}{d_{50}} \quad (4)$$

where

E_p = Probable error

d_{50} = Particle size for which 50% of the feed particles reports to underflow in corrected partition curve

Imperfection provides a relative measure of separation efficiency that is useful for comparing different pieces of equipment.

Besides the above indicators, underflow and overflow particle size distributions were also used to evaluate classification performance.

3. Results

3.1. Feed Characterization

Figure 5 shows the obtained particle size distributions for each one of the two samples used for the testing campaigns. The calculated d_{80} parameter for Sample 1 and Sample 2

are respectively 30 μm and 114 μm . Accordingly, the fraction passing at 10 μm are 58.6% and 29.2% respectively.

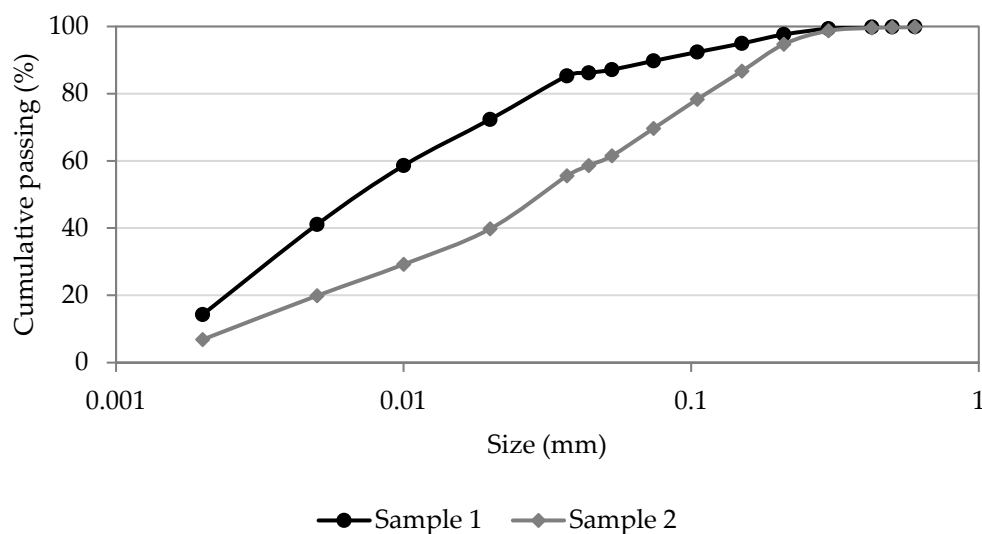


Figure 5. Feed samples particle size distributions.

Sample 1 indicates 10.5% solids by mass, while the corresponding value obtained for Sample 2 is 12.9%. Specific density of solids for sample 1 and 2 is 2.6 g/cm³. Detailed size distributions results are listed in Appendix B.

3.2. Experimental Data Balancing

Experimental data were balanced using JKSImMet V. 6.3 to ensure consistency between the three streams. A parity graph between experimental and balanced data is presented in Figure 6.

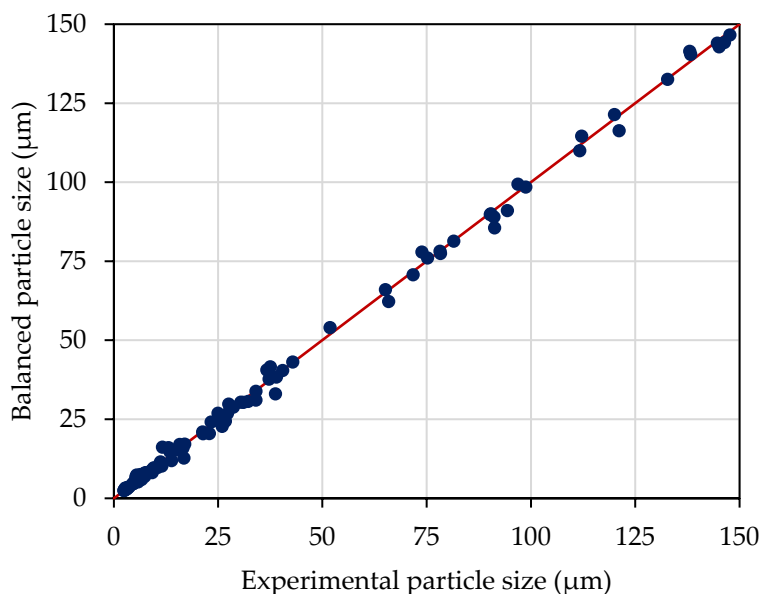


Figure 6. Parity graph between experimental and balanced data—Tests 1 to 12, cyclone feed, underflow and overflow d_{25} , d_{50} and d_{75} .

Figure 6 compares d_{25} , d_{50} and d_{75} from experimental and balanced hydrocyclone feed, underflow and overflow, from the 12 test conditions. There is consistency between the two datasets and balanced data were submitted to cyclone performance evaluation.

3.3. Partition Curve Model Fit

Table 2 shows the error sum between the partition curves modeled using Nageswararao model for hydrocyclones and the mass balanced curves. Figure 7 compares the predicted and mass balanced partition curves for Test 1, which shows the highest sum of errors, and Test 9, which shows the lowest.

Table 2. Error sum associated with partition curve model fitting.

Test	Error Sum
1	4.25
2	3.50
3	0.19
4	4.07
5	3.53
6	4.15
7	0.32
8	0.41
9	0.18
10	0.36
11	0.75
12	0.50

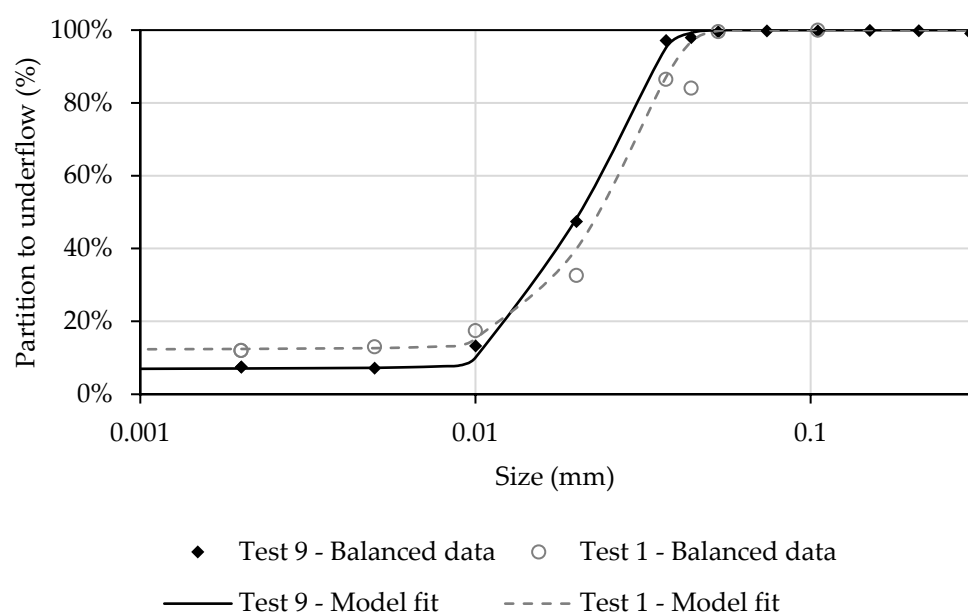


Figure 7. Mass balanced and modeled partition curves of test 1 and 9.

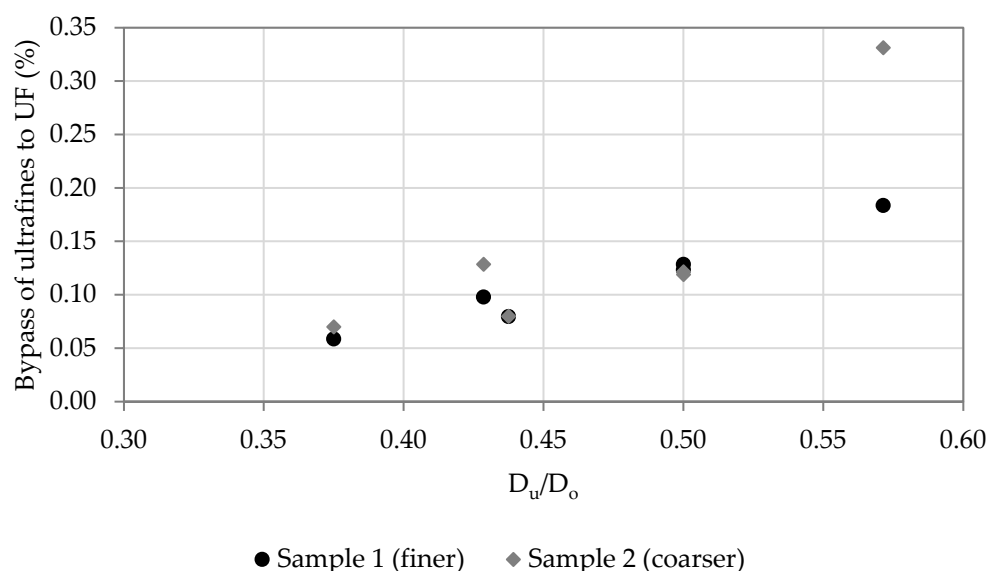
Calibration constants associated with Nageswararao model fitting for the 12 tests are listed in Appendix C.

3.4. Classification Performance

Table 3 and Figure 8 show the bypass of ultrafine particles to underflow (partition of particles smaller than $2\ \mu\text{m}$ to the hydrocyclone underflow) associated with the different testing conditions. Smaller bypass of ultrafines are obtained for smaller apex-to-vortex diameter ratios, here referred to as D_u/D_o .

Table 3. Bypass of ultrafine particles to underflow for the different testing conditions.

Sample	Test	Vortex Diameter (mm)	Apex Diameter (mm)	D_u/D_o	By-Pass of Ultrafines (%)
1	1	102	51	0.50	12.3
	2	102	44	0.44	8.0
	3	102	38	0.38	5.9
	4	89	51	0.57	18.4
	5	89	44	0.50	12.8
	6	89	38	0.43	9.8
2	7	102	51	0.50	12.2
	8	102	44	0.44	8.0
	9	102	38	0.38	7.0
	10	89	51	0.57	33.1
	11	89	44	0.50	11.9
	12	89	38	0.43	12.9

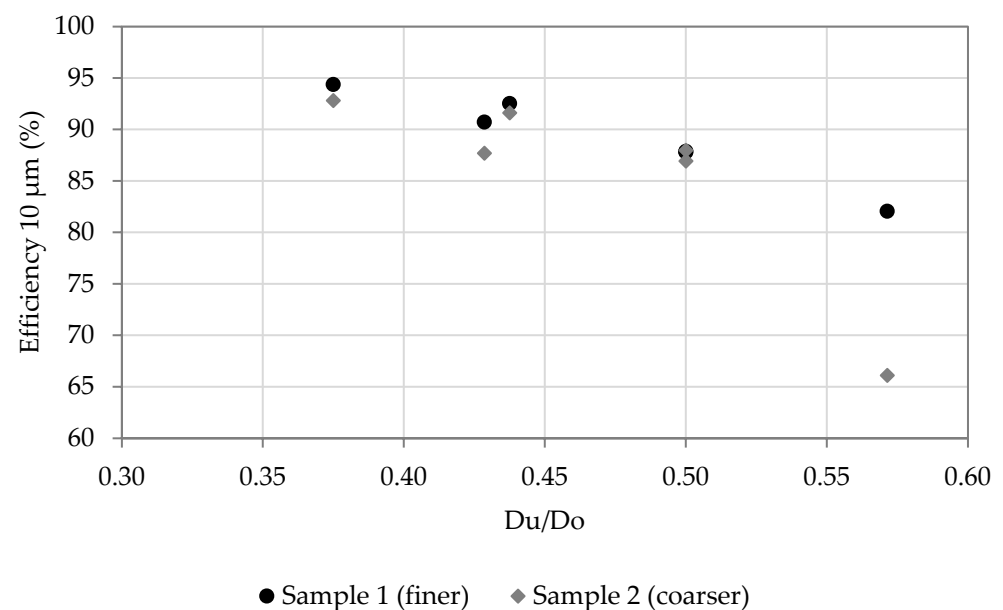
**Figure 8.** Bypass of ultrafines as a function of the apex/vortex (D_u/D_o) diameter ratio.

Similar trends are observed when comparing Sample 1 and Sample 2. Partition to underflow associated with different particle sizes, including the partition to underflow of particles smaller than $2\ \mu\text{m}$, defined in this work as bypass of ultrafines, are listed in Appendix D.

According to Table 4 and Figure 9, smaller apex-to-vortex diameter ratios result in higher classification efficiencies at $10\ \mu\text{m}$, as per Equation (2). In other words, smaller apex-to-vortex diameter ratios result in higher slime removing to overflow. Accordingly, from the greater D_u/D_o Figure (0.57) to the smaller one (0.38), classification efficiency increases from 82% to 94% for Sample 1 (finer), as well as from 66% to 93% for Sample 2 (coarser). The results obtained are listed in Appendix E.

Table 4. Classification efficiency for particles finer than 10 μm for the tested conditions.

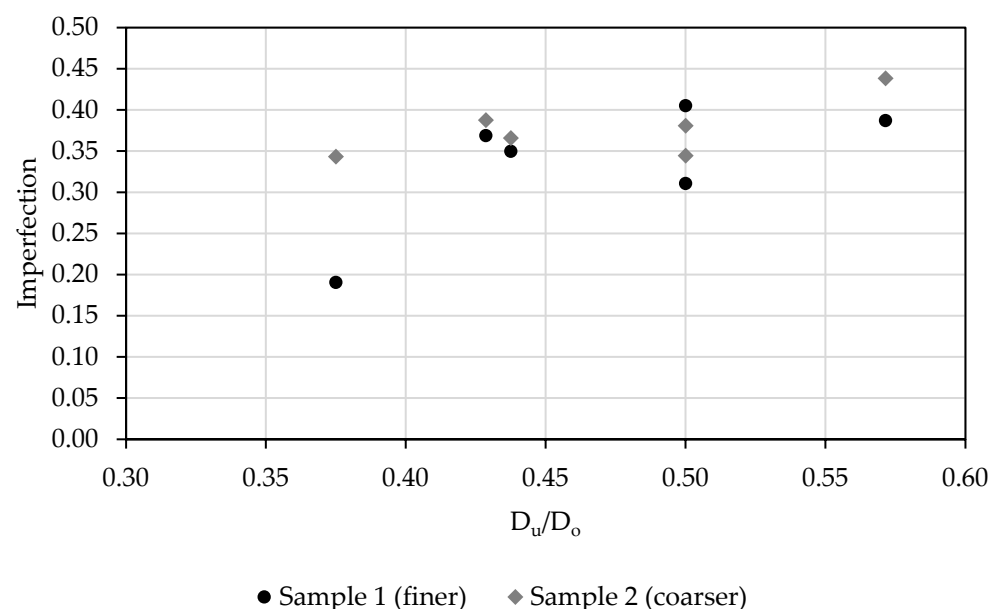
Sample	Test	Vortex Diameter (mm)	Apex Diameter (mm)	D_u/D_o	$\text{Eff}_{10\mu\text{m}}$
1	1	102	51	0.50	87.9
	2	102	44	0.44	92.5
	3	102	38	0.38	94.4
	4	89	51	0.57	82.0
	5	89	44	0.50	87.8
	6	89	38	0.43	90.7
2	7	102	51	0.50	88.0
	8	102	44	0.44	91.6
	9	102	38	0.38	92.8
	10	89	51	0.57	66.1
	11	89	44	0.50	86.9
	12	89	38	0.43	87.7

**Figure 9.** Classification efficiency for particles finer than 10 μm as a function of apex/vortex (D_u/D_o) diameter ratio.

In terms of Imperfection (Equation (4)), better performances are also observed for smaller apex-to-vortex diameter ratios, in both Sample 1 and Sample 2 (Table 5 and Figure 10). For the finer sample, the corrected partition curve Imperfection decreases from 0.39 to 0.19 as D_u/D_o reduces from 0.57 to 0.38. For the coarser sample, it decreases from 0.44 to 0.34. These results indicate that the efficiency improvement at smaller D_u/D_o ratios (Figure 9) arises from two factors: a significant reduction in fine particle bypass to underflow (Figure 8) and sharper classification in the corrected partition curves. The results obtained are listed in Appendix F.

Table 5. Partition curve Imperfection for the different testing conditions.

Sample	Test	Vortex Diameter (mm)	Apex Diameter (mm)	D_u/D_o	Imperfection
1	1	102	51	0.50	0.31
	2	102	44	0.44	0.35
	3	102	38	0.38	0.19
	4	89	51	0.57	0.39
	5	89	44	0.50	0.41
	6	89	38	0.43	0.37
2	7	102	51	0.50	0.34
	8	102	44	0.44	0.37
	9	102	38	0.38	0.34
	10	89	51	0.57	0.44
	11	89	44	0.50	0.38
	12	89	38	0.43	0.39

**Figure 10.** Partition curve Imperfection for the different testing conditions.

An example of the effects of different apex/vortex diameter ratio over the actual and corrected partition curves is illustrated in Figure 11. In this case, extreme values (0.38 and 0.57) of D_u/D_o ratio are selected for Sample 1. Accordingly, while both curves show similar recovery figures to underflow for particles coarser than 40 μm , the curve slope for finer particles is worth noting. Consequently, the bypass associated with the D_u/D_o equals to 0.38 is significantly smaller (6%), as compared with the D_u/D_o equals to 0.57, the latter resulting in a 19% bypass. Corrected partition curve Imperfection associated with the D_u/D_o equals to 0.38 is also significantly smaller (0.19), as compared with 0.39 observed when the D_u/D_o equals to 0.57.

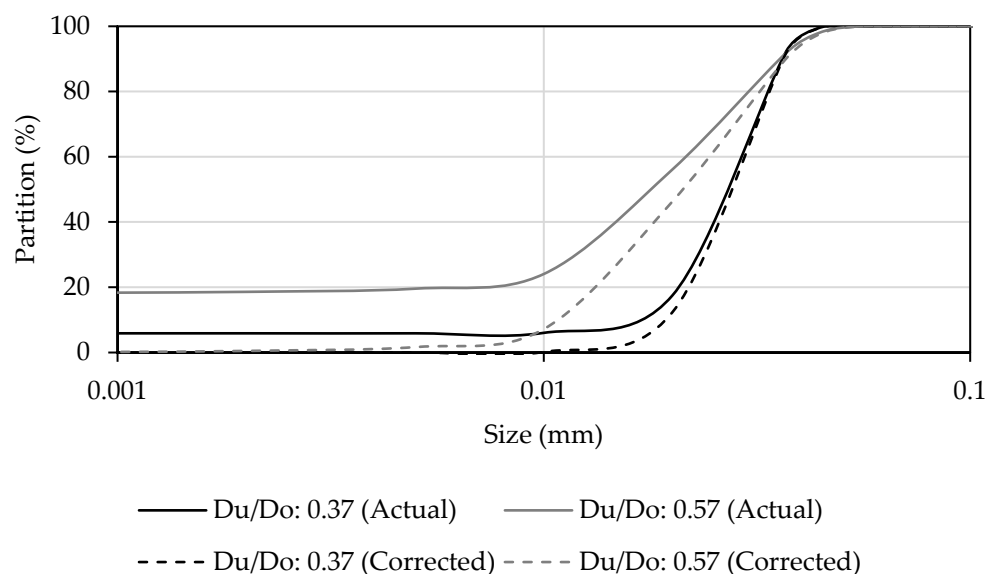


Figure 11. Actual partition curves for Sample 1, D_u/D_o ratios of 0.38 and 0.57.

Regarding hydrocyclone products, Table 6 and Figure 12 show the percent passing at 10 μm in underflow, together with d_{95} in overflow, the latter referred as the 95% passing size. For the tested conditions, reducing apex-to-vortex diameter ratio results in significant reductions in the mass fraction of particles finer than 10 μm in underflow, as referred to values in left-hand vertical axis. Conversely, minor changes in overflow d_{95} are observed for all testing results, as referred to values in right-hand vertical axis. As an example, the reduction in D_u/D_o ratio from 0.57 to 0.38 results in a reduction from 34.5% to 13.9% in the percent passing at 10 μm for Sample 1, while overflow d_{95} slightly increases from 30 μm to 32 μm . The results obtained are listed in Appendix A.

Table 6. Fraction passing at 10 μm in the hydrocyclone underflow and d_{95} in overflow for the different testing conditions.

Sample	Test	Vortex Diameter (mm)	Apex Diameter (mm)	D_u/D_o	Percent Finer than 10 μm UF	d_{95} OF
1	1	102	51	0.50	24.47	30.14
	2	102	44	0.44	18.23	31.05
	3	102	38	0.38	13.94	32.31
	4	89	51	0.57	34.45	29.70
	5	89	44	0.50	27.34	28.96
	6	89	38	0.43	22.73	30.98
2	7	102	51	0.50	6.56	32.16
	8	102	44	0.44	4.27	33.04
	9	102	38	0.38	3.86	32.95
	10	89	51	0.57	14.08	31.76
	11	89	44	0.50	6.13	32.47
	12	89	38	0.43	5.99	32.12

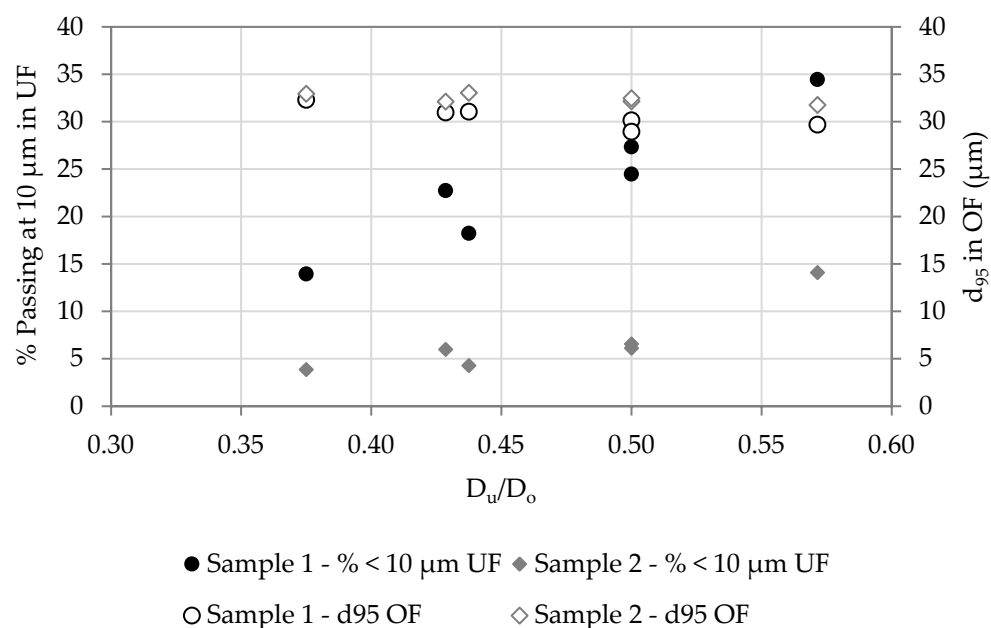


Figure 12. Fraction passing at 10 µm in the hydrocyclone underflow and d_{95} in overflow as a function of D_u/D_o ratio.

3.5. Product Chemical Composition

Tables 7 and 8 show partition to underflow (UF), as well as available alumina (AA) and reactive silica (RS) grades, together with metallurgical recoveries to underflow associated with each D_u/D_o tested for Sample 1 and Sample 2.

Table 7. Product chemical composition and recovery—Sample 1.

Test	1	2	3	4	5	6
Vortex Diameter (mm)	101.6	101.6	101.6	88.9	88.9	88.9
Apex Diameter (mm)	50.8	44.5	38.1	50.8	44.5	38.1
D_u/D_o	0.50	0.44	0.38	0.57	0.50	0.43
UF AA Grade (%)	38.6	41.2	43.0	35.5	37.8	38.9
UF RS Grade (%)	9.3	8.1	7.1	11.2	9.9	9.1
AA Grade/RS Grade (%)	4.1	5.1	6.0	3.2	3.8	4.3
Solids Split to UF (%)	30.8	25.7	24.5	38.8	34.8	26.3
AA Recovery (%)	48.3	42.9	42.8	56.0	53.4	41.6
RS Recovery (%)	15.5	11.2	9.4	23.4	18.6	12.9
RS Recovery/AA Recovery	0.32	0.26	0.22	0.42	0.35	0.31

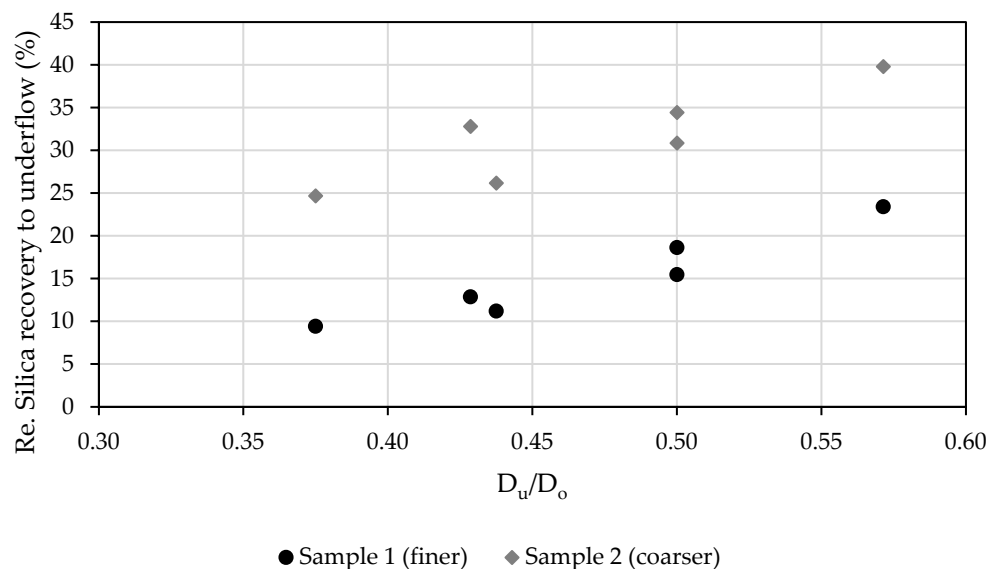
Overall, Table 7 shows that for Sample 1 testing, smaller apex-to-vortex diameter ratios result in lower reactive silica grades and higher available alumina grades in hydrocyclone underflow, as shown by the AA grade/RS grade ratio also listed in Table 7. Similarly, the smaller RS recovery/AA recovery ratio is also obtained for the smaller apex-to-vortex diameter ratio. In absolute terms, the best combined results for RS grade and recovery are obtained for D_u/D_o ratio of 0.38, respectively, 7.1% and 9.4%.

Table 8. Product chemical composition and recovery—Sample 2.

Test	7	8	9	10	11	12
Vortex Diameter (mm)	101.6	101.6	101.6	88.9	88.9	88.9
Apex Diameter (mm)	50.8	44.5	38.1	50.8	44.5	38.1
Du/Do	0.50	0.44	0.38	0.57	0.50	0.43
UF AA Grade (%)	45.2	46.1	46.4	43.4	44.3	44.2
UF RS Grade (%)	5.2	4.7	4.5	6.1	5.6	5.4
AA Grade/RS Grade (%)	8.7	9.8	10.2	7.1	8.0	8.2
Solids Split to UF (%)	61.9	57.7	56.5	67.2	64.3	63.1
AA Recovery (%)	76.3	72.4	71.5	79.6	77.7	76.0
RS Recovery (%)	30.9	26.2	24.7	39.8	34.4	32.8
RS Recovery/AA Recovery	0.40	0.36	0.35	0.50	0.44	0.43

Table 8 shows essentially the same trends observed for Table 7, i.e., for Sample 2 testing, smaller apex-to-vortex diameter ratios result in lower reactive silica grades and higher available alumina grades in hydrocyclone underflow, as shown by the AA grade/RS grade ratio listed in the same Table 8. Similarly, the smaller RS recovery/AA recovery ratio is also obtained for the smaller apex-to-vortex diameter ratio. In absolute terms, the best combined results for RS grade and recovery are obtained for D_u/D_o ratio of 0.38, respectively 4.5% and 24.7%.

The RS recovery to hydrocyclone underflow as a function of D_u/D_o ratio for all 12 tests is shown in Figure 13.

**Figure 13.** RS recovery to hydrocyclone underflow as a function of D_u/D_o ratio.

4. Discussion and Validation

The experimental data result in a consistent relationship between the apex-to-vortex diameter ratio and hydrocyclone classification efficiency. Lower ratios (0.38–0.43) are associated with a significant reduction in the bypass of ultrafines to the underflow and improve the removal of slimes to the overflow. These findings agree with previous studies that emphasizes the role of spigot size and vortex finder geometry in controlling short-circuit flow and the proportion of water reporting to the underflow [1–3].

The bypass of fine particles to the underflow has long been recognized as a key inefficiency in hydrocyclone operation. Kelsall [13] first provided a theoretical basis to explain this phenomenon, attributing it to water entrainment and short-circuiting near the cyclone axis. Subsequent works confirmed that design variables strongly influence this mechanism [5–7]. Results from the present work are consistent with previous studies, as shown in Figure 14. Accordingly, the results obtained for Sample 1 and Sample 2 indicate clear trends, therefore reinforcing the referred preceding works. When comparing the curves obtained for the two tested samples, a difference in their slopes can be observed. Most data points follow a similar trend, except for the point corresponding to the highest underflow bypass observed for Sample 2. This point is associated with Test 10, which presents the highest D_u/D_o ratio among the evaluated conditions, equal to 0.57. Additional tests are required to confirm whether there is a statistically and practically significant difference in the slope of the curves.

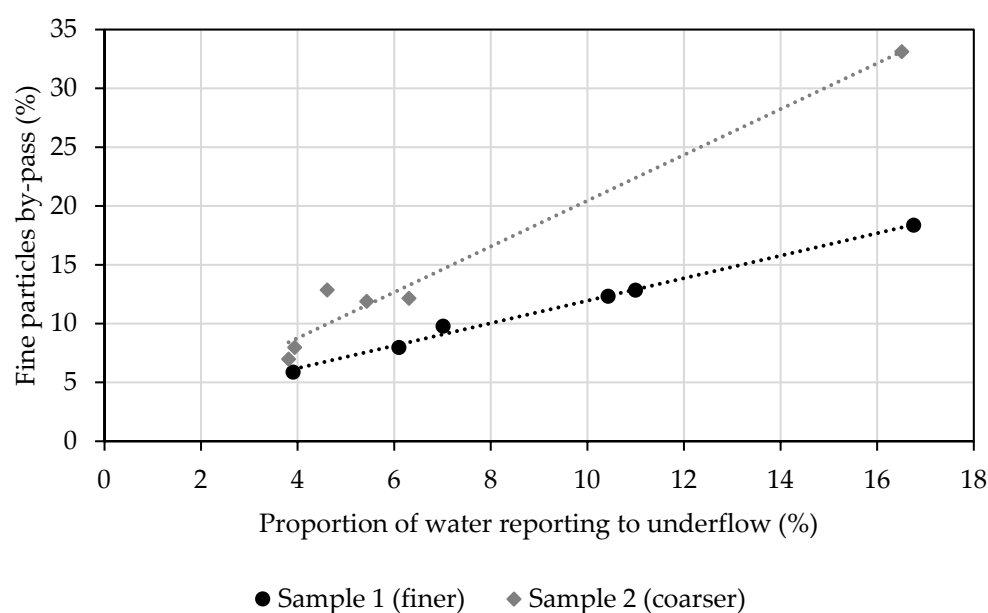


Figure 14. Bypass of ultrafines to underflow as a function of the proportion of water reporting to underflow, with linear regression curves in dashed line.

The observed improvement in corrected partition curve sharpness with smaller D_u/D_o ratios further emphasize the above-described results. In this case, for both feed Sample 1 and Sample 2, classification Imperfection is reduced, leading to sharper separation and improved cut size control for smaller D_u/D_o ratios. Importantly, these benefits are obtained without significant deterioration of the overflow d_{95} . This is a relevant finding for bauxite beneficiation, where minimizing fines in the underflow reduces operational costs and residue generation in downstream processing [7,9].

Based on the experimental test results and assessments, apex and vortex modifications were carried out in selected industrial hydrocyclones installed at both the first and second desliming stages, the latter referred to in the flow sheet shown in Figure 2. Accordingly, standard hydrocyclone orifice diameters at Mineração Paragominas' desliming circuit were 89 mm vortex finder and 51 mm apex on both primary and secondary classification stages, while the change included installing 102 mm vortex finder and 38 mm apex in selected industrial hydrocyclones. The latter combination followed here was considered the best experimental results, thus corresponding to Test 3 (primary stage) and Test 9 (second stage). The results obtained are summarized in Table 9.

Table 9. Summary of selected experimental tests and industrial operation results.

	Variable	Experimental Tests	Industrial Operation
Vortex and Apex	Vortex diameter (mm)	102	102
	Apex diameter (mm)	38	38
	Apex/Vortex ratio	0.38	0.38
Stage 1 and Stage 2 Combined	AA recovery (%)	42.5	43.8
	RS recovery (%)	4.6	4.7
	Increase in AA grade (%)	2.7	2.4
	Reduction in RS grade (%)	1.6	1.6

This setup with lower bypass was submitted to industrial operation, initially in one of the processing lines, to allow a performance evaluation, comparing the two processing lines with different cyclone setups. It resulted in improved bauxite quality in desliming circuit underflow, increasing AA grade by 2.4% and reducing RS grade by 1.6%, supporting the decision to run both processing lines with the new setup. Those results from industrial operation are consistent with the quality improvement potentials estimated after experimental tests, 2.7% for AA grade and 1.6% for RS grade. Regarding recovery in desliming circuit, the setup with lower bypass resulted in 43.8% AA recovery, consistent with the estimate based on experimental tests of 42.5%. Similarly, RS recovery measured in industrial operation, 4.7% show consistency with the estimate based on experimental results, of 4.6%.

Regarding the industrial beneficiation plant product, AA grade is increased by 0.4% and RS grade is reduced by 0.26%. This improves bauxite quality resulting in better performance on the bauxite refining process, carried out at Alunorte alumina refinery, located in Barcarena, PA. The combined benefits are (a) reduction in bauxite residue generation by 75 kt/year, (b) reduction in caustic soda consumption by 23 kt/year and (c) reduction in CO₂ emissions by 28 kt/year.

5. Conclusions

Six different apex-to-vortex diameter ratios were tested for two different bauxite samples seeking the optimization on fine particles removal to overflow. Better classification performance was obtained with a smaller apex-to-vortex diameter ratio for most of the parameters evaluated. The main conclusions are:

- Bypass of ultrafines to underflow: Reducing D_u/D_o from 0.57 to 0.38 results in a reduction in fine particles bypass to underflow from 18% to 6% on the finer sample, and from 33% to 7% on the coarser one.
- Classification efficiency: Efficiency on removing particles finer than 10 μm to overflow increases as D_u/D_o decreases. Based on a D_u/D_o reduction from 0.57 to 0.38, classification efficiency increases from 82% to 94% on the finer sample and from 66% to 93% on the coarser one.
- Partition curve Imperfection: Corrected partition curve Imperfection is smaller for reduced apex-to-vortex diameter ratios, indicating that in those conditions, sharper classification is achieved. For the finer sample, partition curve Imperfection reduces from 0.39 to 0.19 as D_u/D_o reduces from 0.57 to 0.38. For the coarser sample, it reduces from 0.44 to 0.34.
- Hydrocyclone products particle size: Reducing apex-to-vortex diameter ratio improves bauxite desliming performance. Accordingly, the amount of particles finer than 10 μm on hydrocyclone underflow is significantly reduced with smaller D_u/D_o . Interestingly,

the overflow d_{95} shows no significant change. The effect of reducing D_u/D_o on the finer sample is a reduction from 34% to 14% of particles finer than 10 μm in the underflow, while overflow d_{95} slightly increases from 30 μm to 32 μm . For the coarser sample D_u/D_o reduction results in a reduction from 14.1% to 3.9% of particles finer than 10 μm in the underflow. Here too, the overflow d_{95} slightly increases from 31 μm to 33 μm .

- The obtained results are particularly relevant on bauxite beneficiation context as the fraction finer than 10 μm shows relatively small available alumina, AA, grades together with high grades of reactive silica, RS. The opposite occurs with fractions coarser than 10 μm , i.e., greater AA and smaller RS grades [9].
- The implementation of vortex and apex in industrial hydrocyclones following the best tested combination of D_u/D_o ratio results in significant improvements in desliming combined product, increasing AA grade by 2.4% while reducing RS grade by 1.6%.
- The optimized hydrocyclone operation results in a combination of three significant benefits at the alumina refinery: (a) reduction in bauxite residue generation by 75 kt/year, (b) reduction in caustic soda consumption by 23 kt/year and (c) reduction in CO_2 emissions by 28 kt/year.

When compared with alternative strategies such as water injection systems (Cyclowash) [11–14] or structural modifications of the cyclone body [16], the present approach shows that simpler hydrocyclone modifications, requiring no additional auxiliary systems, effectively improve the classification performance. Nonetheless, further investigation combining apex/vortex optimization with controlled water injection will be assessed in Part II of the present work.

Author Contributions: Conceptualization, A.S.R. and H.D.J.; method and validation, A.S.R. and H.D.J. formal analysis, A.S.R. and H.D.J.; writing—original draft preparation, A.S.R.; writing—review and editing H.D.J.; supervision, H.D.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data are contained within the article.

Acknowledgments: Special thanks to Norsk Hydro Brazil for allowing and strongly supporting the development of this study.

Conflicts of Interest: Allan Suhett Reis is an employee of Norsk Hydro Brasil. The views expressed in the paper are those of the scientists and do not necessarily represent the company.

Appendix A

Table A1. Balanced feed, underflow (UF) and overflow (OF) particle size distributions—Tests 1 to 3.

Sample	1	1	1	1	1	1	1	1	1
Test	1	1	1	2	2	2	3	3	3
Vortex diameter (mm)	102	102	102	102	102	102	102	102	102
Apex diameter (mm)	51	51	51	44	44	44	38	38	38
Stream	feed	UF	OF	feed	UF	OF	feed	UF	OF
Mass distribution (%)	100.00	28.99	71.01	100.00	24.46	75.54	100.00	22.26	77.74
% solids	10.50	25.01	8.49	10.50	32.00	8.62	10.50	39.84	8.67

Table A1. *Cont.*

Size (mm)	Cumulative passing (%)								
0.600	99.93	99.77	99.99	99.94	99.78	99.99	99.94	99.77	99.99
0.500	99.89	99.62	99.99	99.89	99.62	99.98	99.89	99.58	99.98
0.425	99.83	99.41	99.99	99.79	99.34	99.94	99.83	99.32	99.98
0.300	99.4	97.90	99.99	99.32	97.41	99.94	99.32	97.03	99.98
0.210	97.67	91.90	99.98	97.57	90.25	99.93	97.57	89.18	99.97
0.150	94.72	81.65	99.96	94.80	78.87	99.92	94.83	76.87	99.95
0.105	91.95	72.10	99.92	92.10	67.79	99.91	92.19	65.01	99.93
0.074	89.16	62.42	99.90	89.39	56.68	99.89	89.38	52.39	99.91
0.053	86.16	52.12	99.83	86.58	45.24	99.85	86.72	40.57	99.86
0.044	85.37	49.83	99.78	85.55	41.11	99.81	85.68	35.97	99.83
0.037	84.29	46.61	99.70	84.71	37.87	99.74	84.72	31.93	99.75
0.020	72.02	32.81	88.05	71.35	25.10	86.19	68.49	19.13	82.54
0.010	58.16	24.47	71.97	56.46	18.23	68.73	53.06	13.94	64.20
0.005	40.43	16.53	50.31	38.74	12.06	47.30	35.65	9.56	43.08
0.002	13.99	5.60	17.50	12.86	3.92	15.74	11.57	3.27	13.93

Table A2. Balanced feed, underflow (UF) and overflow (OF) particle size distributions—Tests 4 to 6.

Sample	1	1	1	1	1	1	1	1	1
Test	4	4	4	5	5	5	6	6	6
Vortex diameter (mm)	89	89	89	89	89	89	89	89	89
Apex diameter (mm)	51	51	51	44	44	44	38	38	38
Stream	feed	UF	OF	feed	UF	OF	feed	UF	OF
Mass distribution (%)	100.00	32.11	67.89	100.00	28.06	71.94	100.00	25.33	74.67
% solids	10.50	18.00	8.77	10.50	22.64	8.68	10.50	29.40	8.62
Size (mm)	Cumulative passing (%)								
0.600	99.92	99.84	99.96	99.93	99.76	99.99	99.91	99.72	99.98
0.500	99.87	99.71	99.95	99.88	99.62	99.98	99.87	99.56	99.98
0.425	99.81	99.52	99.95	99.82	99.39	99.98	99.80	99.30	99.98
0.300	99.29	97.90	99.95	99.38	97.84	99.98	99.27	97.27	99.96
0.210	97.69	92.96	99.92	97.74	92.05	99.97	97.61	90.77	99.94
0.150	95.13	85.01	99.90	95.21	83.12	99.95	95.19	81.34	99.89
0.105	92.79	77.75	99.87	92.77	74.54	99.92	92.53	71.12	99.79
0.074	90.39	70.36	99.82	90.22	65.56	99.88	90.09	61.80	99.68
0.053	88.05	63.17	99.77	87.69	56.73	99.82	87.68	52.30	99.53
0.044	87.17	60.48	99.74	86.79	53.62	99.78	86.70	48.61	99.46
0.037	86.37	58.09	99.69	85.95	50.77	99.73	85.86	45.48	99.39
0.020	74.68	44.76	88.78	74.73	36.42	89.73	72.90	30.82	87.00
0.010	61.36	34.45	74.04	62.12	27.34	75.74	60.68	22.73	73.39
0.005	43.60	23.61	53.01	44.63	18.70	54.79	43.60	15.41	53.00
0.002	15.49	8.27	18.88	15.83	6.47	19.51	15.75	5.29	19.16

Table A3. Balanced feed, underflow (UF) and overflow (OF) particle size distributions—Tests 7 to 9.

Sample	2	2	2	2	2	2	2	2	2
Test	7	7	7	8	8	8	9	9	9
Vortex diameter (mm)	102	102	102	102	102	102	102	102	102
Apex diameter (mm)	51	51	51	44	44	44	38	38	38
Stream	feed	UF	OF	feed	UF	OF	feed	UF	OF
Mass distribution (%)	100.00	57.23	42.77	100.00	55.94	44.06	100.00	54.70	45.30
% solids	12.88	56.32	6.34	12.88	67.76	6.35	12.88	68.12	6.51
Size (mm)	Cumulative passing (%)								
0.600	99.84	99.72	99.99	99.81	99.67	100.00	99.84	99.71	100.00
0.500	99.74	99.55	99.99	99.73	99.53	100.00	99.74	99.53	99.99
0.425	99.59	99.30	99.98	99.60	99.32	99.95	99.61	99.30	99.99
0.300	98.94	98.14	99.98	98.74	97.86	99.85	98.80	97.83	99.97
0.210	95.15	91.63	99.98	95.17	91.49	99.85	94.80	90.53	99.96
0.150	87.65	78.38	99.97	87.30	77.47	99.79	86.96	76.21	99.95
0.105	79.09	63.34	99.96	79.32	63.22	99.77	79.12	61.90	99.92
0.074	70.62	48.57	99.94	70.95	48.29	99.72	70.74	46.61	99.88
0.053	61.79	33.22	99.86	62.41	33.09	99.64	62.16	30.99	99.80
0.044	58.86	28.10	99.78	59.43	27.85	99.53	59.45	26.14	99.68
0.037	55.96	23.28	99.61	56.67	23.09	99.33	56.30	20.54	99.48
0.020	41.90	10.68	83.42	39.59	7.19	80.73	40.06	6.46	80.65
0.010	30.02	6.56	61.41	28.90	4.27	60.17	29.35	3.86	60.14
0.005	20.02	4.47	40.99	19.50	2.99	40.46	19.71	2.59	40.37
0.002	6.86	1.43	14.16	6.52	0.91	13.65	6.54	0.79	13.48

Table A4. Balanced feed, underflow (UF) and overflow (OF) particle size distributions—Tests 10 to 12.

Sample	2	2	2	2	2	2	2	2	2
Test	10	10	10	11	11	11	12	12	12
Vortex diameter (mm)	89	89	89	89	89	89	89	89	89
Apex diameter (mm)	51	51	51	44	44	44	38	38	38
Stream	feed	UF	OF	feed	UF	OF	feed	UF	OF
Mass distribution (%)	100.00	71.34	28.66	100.00	58.51	41.49	100.00	62.41	37.59
% solids	12.88	40.41	4.78	12.88	60.45	6.10	12.88	66.73	5.50
Size (mm)	Cumulative passing (%)								
0.600	99.82	99.75	100.00	99.81	99.67	100.00	99.78	99.66	99.99
0.500	99.71	99.59	100.00	99.70	99.49	99.99	99.66	99.47	99.98
0.425	99.57	99.39	99.99	99.56	99.26	99.99	99.51	99.22	99.98
0.300	98.66	98.13	99.99	98.77	97.91	99.98	98.48	97.58	99.97
0.210	94.35	92.10	99.97	94.95	91.40	99.97	94.06	90.51	99.96
0.150	85.74	80.03	99.95	87.01	77.83	99.95	85.64	77.04	99.92
0.105	76.72	67.39	99.94	78.81	63.83	99.94	76.86	63.00	99.88
0.074	67.23	54.10	99.91	70.34	49.38	99.90	68.03	48.88	99.84

Table A4. *Cont.*

0.053	62.32	47.24	99.83	61.68	34.62	99.83	58.76	34.08	99.74
0.044	59.06	42.70	99.76	59.05	30.19	99.75	55.86	29.48	99.66
0.037	55.69	38.03	99.62	55.86	24.83	99.63	52.81	24.68	99.52
0.020	39.39	21.21	84.62	40.16	10.31	82.26	37.60	9.78	83.79
0.010	28.98	14.08	66.07	29.83	6.13	63.24	28.19	5.99	65.05
0.005	19.99	9.62	45.80	20.44	4.29	43.23	19.70	4.34	45.20
0.002	6.85	3.17	16.01	7.13	1.44	15.15	7.02	1.55	16.10

Appendix B

Table A5. Feed samples particle size distributions—Average from six tests carried out with sample.

Size (mm)	Cumulative Passing (%)	
	Sample 1	Sample 2
0.600	99.93	99.82
0.500	99.88	99.71
0.425	99.81	99.57
0.300	99.33	98.73
0.210	97.64	94.75
0.150	94.98	86.72
0.105	92.39	78.32
0.074	89.77	69.65
0.053	87.15	61.52
0.044	86.21	58.62
0.037	85.32	55.55
0.020	72.36	39.78
0.010	58.64	29.21
0.005	41.11	19.89
0.002	14.25	6.82

Appendix C

Equations (A1) and (A2) from the Nageswararao hydrocyclone model [27] were used for partition curve modeling.

$$\frac{d_{50c}}{D_c} = K_{D_o} \left\{ D_c^{-0.65} \right\} \left(\frac{D_o}{D_c} \right)^{0.52} \left(\frac{D_u}{D_c} \right)^{-0.5} \left(\frac{D_i}{D_c} \right)^{0.2} \left(\frac{L_c}{D_c} \right)^{0.2} \theta^{0.15} \left(\frac{P}{\rho_p g D_c} \right)^{-0.22} \lambda^{0.93} \quad (\text{A1})$$

where

d_{50c} = corrected classification size, μm

D_c, D_o, D_u, D_i = diameter of the cyclone, vortex finder, spigot and inlet

L_c = length of the cylindrical section of the cyclone

K_d = material dependent constant in the model

θ = full cone angle, degrees

P = cyclone feed pressure

ρ_p = density of feed slurry

g = acceleration due to gravity

λ = hindered settling factor, $8.05 \times \frac{10^{1.82C_v}}{(1-C_v)^2}$

C_v = volumetric fraction of feed solids

$$E_{ui} = 1 - C \left[\frac{e^\alpha - 1}{e^{\alpha \frac{d_i}{d_{50c}}} + e^\alpha - 2} \right] \quad (A2)$$

where

E_{ui} = actual cyclone split to underflow associated with particle size “i”

C = cyclone water split to overflow

d_i = size of the particle, μm

α = cyclone efficiency curve shape parameter

Table A6. Nageswararao model calibration constants for tests 1 to 12.

Sample	1	1	1	1	1	1	2	2	2	2	2	2
Test	1	2	3	4	5	6	7	8	9	10	11	12
Vortex Diameter (mm)	102	102	102	89	89	89	102	102	102	89	89	89
Apex Diameter (mm)	51	44	38	51	44	38	51	44	38	51	44	38
Du/Do	0.50	0.44	0.38	0.57	0.50	0.43	0.50	0.44	0.38	0.57	0.50	0.43
Kd ($\times 10^{-4}$)	2.37	2.10	2.20	2.30	2.38	2.28	2.03	1.84	1.76	1.77	1.98	1.74
Kq	547.0	547.0	547.0	547.0	547.0	547.0	547.0	547.0	547.0	547.0	547.0	547.0
Kv	5.22	4.15	3.77	7.16	6.16	5.50	4.17	4.05	5.24	8.18	4.29	5.45
Kw	39.31	34.70	37.56	49.95	47.32	52.26	38.33	34.41	43.68	87.46	43.97	68.32
α	6.00	5.50	11.02	5.00	4.00	4.50	6.78	6.27	6.69	3.62	5.81	5.49

Appendix D

Table A7. Partition to underflow associated with the different particle sizes for tests 1 to 12.

Sample	1	1	1	1	1	1	2	2	2	2	2	2
Test	1	2	3	4	5	6	7	8	9	10	11	12
Vortex diameter (mm)	102	102	102	89	89	89	102	102	102	89	89	89
Apex diameter (mm)	51	44	38	51	44	38	51	44	38	51	44	38
Du/Do	0.50	0.44	0.38	0.57	0.50	0.43	0.50	0.44	0.38	0.57	0.50	0.43
Size (mm)	Partition to underflow (%)											
0.105	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.074	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.053	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
0.044	0.97	0.97	0.99	0.98	0.92	0.92	0.99	0.99	0.99	0.99	0.99	0.99
0.037	0.87	0.89	0.92	0.92	0.80	0.79	0.96	0.95	0.95	0.95	0.95	0.96
0.020	0.40	0.44	0.17	0.56	0.46	0.40	0.52	0.53	0.48	0.76	0.57	0.62
0.010	0.15	0.13	0.06	0.24	0.20	0.16	0.15	0.12	0.10	0.45	0.17	0.19
0.005	0.13	0.09	0.06	0.20	0.15	0.11	0.13	0.09	0.07	0.36	0.13	0.14
0.002	0.12	0.08	0.06	0.19	0.13	0.10	0.12	0.08	0.07	0.34	0.12	0.13
<0.002	0.12	0.08	0.06	0.18	0.13	0.10	0.12	0.08	0.07	0.33	0.12	0.13

Appendix E

Table A8. Classification efficiency for particles finer than 10 µm for the tested conditions.

Sample	Test	Vortex Diameter (mm)	Apex Diameter (mm)	D _u /D _o	Solids Split to Overflow (%)	Percent < 10 µm Overflow (%)	Percent < 10 µm Feed (%)	Eff _{10µm}
1	1	102	51	0.50	71.0	72.0	58.2	87.9
1	2	102	44	0.44	76.0	68.7	56.5	92.5
1	3	102	38	0.38	78.0	64.2	53.1	94.4
1	4	89	51	0.57	68.0	74.0	61.4	82.0
1	5	89	44	0.50	72.0	75.7	62.1	87.8
1	6	89	38	0.43	75.0	73.4	60.7	90.7
2	7	102	51	0.50	43.0	61.4	30.0	88.0
2	8	102	44	0.44	44.0	60.2	28.9	91.6
2	9	102	38	0.38	45.3	60.1	29.4	92.8
2	10	89	51	0.57	29.0	66.1	29.0	66.1
2	11	89	44	0.50	41.0	63.2	29.8	86.9
2	12	89	38	0.43	38.0	65.0	28.2	87.7

Appendix F

Table A9. Partition curve imperfection for the different testing conditions.

Sample	Test	Vortex Diameter (mm)	Apex Diameter (mm)	D _u /D _o	d ₇₅	d ₅₀	d ₂₅	Ep	I
1	1	102	51	0.50	0.034	0.026	0.018	0.008	0.31
1	2	102	44	0.44	0.032	0.024	0.016	0.008	0.35
1	3	102	38	0.38	0.033	0.028	0.023	0.005	0.19
1	4	89	51	0.57	0.031	0.021	0.015	0.008	0.39
1	5	89	44	0.50	0.036	0.025	0.016	0.010	0.41
1	6	89	38	0.43	0.036	0.026	0.017	0.010	0.37
2	7	102	51	0.50	0.030	0.022	0.015	0.007	0.34
2	8	102	44	0.44	0.030	0.020	0.015	0.008	0.37
2	9	102	38	0.38	0.030	0.022	0.015	0.008	0.34
2	10	89	51	0.57	0.027	0.017	0.012	0.007	0.44
2	11	89	44	0.50	0.029	0.020	0.014	0.008	0.38
2	12	89	38	0.43	0.028	0.019	0.014	0.007	0.39

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