

This supplementary file includes:

1. A HeFTy file with inversion modelling results for the monotonic cooling scenario
2. A HeFTy file with inversion modelling results for the reheating scenario from one of the computers. Note that the a priori t-T constraints in this file are slightly different from those shown in Fig. 4 of our paper. This is because Fig. 4 shows t-T constraints from another HeFTy file which was executed on the second computer.
3. A pdf file showing t-T envelopes for one of the runs with inflated uncertainties. The a priori t-T constraints in this run were very broad and would allow any t-T path. Nevertheless, the obtained envelopes remain well below 500°C, which is the peak temperature of the forward-modelled reheating event. We could not find any good or acceptable t-T paths using the same broad a priori t-T constraints in runs that assumed 1σ uncertainties of 1%.