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Merger and Acquisition Target Selection Based on Interval Neutrosophic Multigranulation Rough Sets over Two Universes

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Abstract: As a significant business activity, merger and acquisition (M&A) generally means transactions in which the ownership of companies, other business organizations or their operating units are transferred or combined. In a typical M&A procedure, M&A target selection is an important issue that tends to exert an increasingly significant impact on different business areas. Although some research works based on fuzzy methods have been explored on this issue, they can only deal with incomplete and uncertain information, but not inconsistent and indeterminate information that exists universally in the decision making process. Additionally, it is advantageous to solve M&A problems under the group decision making context. In order to handle these difficulties in M&A target selection background, we introduce a novel rough set model by combining interval neutrosophic sets (INSs) with multigranulation rough sets over two universes, called an interval neutrosophic (IN) multigranulation rough set over two universes. Then, we discuss the definition and some fundamental properties of the proposed model. Finally, we establish decision making rules and computing approaches for the proposed model in M&A target selection background, and the effectiveness of the decision making approach is demonstrated by an illustrative case analysis.

Keywords: merger and acquisition (M&A) target selection; group decision making; interval neutrosophic sets; multigranulation rough sets over two universes

1. Introduction

In the era of business intelligence, the development of computational intelligence approaches has far-reaching effects on business organizations' daily activities, including project management, human resource allocation optimization, merger and acquisition (M&A), and so on. As a key business activity in many organizations, M&A requires business administrators to make effective decisions by analyzing massive business data. Additionally, in M&A, one of the most important elements that determines the success ratio of business organizations is M&A target selection [1]. To deal with this issue, some efforts have been made through combining fuzzy approaches with the classical M&A research [2–5]. On the basis of the fuzzy set (FS) theory [6], fuzzy approaches have been widely used in realistic decision making problems. However, there is much uncertain information induced from various vague sources, and this often leads to some limitations when analyzing information systems through using FSs. Consequently, many new concepts of high-order FSs were established over the past decades. Among them, as a typical representative of generalized FSs, intuitionistic fuzzy sets (IFSs) [7] are characterized by a membership degree and a non-membership degree that describe whether one element belongs to a certain set or not, which provide a flexible framework to handle imprecise data, both complete and incomplete in nature. However, IFSs cannot cope with all kinds of uncertainties perfectly, such as problems including inconsistent and indeterminate information. Hence, it is necessary to develop some new theories.

Smarandache [8,9] presented the concept of neutrosophic logic and neutrosophic sets (NSs) from philosophical standpoints. Additionally, an NS is characterized by each element of the universe owning a degree of truth, indeterminacy and falsity, respectively. However, NSs can only be applied in philosophical problems. In order to utilize NSs easily in real-world situations, Wang et al. [10] constructed the definition and some operational laws of interval neutrosophic sets (INSs). Ever since the establishment of INSs, many scholars have studied INSs from different viewpoints and obtained an increasing number of academic achievements [11–21]. In light of the above, M&A target selection using IN information could handle uncertain situations and indeterminate information well and provide corporate acquirers with more exemplary and flexible access to convey their understandings about the M&A knowledge base.

As a typical model in the granular computing paradigm [22], the multigranulation rough set model [23,24] aims to analyze complicated problems from multiple views and levels, and it is seen as an efficient way to integrate and analyze information in group decision making procedures. Specifically, the advantages of utilizing multigranulation rough sets to solve group decision making problems can be summed up as follows:

1. In group decision making procedures, according to actual requirements of realistic problems, multigranulation rough set model-based computations consider multiple binary relations at the same time, which could increase the efficiency of the whole knowledge discovery process for multi-source information systems.
2. In light of different risk attitudes, the multigranulation rough set model can be divided into two parts, i.e., optimistic multigranulation rough sets [23] based on the "seeking common ground while reserving differences" strategy and pessimistic multigranulation rough sets [24] based on the "seeking common ground while eliminating differences" strategy. Thus, the multigranulation rough set model is suitable for solving risk-based group decision making problems.

Additionally, the classical rough set [25] is usually expressed and computed based on a single universe; this may lead to a limitation when describing group decision making information that is made up of multiple aspects. Through extending a single universe to two universes, it is beneficial to express a complicated group decision making knowledge base. Hence, Pei and Xu [26] studied the concept of rough sets over two universes systematically. Since then, several scholars have researched rough sets over two universes widely according to numerous practical requirements [27–33]. Recently, in order to expand the application scopes of rough sets over two universes from the ideas of granular computing, Sun and Ma [34] introduced multigranulation rough sets over two universes, which could not only describe real-life decision making information effectively and reasonably through different universes of discourse, but also integrate each expert's opinion to form an ultimate conclusion by aggregating multiple binary relations. Therefore, multigranulation rough sets over two universes constitute another approach to aid group decision making [35–40].

In this article, in order to handle the problems of IN data analysis and group decision making, it is necessary to introduce IN multigranulation rough sets over two universes through fusing multigranulation rough sets over two universes with INSs; both the general definition and some main properties of the proposed model are discussed. Then, we construct a new decision making method for M&A target selection problems by utilizing the proposed rough set model. Moreover, we give an illustrative case to interpret fundamental steps and a practical application to M&A target selection.

The rest of the article is structured as follows. In Section 2, we briefly introduce some concepts such as NSs, INSs and IN rough sets over two universes. Section 3 introduces IN multigranulation rough sets over two universes and some related properties. In Section 4, we establish decision making rules and an algorithm for M&A target selection problems. In Section 5, we give the steps of the proposed decision making approach by a case study, and conclusions and future research directions are illustrated in Section 6.

2. Preliminaries

In this section, we first review some fundamental concepts such as NSs, INSs and their properties. Next, we develop the definition of IN rough sets over two universes.

2.1. Neutrosophic Sets

NSs were defined by Smarandache [8] from philosophical standpoints. According to [8], NSs derive their origin from neutrosophy. In what follows, we review the concept of NSs.

Definition 1. *Let U be the universe of discourse, then an NS A can be expressed as the form $A = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x)\rangle | x \in U\}$, where the functions $\mu_A(x)$, $\nu_A(x)$ and $\omega_A(x)$ denote the degree of membership, the degree of indeterminacy and the degree of non-membership of the element $x \in U$ to the set A. Additionally, the functions $\mu_A(x)$, $\nu_A(x)$ and $\omega_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is $\mu_A(x) : U \to]0^-, 1^+[$, $\nu_A(x) : U \to]0^-, 1^+[$ and $\omega_A(x) : U \to]0^-, 1^+[$. There is no restriction on the sum of $\mu_A(x)$, $\nu_A(x)$ and $\omega_A(x)$. Thus, $0^- \le \sup \mu_A(x) + \sup \nu_A(x) + \sup \omega_A(x) \le 3^+$ [8].*

2.2. Interval Neutrosophic Sets

Since it is hard to utilize NSs in various practical situations, Wang et al. [10] developed the concept of INSs, which can be regarded as a subclass of NSs and a powerful structure in reflecting an expert's inconsistent and indeterminate preferences in real-life decision making procedures. In what follows, we present the concept of INSs.

Definition 2. *[10] Let U be the universe of discourse; an INS A is characterized by a truth-membership function $\mu_A(x)$, an indeterminacy-membership function $\nu_A(x)$ and a falsity-membership function $\omega_A(x)$. Then, an INS A can be denoted as the following mathematical symbol:*

$$A = \left\{ \left\langle x, \left[\mu_A^L(x), \mu_A^U(x)\right], \left[\nu_A^L(x), \nu_A^U(x)\right], \left[\omega_A^L(x), \omega_A^U(x)\right] \right\rangle | x \in U \right\},$$

where $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)] \subseteq [0,1]$ for all $x \in U$ to the set A. Thus, the sum of $\mu_A^U(x)$, $\nu_A^U(x)$ and $\omega_A^U(x)$ satisfies the condition: $0 \le \mu_A^U(x) + \nu_A^U(x) + \omega_A^U(x) \le 3$.

Suppose that U is the universe of discourse, then the set of all INSs on U is represented by $IN(U)$. Moreover, $\forall A \in IN(U)$. Based on the above definition, Wang et al. [10] defined the following operational laws on INSs.

Definition 3. *Let U be the universe of discourse, $\forall A, B \in IN(U)$, then [10]:*

1. *the complement of A is denoted by A^c such that $\forall x \in U$,* $A^c = \{\langle x, [\omega_A^L(x), \omega_A^U(x)], [1 - \nu_A^U(x), 1 - \nu_A^L(x)], [\mu_A^L(x), \mu_A^U(x)]\rangle | x \in U\}$;
2. *the intersection of A and B is denoted by $A \cap B$ such that $\forall x \in U$,* $A \cap B = \{\langle x, [\min(\mu_A^L(x), \mu_B^L(x)), \min(\mu_A^U(x), \mu_B^U(x))], [\max(\nu_A^L(x), \nu_B^L(x)), \max(\nu_A^U(x), \nu_B^U(x))], [\max(\omega_A^L(x), \omega_B^L(x)), \max(\omega_A^U(x), \omega_B^U(x))]\rangle | x \in U\}$;
3. *the union of A and B is denoted by $A \cup B$ such that $\forall x \in U$,* $A \cup B = \{\langle x, [\max(\mu_A^L(x), \mu_B^L(x)), \max(\mu_A^U(x), \mu_B^U(x))], [\min(\nu_A^L(x), \nu_B^L(x)), \min(\nu_A^U(x), \nu_B^U(x))], [\min(\omega_A^L(x), \omega_B^L(x)), \min(\omega_A^U(x), \omega_B^U(x))]\rangle | x \in U\}$;
4. *inclusion: $A \subseteq B$ if and only if $\mu_A^L(x) \le \mu_B^L(x)$, $\mu_A^U(x) \le \mu_B^U(x)$, $\nu_A^L(x) \ge \nu_B^L(x)$, $\nu_A^U(x) \ge \nu_B^U(x)$, $\omega_A^L(x) \ge \omega_B^L(x)$ and $\omega_A^U(x) \ge \omega_B^U(x)$ for any x in U;*

5. *equality:* $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$.

Next, we introduce some fundamental properties of the above operations, which state some important algebraic properties of the operations defined on INSs.

Theorem 1. *Let U be the universe of discourse; suppose that A, B and C are three INSs. Then, the followings are true [10]:*

1. *Double negation law:* $(A^c)^c = A$;
2. *De Morgan's laws:* $(A \cup B)^c = A^c \cap B^c$, $(A \cap B)^c = A^c \cup B^c$;
3. *Commutativity:* $A \cup B = B \cup A$, $A \cap B = B \cap A$;
4. *Associativity:* $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$;
5. *Distributivity:* $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

To compare the magnitude of different interval neutrosophic numbers (INNs), Zhang et al. [15] introduced the following comparison laws.

Definition 4. *Let* $x = \langle [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)] \rangle$ *be an INN; the score function of x is defined as follows [15]:*

$$s(x) = \left[\mu_A^L(x) + 1 - \nu_A^U(x) + 1 - \omega_A^U(x), \mu_A^U(x) + 1 - \nu_A^L(x) + 1 - \omega_A^L(x)\right].$$

It is noted that the score value is a significant index in ranking INNs. For an INN x, the bigger truth-membership $\mu_A(x)$ is, the less indeterminacy-membership $\nu_A(x)$ is, and the less falsity-membership $\omega_A(x)$ is, the greater INN x is.

2.3. Interval Neutrosophic Rough Sets over Two Universes

In this subsection, we introduce IN relations over two universes firstly.

Definition 5. *Let U, V be two non-empty and finite universes of discourse; an IN relation R from U to V can be denoted as the following form:*

$$R = \left\{ \left\langle (x,y), \left[\mu_R^L(x,y), \mu_R^U(x,y)\right], \left[\nu_R^L(x,y), \nu_R^U(x,y)\right], \left[\omega_R^L(x,y), \omega_R^U(x,y)\right] \right\rangle | (x,y) \in U \times V \right\},$$

where $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)] \subseteq [0,1]$, *denoting the truth-membership, the indeterminacy-membership and the falsity-membership for all* $(x,y) \in U \times V$, *respectively. Furthermore, the family of all IN relations on* $U \times V$ *is represented by* $INR(U \times V)$.

According to IN relations over two universes, we present the definition of IN rough sets over two universes below.

Definition 6. *Let U, V be two non-empty and finite universes of discourse, and $R \in INR(U \times V)$, the pair (U, V, R) is called an IN approximation space over two universes. For any $A \in IN(V)$, the IN rough lower and upper approximations of A with respect to (U, V, R), denoted by $\underline{R}(A)$ and $\overline{R}(A)$, are defined as follows:*

$$\underline{R}(A) = \left\{ \left\langle x, \left[\mu_{\underline{R}(A)}^L(x), \mu_{\underline{R}(A)}^U(x)\right], \left[\nu_{\underline{R}(A)}^L(x), \nu_{\underline{R}(A)}^U(x)\right], \left[\omega_{\underline{R}(A)}^L(x), \omega_{\underline{R}(A)}^U(x)\right] \right\rangle | x \in U \right\},$$

$$\overline{R}(A) = \left\{ \left\langle x, \left[\mu_{\overline{R}(A)}^L(x), \mu_{\overline{R}(A)}^U(x)\right], \left[\nu_{\overline{R}(A)}^L(x), \nu_{\overline{R}(A)}^U(x)\right], \left[\omega_{\overline{R}(A)}^L(x), \omega_{\overline{R}(A)}^U(x)\right] \right\rangle | x \in U \right\},$$

where

$$\mu^{L}_{\underline{R}(A)}(x) = \wedge_{y\in V}\left\{\omega^{L}_{R}(x,y) \vee \mu^{L}_{A}(y)\right\}, \mu^{U}_{\underline{R}(A)}(x) = \wedge_{y\in V}\left\{\omega^{U}_{R}(x,y) \vee \mu^{U}_{A}(y)\right\},$$
$$\nu^{L}_{\underline{R}(A)}(x) = \vee_{y\in V}\left\{\left(1-\nu^{U}_{R}(x,y)\right) \wedge \nu^{L}_{A}(y)\right\}, \nu^{U}_{\underline{R}(A)}(x) = \vee_{y\in V}\left\{\left(1-\nu^{L}_{R}(x,y)\right) \wedge \nu^{U}_{A}(y)\right\},$$
$$\omega^{L}_{\underline{R}(A)}(x) = \vee_{y\in V}\left\{\mu^{L}_{R}(x,y) \wedge \omega^{L}_{A}(y)\right\}, \omega^{U}_{\underline{R}(A)}(x) = \vee_{y\in V}\left\{\mu^{U}_{R}(x,y) \wedge \omega^{U}_{A}(y)\right\},$$
$$\mu^{L}_{\overline{R}(A)}(x) = \vee_{y\in V}\left\{\mu^{L}_{R}(x,y) \wedge \mu^{L}_{A}(y)\right\}, \mu^{U}_{\overline{R}(A)}(x) = \vee_{y\in V}\left\{\mu^{U}_{R}(x,y) \wedge \mu^{U}_{A}(y)\right\},$$
$$\nu^{L}_{\overline{R}(A)}(x) = \wedge_{y\in V}\left\{\nu^{L}_{R}(x,y) \vee \nu^{L}_{A}(y)\right\}, \nu^{U}_{\overline{R}(A)}(x) = \wedge_{y\in V}\left\{\nu^{U}_{R}(x,y) \vee \nu^{U}_{A}(y)\right\},$$
$$\omega^{L}_{\overline{R}(A)}(x) = \wedge_{y\in V}\left\{\omega^{L}_{R}(x,y) \vee \omega^{L}_{A}(y)\right\}, \omega^{U}_{\overline{R}(A)}(x) = \wedge_{y\in V}\left\{\omega^{U}_{R}(x,y) \vee \omega^{U}_{A}(y)\right\}.$$

The pair $\left(\underline{R}(A), \overline{R}(A)\right)$ *is called an IN rough set over two universes of A.*

3. Interval Neutrosophic Multigranulation Rough Sets over Two Universes

In this section, we present IN relations over two universes from a single granulation to multiple granulations, both the definition and properties of IN multigranulation rough sets over two universes will be elaborated on.

3.1. Optimistic in Multigranulation Rough Sets over Two Universes

Definition 7. *Let U, V be two non-empty and finite universes of discourse and $R_i \in INR(U\times V)\,(i=1,2,\ldots,m)$ be m IN relations over $U\times V$; the pair (U,V,R_i) is called an IN multigranulation approximation space over two universes. For any $A \in IN(V)$, the optimistic IN multigranulation rough lower and upper approximations of A with respect to (U,V,R_i), denoted by $\underline{\sum_{i=1}^{m} R_i}^{O}(A)$ and $\overline{\sum_{i=1}^{m} R_i}^{O}(A)$, are defined as follows:*

$$\underline{\sum_{i=1}^{m} R_i}^{O}(A) = \left\{\left\langle x, \left[\mu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \mu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right], \left[\nu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \nu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right],\right.\right.$$
$$\left.\left.\left[\omega^{L}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \omega^{U}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right]\right\rangle | x\in U\right\},$$

$$\overline{\sum_{i=1}^{m} R_i}^{O}(A) = \left\{\left\langle x, \left[\mu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \mu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right], \left[\nu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \nu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right],\right.\right.$$
$$\left.\left.\left[\omega^{L}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x), \omega^{U}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x)\right]\right\rangle | x\in U\right\},$$

where:

$$
\begin{aligned}
&\mu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\vee}_{i=1}^{m} \wedge_{y\in V}\left\{\omega^{L}_{R_i}(x,y) \vee \mu^{L}_{A}(y)\right\}, \mu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\vee}_{i=1}^{m} \wedge_{y\in V}\left\{\omega^{U}_{R_i}(x,y) \vee \mu^{U}_{A}(y)\right\},\\
&\nu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\wedge}_{i=1}^{m} \vee_{y\in V}\left\{\left(1-\nu^{U}_{R_i}(x,y)\right) \wedge \nu^{L}_{A}(y)\right\},\\
&\nu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\wedge}_{i=1}^{m} \vee_{y\in V}\left\{\left(1-\nu^{L}_{R_i}(x,y)\right) \wedge \nu^{U}_{A}(y)\right\},\\
&\omega^{L}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\wedge}_{i=1}^{m} \vee_{y\in V}\left\{\mu^{L}_{R_i}(x,y) \wedge \omega^{L}_{A}(y)\right\}, \omega^{U}_{\underline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\wedge}_{i=1}^{m} \vee_{y\in V}\left\{\mu^{U}_{R_i}(x,y) \wedge \omega^{U}_{A}(y)\right\},\\
&\mu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\wedge}_{i=1}^{m} \vee_{y\in V}\left\{\mu^{L}_{R_i}(x,y) \wedge \mu^{L}_{A}(y)\right\}, \mu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\wedge}_{i=1}^{m} \vee_{y\in V}\left\{\mu^{U}_{R_i}(x,y) \wedge \mu^{U}_{A}(y)\right\},\\
&\nu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\vee}_{i=1}^{m} \wedge_{y\in V}\left\{\nu^{L}_{R_i}(x,y) \vee \nu^{L}_{A}(y)\right\}, \nu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\vee}_{i=1}^{m} \wedge_{y\in V}\left\{\nu^{U}_{R_i}(x,y) \vee \nu^{U}_{A}(y)\right\},\\
&\omega^{L}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\vee}_{i=1}^{m} \wedge_{y\in V}\left\{\omega^{L}_{R_i}(x,y) \vee \omega^{L}_{A}(y)\right\}, \omega^{U}_{\overline{\sum_{i=1}^{m} R_i}^{O}(A)}(x) = \mathop{\vee}_{i=1}^{m} \wedge_{y\in V}\left\{\omega^{U}_{R_i}(x,y) \vee \omega^{U}_{A}(y)\right\}.
\end{aligned}
$$

The pair $\left(\underline{\sum_{i=1}^{m} R_i}^{O}(A), \overline{\sum_{i=1}^{m} R_i}^{O}(A)\right)$ *is called an optimistic IN multigranulation rough set over two universes of A.*

Theorem 2. *Let U, V be two non-empty and finite universes of discourse and $R_i \in INR(U \times V)$ $(i = 1, 2, \ldots, m)$ be m IN relations over $U \times V$. Then, for any $A, A' \in IN(V)$, the following properties hold:*

1. $\underline{\sum_{i=1}^{m} R_i}^{O}(A^c) = \left(\overline{\sum_{i=1}^{m} R_i}^{O}(A)\right)^c$,
   $\overline{\sum_{i=1}^{m} R_i}^{O}(A^c) = \left(\underline{\sum_{i=1}^{m} R_i}^{O}(A)\right)^c$;
2. $A \subseteq A' \Rightarrow \underline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A')$,
   $A \subseteq A' \Rightarrow \overline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A')$;
3. $\underline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') = \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \underline{\sum_{i=1}^{m} R_i}^{O}(A')$,
   $\overline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') = \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \overline{\sum_{i=1}^{m} R_i}^{O}(A')$;
4. $\underline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') \supseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \underline{\sum_{i=1}^{m} R_i}^{O}(A')$,
   $\overline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \overline{\sum_{i=1}^{m} R_i}^{O}(A')$.

Proof.

1. For all $x \in U$, we have:

$$
\begin{aligned}
&\underline{\sum_{i=1}^{m} R_i}^{O}(A^c) = \\
&\Big\{\Big\langle x, \Big[\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^L_{R_i}(x,y)\vee \mu^L_{A^c}(y)\Big\}, \bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^U_{R_i}(x,y)\vee \mu^U_{A^c}(y)\Big\}\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\Big(1-\nu^U_{R_i}(x,y)\Big)\wedge \nu^L_{A^c}(y)\Big\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\Big(1-\nu^L_{R_i}(x,y)\Big)\wedge \nu^U_{A^c}(y)\Big\}\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^L_{R_i}(x,y)\wedge \omega^L_{A^c}(y)\Big\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^U_{R_i}(x,y)\wedge \omega^U_{A^c}(y)\Big\}\Big]\Big\rangle \,|\, x\in U\Big\} \\
&= \Big\{\Big\langle x, \Big[\bigvee_{i=1}^{m} \wedge_{y\in V}\Big(\sim\Big\{\mu^L_{R_i}(x,y)\wedge \mu^L_{A}(y)\Big\}\Big), \bigvee_{i=1}^{m} \wedge_{y\in V}\Big(\sim\Big\{\mu^U_{R_i}(x,y)\wedge \mu^U_{A}(y)\Big\}\Big)\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big(\sim\Big\{\nu^L_{R_i}(x,y)\vee \nu^L_{A}(y)\Big\}\Big), \bigwedge_{i=1}^{m} \vee_{y\in V}\Big(\sim\Big\{\nu^U_{R_i}(x,y)\vee \nu^U_{A}(y)\Big\}\Big)\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big(\sim\Big\{\omega^L_{R_i}(x,y)\vee \omega^L_{A}(y)\Big\}\Big), \bigwedge_{i=1}^{m} \vee_{y\in V}\Big(\sim\Big\{\omega^U_{R_i}(x,y)\vee \omega^U_{A}(y)\Big\}\Big)\Big]\Big\rangle \,|\, x\in U\Big\} \\
&= \Big\{\Big\langle x, \Big[\sim\Big(\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^L_{R_i}(x,y)\wedge \mu^L_{A}(y)\Big\}\Big), \sim\Big(\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^U_{R_i}(x,y)\wedge \mu^U_{A}(y)\Big\}\Big)\Big], \\
&\Big[\sim\Big(\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\nu^L_{R_i}(x,y)\vee \nu^L_{A}(y)\Big\}\Big), \sim\Big(\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\nu^U_{R_i}(x,y)\vee \nu^U_{A}(y)\Big\}\Big)\Big], \\
&\Big[\sim\Big(\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^L_{R_i}(x,y)\vee \omega^L_{A}(y)\Big\}\Big), \sim\Big(\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^U_{R_i}(x,y)\vee \omega^U_{A}(y)\Big\}\Big)\Big]\Big\rangle \,|\, x\in U\Big\} \\
&= \Big(\overline{\sum_{i=1}^{m} R_i}^{O}(A)\Big)^c.
\end{aligned}
$$

Thus, we obtain $\underline{\sum_{i=1}^{m} R_i}^{O}(A^c) = \Big(\overline{\sum_{i=1}^{m} R_i}^{O}(A)\Big)^c$. Then, $\overline{\sum_{i=1}^{m} R_i}^{O}(A^c) = \Big(\underline{\sum_{i=1}^{m} R_i}^{O}(A)\Big)^c$ is obtained in an identical fashion.

2. Since $A \subseteq A'$, we have $\mu^L_A(y) \le \mu^L_{A'}(y)$, $\mu^U_A(y) \le \mu^U_{A'}(y)$, $\nu^L_A(y) \ge \nu^L_{A'}(y)$, $\nu^U_A(y) \ge \nu^U_{A'}(y)$, $\omega^L_A(y) \ge \omega^L_{A'}(y)$ and $\omega^U_A(y) \ge \omega^U_{A'}(y)$.
So it follows that:

$$
\begin{aligned}
&\Big\{\Big\langle x, \Big[\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^L_{R_i}(x,y)\vee \mu^L_{A}(y)\Big\}, \bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^U_{R_i}(x,y)\vee \mu^U_{A}(y)\Big\}\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\Big(1-\nu^U_{R_i}(x,y)\Big)\wedge \nu^L_{A}(y)\Big\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\Big(1-\nu^L_{R_i}(x,y)\Big)\wedge \nu^U_{A}(y)\Big\}\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^L_{R_i}(x,y)\wedge \omega^L_{A}(y)\Big\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^U_{R_i}(x,y)\wedge \omega^U_{A}(y)\Big\}\Big]\Big\rangle \,|\, x\in U\Big\} \le \\
&\Big\{\Big\langle x, \Big[\bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^L_{R_i}(x,y)\vee \mu^L_{A'}(y)\Big\}, \bigvee_{i=1}^{m} \wedge_{y\in V}\Big\{\omega^U_{R_i}(x,y)\vee \mu^U_{A'}(y)\Big\}\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\Big(1-\nu^U_{R_i}(x,y)\Big)\wedge \nu^L_{A'}(y)\Big\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\Big(1-\nu^L_{R_i}(x,y)\Big)\wedge \nu^U_{A'}(y)\Big\}\Big], \\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^L_{R_i}(x,y)\wedge \omega^L_{A'}(y)\Big\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\Big\{\mu^U_{R_i}(x,y)\wedge \omega^U_{A'}(y)\Big\}\Big]\Big\rangle \,|\, x\in U\Big\}.
\end{aligned}
$$

Therefore, we have $A \subseteq A' \Rightarrow \underline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A')$. Then, $A \subseteq A' \Rightarrow \overline{\sum_{i=1}^{m} R_i}^{O}(A) \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A')$ is obtained in a similar manner.

3. In light of the previous results, it is easy to obtain:
$\underline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') = \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \underline{\sum_{i=1}^{m} R_i}^{O}(A')$,
$\overline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') = \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \overline{\sum_{i=1}^{m} R_i}^{O}(A')$.
4. In light of the previous results, it is easy to obtain:
$\underline{\sum_{i=1}^{m} R_i}^{O}(A \cup A') \supseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A) \cup \underline{\sum_{i=1}^{m} R_i}^{O}(A')$,
$\overline{\sum_{i=1}^{m} R_i}^{O}(A \cap A') \subseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A) \cap \overline{\sum_{i=1}^{m} R_i}^{O}(A')$.

□

Theorem 3. *Let U, V be two non-empty and finite universes of discourse and $R_i, R_i' \in INR(U \times V)$ $(i = 1, 2, \ldots, m)$ be two IN relations over $U \times V$. If $R_i \subseteq R_i'$, for any $A \in IN(V)$, the following properties hold:*

1. $\underline{\sum_{i=1}^{m} R_i'}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$, *for all $A \in IN(V)$;*
2. $\overline{\sum_{i=1}^{m} R_i'}^{O}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$, *for all $A \in IN(V)$.*

Proof. Since $R_i \subseteq R_i'$, we have $\mu^L_{R_i}(x,y) \le \mu^L_{R_i'}(x,y)$, $\mu^U_{R_i}(x,y) \le \mu^U_{R_i'}(x,y)$, $\nu^L_{R_i}(x,y) \ge \nu^L_{R_i'}(x,y)$, $\nu^U_{R_i}(x,y) \ge \nu^U_{R_i'}(x,y)$, $\omega^L_{R_i}(x,y) \ge \omega^L_{R_i'}(x,y)$ and $\omega^U_{R_i}(x,y) \ge \omega^U_{R_i'}(x,y)$.

Therefore, it follows that:

$$
\begin{aligned}
&\Big\{\Big\langle x, \Big[\bigvee_{i=1}^{m} \wedge_{y\in V}\left\{\omega^L_{R_i}(x,y) \vee \mu^L_A(y)\right\}, \bigvee_{i=1}^{m} \wedge_{y\in V}\left\{\omega^U_{R_i}(x,y) \vee \mu^U_A(y)\right\}\Big],\\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\left(1-\nu^U_{R_i}(x,y)\right) \wedge \nu^L_A(y)\right\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\left(1-\nu^L_{R_i}(x,y)\right) \wedge \nu^U_A(y)\right\}\Big],\\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\mu^L_{R_i}(x,y) \wedge \omega^L_A(y)\right\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\mu^U_{R_i}(x,y) \wedge \omega^U_A(y)\right\}\Big]\Big\rangle \mid x \in U\Big\} \ge\\
&\Big\{\Big\langle x, \Big[\bigvee_{i=1}^{m} \wedge_{y\in V}\left\{\omega^L_{R_i'}(x,y) \vee \mu^L_A(y)\right\}, \bigvee_{i=1}^{m} \wedge_{y\in V}\left\{\omega^U_{R_i'}(x,y) \vee \mu^U_A(y)\right\}\Big],\\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\left(1-\nu^U_{R_i'}(x,y)\right) \wedge \nu^L_A(y)\right\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\left(1-\nu^L_{R_i'}(x,y)\right) \wedge \nu^U_A(y)\right\}\Big],\\
&\Big[\bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\mu^L_{R_i'}(x,y) \wedge \omega^L_A(y)\right\}, \bigwedge_{i=1}^{m} \vee_{y\in V}\left\{\mu^U_{R_i'}(x,y) \wedge \omega^U_A(y)\right\}\Big]\Big\rangle \mid x \in U\Big\}.
\end{aligned}
$$

Thus, we obtain $\underline{\sum_{i=1}^{m} R_i'}^{O}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$. Then, $\overline{\sum_{i=1}^{m} R_i'}^{O}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ is obtained in an identical fashion. □

3.2. Pessimistic in Multigranulation Rough Sets over Two Universes

Definition 8. *Let U, V be two non-empty and finite universes of discourse and $R_i \in INR(U \times V)$ $(i = 1, 2, \ldots, m)$ be m IN relations over $U \times V$; the pair (U, V, R_i) is called an IN multigranulation approximation space over two universes. For any $A \in IN(V)$, the pessimistic IN* *multigranulation rough lower and upper approximations of A with respect to* (U, V, R_i)*, denoted by* $\underline{\sum_{i=1}^{m} R_i}^{P}(A)$ *and* $\overline{\sum_{i=1}^{m} R_i}^{P}(A)$*, are defined as follows:*

$$\underline{\sum_{i=1}^{m} R_i}^{P}(A) = \left\{ \left\langle x, \left[\mu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \mu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right], \left[\nu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \nu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right], \left[\omega^{L}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \omega^{U}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right] \right\rangle | x \in U \right\},$$

$$\overline{\sum_{i=1}^{m} R_i}^{P}(A) = \left\{ \left\langle x, \left[\mu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \mu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right], \left[\nu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \nu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right], \left[\omega^{L}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x), \omega^{U}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x)\right] \right\rangle | x \in U \right\},$$

where:

$$\mu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigwedge_{i=1}^{m} \wedge_{y\in V} \left\{\omega^{L}_{R_i}(x,y) \vee \mu^{L}_{A}(y)\right\}, \mu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigwedge_{i=1}^{m} \wedge_{y\in V} \left\{\omega^{U}_{R_i}(x,y) \vee \mu^{U}_{A}(y)\right\},$$
$$\nu^{L}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigvee_{i=1}^{m} \vee_{y\in V} \left\{\left(1-\nu^{U}_{R_i}(x,y)\right) \wedge \nu^{L}_{A}(y)\right\},$$
$$\nu^{U}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigvee_{i=1}^{m} \vee_{y\in V} \left\{\left(1-\nu^{L}_{R_i}(x,y)\right) \wedge \nu^{U}_{A}(y)\right\},$$
$$\omega^{L}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigvee_{i=1}^{m} \vee_{y\in V} \left\{\mu^{L}_{R_i}(x,y) \wedge \omega^{L}_{A}(y)\right\}, \omega^{U}_{\underline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigvee_{i=1}^{m} \vee_{y\in V} \left\{\mu^{U}_{R_i}(x,y) \wedge \omega^{U}_{A}(y)\right\},$$
$$\mu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigvee_{i=1}^{m} \vee_{y\in V} \left\{\mu^{L}_{R_i}(x,y) \wedge \mu^{L}_{A}(y)\right\}, \mu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigvee_{i=1}^{m} \vee_{y\in V} \left\{\mu^{U}_{R_i}(x,y) \wedge \mu^{U}_{A}(y)\right\},$$
$$\nu^{L}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigwedge_{i=1}^{m} \wedge_{y\in V} \left\{\nu^{L}_{R_i}(x,y) \vee \nu^{L}_{A}(y)\right\}, \nu^{U}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigwedge_{i=1}^{m} \wedge_{y\in V} \left\{\nu^{U}_{R_i}(x,y) \vee \nu^{U}_{A}(y)\right\},$$
$$\omega^{L}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigwedge_{i=1}^{m} \wedge_{y\in V} \left\{\omega^{L}_{R_i}(x,y) \vee \omega^{L}_{A}(y)\right\}, \omega^{U}_{\overline{\sum_{i=1}^{m} R_i}^{P}(A)}(x) = \bigwedge_{i=1}^{m} \wedge_{y\in V} \left\{\omega^{U}_{R_i}(x,y) \vee \omega^{U}_{A}(y)\right\}.$$

The pair $\left(\underline{\sum_{i=1}^{m} R_i}^{P}(A), \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$ *is called a pessimistic IN multigranulation rough set over two universes of A.*

Theorem 4. *Let* U*,* V *be two non-empty and finite universes of discourse and* $R_i \in INR(U \times V)$ $(i = 1, 2, \ldots, m)$ *be m IN relations over* $U \times V$*. Then, for any* $A, A' \in IN(V)$*, the following properties hold:*

1. $\underline{\sum_{i=1}^{m} R_i}^{P}(A^c) = \left(\overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)^c$,
$\overline{\sum_{i=1}^{m} R_i}^{P}(A^c) = \left(\underline{\sum_{i=1}^{m} R_i}^{P}(A)\right)^c$;
2. $A \subseteq A' \Rightarrow \underline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{P}(A')$,
$A \subseteq A' \Rightarrow \overline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \overline{\sum_{i=1}^{m} R_i}^{P}(A')$;
3. $\underline{\sum_{i=1}^{m} R_i}^{P}(A \cap A') = \underline{\sum_{i=1}^{m} R_i}^{P}(A) \cap \underline{\sum_{i=1}^{m} R_i}^{P}(A')$,
$\overline{\sum_{i=1}^{m} R_i}^{P}(A \cup A') = \overline{\sum_{i=1}^{m} R_i}^{P}(A) \cup \overline{\sum_{i=1}^{m} R_i}^{P}(A')$;
4. $\underline{\sum_{i=1}^{m} R_i}^{P}(A \cup A') \supseteq \underline{\sum_{i=1}^{m} R_i}^{P}(A) \cup \underline{\sum_{i=1}^{m} R_i}^{P}(A')$,
$\overline{\sum_{i=1}^{m} R_i}^{P}(A \cap A') \subseteq \overline{\sum_{i=1}^{m} R_i}^{P}(A) \cap \overline{\sum_{i=1}^{m} R_i}^{P}(A')$.

Theorem 5. *Let U, V be two non-empty and finite universes of discourse and $R_i, R_i' \in INR(U \times V)$ $(i = 1, 2, \ldots, m)$ be two IN relations over $U \times V$. If $R_i \subseteq R_i'$, for any $A \in IN(V)$, the following properties hold:*

1. $\underline{\sum_{i=1}^{m} R_i'}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{P}(A)$, *for all* $A \in IN(V)$;
2. $\overline{\sum_{i=1}^{m} R_i'}^{P}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{P}(A)$, *for all* $A \in IN(V)$.

3.3. Relationships between Optimistic and Pessimistic in Multigranulation Rough Sets over Two Universes

Theorem 6. *Let U, V be two non-empty and finite universes of discourse and $R_i \in INR(U \times V)$ $(i = 1, 2, \ldots, m)$ be m IN relations over $U \times V$. Then, for any $A \in IN(V)$, the following properties hold:*

1. $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$;
2. $\overline{\sum_{i=1}^{m} R_i}^{P}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$.

Proof. For any $x \in U$, we have:

$$\begin{aligned}
&\underline{\sum_{i=1}^{m} R_i}^{O}(A) = \Bigg\{\Bigg\langle x, \left[\bigvee_{i=1}^{m} \wedge_{y \in V}\left\{\omega_{R_i}^{L}(x,y) \vee \mu_A^L(y)\right\}, \bigvee_{i=1}^{m} \wedge_{y \in V}\left\{\omega_{R_i}^{U}(x,y) \vee \mu_A^U(y)\right\}\right], \\
&\left[\bigwedge_{i=1}^{m} \vee_{y \in V}\left\{\left(1 - \nu_{R_i}^{U}(x,y)\right) \wedge \nu_A^L(y)\right\}, \bigwedge_{i=1}^{m} \vee_{y \in V}\left\{\left(1 - \nu_{R_i}^{L}(x,y)\right) \wedge \nu_A^U(y)\right\}\right], \\
&\left[\bigwedge_{i=1}^{m} \vee_{y \in V}\left\{\mu_{R_i}^{L}(x,y) \wedge \omega_A^L(y)\right\}, \bigwedge_{i=1}^{m} \vee_{y \in V}\left\{\mu_{R_i}^{U}(x,y) \wedge \omega_A^U(y)\right\}\right]\Bigg\rangle \mid x \in U\Bigg\} \\
&\geq \Bigg\{\Bigg\langle x, \left[\bigwedge_{i=1}^{m} \wedge_{y \in V}\left\{\omega_{R_i}^{L}(x,y) \vee \mu_A^L(y)\right\}, \bigwedge_{i=1}^{m} \wedge_{y \in V}\left\{\omega_{R_i}^{U}(x,y) \vee \mu_A^U(y)\right\}\right], \\
&\left[\bigvee_{i=1}^{m} \vee_{y \in V}\left\{\left(1 - \nu_{R_i}^{U}(x,y)\right) \wedge \nu_A^L(y)\right\}, \bigvee_{i=1}^{m} \vee_{y \in V}\left\{\left(1 - \nu_{R_i}^{L}(x,y)\right) \wedge \nu_A^U(y)\right\}\right], \\
&\left[\bigvee_{i=1}^{m} \vee_{y \in V}\left\{\mu_{R_i}^{L}(x,y) \wedge \omega_A^L(y)\right\}, \bigvee_{i=1}^{m} \vee_{y \in V}\left\{\mu_{R_i}^{U}(x,y) \wedge \omega_A^U(y)\right\}\right]\Bigg\rangle \mid x \in U\Bigg\} = \underline{\sum_{i=1}^{m} R_i}^{P}(A).
\end{aligned}$$

Therefore, we have $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \subseteq \underline{\sum_{i=1}^{m} R_i}^{O}(A)$. Then, $\overline{\sum_{i=1}^{m} R_i}^{P}(A) \supseteq \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ is obtained in an identical fashion. $\square$

4. The Model and Approach of Merger and Acquisition Target Selection

In this section, in order to explore M&A target selection, we develop a new method by utilizing IN multigranulation rough sets over two universes. Specifically, some main points of the established decision making method can be summarized in the following subsections.

4.1. The Application Model

Suppose that $U = \{x_1, x_2, \ldots, x_j\}$ is a set of optional M&A targets and $V = \{y_1, y_2, \ldots, y_k\}$ is a set of evaluation factors. Let $R_i \in INR\,(U \times V)\,(i = 1, 2, \ldots, m)$ be m IN relations, which reflect the related degree between some optional M&A targets and evaluation factors provided by m decision makers. In what follows, suppose that $A \in IN\,(V)$ is a standard set, representing some requirements of corporate acquirers. Thus, we construct an IN information system that is denoted as (U, V, R_i, A). On the basis of the above descriptions, we calculate the optimistic and pessimistic IN multigranulation rough lower and upper approximations of A according to Definitions 7 and Definition 8, respectively. Then, with reference to operational laws developed in [15]: $\forall A, B \in IN\,(U)$, we have $A \oplus B = \{\langle x, [\mu_A^L(x) + \mu_B^L(x) - \mu_A^L(x)\,\mu_B^L(x), \mu_A^U(x) + \mu_B^U(x) - \mu_A^U(x)\,\mu_B^U(x)], [\nu_A^L(x)\,\nu_B^L(x),\ \nu_A^U(x)\,\nu_B^U(x)], [\omega_A^L(x)\,\omega_B^L(x), \omega_A^U(x)\,\omega_B^U(x)]\rangle\,|x \in U\}$, $\lambda A = \left\{\left\langle x, \left[1 - (1 - \mu_A^L(x))^{\lambda}, 1 - (1 - \mu_A^U(x))^{\lambda}\right], \left[(\nu_A^L(x))^{\lambda}, (\nu_A^U(x))^{\lambda}\right], \left[(\omega_A^L(x))^{\lambda}, (\omega_A^U(x))^{\lambda}\right]\right\rangle |x \in U\right\}$.

By virtue of the above operational laws, we further calculate the sets $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ and $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$. Since optimistic multigranulation rough sets are based on the "seeking common ground while reserving differences" strategy, which implies that one reserves both common decisions and inconsistent decisions at the same time, thus, the set $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ can be seen as a risk-seeking decision strategy. Similarly, pessimistic multigranulation rough sets are based on the "seeking common ground while eliminating differences" strategy, which implies that one reserves common decisions while deleting inconsistent decisions. Hence, the set $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ can be seen as a risk-averse decision strategy.

In realistic M&A target selection procedures, the enterprise's person in charge can choose the optimal M&A target through referring to the above mentioned risk-seeking and risk-averse decision strategies. In what follows, in order to make better use of those decision strategies, we present a compromise decision strategy with a risk coefficient of decision makers, and the risk coefficient is denoted as $\lambda\,(\lambda \in [0, 1])$. Based on this, the compromise decision strategy can be described as $\lambda \left(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)\right) \oplus (1 - \lambda) \left(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$. Additionally, it is noted that the compromise decision strategy reduces to the risk-seeking decision strategy and the risk-averse decision strategy when $\lambda = 1$ and $\lambda = 0$, respectively. Moreover, the larger λ is, business administrators are more prone to seek the maximum of risk, while the smaller λ is, business administrators are more prone to seek the minimum of risk. Finally, by virtue of the risk coefficient, the optimal decision result can be obtained by selecting the M&A target with the largest score value in $\lambda \left(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)\right) \oplus (1 - \lambda) \left(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$.

4.2. Algorithm for Merger and Acquisition Target Selection Used IN Multigranulation Rough Sets over Two Universes

In what follows, we present an algorithm for M&A target selection based on IN multigranulation rough sets over two universes.

Algorithm 1 M&A target selection based on IN multigranulation rough sets over two universes

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Require: The relations between the universes U and V provided by multiple experts (U, V, R_i) and a standard set A.

Ensure: The determined M&A target.

Step 1. Calculate the following sets: $\underline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\overline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\underline{\sum_{i=1}^{m} R_i}^{P}(A)$ and $\overline{\sum_{i=1}^{m} R_i}^{P}(A)$, respectively;

Step 2. Calculate the following sets: $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$, $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$ and $\lambda\left(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)\right) \oplus (1-\lambda)\left(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$, respectively;

Step 3. Determine the score values for optional M&A targets in compromise decision strategy $\lambda\left(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)\right) \oplus (1-\lambda)\left(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$;

Step 4. The optimal solution is the M&A target with the largest score value in $\lambda\left(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)\right) \oplus (1-\lambda)\left(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)\right)$.

---

5. Numerical Example

In this section, by utilizing an illustrative case analysis that was previously modeled in [2], we show fundamental steps of the proposed decision making method in M&A target selection background.

5.1. Case Description

Suppose there is a steel corporate acquirer who aims to evaluate which target organization is suitable for the acquiring firm. In order to reflect the fairness of the procedure, the acquiring firm invites three specialists to establish M&A target selection information systems, denoted as (U, V, R_i, A). Suppose that $U = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of optional M&A targets and another universe $V = \{y_1, y_2, y_3, y_4, y_5\}$ is a set of evaluation factors, where the evaluation factor represents mineral output, mining difficulty, proved reserves, reserve-production ratio, science and technology contribution rate, respectively. Based on the above, the M&A knowledge base can be composed of two kinds of objections related to M&A target selection, i.e., the optional M&A targets set and the evaluation factors set. In order to solve this group decision making problem, each decision maker provides his or her own thought about the M&A knowledge base that is shown as the following Tables 1–3.

Moreover, assume that decision makers provide a standard set that is represented by the following INS:

$$
\begin{aligned}
A = \; & \{\langle y_1, \langle [0.7, 0.8], [0.4, 0.6], [0.2, 0.3] \rangle\rangle, \langle y_2, \langle [0.5, 0.6], [0.3, 0.4], [0.6, 0.7] \rangle\rangle, \\
& \langle y_3, \langle [0.7, 0.8], [0.2, 0.3], [0.4, 0.5] \rangle\rangle, \langle y_4, \langle [0.2, 0.3], [0.5, 0.7], [0.6, 0.7] \rangle\rangle, \\
& \langle y_5, \langle [0.4, 0.5], [0.2, 0.3], [0.8, 0.9] \rangle\rangle\}.
\end{aligned}
$$

5.2. Decision Making Process

According to Algorithm 1, we aim to seek the determined M&A target by utilizing the proposed model. At first, by virtue of Definitions 7 and 8, we calculate the optimistic and pessimistic IN multigranulation rough lower and upper approximations of A:

$$\begin{aligned}
\underline{\sum_{i=1}^{3} R_i}^{O}(A) = \{ & \langle x_1, \langle [0.4,0.5], [0.5,0.6], [0.6,0.7] \rangle\rangle, \langle x_2, \langle [0.4,0.5], [0.5,0.7], [0.7,0.8] \rangle\rangle, \\
& \langle x_3, \langle [0.5,0.6], [0.4,0.6], [0.6,0.7] \rangle\rangle, \langle x_4, \langle [0.4,0.5], [0.5,0.6], [0.6,0.7] \rangle\rangle, \\
& \langle x_5, \langle [0.2,0.3], [0.5,0.6], [0.7,0.8] \rangle\rangle \}; \\
\overline{\sum_{i=1}^{3} R_i}^{O}(A) = \{ & \langle x_1, \langle [0.7,0.8], [0.3,0.4], [0.4,0.5] \rangle\rangle, \langle x_2, \langle [0.5,0.6], [0.3,0.5], [0.6,0.7] \rangle\rangle, \\
& \langle x_3, \langle [0.7,0.8], [0.2,0.3], [0.5,0.6] \rangle\rangle, \langle x_4, \langle [0.7,0.8], [0.2,0.3], [0.5,0.6] \rangle\rangle, \\
& \langle x_5, \langle [0.7,0.8], [0.3,0.4], [0.2,0.3] \rangle\rangle \}; \\
\underline{\sum_{i=1}^{3} R_i}^{P}(A) = \{ & \langle x_1, \langle [0.4,0.5], [0.5,0.7], [0.6,0.7] \rangle\rangle, \langle x_2, \langle [0.4,0.5], [0.5,0.7], [0.8,0.9] \rangle\rangle, \\
& \langle x_3, \langle [0.5,0.6], [0.4,0.6], [0.6,0.7] \rangle\rangle, \langle x_4, \langle [0.3,0.4], [0.5,0.7], [0.7,0.8] \rangle\rangle, \\
& \langle x_5, \langle [0.2,0.3], [0.5,0.7], [0.8,0.9] \rangle\rangle \}; \\
\overline{\sum_{i=1}^{3} R_i}^{P}(A) = \{ & \langle x_1, \langle [0.7,0.8], [0.2,0.3], [0.4,0.5] \rangle\rangle, \langle x_2, \langle [0.5,0.7], [0.3,0.4], [0.5,0.6] \rangle\rangle, \\
& \langle x_3, \langle [0.7,0.8], [0.2,0.3], [0.4,0.5] \rangle\rangle, \langle x_4, \langle [0.7,0.8], [0.2,0.3], [0.4,0.5] \rangle\rangle, \\
& \langle x_5, \langle [0.7,0.8], [0.3,0.4], [0.2,0.3] \rangle\rangle \}.
\end{aligned}$$

By virtue of the above results, we further calculated the sets $\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A)$ and $\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A)$.

$$\begin{aligned}
\underline{\sum_{i=1}^{3} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{O}(A) = \{ & \langle x_1, \langle [0.82,0.90], [0.15,0.24], [0.24,0.35] \rangle\rangle, \\
& \langle x_2, \langle [0.70,0.80], [0.15,0.35], [0.42,0.56] \rangle\rangle, \langle x_3, \langle [0.85,0.92], [0.08,0.18], [0.30,0.42] \rangle\rangle, \\
& \langle x_4, \langle [0.82,0.90], [0.10,0.18], [0.30,0.42] \rangle\rangle, \langle x_5, \langle [0.76,0.86], [0.15,0.24], [0.14,0.24] \rangle\rangle \}, \\
\underline{\sum_{i=1}^{3} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{P}(A) = \{ & \langle x_1, \langle [0.82,0.90], [0.10,0.21], [0.24,0.35] \rangle\rangle, \\
& \langle x_2, \langle [0.70,0.85], [0.15,0.28], [0.40,0.54] \rangle\rangle, \langle x_3, \langle [0.85,0.92], [0.08,0.18], [0.24,0.35] \rangle\rangle, \\
& \langle x_4, \langle [0.79,0.88], [0.10,0.21], [0.28,0.40] \rangle\rangle, \langle x_5, \langle [0.76,0.86], [0.15,0.28], [0.16,0.27] \rangle\rangle \}.
\end{aligned}$$

Next, suppose business managers take $\lambda = 0.6$, then $\lambda \left(\underline{\sum_{i=1}^{m} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{O}(A) \right) \oplus (1-\lambda) \left(\underline{\sum_{i=1}^{m} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{m} R_i}^{P}(A) \right)$ can be obtained as follows:

$$\begin{aligned}
& 0.6 \left(\underline{\sum_{i=1}^{3} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{O}(A) \right) \oplus (1-0.6) \left(\underline{\sum_{i=1}^{3} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{P}(A) \right) = \\
& \{ \langle x_1, \langle [0.82,0.90], [0.13,0.23], [0.24,0.35] \rangle\rangle, \langle x_2, \langle [0.70,0.82], [0.15,0.32], [0.41,0.55] \rangle\rangle, \\
& \langle x_3, \langle [0.85,0.92], [0.08,0.18], [0.28,0.39] \rangle\rangle, \langle x_4, \langle [0.81,0.89], [0.10,0.19], [0.29,0.41] \rangle\rangle, \\
& \langle x_5, \langle [0.76,0.86], [0.15,0.25], [0.15,0.25] \rangle\rangle \}.
\end{aligned}$$

Table 1. The knowledge of merger and acquisition (M&A) target selection given by Expert 1.

R_1	y_1	y_2	y_3	y_4	y_5
x_1	$\langle[0.8,0.9],[0.2,0.3],[0.4,0.5]\rangle$	$\langle[0.3,0.4],[0.3,0.4],[0.8,0.9]\rangle$	$\langle[0.6,0.7],[0.5,0.6],[0.3,0.4]\rangle$	$\langle[0.6,0.7],[0.4,0.5],[0.4,0.5]\rangle$	$\langle[0.2,0.3],[0.2,0.3],[0.8,0.9]\rangle$
x_2	$\langle[0.4,0.5],[0.3,0.4],[0.7,0.8]\rangle$	$\langle[0.3,0.4],[0.4,0.5],[0.7,0.8]\rangle$	$\langle[0.5,0.6],[0.4,0.5],[0.6,0.7]\rangle$	$\langle[0.6,0.7],[0.3,0.4],[0.5,0.6]\rangle$	$\langle[0.7,0.8],[0.3,0.5],[0.3,0.4]\rangle$
x_3	$\langle[0.7,0.8],[0.4,0.5],[0.5,0.6]\rangle$	$\langle[0.7,0.9],[0.3,0.4],[0.4,0.5]\rangle$	$\langle[0.3,0.4],[0.5,0.6],[0.6,0.7]\rangle$	$\langle[0.2,0.3],[0.6,0.7],[0.6,0.7]\rangle$	$\langle[0.5,0.6],[0.2,0.3],[0.7,0.8]\rangle$
x_4	$\langle[0.8,0.9],[0.2,0.4],[0.4,0.6]\rangle$	$\langle[0.5,0.6],[0.3,0.4],[0.6,0.7]\rangle$	$\langle[0.4,0.5],[0.2,0.3],[0.7,0.8]\rangle$	$\langle[0.6,0.7],[0.4,0.5],[0.3,0.5]\rangle$	$\langle[0.6,0.7],[0.2,0.3],[0.7,0.8]\rangle$
x_5	$\langle[0.8,0.9],[0.2,0.3],[0.1,0.2]\rangle$	$\langle[0.4,0.5],[0.1,0.2],[0.8,0.9]\rangle$	$\langle[0.6,0.7],[0.5,0.6],[0.2,0.3]\rangle$	$\langle[0.7,0.8],[0.4,0.5],[0.1,0.2]\rangle$	$\langle[0.7,0.9],[0.5,0.6],[0.1,0.2]\rangle$

Table 2. The knowledge of M&A target selection given by Expert 2.

R_2	y_1	y_2	y_3	y_4	y_5
x_1	$\langle[0.7,0.8],[0.2,0.3],[0.4,0.6]\rangle$	$\langle[0.3,0.4],[0.2,0.3],[0.7,0.8]\rangle$	$\langle[0.6,0.8],[0.4,0.6],[0.3,0.4]\rangle$	$\langle[0.6,0.7],[0.3,0.4],[0.4,0.5]\rangle$	$\langle[0.2,0.4],[0.3,0.4],[0.7,0.9]\rangle$
x_2	$\langle[0.4,0.6],[0.2,0.3],[0.7,0.8]\rangle$	$\langle[0.3,0.5],[0.4,0.5],[0.6,0.7]\rangle$	$\langle[0.5,0.6],[0.3,0.4],[0.5,0.6]\rangle$	$\langle[0.5,0.6],[0.2,0.3],[0.5,0.6]\rangle$	$\langle[0.8,0.9],[0.4,0.5],[0.3,0.4]\rangle$
x_3	$\langle[0.7,0.8],[0.3,0.4],[0.4,0.5]\rangle$	$\langle[0.8,0.9],[0.3,0.4],[0.3,0.4]\rangle$	$\langle[0.3,0.5],[0.4,0.5],[0.5,0.6]\rangle$	$\langle[0.2,0.4],[0.5,0.6],[0.5,0.7]\rangle$	$\langle[0.4,0.6],[0.2,0.3],[0.7,0.8]\rangle$
x_4	$\langle[0.7,0.9],[0.2,0.3],[0.5,0.6]\rangle$	$\langle[0.5,0.7],[0.2,0.3],[0.5,0.6]\rangle$	$\langle[0.4,0.5],[0.3,0.4],[0.7,0.8]\rangle$	$\langle[0.5,0.7],[0.2,0.4],[0.4,0.5]\rangle$	$\langle[0.7,0.8],[0.2,0.3],[0.6,0.7]\rangle$
x_5	$\langle[0.7,0.9],[0.3,0.4],[0.2,0.3]\rangle$	$\langle[0.4,0.6],[0.1,0.2],[0.7,0.8]\rangle$	$\langle[0.6,0.7],[0.4,0.6],[0.3,0.4]\rangle$	$\langle[0.7,0.8],[0.3,0.4],[0.1,0.2]\rangle$	$\langle[0.7,0.8],[0.3,0.6],[0.1,0.2]\rangle$

Table 3. The knowledge of M&A target selection given by Expert 3.

R_3	y_1	y_2	y_3	y_4	y_5
x_1	$\langle[0.7,0.8],[0.3,0.4],[0.5,0.6]\rangle$	$\langle[0.3,0.5],[0.2,0.4],[0.7,0.8]\rangle$	$\langle[0.7,0.8],[0.4,0.5],[0.4,0.5]\rangle$	$\langle[0.7,0.8],[0.3,0.5],[0.4,0.5]\rangle$	$\langle[0.3,0.4],[0.2,0.4],[0.7,0.8]\rangle$
x_2	$\langle[0.5,0.6],[0.2,0.3],[0.8,0.9]\rangle$	$\langle[0.4,0.5],[0.4,0.6],[0.6,0.7]\rangle$	$\langle[0.5,0.7],[0.3,0.5],[0.5,0.7]\rangle$	$\langle[0.5,0.7],[0.2,0.4],[0.5,0.6]\rangle$	$\langle[0.7,0.8],[0.3,0.4],[0.3,0.4]\rangle$
x_3	$\langle[0.7,0.8],[0.3,0.5],[0.4,0.5]\rangle$	$\langle[0.8,0.9],[0.4,0.5],[0.3,0.5]\rangle$	$\langle[0.4,0.5],[0.4,0.5],[0.5,0.7]\rangle$	$\langle[0.3,0.4],[0.5,0.7],[0.5,0.6]\rangle$	$\langle[0.4,0.5],[0.2,0.3],[0.8,0.9]\rangle$
x_4	$\langle[0.7,0.8],[0.2,0.3],[0.4,0.5]\rangle$	$\langle[0.6,0.7],[0.2,0.4],[0.5,0.6]\rangle$	$\langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle$	$\langle[0.5,0.6],[0.3,0.4],[0.3,0.4]\rangle$	$\langle[0.6,0.7],[0.2,0.3],[0.6,0.8]\rangle$
x_5	$\langle[0.7,0.8],[0.1,0.2],[0.1,0.3]\rangle$	$\langle[0.5,0.6],[0.2,0.3],[0.7,0.8]\rangle$	$\langle[0.6,0.7],[0.4,0.5],[0.3,0.4]\rangle$	$\langle[0.7,0.8],[0.3,0.5],[0.1,0.2]\rangle$	$\langle[0.8,0.9],[0.3,0.5],[0.1,0.2]\rangle$

At last, we determine the score values for five optional target organizations in $0.6\left(\underline{\sum_{i=1}^{3} R_i}^{O}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{O}(A)\right) \oplus (1-0.6)\left(\underline{\sum_{i=1}^{3} R_i}^{P}(A) \oplus \overline{\sum_{i=1}^{3} R_i}^{P}(A)\right)$; it is not difficult to obtain the ordering result for M&A targets in the compromise decision strategy: $x_3 \succ x_5 \succ x_1 \succ x_4 \succ x_2$. Thus, we can see that the optimal selection is the third target enterprise.

5.3. Comparative Analysis

The above subsection shows basic steps and procedures of the proposed algorithm based on IN multigranulation rough sets over two universes in the M&A target selection background. In order to illustrate the effectiveness of the proposed decision making approach, we compare the newly-proposed decision making rules with the decision making method based on IN aggregation operators in this subsection. In the literature [15], Zhang et al. developed several common IN aggregation operators and applied them to a multiple criteria decision making problem. In what follows, a comparison analysis is conducted by utilizing the proposed M&A target selection approach with the interval neutrosophic number weighted averaging (INNWA) operator and the interval neutrosophic number weighted geometric (INNWG) operator presented in [15]. Prior to the specific comparison analysis, we review the above-mentioned IN aggregation operators. Let $A_j = \left\langle \left[T_{A_j}^L, T_{A_j}^U\right], \left[I_{A_j}^L, I_{A_j}^U\right], \left[F_{A_j}^L, F_{A_j}^U\right] \right\rangle (j = 1, 2, \ldots, n)$ be a collection of INNs and $w = (1/n, 1/n, \ldots, 1/n)^T$ be the weight of A_j with equal weight, then the INNWA operator and the INNWG operator are presented below.

1. The interval neutrosophic number weighted averaging (INNWA) operator:

$$INNWA_w(A_1, A_2, \ldots, A_n) = \sum_{j=1}^{n}\left(\frac{1}{n}A_j\right) =$$
$$\left\langle \begin{array}{l} \left[1-\prod_{j=1}^{n}\left(1-T_{A_j}^L\right)^{1/n}, 1-\prod_{j=1}^{n}\left(1-T_{A_j}^U\right)^{1/n}\right], \\ \left[\prod_{j=1}^{n}\left(I_{A_j}^L\right)^{1/n}, \prod_{j=1}^{n}\left(I_{A_j}^U\right)^{1/n}\right], \left[\prod_{j=1}^{n}\left(F_{A_j}^L\right)^{1/n}, \prod_{j=1}^{n}\left(F_{A_j}^U\right)^{1/n}\right] \end{array} \right\rangle,$$

2. The interval neutrosophic number weighted geometric (INNWG) operator:

$$INNWG_w(A_1, A_2, \ldots, A_n) = \prod_{j=1}^{n}\left(A_j\right)^{1/n} =$$
$$\left\langle \begin{array}{l} \left[\prod_{j=1}^{n}\left(T_{A_j}^L\right)^{1/n}, \prod_{j=1}^{n}\left(T_{A_j}^U\right)^{1/n}\right], \left[1-\prod_{j=1}^{n}\left(1-I_{A_j}^L\right)^{1/n}, 1-\prod_{j=1}^{n}\left(1-I_{A_j}^U\right)^{1/n}\right], \\ \left[1-\prod_{j=1}^{n}\left(1-F_{A_j}^L\right)^{1/n}, 1-\prod_{j=1}^{n}\left(1-F_{A_j}^U\right)^{1/n}\right] \end{array} \right\rangle.$$

On the basis of the INNWA and INNWG operators introduced above, the knowledge of M&A target selection given by Expert 1, Expert 2 and Expert 3 could be aggregated by using INNWA and INNWG operators, i.e., we aggregate IN relations over two universes R_1, R_2 and R_3 to a single IN relation over two universes, represented by R_{INNWA} and R_{INNWG}, which is shown as the following Tables 4 and 5.

Next, by virtue of IN rough sets over two universes presented in Definition 6, we calculate the IN rough lower and upper approximations of A with respect to (U, V, R_{INNWA}) and (U, V, R_{INNWG}), respectively.

$$\begin{aligned}
\underline{R_{INNWA}}(A) = \{&\langle x_1, \langle [0.40, 0.50], [0.50, 0.67], [0.60, 0.70] \rangle\rangle, \langle x_2, \langle [0.40, 0.50], [0.50, 0.70], [0.74, 0.85] \rangle\rangle, \\
&\langle x_3, \langle [0.50, 0.60], [0.40, 0.60], [0.60, 0.70] \rangle\rangle, \langle x_4, \langle [0.33, 0.46], [0.50, 0.70], [0.63, 0.74] \rangle\rangle, \\
&\langle x_5, \langle [0.20, 0.30], [0.50, 0.67], [0.74, 0.88] \rangle\rangle\}; \\
\overline{R_{INNWA}}(A) = \{&\langle x_1, \langle [0.70, 0.80], [0.23, 0.37], [0.33, 0.46] \rangle\rangle, \langle x_2, \langle [0.52, 0.65], [0.33, 0.47], [0.60, 0.70] \rangle\rangle, \\
&\langle x_3, \langle [0.70, 0.80], [0.20, 0.30], [0.43, 0.52] \rangle\rangle, \langle x_4, \langle [0.70, 0.80], [0.20, 0.30], [0.43, 0.56] \rangle\rangle, \\
&\langle x_5, \langle [0.70, 0.80], [0.30, 0.40], [0.20, 0.30] \rangle\rangle\}; \\
\underline{R_{INNWG}}(A) = \{&\langle x_1, \langle [0.40, 0.50], [0.50, 0.67], [0.60, 0.70] \rangle\rangle, \langle x_2, \langle [0.40, 0.50], [0.50, 0.70], [0.72, 0.84] \rangle\rangle, \\
&\langle x_3, \langle [0.50, 0.60], [0.40, 0.60], [0.60, 0.70] \rangle\rangle, \langle x_4, \langle [0.33, 0.48], [0.50, 0.70], [0.62, 0.72] \rangle\rangle, \\
&\langle x_5, \langle [0.20, 0.30], [0.50, 0.67], [0.72, 0.88] \rangle\rangle\}; \\
\overline{R_{INNWG}}(A) = \{&\langle x_1, \langle [0.70, 0.80], [0.23, 0.37], [0.40, 0.50] \rangle\rangle, \langle x_2, \langle [0.50, 0.62], [0.33, 0.48], [0.54, 0.67] \rangle\rangle, \\
&\langle x_3, \langle [0.70, 0.80], [0.20, 0.30], [0.44, 0.54] \rangle\rangle, \langle x_4, \langle [0.70, 0.80], [0.20, 0.30], [0.44, 0.57] \rangle\rangle, \\
&\langle x_5, \langle [0.70, 0.80], [0.30, 0.40], [0.20, 0.30] \rangle\rangle\}.
\end{aligned}$$

Then, in a single granulation context, we further calculate the sets $\lambda\left(\underline{R_{INNWA}}(A)\right) \oplus (1-\lambda)\left(\overline{R_{INNWA}}(A)\right)$ and $\lambda\left(\underline{R_{INNWG}}(A)\right) \oplus (1-\lambda)\left(\overline{R_{INNWG}}(A)\right)$ when $\lambda = 0.6$:

$$\begin{aligned}
0.6\left(\underline{R_{INNWA}}(A)\right) \oplus (1-0.6)\left(\overline{R_{INNWA}}(A)\right) = \{&\langle x_1, \langle [0.54, 0.65], [0.37, 0.53], [0.51, 0.62] \rangle\rangle, \\
&\langle x_2, \langle [0.44, 0.56], [0.42, 0.59], [0.64, 0.76] \rangle\rangle, \langle x_3, \langle [0.59, 0.69], [0.31, 0.46], [0.53, 0.62] \rangle\rangle, \\
&\langle x_4, \langle [0.51, 0.63], [0.35, 0.50], [0.54, 0.66] \rangle\rangle, \langle x_5, \langle [0.46, 0.57], [0.41, 0.55], [0.44, 0.58] \rangle\rangle\}, \\
0.6\left(\underline{R_{INNWG}}(A)\right) \oplus (1-0.6)\left(\overline{R_{INNWG}}(A)\right) = \{&\langle x_1, \langle [0.54, 0.65], [0.37, 0.53], [0.51, 0.62] \rangle\rangle, \\
&\langle x_2, \langle [0.44, 0.55], [0.42, 0.61], [0.64, 0.77] \rangle\rangle, \langle x_3, \langle [0.59, 0.69], [0.31, 0.46], [0.53, 0.63] \rangle\rangle, \\
&\langle x_4, \langle [0.51, 0.64], [0.35, 0.50], [0.54, 0.66] \rangle\rangle, \langle x_5, \langle [0.46, 0.57], [0.41, 0.55], [0.43, 0.58] \rangle\rangle\}.
\end{aligned}$$

Table 4. The aggregated knowledge of M&A target selection by using the interval neutrosophic number weighted averaging (INNWA) operator.

R_{INNWA}	y_1	y_2	y_3	y_4	y_5
x_1	$\langle [0.74, 0.85], [0.23, 0.33], [0.43, 0.56] \rangle$	$\langle [0.30, 0.44], [0.23, 0.37], [0.72, 0.84] \rangle$	$\langle [0.63, 0.77], [0.43, 0.56], [0.33, 0.43] \rangle$	$\langle [0.63, 0.74], [0.33, 0.46], [0.40, 0.50] \rangle$	$\langle [0.23, 0.37], [0.23, 0.37], [0.72, 0.88] \rangle$
x_2	$\langle [0.44, 0.57], [0.23, 0.33], [0.72, 0.84] \rangle$	$\langle [0.33, 0.48], [0.40, 0.52], [0.62, 0.72] \rangle$	$\langle [0.50, 0.63], [0.33, 0.46], [0.52, 0.65] \rangle$	$\langle [0.54, 0.67], [0.23, 0.37], [0.50, 0.60] \rangle$	$\langle [0.74, 0.85], [0.33, 0.46], [0.30, 0.40] \rangle$
x_3	$\langle [0.70, 0.80], [0.33, 0.46], [0.43, 0.52] \rangle$	$\langle [0.77, 0.90], [0.33, 0.43], [0.33, 0.46] \rangle$	$\langle [0.33, 0.48], [0.43, 0.52], [0.52, 0.65] \rangle$	$\langle [0.23, 0.37], [0.52, 0.65], [0.52, 0.65] \rangle$	$\langle [0.44, 0.57], [0.20, 0.30], [0.72, 0.84] \rangle$
x_4	$\langle [0.74, 0.88], [0.20, 0.33], [0.43, 0.56] \rangle$	$\langle [0.54, 0.67], [0.23, 0.37], [0.52, 0.62] \rangle$	$\langle [0.40, 0.50], [0.23, 0.37], [0.70, 0.84] \rangle$	$\langle [0.53, 0.67], [0.29, 0.43], [0.33, 0.46] \rangle$	$\langle [0.63, 0.74], [0.20, 0.30], [0.62, 0.76] \rangle$
x_5	$\langle [0.74, 0.88], [0.18, 0.29], [0.12, 0.26] \rangle$	$\langle [0.44, 0.57], [0.12, 0.23], [0.72, 0.84] \rangle$	$\langle [0.60, 0.70], [0.43, 0.56], [0.26, 0.37] \rangle$	$\langle [0.70, 0.80], [0.33, 0.46], [0.10, 0.20] \rangle$	$\langle [0.74, 0.88], [0.35, 0.56], [0.10, 0.20] \rangle$

Table 5. The aggregated knowledge of M&A target selection by using the interval neutrosophic number weighted geometric (INNWG) operator.

R_{INNWG}	y_1	y_2	y_3	y_4	y_5
x_1	$\langle [0.72, 0.84], [0.23, 0.33], [0.44, 0.57] \rangle$	$\langle [0.30, 0.43], [0.23, 0.37], [0.74, 0.85] \rangle$	$\langle [0.62, 0.76], [0.44, 0.57], [0.33, 0.44] \rangle$	$\langle [0.62, 0.72], [0.33, 0.48], [0.40, 0.50] \rangle$	$\langle [0.23, 0.37], [0.23, 0.37], [0.74, 0.88] \rangle$
x_2	$\langle [0.43, 0.56], [0.23, 0.33], [0.74, 0.85] \rangle$	$\langle [0.33, 0.46], [0.40, 0.54], [0.63, 0.74] \rangle$	$\langle [0.50, 0.62], [0.33, 0.48], [0.54, 0.67] \rangle$	$\langle [0.52, 0.65], [0.23, 0.37], [0.50, 0.60] \rangle$	$\langle [0.72, 0.84], [0.33, 0.48], [0.30, 0.40] \rangle$
x_3	$\langle [0.70, 0.80], [0.33, 0.48], [0.44, 0.54] \rangle$	$\langle [0.76, 0.90], [0.33, 0.44], [0.33, 0.48] \rangle$	$\langle [0.33, 0.46], [0.44, 0.54], [0.54, 0.67] \rangle$	$\langle [0.23, 0.37], [0.54, 0.67], [0.54, 0.67] \rangle$	$\langle [0.43, 0.56], [0.20, 0.30], [0.74, 0.85] \rangle$
x_4	$\langle [0.72, 0.88], [0.20, 0.33], [0.44, 0.57] \rangle$	$\langle [0.52, 0.65], [0.23, 0.37], [0.54, 0.63] \rangle$	$\langle [0.40, 0.50], [0.23, 0.37], [0.70, 0.85] \rangle$	$\langle [0.53, 0.65], [0.30, 0.44], [0.33, 0.48] \rangle$	$\langle [0.62, 0.72], [0.20, 0.30], [0.63, 0.77] \rangle$
x_5	$\langle [0.72, 0.88], [0.20, 0.30], [0.12, 0.26] \rangle$	$\langle [0.43, 0.56], [0.12, 0.23], [0.74, 0.85] \rangle$	$\langle [0.60, 0.70], [0.44, 0.57], [0.26, 0.37] \rangle$	$\langle [0.70, 0.80], [0.33, 0.48], [0.10, 0.20] \rangle$	$\langle [0.72, 0.88], [0.37, 0.57], [0.10, 0.20] \rangle$

Finally, we compute the score values for five optional target organizations in $0.6\left(\underline{R_{INNWA}}(A)\right) \oplus (1-0.6)\left(\overline{R_{INNWA}}(A)\right)$ and $0.6\left(\underline{R_{INNWG}}(A)\right) \oplus (1-0.6)\left(\overline{R_{INNWG}}(A)\right)$ and further determine the M&A target with the largest score value. Then, the ranking results by utilizing INNWA and INNWG operators show that the best alternative is x_3, namely the third target enterprise, which is consistent with the decision result determined by Algorithm 1.

5.4. Result Analysis and Discussion

From the above ranking results, we can see that the proposed model takes full advantage of INSs and multigranulation rough sets over two universes in M&A target selection procedures, and the superiorities of utilizing INSs and multigranulation rough sets over two universes to deal with group decision making problems can be summarized as follows:

1. In the process of describing decision making information, INSs provide experts with more exemplary and flexible access to convey their understandings about the M&A knowledge base. Specifically, it is worth noting that a variety of decision making information systems based on INSs outperform some common extended forms of classical FSs, such as IFSs, interval-valued fuzzy sets (IVFSs), interval-valued intuitionistic fuzzy sets (IVIFSs) and single-valued neutrosophic sets (SVNSs). In the concept of INSs, by introducing the degree of indeterminacy and the degree of non-membership of an element to a set, decision makers could express their incomplete, indeterminate and inconsistent preferences more precisely, as well as avoid the loss of decision making information through considering the truth, indeterminacy and falsity membership functions in various decision making processes. Additionally, the expression of the degree of membership, the degree of indeterminacy and the degree of non-membership using an interval number enables decision makers to better model insufficiency in available information. Thus, INSs could effectively deal with the more uncertain information in M&A target selection.
2. In group decision making procedures, multigranulation rough sets over two universes can be seen as an efficient information fusion approach that could integrate each expert's preference to form an ultimate conclusion by considering optimistic and pessimistic decision making strategies. Moreover, compared with classical IN group decision making approaches based on INNWA and INNWG operators, it is noted that INNWA and INNWG operators could only offer a one-fold information fusion strategy, i.e., the information fusion strategy based on averaging operators or geometric operators without considering the risk appetite of experts, which may cause the loss of risk-based information and further preclude the solution of risk-based group decision making problems. However, the proposed decision making approach based on multigranulation rough sets over two universes could not only provide risk-seeking and risk-averse decision strategies simultaneously, but also provide a compromise decision strategy that considers the risk preference of decision makers. Hence, multigranulation rough sets over two universes can be regarded as a multiple information fusion strategy that is suitable for solving risk-based group decision making problems.

As discussed previously, the advantages of the proposed model could provide a reasonable way to express some complicated decision making information, utilize risk-seeking and risk-averse decision strategies through considering the risk preference of decision makers and also increase the efficiency of M&A target selection.

6. Conclusions

In this article, in order to conduct group decision making from the granular computing paradigm, by combining multigranulation rough sets over two universes with INSs, a novel rough set model named IN multigranulation rough sets over two universes is developed. The definition and some properties of optimistic and pessimistic IN multigranulation rough sets over two universes are studied systematically. Then, by virtue of IN multigranulation rough sets over two universes, we further construct decision making rules and computing approaches for M&A target selection problems. At last, we illustrate the newly-proposed decision making approach on the basis of a practical M&A case study. It is desirable to study attribute reduction algorithms and uncertainty measures based on IN multigranulation rough sets over two universes in the future. Another future research direction is to apply the proposed decision making approach to other business intelligence issues.

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References

1. Yilmaz, I.S.; Tanyeri, B. Global merger and acquisition (M&A) activity: 1992–2011. *Financ. Res. Lett.* **2016**, *17*, 110–117.
2. He, M.H.; Guo, W. The fuzzy decision-making model based on rough set of the M&A of resource-typed enterprises. In Proceedings of the IEEE International Conference on Fuzzy Systems and Knowledge Discovery, Haikou, China, 24–27 August 2007; pp. 218–223.
3. Tsao, C.T. Applying a fuzzy multiple criteria decision-making approach to the M&A due diligence. *Expert. Syst. Appl.* **2009**, *36*, 1559–1568.
4. Liang, T.; Chu, Y. The study on the multi-object fuzzy decision-making of the M&A of resource-typed enterprises based on rough sets. In *Future Communication, Computing, Control and Management*; Zhang, Y., Ed.; Springer: Berlin, Germany, 2012; pp. 479–486.
5. Li, G.R.; Li, C.H.; Niu, X.H.; Yang, L.P. Risk assessment of enterprise merger and acquisition based on event tree method and fuzzy set theory. *J.Appl. Sci.* **2013**, *21*, 4819–4825.
6. Zadeh, L.A. Fuzzy sets. *Inform. Control.* **1965**, *8*, 338–353.
7. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Set. Syst.* **1986**, *20*, 87–96.
8. Smarandache, F. A unifying field in logics: Neutrosophic logic. *Mult-Valued. Log.* **1999**, *8*, 489–503.
9. Smarandache, F. *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability*; American Research Press: Rehoboth, DE, USA, 2003; pp. 1–141.
10. Wang, H.B.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*; Hexis: Phoenix, AZ, USA, 2005; pp. 1–87.
11. Kraipeerapun, P.; Fung, C.C. Binary classification using ensemble neural networks and interval neutrosophic sets. *Neurocomputing* **2009**, *72*, 2845–2856.
12. Broumi, S.; Smarandache, F. Correlation coefficient of interval neutrosophic set. *Appl. Mech. Mater.* **2013**, *436*, 511–517.
13. Broumi, S.; Deli, I.; Smarandache, F. Interval valued neutrosophic parameterized soft set theory and its decision making. *Appl. Soft. Comput.* **2014**, *28*, 109–113.
14. Broumi, S.; Talea, M.; Smarandache, F.; Bakali, A. Decision-making method based on the interval valued neutrosophic graph. In Proceedings of the Future Technologies Conference, San Francisco, CA, USA, 6–7 December 2016; pp. 44–50.
15. Zhang, H.Y.; Wang, J.Q.; Chen, X.H. Interval neutrosophic sets and their application in multicriteria decision making problems. *Sci. World J.* **2014**, *2014*; doi:10.1155/2014/645953.
16. Ye, J. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *J. Intell. Fuzzy Syst.* **2014**, *26*, 165–172.
17. Zhang, H.Y.; Ji, P.; Wang, J.Q.; Chen, X.H. An improved weighted correlation coefficient based on integrated weight for interval neutrosophic sets and its application in multi-criteria decision-making problems. *Int. J. Comput. Intell. Syst.* **2015**, *8*, 1027–1043.

18. Ye, J. Interval neutrosophic multiple attribute decision-making method with credibility information. *Int. J. Fuzzy Syst.* **2016**, *18*, 914–923.
19. Ye, J. Exponential operations and aggregation operators of interval neutrosophic sets and their decision making methods. *SpringerPlus* **2016**, *5*, 1–18; doi: 10.1186/s40064-016-3143-z.
20. Ye, J. Multiple attribute group decision making based on interval neutrosophic uncertain linguistic variables. *Int. J. Mach. Learn. Cyb.* **2017**, *8*, 837–848.
21. Deli, I. Interval-valued neutrosophic soft sets and its decision making. *Int. J. Mach. Learn. Cyb.* **2017**, *8*, 665–676.
22. Yao, J.T.; Vasilakos, A.V.; Pedrycz, W. Granular computing: Perspectives and challenges. *IEEE Trans. Cybern.* **2013**, *43*, 1977–1989.
23. Qian, Y.H.; Liang, J.Y.; Yao, Y.Y.; Dang, C.Y. MGRS: A multi-granulation rough set. *Inform. Sci.* **2010**, *180*, 949–970.
24. Qian, Y.H.; Li, S.Y.; Liang, J.Y.; Shi, Z.Z.; Wang, F. Mgrs: Pessimistic rough set based decisions: A multigranulation fusion strategy. *Inf. Sci.* **2014**, *264*, 196–210.
25. Pawlak, Z. Rough sets. *Int. J. Comput. Inf. Sci.* **1982**, *11*, 341–356.
26. Pei, D.W.; Xu, Z.B. Rough set models on two universes. *Int. J. Gen. Syst.* **2004**, *33*, 569–581.
27. Sun, B.Z.; Ma, W.M. Fuzzy rough set model on two different universes and its application. *Appl. Math. Model.* **2011**, *35*, 1798–1809.
28. Yang, H.L.; Liao, X.W.; Wang, S.Y.; Wang, J. Fuzzy probabilistic rough set model on two universes and its applications. *Int. J. Approx. Reason.* **2013**, *54*, 1410–1420.
29. Sun, B.Z.; Ma, W.M.; Zhao, H.Y. A fuzzy rough set approach to emergency material demand prediction over two universes. *Appl. Math. Model.* **2013**, *37*, 7062–7070.
30. Xu, W.H.; Sun, W.X.; Liu, Y.F.; Zhang, W.X. Fuzzy rough set models over two universes. *Int. J. Mach. Learn. Cyb.* **2013**, *4*, 631–645.
31. Sun, B.Z.; Ma, W.M.; Chen, X.T. Fuzzy rough set on probabilistic approximation space over two universes and its application to emergency decision-making. *Expert. Syst.* **2015**, *32*, 507–521.
32. Sun, B.Z.; Ma, W.M.; Zhao, H.Y. An approach to emergency decision making based on decision-theoretic rough set over two universes. *Soft Comput.* **2016**, *20*, 3617–3628.
33. Zhang, C.; Li, D.Y.; Liang, J.Y. Hesitant fuzzy linguistic rough set over two universes model and its applications. *Int. J. Mach. Learn. Cyb.* **2016**, 1–12, doi:10.1007/s13042-016-0541-z.
34. Sun, B.Z.; Ma, W.M. Multigranulation rough set theory over two universes. *J. Intell. Fuzzy Syst.* **2015**, *28*, 1251–1269.
35. Zhang, C.; Li, D.Y.; Ren, R. Pythagorean fuzzy multigranulation rough set over two universes and its applications in merger and acquisition. *Int. J. Intell. Syst.* **2016**, *31*, 921–943.
36. Zhang, C.; Li, D.Y.; Yan, Y. A dual hesitant fuzzy multigranulation rough set over two-universe model for medical diagnoses. *Comput. Math. Method. Med.* **2015**, *2015*; doi:10.1155/2015/292710.
37. Zhang, C.; Zhai, Y.H.; Li, D.Y.; Mu, Y.M. Steam turbine fault diagnosis based on single-valued neutrosophic multigranulation rough sets over two universes. *J. Intell. Fuzzy. Syst.* **2016**, *31*, 2829–2837.
38. Sun, B.Z.; Ma, W.M.; Xiao, X. Three-way group decision making based on multigranulation fuzzy decision-theoretic rough set over two universes. *Int. J. Approx. Reason.* **2017**, *81*, 87–102.
39. Sun, B.Z.; Ma, W.M.; Qian, Y.H. Multigranulation fuzzy rough set over two universes and its application to decision making. *Knowl.-Based Syst.* **2017**, *123*, 61–74.
40. Zhang, C.; Li, D.Y.; Mu, Y.M.; Song, D. An interval-valued hesitant fuzzy multigranulation rough set over two universes model for steam turbine fault diagnosis. *Appl. Math. Model.* **2017**, *42*, 693–704.