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# Prospect–Regret Theory-Based MCDM Method Integrating Novel Score Function and Similarity Measure for q-Rung Orthopair Fuzzy Z-Number in Medical Diagnosis

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**Abstract**

To address the impact of uncertain information reliability in decision-making environments and decision-makers' psychology on decision results, this paper proposes a multi-criteria decision-making (MCDM) method for q-rung orthopair fuzzy Z-number (qROFZN) based on prospect–regret theory. First, a novel score function for qROFZN is developed, accompanied by a comprehensive discussion of its fundamental properties. Second, the Jaccard similarity distance measure is innovatively proposed for qROFZN, along with an analysis of its essential characteristics. Subsequently, on this basis, the prospect–regret theory is extended to the qROFZN environment, and a novel MCDM method is established. In this method, the criteria importance through inter-criteria correlation (CRITIC) weighting method and prospect theory's weighting function are employed to determine comprehensive weights, which are then integrated with prospect–regret theory to identify the optimal solution, thereby considering both subjective and objective factors and enhancing the practical relevance of the decision results. Finally, a case study on medical diagnosis is conducted, along with comparative analyses. The experimental results are consistent with the outcomes of the referenced case, which validates the practicality and effectiveness of the proposed method, demonstrating that the qROFZN-based MCDM method can effectively capture fuzziness and reliability, thus enabling accurate decision analysis under uncertainty.

**Keywords:** q-rung orthopair fuzzy sets; Z-number; Jaccard similarity distance; prospect–regret theory; multi-criteria decision making

## 1. Introduction

As a core research direction within the sphere of decision-making research, multi-criteria decision-making (MCDM) aims to systematically analyze the multi-dimensional objectives or criteria involved in decision-making processes and to make trade-offs among them in order to select the optimal alternative. However, the pervasive uncertainty in decision-making scenarios often leads to data exhibiting characteristics of imprecision and incompleteness. The universal presence of fuzziness and uncertainty further increases the difficulty for decision-makers in making effective trade-offs, rendering traditional crisp set theories significantly limited in solving such problems. To address this issue, Zadeh [1] first proposed the concept of fuzzy sets, innovatively extending classical set theory by assigning membership degree values within the $[0, 1]$ interval to each element in the set. This theoretical framework has subsequently spawned a series of extended models,

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including intuitionistic fuzzy sets (IFSs) [2], Pythagorean fuzzy sets (PFSs) [3], Fermatean fuzzy sets (FFSs) [4], q-rung orthopair fuzzy sets (q-ROFSs) [5] and so on. In particular, q-ROFSs can characterize uncertain information more comprehensively and flexibly due to the introduction of parameter $q$. Therefore, q-ROFSs exhibit prominent advantages especially in addressing multi-criteria decision-making problems.

Fuzzy decision-making has been widely applied in the field of medical diagnosis. Huang et al. [6] proposed a three-way decision model based on Gaussian spherical fuzzy information granularity, achieving a precise assessment of an individual's health status, disease probability, and severity levels. Asghar et al. [7] developed interval-valued Pythagorean fuzzy linear Diophantine Aczel Alsina aggregation operators, and constructed a multi-attribute decision-making framework based on these operators. This assists doctors in making more precise diagnostic judgments and treatment plan formulations amid complex clinical information. Luo et al. [8] put forward a fuzzy reasoning method based on novel similarity distance measurement and applied it to practical medical diagnosis scenarios. Liao et al. [9] innovatively proposed new formulas for Pythagorean fuzzy information measures, discussing their validity and superiority. Based on this, they further constructed an MCDM model, offering a novel solution for decision-making in complex medical diagnostic scenarios.

In the field of medical diagnosis, which demands high reliability, the handling of information fuzziness not only pertains to the scientificity of medical diagnoses but also directly influences the decision-making confidence of both clinicians and patients. While the aforementioned studies have made significant advancements in addressing uncertainty within decision-making models, there still exist some limitations as follows:

(1) The diversity of medical data sources may lead to discrepancies in their credibility, while these models generally lack the assessment and quantification of the reliability of fuzzy information. This indicates that the information may not be effectively discerned, which could potentially influence the efficiency and accuracy of the diagnostic decision-making process. Therefore, in fuzzy decision-making, score functions and distance measures are important tools for quantifying fuzzy information. However, research on both concepts within the qROFZN framework has been rather scarce to date. When two qROFZNs have identical scoring values, traditional score functions are unable to effectively distinguish between them, whereas existing similarity distances fail to differentiate feature sets that are highly similar yet not identical.

(2) In medical diagnosis, certain symptoms may be more critical for identifying specific diseases, which makes the assignment of symptom weights particularly significant. However, the aforementioned studies adopt relatively simplistic approaches to determining weights, relying either on subjective assignment or purely objective data-driven methods. This fails to effectively integrate clinical experience with data evidence. This may lead to a failure to adequately highlight the discriminative signals of certain critical symptoms, thereby compromising the accuracy of symptom assessment and potentially interfering with the timeliness of subsequent diagnostic and therapeutic planning. Therefore, the weight calculation method is of critical importance in MCDM.

(3) Medical diagnosis essentially involves a classification decision-making process under conditions of incomplete information, which requires clinicians to navigate the trade-off between overdiagnosis and underdiagnosis. This process is often accompanied by the potential risk of decision regret. However, existing frameworks have yet to incorporate such significant behavioral cognitive factors into model considerations. Such neglect may cause the final decision to deviate from the actual risk prefer-

ences of physicians and patients, reduce treatment adherence, and even exacerbate doctor-patient conflicts.

To address the aforementioned limitations and construct an intelligent decision-making model capable of integrating information credibility assessment, dynamic integration of subjective and objective weights, and a regret-avoidance mechanism, this paper is dedicated to developing a novel MCDM method for qROFZN based on the CRITIC method and prospect–regret theory. The main contributions and innovations of this study are summarized as follows:

(1) A score function specifically designed for q-rung orthopair fuzzy Z-number is developed. In contrast to existing score functions that are based on product conditions, the proposed function yields ties under sum conditions, and the two tie sets are mutually non-inclusive. The boundedness and monotonicity of the proposed function are systematically established. A theoretical comparison precisely characterizes its complementary relationship with existing functions, thereby clarifying its distinct applicability in scenarios involving ties under product conditions. Consequently, this work provides a more targeted alternative metric for the ranking of fuzzy information.

(2) A novel similarity distance measure based on the Jaccard index is proposed for q-rung orthopair fuzzy Z-numbers. This measure directly quantifies similarity as the degree of overlap between sets. Therefore, it can effectively mitigate interference from feature sparsity and differences in numerical scales, thereby improving decision-making accuracy. Moreover, the relevant properties of this extended metric were thoroughly analyzed, thereby refining similarity distance measurement for qROFZN.

(3) An integrated weighting calculation method is proposed. This method effectively combines the objective weights derived from the CRITIC method with subjective weights grounded in prospect theory. It concurrently addresses inherent data patterns and decision-maker preferences, significantly enhancing the adaptability and robustness of the fuzzy decision-making method in complex scenarios.

(4) Prospect–regret theory is incorporated into the multi-criteria decision-making method. By quantifying decision-makers' loss aversion psychology and reference point dependence, this approach embeds subjective psychological biases into the MCDM process, augmenting the MCDM method's realism regarding human judgment. It is worth noting that the mathematical structure of qROFZN inherently implies an information symmetry, in which membership and non-membership degrees share a unified information budget within the $q$-power space. The proposed prospect–regret integrated decision framework is built precisely upon this symmetric information space, while introducing asymmetric psychological preferences of decision makers, namely loss aversion and regret aversion.

(5) The developed MCDM method is applied to a medical diagnosis case study. Parameter sensitivity analysis is conducted and comparative analysis with existing decision-making methods is performed. The results validate the effectiveness and stability of the proposed method.

The remainder of this paper is structured as follows. Section 2 provides a literature review. Section 3 introduces the fundamental definitions and theoretical background. Section 4 proposes a novel score function, an improved Jaccard similarity distance measure, and discusses their related properties. Section 5 details the specific steps of the proposed method for solving multi-criteria decision-making problems. Section 6 demonstrates the practicality and effectiveness of the proposed method through a medical diagnosis case study. The stability of the proposed method is validated via sensitivity analysis, and comparative analysis and discussion with other decision-making methods highlight its superiority. Finally, Section 7 concludes the paper.

## 2. Literature Review

Intuitionistic fuzzy sets [2], put forward by Atanassov, importantly extend the single-dimensional membership framework of traditional fuzzy sets by introducing two independent dimensions: non-membership degree and hesitation degree. This theoretical innovation enables IFSs to more finely characterize the uncertainty and hesitancy in decision-making processes. Yager [3] proposed Pythagorean fuzzy sets (PFSs), which significantly expand the effective expression range of fuzzy sets by optimizing the constraint to require that the sum of squared membership degree and squared non-membership degree does not exceed 1. To further expand the membership space, Senapati and Yager [4] proposed Fermatean fuzzy sets (FFSs). FFSs realize the ability to describe a broader space of uncertain information by relaxing the constraint to require that the sum of the cubes of the membership degree and non-membership degree is less than 1. q-rung orthopair fuzzy sets (qROFSs), proposed by Yager [5] in 2017, introduce a parameter $q$, generalizing the constraint between membership degree and non-membership degree from traditional linear sums or squared sums to a general $q$-power sum form. By adjusting the value of parameter $q$, the size of the membership space can be flexibly changed. And the higher the parameter value, the wider the corresponding membership space. This powerful parameterized expression capability gives q-rung orthopair fuzzy sets higher flexibility and adaptability in characterizing membership spaces, thus demonstrating broad development prospects in both theoretical research and practical applications, thereby making qROFSs become a significant research direction in the current field of fuzzy set theory.

Zadeh [10] extended classical fuzzy set theory to Z-numbers, introducing a reliability assessment into classical fuzzy sets. A Z-number is defined as ordered pairs $Z = (A, B)$, where $A$ characterizes the membership distribution of a fuzzy variable and $B$ represents the reliability level of the membership evaluation. This innovation provides a formal tool for decision-makers to quantify the confidence degree of information uncertainty, significantly enhancing the reliability of fuzzy set theory in real-world decision-making. Due to their emphasis on information reliability, the concept of Z-numbers has sparked extensive research interest in academia. Studies primarily focus on constructing complex fuzzy sets, which form a more expressive decision-making information representation system through deep integration with different types of fuzzy sets. This integration model overcomes the descriptive limitations of single fuzzy sets, effectively addressing real-world complex decision-making problems. In recent years, composite fuzzy models based on Z-numbers have attracted significant attention due to their unique advantages in information reliability. For example, Peng and Wang [11] established a hesitant uncertain linguistic Z-number framework, constructing a decision-making framework adaptable to multi-granular linguistic uncertainty by integrating hesitant fuzzy sets, interval-valued fuzzy sets and linguistic Z-numbers. Chen et al. [12] developed a Fermatean fuzzy Z-number-based method for Failure Mode and Effect Analysis by using weighted Muirhead mean operators. Liu et al. [13] developed a linguistic q-rung orthopair fuzzy Z-number model for multi-criteria decision-making problems. Hameed et al. [14] proposed aggregation operators for qROFZN, applied to smart city air quality monitoring. Therefore, this paper introduces q-rung orthopair fuzzy Z-number. Leveraging the expanded representation space and adaptable q-parameter tuning of qROFSs, combined with the reliability measure of Z-number to quantify the credibility of uncertain information, this method integrates both uncertainty and reliability levels. This integration comprehensively captures the essential characteristics of data and significantly enhances decision reliability, providing a more versatile and precise theoretical framework for addressing decision-making problems in complex systems.

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In MCDM, determining attribute weights is a crucial step, as their rationality directly dictates the reliability of the final alternative ranking and the effectiveness of the decision. Consequently, accurately determining attribute weights constitutes a core challenge in decision problems. The CRITIC method [15], aims to calculate the relative importance of criteria and alternatives within the decision-making process. This method evaluates the influence degree of each criterion on the final decision outcome by considering the intercorrelations among the criteria. The CRITIC method is widely applied in multi-criteria decision analysis. Akram et al. [16] integrate Pythagorean fuzzy rough numbers into the CRITIC framework to develop an enhanced CRITIC-REGIME decision-making approach. Verma et al. [17] propose a CRITIC-WASPAS multi-attribute group decision-making model within a two-dimensional linguistic intuitionistic fuzzy environment combined with power aggregation operators. Menekşe et al. [18] apply spherical fuzzy sets combined with the CRITIC-WASPAS method to medical waste treatment planning problems. However, the weight calculations in these examples yield solely objective weights, which may fail to capture decision context-specific preferences. Therefore, this paper proposes a comprehensive weighting approach. This approach effectively integrates the objective weights calculated based on the CRITIC method with subjective weights derived from prospect theory, aiming to obtain more holistic and practical attribute weights that better align with actual decision-making needs within uncertain environments.

In most decision-making models, decision-makers are typically postulated to possess complete rationality. However, this assumption exhibits significant deviations from real-world decision-making contexts. In fact, decision-makers' risk preferences and psychological characteristics are significantly heterogeneous, and these irrational factors often exert critical influences on decision outcomes. To overcome the limitations of the complete rationality assumption, Kahneman and Tversky [19] pioneered the prospect theory, which innovatively combines research methodologies with decision-makers' psychological and behavioral characteristics. Given that decision-makers often experience regret over non-optimal choices during risk perception, Bell [20] further proposed regret theory. Zhang et al. [21] developed a regret–joy function to achieve quantitative measurement of decision-makers' subjective perceived utility. The core advantage of these two theories lies in their ability to accurately describe decision-makers' irrational behavioral characteristics, and their theoretical integration forms an analytical framework that is more aligned with real-world decision-making processes. In recent years, this theoretical system has demonstrated broad applicability in complex decision-making problems. For example, Bisht et al. [22] proposed a decision-making method under incomplete probabilistic fuzzy environments based on prospect regret theory, successfully applying this theory to the field of medical data analysis.

## 3. Preliminaries

This section introduces some basic concepts. The concept of q-rung orthopair fuzzy Z-number and its properties are given. The basic knowledge about Jaccard index, prospect theory and regret theory are also presented.

**Definition 1** ([5]). *Let $X$ be a finite universal set, the q-rung orthopair fuzzy set $F$ on $X$ can be defined as:*

$$F = \{(x_i, \mu(x_i), \nu(x_i))|x_i \in X\},$$

*where $\mu(x) : X \to [0,1], \nu(x) : X \to [0,1]$, $\mu(x)$ and $\nu(x)$ denote the membership and non-membership degrees of an element $x$ in a set $X$, respectively, and both satisfy $0 \leq \mu(x), \nu(x) \leq 1$. They also need to comply with the following relation: $0 \leq \mu^q(x) + \nu^q(x) \leq 1$, where $q$ rep-*

*resents an arbitrary positive integer satisfying* $q \geq 1$. *Furthermore, the degree of hesitation is* $\pi(x) = \sqrt[q]{1 - \mu^q(x) - \nu^q(x)}$.

**Definition 2** ([10]). *Let Z = (A, B) be a Z-number. A represents a fuzzy constraint on an event X, encapsulating the vagueness associated with the event. B signifies the reliability of A, typically quantified as a value between 0 and 1.*

**Definition 3** ([14]). *Let X be a universal set. The q-rung orthopair fuzzy Z-number can be defined as:*

$$Z_Q = \{((\mu(x_i), \mu_c(x_i)), (\nu(x_i), \nu_c(x_i)))|x_i \in X\},$$

*where* $\mu(x), \nu(x), \mu_c(x), \nu_c(x) : X \to [0,1]$, $\mu(x)$ *and* $\nu(x)$ *denote the membership and non-membership degrees of an element x in a set X, respectively. Furthermore,* $\mu_c(x)$ *and* $\nu_c(x)$ *represent the reliability of* $\mu(x)$ *and* $\nu(x)$, *respectively. They satisfy the following requirements:* $0 \leq \mu^q(x) + \nu^q(x) \leq 1$, $0 \leq \mu_c^q(x) + \nu_c^q(x) \leq 1$. *The degree of hesitation is defined as* $\pi(x_i) = \sqrt[q]{1 - (\mu_i^q + \mu_{c_i}^q - \mu_i^q \mu_{c_i}^q) - \nu_i^q \nu_{c_i}^q}$. *Moreover,* $Z_Q$ *can be simplified to a simpler form* $Z_Q = ((\mu_i, \mu_{c_i}), (\nu_i, \nu_{c_i}))$.

**Definition 4** ([14]). *Let* $Z_Q = ((\mu, \mu_c), (\nu, \nu_c))$ *be a qROFZN, then the score function and accuracy function are as follows:*

$$S(Z_Q) = \mu^q \mu_c^q - \nu^q \nu_c^q, \tag{1}$$

$$H(Z_Q) = \mu^q \mu_c^q + \nu^q \nu_c^q, \tag{2}$$

**Definition 5** ([14]). *Let* $Z_Q^1 = ((\mu_1, \mu_{c_1}), (\nu_1, \nu_{c_1}))$ *and* $Z_Q^2 = ((\mu_2, \mu_{c_2}), (\nu_2, \nu_{c_2}))$ *be any two qROFZNs, then the comparative relationships of the score and accuracy function are as follows:*

*(1) if* $S(Z_Q^1) > S(Z_Q^2)$, *then* $Z_Q^1 > Z_Q^2$;
*(2) if* $S(Z_Q^1) < S(Z_Q^2)$, *then* $Z_Q^1 < Z_Q^2$;
*(3) if* $S(Z_Q^1) = S(Z_Q^2)$, *then*
  *(a) if* $H(Z_Q^1) > H(Z_Q^2)$, *then* $Z_Q^1 > Z_Q^2$;
  *(b) if* $H(Z_Q^1) < H(Z_Q^2)$, *then* $Z_Q^1 < Z_Q^2$;
  *(c) if* $H(Z_Q^1) = H(Z_Q^2)$, *then* $Z_Q^1 = Z_Q^2$.

**Definition 6** ([23]). *Let A and B be two finite sets, the Jaccard index is expressed as*

$$J(A, B) = \frac{|A \cap B|}{|A \cup B|}, \tag{3}$$

*where the Jaccard index falls within the range of 0 to 1. A value of 0 denotes complete dissimilarity, indicating no shared elements between the two sets, while a value of 1 signifies that the two sets are identical.*

**Definition 7** ([19]). *The value function can be expressed as*

$$v(\Delta x) = \begin{cases} (\Delta x)^\alpha & \Delta x \geq 0, \\ -\theta(-\Delta x)^\beta & \Delta x < 0, \end{cases} \tag{4}$$

*where* $\Delta x$ *represents the size of the deviation of x from a reference point* $x_0$. *When* $\Delta x \geq 0$, *it indicates gains, and when* $\Delta x < 0$, *it indicates losses.* $\alpha, \beta \in (0,1)$, *represent the gain sensitivity parameter and loss sensitivity parameter respectively, and* $\theta > 0$, *is the loss aversion coefficient.*

**Remark 1.** *The optimal values of parameters* $\alpha, \beta, \theta$ *are given as* $\alpha = \beta = 0.88,\ \theta = 2.25$.

**Definition 8** ([19]). *The weight function can be expressed as*

$$P(\omega_j) = \begin{cases} P(\omega_j)^+ = \frac{\omega_j^{\varepsilon}}{(\omega_j^{\varepsilon} + (1-\omega_j)^{\varepsilon})^{1/\varepsilon}}, \\ P(\omega_j)^- = \frac{\omega_j^{\delta}}{(\omega_j^{\delta} + (1-\omega_j)^{\delta})^{1/\delta}}, \end{cases} \tag{5}$$

*where* $P(\omega_j)^+$ *and* $P(\omega_j)^-$ *represent the benefit state and loss state, respectively.* $\varepsilon$ *and* $\delta$ *represent the attitude of decision-makers towards risk-benefit and risk-loss, with* $\varepsilon, \delta \in (0, 1)$.

**Remark 2.** *Kahneman and Tversky show the optimal values of parameters, which are given:* $\varepsilon = 0.61,\ \delta = 0.69$.

**Definition 9** ([21]). *The regret–rejoice function is defined below.*

$$R(\Delta x) = 1 - e^{-\sigma \Delta x}, \tag{6}$$

*where* $\Delta x$ *represent the difference in utility values of the two alternatives.* $\sigma$ *represents the regret aversion coefficient of decision-makers. When* $R(\Delta x) > 0$, *it signifies that the decision outcome surpasses anticipations, thereby inducing a sense of elation in the decision-maker; conversely, if otherwise, regret is experienced.*

## 4. Novel Score Function and Similar Distance Measure for q-Rung Orthopair Fuzzy Z-Number

In this section, a detailed discussion is conducted on novel score functions based on qROFZN and the newly improved similarity measures extended from the Jaccard index for qROFZN. Additionally, the associated essential properties and their rigorous proofs are elaborated, demonstrating the theoretical soundness and logical consistency of the proposed methodologies. In addition, comprehensive comparative analyses are conducted to demonstrate the innovation, superiority and effectiveness of the proposed score function and similar distance measure.

### *4.1. Proposed Score Function for qROFZN*

The score function is a measurement method that transforms the fuzzy membership relations in qROFZN into crisp values, thereby quantifying the membership degree to a fuzzy set. Through crisp values, qROFZN can be ranked and compared. However, existing score functions cannot perform accurate ranking in some special cases, requiring the use of accuracy functions, which leads to overly complex calculations. For example, for the score function (1), if $Z_Q^1 = ((0.6, 0.2), (0.3, 0.4))$ and $Z_Q^2 = ((0.8, 0.5), (0.5, 0.8))$, then obviously, their score values can be calculated, $S(Z_Q^1) = S(Z_Q^2) = 0$, then accuracy function (2) need be introduced to further determine the relationship between the two $Z_Q^1$ and $Z_Q^2$. So, to address the limitations of existing score functions, this section introduces a novel score function.

**Definition 10.** *Let* $Z_Q = ((\mu, \mu_c), (\nu, \nu_c))$ *be a qROFZN, then the score function is as follows:*

$$S(Z_Q) = \frac{1}{2e^2 - 2}\left(e^{\mu^q + \mu_c^q} - e^{\nu^q + \nu_c^q} + e^2 - 1\right). \tag{7}$$

Continuing with the same example as above, if $q$ is taken as three, the proposed score function can be used to calculate the following results: $S(Z_Q^1) = 0.5122, S(Z_Q^2) = 0.5000$, so it is easy to conclude that $Z_Q^2$ is superior to $Z_Q^1$.

Subsequently, the properties satisfied by the proposed score function are discussed.

**Property 1.** *The score function exhibits a monotonically increasing trend with respect to the variables, $\mu$ and $\mu_c$, and demonstrates a monotonically decreasing tendency with respect to the variables, $\nu$ and $\nu_c$.*

**Proof.** Conducting partial differentiation of the score function with respect to each of the variables $\mu, \mu_c, \nu, \nu_c$ separately,

$$\frac{\partial S(Z_Q)}{\partial u} = \frac{q\mu^{q-1}e^{\mu^q+\mu_c^q}}{2e^2-2} \geq 0, \frac{\partial S(Z_Q)}{\partial u_c} = \frac{q\mu_c^{q-1}e^{\mu^q+\mu_c^q}}{2e^2-2} \geq 0,$$
$$\frac{\partial S(Z_Q)}{\partial v} = \frac{-q\nu^{q-1}e^{\nu^q+\nu_c^q}}{2e^2-2} \leq 0, \frac{\partial S(Z_Q)}{\partial v_c} = \frac{-q\nu_c^{q-1}e^{\nu^q+\nu_c^q}}{2e^2-2} \leq 0.$$

Therefore, $S(Z_Q)$ presents a monotonically increasing trend in terms of $\mu$ and $\mu_c$, while it is monotonically decreasing with respect to $\nu$ and $\nu_c$. □

**Property 2.** *Let $Z_Q = ((\mu, \mu_c), (\nu, \nu_c))$ be a qROFZN, then the score function satisfies $0 \leq S(Z_Q) \leq 1$.*

**Proof.** Since $\mu, \mu_c, \nu, \nu_c$ belong to the interval $[0, 1]$, and $S(Z_Q)$ is monotonically increasing with respect to $\mu$ and $\mu_c$, and monotonically decreasing with respect to $\nu$ and $\nu_c$. So, by taking $Z_Q = ((1, 1), (0, 0))$ and $Z_Q = ((0, 0), (1, 1))$, the maximum and minimum values of the score function can be obtained, $S(Z_Q)_{max} = 1, S(Z_Q)_{min} = 0$.

Obviously, $0 \leq S(Z_Q) \leq 1$. □

**Property 3.** *If there exist two qROFZNs, $Z_Q^1 = ((\mu_1, \mu_{c_1}), (\nu_1, \nu_{c_1}))$ and $Z_Q^2 = ((\mu_2, \mu_{c_2}), (\nu_2, \nu_{c_2}))$, and $\mu_1 \geq \mu_2, \mu_{c_1} \geq \mu_{c_2}, \nu_1 \leq \nu_2, \nu_{c_1} \leq \nu_{c_2}$, then $S(Z_Q^1) \geq S(Z_Q^2)$.*

**Proof.** Since the score function $S(Z_Q)$ is monotonically increasing with respect to $\mu$ and $\mu_c$, and monotonically decreasing with respect to $\nu$ and $\nu_c$, it is obvious that, when $\mu_1 \geq \mu_2$, $\mu_{c_1} \geq \mu_{c_2}, \nu_1 \leq \nu_2, \nu_{c_1} \leq \nu_{c_2}$, then $S(Z_Q^1) \geq S(Z_Q^2)$. □

### *4.2. Tie Set Analysis and Comparative Study*

To precisely evaluate the performance of the novel score function, this section rigorously characterizes the tie conditions of both the classical score function (1) and the proposed score function (7), and demonstrates that the two tie sets are complementary in a mutually non-inclusive manner, rather than standing in a simple substitution or inclusion relationship.

**Proposition 1.** *Let $Z_Q = ((\mu, \mu_c), (\nu, \nu_c))$ be a qROFZN, the tie set of the classical score function (1) can be defined as*

$$T_1 = \left\{ Z_Q | \mu^q \mu_c^q = \nu^q \nu_c^q \right\}.$$

*All elements in this set satisfy that $\mu^q \mu_c^q = \nu^q \nu_c^q$, which means that $S(Z_Q) = 0$.*

**Proposition 2.** *Let $Z_Q = ((\mu, \mu_c), (\nu, \nu_c))$ be a qROFZN, the tie set of the proposed score function (10) can be defined as*

$$T_2 = \left\{ Z_Q | \mu^q + \mu_c^q = \nu^q + \nu_c^q \right\}.$$

*All elements in this set satisfy that $\mu^q + \mu_c^q = \nu^q + \nu_c^q$, which means that $S(Z_Q) = 0.5$.*

**Proposition 3.** *The two sets $T_1$ and $T_2$ are mutually non-inclusive, i.e., $T_1 \nsubseteq T_2, T_2 \nsubseteq T_1$.*

**Proof.**

(1) To prove $T_1 \nsubseteq T_2$, suppose $T_1 \subseteq T_2$ and then a q-rung orthopair fuzzy Z-number is constructed by letting $q = 2$ and taking $Z_Q = ((0.8, 0.5), (0.4, 1.0))$.
Under this assumption, $\mu^q \mu_c^q = 0.8^2 * 0.5^2 = 1.6, \nu^q \nu_c^q = 0.4^2 * 1.0^2 = 1.6$, therefore $Z_Q \in T_1$. Since $T_1 \subseteq T_2$, $Z_Q \in T_2$.
However, $\mu^q + \mu_c^q = 0.8^2 + 0.5^2 = 0.89, \nu^q + \nu_c^q = 0.4^2 + 1.0^2 = 1.16$. Obviously, $Z_Q \notin T_2$.
This yields a contradiction, and therefore $T_1 \nsubseteq T_2$.

(2) To prove $T_2 \nsubseteq T_1$, suppose $T_2 \subseteq T_1$ and then a q-rung orthopair fuzzy Z-number is constructed by letting $q = 3$ and taking $Z_Q = ((0.7368, 0.5848), (0.6694, 0.6694))$.
Under this assumption, $\mu^q + \mu_c^q = \nu^q + \nu_c^q = 0.6$. therefore $Z_Q \in T_2$. Since $T_1 \subseteq T_1$, $Z_Q \in T_1$.
However, $\mu^q \mu_c^q = 0.4 * 0.2 = 0.08, \nu^q \nu_c^q = 0.3 * 0.3 = 0.09$, so $\mu^q \mu_c^q \neq \nu^q \nu_c^q$. Therefore, $Z_Q \notin T_1$.
This yields a contradiction, and therefore $T_2 \nsubseteq T_1$.

In summary, $T_1$ and $T_2$ are two mutually non-inclusive sets. □

To clearly present the differences between the two score functions, the comparison results are summarized in Table 1. A brief summary of the differences and theoretical implications is provided below.

**Table 1.** Comparison of the two score functions.

| | **Classical Score Function (1)** | **Proposed Score Function (7)** |
|---|---|---|
| Mathematical expression | $\mu^q \mu_c^q - \nu^q \nu_c^q$ | $\frac{1}{2e^2-2}\left(e^{\mu^q+\mu_c^q} - e^{\nu^q+\nu_c^q} + e^2 - 1\right)$ |
| Structural construction | multiplicative coupling of $\mu$ and $\nu$ | additive coupling of $\mu$ and $\nu$ |
| Range of valuess | $[-1, 1]$ | $[0, 1]$ |
| Tie-triggering condition | $\mu^q \mu_c^q = \nu^q \nu_c^q$ | $\mu^q + \mu_c^q = \nu^q + \nu_c^q$ |
| Numerical characteristics under tie conditions | collapse of scoring value to 0 | collapse of scoring value to 0.5 |
| Advantageous scenarios | equal sums but unequal products | equal products but unequal sums |

(1) The essential difference between the two score functions lies in their information aggregation approaches. The classical score function (1) adopts a multiplicative coupling structure, associating the membership and non-membership degrees through multiplication, with the tie-triggering condition given by $\mu^q \mu_c^q = \nu^q \nu_c^q$. In contrast, The proposed score function (7) employs an additive coupling structure, with the tie-triggering condition being $\mu^q + \mu_c^q = \nu^q + \nu_c^q$. The tie sets of the two functions are mutually non-inclusive (Proposition 3), indicating that the functions are mathematically complementary. Notably, under tie conditions, the proposed score function (7) collapses the scoring value to 0.5, whereas the score function (1) collapses it to 0. The

former avoids confusion with the "zero-information" state and thus offers superior numerical discriminability.

(2) The above complementarity analysis reveals a fact: within the q-ROFZN framework, there exists no single score function that maintains full resolution across all information structures. The paper formally characterizes the inherent "multiplicative-additive duality"—namely, that the failure modes of score functions naturally fall into two categories, product-dominated and sum-dominated, which are orthogonal to each other and exhaust the basic forms of linear aggregation. This finding provides a clear direction for subsequent research: to construct composite scoring aggregators with broader discriminative capability by designing adaptive weights or hybrid strategies between the two paradigms.

### *4.3. Novel Similarity Distance for qROFZN Based on Jaccard Index*

Due to the limited research on distance measurement for qROFZN at present, this section proposes a novel similarity distance based on the Jaccard index.

**Definition 11.** *Let* $X = \{x_1, x_2, ..., x_n\}$, $Z_Q^1 = ((\mu_1, \mu_{c_1}), (\nu_1, \nu_{c_1}))$ *and* $Z_Q^2 = ((\mu_2, \mu_{c_2}), (\nu_2, \nu_{c_2}))$ *be any two qROFZNs on the set X, then the similarity distance is as follows:*

$$\begin{aligned} & D(Z_Q^1, Z_Q^2) \\ = & \frac{1}{n}\sum_{j=1}^{n}\left(\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q\mu_{c_{1j}}^q\right)\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q\mu_{c_{2j}}^q\right) + \left(\nu_{1j}\nu_{c_{1j}}\nu_{2j}\nu_{c_{2j}}\right)^q\right) \\ & \Big/\left(\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q\mu_{c_{1j}}^q\right)^2 + \left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q\mu_{c_{2j}}^q\right)^2\right. \\ & - \left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q\mu_{c_{1j}}^q\right)\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q\mu_{c_{2j}}^q\right) \\ & \left. + \nu_{1j}^{2q}\nu_{c_{1j}}^{2q} + \nu_{2j}^{2q}\nu_{c_{2j}}^{2q} - \left(\nu_{1j}\nu_{c_{1j}}\nu_{2j}\nu_{c_{2j}}\right)^q\right). \end{aligned} \tag{8}$$

**Theorem 1.** *The proposed similarity distance* (8) *satisfies the following properties:*

*(1)* $0 \leqslant D(Z_Q^1, Z_Q^2) \leqslant 1$;

*(2)* $D(Z_Q^1, Z_Q^2) = D(Z_Q^2, Z_Q^1)$;

*(3)* $D(Z_Q^1, Z_Q^2) = 1, if\, Z_Q^1 = Z_Q^2$.

**Proof.**

(1) Since, for any $q \geqslant 1$ and for all $j = 1, 2, ..., n$, it holds that

$$\left(\left(u_{1j}^q + u_{c_{1j}}^q - u_{1j}^q u_{c_{1j}}^q\right) - \left(u_{2j}^q + u_{c_{2j}}^q - u_{2j}^q u_{c_{2j}}^q\right)\right)^2 \geq 0,$$

$$\left(v_{1j}^q v_{c_{1j}}^q - v_{2j}^q v_{c_{2j}}^q\right)^2 \geq 0,$$

and thus,

$$\begin{aligned} & \left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q\mu_{c_{1j}}^q\right)^2 + \left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q\mu_{c_{2j}}^q\right)^2 \\ \geq & 2\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q\mu_{c_{1j}}^q\right)\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q\mu_{c_{2j}}^q\right), \\ & \left(\nu_{1j}^q\nu_{c_{1j}}^q\right)^2 + \left(\nu_{2j}^q\nu_{c_{2j}}^q\right)^2 \geq 2\left(\nu_{1j}\nu_{c_{1j}}\nu_{2j}\nu_{c_{2j}}\right)^{2q}. \end{aligned}$$

Hence,

$$D(Z_Q^1, Z_Q^2) \leqslant \frac{1}{n}\sum_{j=1}^{n} 1 = \frac{1}{n} \times n = 1.$$

Next, since $\mu_{ij}, \mu_{c_{ij}}, \nu_{ij}, \nu_{c_{ij}} \in [0,1]$, it is easy to know that for any $q \geq 1$, $\mu_{ij}^q, \mu_{c_{ij}}^q, \nu_{ij}^q, \nu_{c_{ij}}^q \in [0,1]$. So,

$$\left(\mu_{ij}^q + \mu_{c_{ij}}^q - \mu_{ij}^q \mu_{c_{ij}}^q\right) = \left(\mu_{ij}^q + \mu_{c_{ij}}^q\left(1 - \mu_{ij}^q\right)\right) \geq 0,$$

and thus,

$$\begin{aligned}
&\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)^2 + \left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right)^2 \\
&- \left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right) \\
=&\left(\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right) - \left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right)\right)^2 \\
&+ \left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right) \geq 0, \\
&\nu_{1j}^{2q}\nu_{c_{1j}}^{2q} + \nu_{2j}^{2q}\nu_{c_{2j}}^{2q} - \left(\nu_{1j}\nu_{c_{1j}}\nu_{2j}\nu_{c_{2j}}\right)^q \\
=&\left(\nu_{1j}^q\nu_{c_{1j}}^q - \nu_{2j}^q\nu_{c_{2j}}^q\right)^2 + \left(\nu_{1j}\nu_{c_{1j}}\nu_{2j}\nu_{c_{2j}}\right)^q \geq 0.
\end{aligned}$$

Hence, $D(Z_Q^1, Z_Q^2) \geq 0$. Combining all the above, $0 \leq D(Z_Q^1, Z_Q^2) \leq 1$.

(2) It can be readily obtained that

$$\begin{aligned}
&D(Z_Q^2, Z_Q^1) \\
=& \frac{1}{n}\sum_{j=1}^{n}\left(\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right)\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right) + (\nu_{2j}\nu_{c_{2j}}\nu_{1j}\nu_{c_{1j}})^q\right) \\
&\Big/\left(\left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right)^2 + \left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)^2\right. \\
&- \left(\mu_{2j}^q + \mu_{c_{2j}}^q - \mu_{2j}^q \mu_{c_{2j}}^q\right)\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right) \\
&\left.+ \nu_{2j}^{2q}\nu_{c_{2j}}^{2q} + \nu_{1j}^{2q}\nu_{c_{1j}}^{2q} - \left(\nu_{2j}\nu_{c_{2j}}\nu_{1j}\nu_{c_{1j}}\right)^q\right).
\end{aligned}$$

Obviously, $D(Z_Q^1, Z_Q^2) = D(Z_Q^2, Z_Q^1)$.

(3) If $Z_Q^1 = Z_Q^2$, then $\mu_{1j} = \mu_{2j}, \mu_{c_{1j}} = \mu_{c_{2j}}, \nu_{1j} = \nu_{2j}, \nu_{c_{1j}} = \nu_{c_{2j}}$, and thus,

$$\begin{aligned}
D(Z_Q^1, Z_Q^2) =& \frac{1}{n}\sum_{j=1}^{n}\left(\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)^2 + \left(\nu_{1j}\nu_{c_{1j}}\right)^{2q}\right) \\
&\Big/\left(2\left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)^2 - \left(\mu_{1j}^q + \mu_{c_{1j}}^q - \mu_{1j}^q \mu_{c_{1j}}^q\right)^2\right. \\
&\left.+ 2\nu_{1j}^{2q}\nu_{c_{1j}}^{2q} - \left(\nu_{1j}\nu_{c_{1j}}\right)^{2q}\right) \\
=& \frac{1}{n}\sum_{j=1}^{n} 1 = \frac{1}{n} \times n \\
=& 1.
\end{aligned}$$

This ends the proof. □

Table 2 presents a comparison of some existing distance measures in terms of set forms and core properties. The proposed similarity distance possesses the following contributions and advantages:

**Table 2.** Comparison of the measures.

| Distance Measure | Decision Information | Core Property |
| --- | --- | --- |
| Euclidean distance [14] | q-ROFZN | L2 norm, geometric straight-line distance |
| Hamming distance [24] | IFS | L1 norm, equally weighted linear superposition |
| Dice distance [25] | PFS | set overlap ratio, mean-normalized |
| Cosine-based distance [26] | p,q-ROFS | angle between directions, scale invariance |
| Proposed distance | q-ROFZN | ratio of feature interaction terms to normalization terms |

(1) To date, research on q-ROFZNs remains limited, with only Reference [14] extending the Euclidean distance to q-ROFZNs. Therefore, this paper introduces the Jaccard index and proposes a novel similarity distance measure. The Euclidean distance measure is greatly influenced by the absolute scale of numerical values. Compared with the Euclidean distance [14], the proposed distance calculates the ratio of shared features to overall features. By calculation, this focuses on the core attributes common to the entities, effectively mitigating interference from feature sparsity and differences in numerical scales. As a result, it enhances both the comparative clarity and the robustness of outcomes in fuzzy decision-making.

(2) The Hamming distance is a linear computation based on the L1 norm and is overly sensitive to sparse zero-valued features. The proposed distance, by contrast, employs a normalized-ratio structure that overcomes the interference of numerical scale differences and zero-value sparsity across feature vectors. Consequently, the resulting evaluations achieve higher discriminative clarity and robustness when applied to extensive real-world data.

(3) Although the Dice distance and cosine-based distance possess ratio-based properties, they are unable to distinguish highly similar yet distinct sets and fail to capture the interactions among features. The proposed similarity distance measure places greater emphasis on characterizing intrinsic correlations among features rather than purely geometric or algebraic differences.

(4) It is worth emphasizing that, although the proposed distance satisfies commutativity in numerical computation (i.e., $D(Z_Q^1, Z_Q^2) = D(Z_Q^2, Z_Q^1)$), this merely represents the static geometric symmetry of information overlap. In higher-order fuzzy decision optimization, the deeper value of this metric lies in its asymmetry within dynamic applications. On the one hand, the introduction of the parameter $q$ breaks the linear compensatory relationship between membership and non-membership degrees, causing the distance to exhibit a parameter-driven asymmetric sensitivity with respect to the membership and non-membership spaces. On the other hand, when the distance is employed in ranking frameworks such as TOPSIS, the degrees of proximity to the positive ideal solution and remoteness from the negative ideal solution tend to be assigned different utility weights depending on whether the decision maker faces gains or losses (i.e., loss aversion in prospect theory). This asymmetric weighting in decision psychology is precisely formalized through the reliability dimension inherent in Z-numbers.

## 5. MCDM Method for q-Rung Orthopair Fuzzy Z-Number

In recent years, methods for handling multi-criteria decision-making problems within the q-rung orthopair fuzzy environment have been extensively and successfully applied to numerous complex real-world issues. However, the reliability of information, which significantly impacts the accuracy of decision-making results, is frequently overlooked. To address this problem, this section presents a novel MCDM method within the qROFZN environment, based on the proposed score function and similarity distance measure. Figure 1 illustrates the proposed MCDM method, which is described in three phases in this section: problem description, criterion weight calculation, and ranking approach.

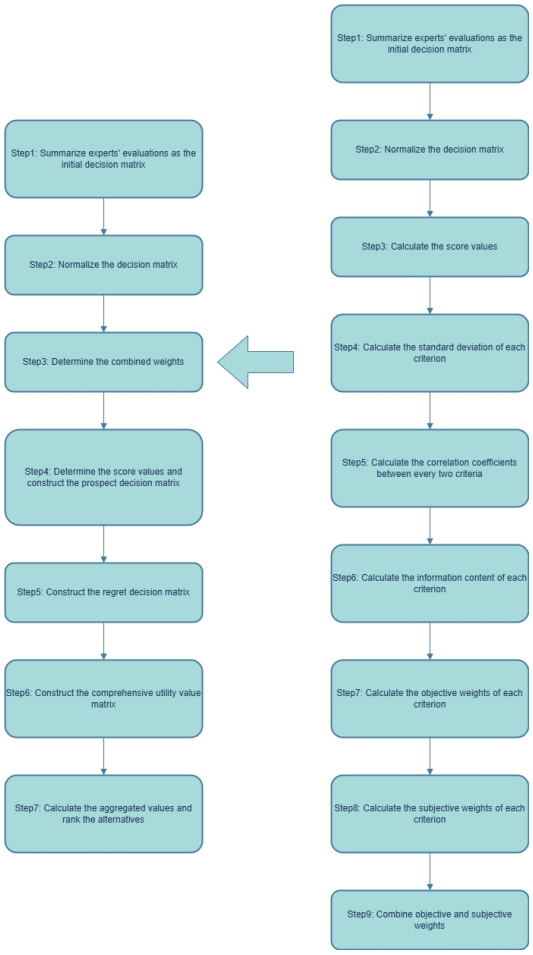



**Figure 1.** (**Left**) The proposed MCDM method for qROFZN. (**Right**) The calculation process of comprehensive weights.

### *5.1. The Description of MCDM Problem*

A multi-criteria decision-making problem has the following essential elements: m alternatives$\{A_1, A_2, ..., A_m\}$, n criteria$\{C_1, C_2, ..., C_n\}$, and the vector of criteria weights is $\omega = \{\omega_1, \omega_2, ..., \omega_n\}$, where $\omega_k \geq 0,\ k = 1, 2, ..., n, \sum_{k=1}^{n} \omega_k = 1$. The decision maker gives the evaluation of each alternative for the criteria, and the decision matrix in the form of qROFZN $Z_{ij} = ((\mu_{ij}, \mu_{c_{ij}}), (\nu_{ij}, \nu_{c_{ij}}))$ can be expressed as

$$Z = \begin{pmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mn} \end{pmatrix}. \tag{9}$$

### *5.2. The Determination of Criteria Weights*

In MCDM, the determination of criterion weights must balance the objective data characteristics and the decision-makers' subjective preferences, so as to enhance the scientificity and reliability of weights. The prospect theory primarily captures decision-makers' irrational perceptions of risk and loss through the value function and the decision weighting function. However, the weighting function in prospect theory predominantly reflects subjective attitudes toward probabilities or risks, yet fails to effectively utilize the objective information inherent in the evaluation indicators. Relying solely on subjective weights may lead to decisions that deviate from the actual data distribution.

To address this gap, the CRITIC method is introduced to determine indicator weights. By analyzing the standard deviation and correlation coefficients among indicators, CRITIC quantifies the information content and independence of each criterion, thereby objectively extracting relative importances from the raw decision matrix and effectively avoiding the arbitrariness of subjective assignments.

Therefore, the proposed MCDM not only presents a novel CRITIC method based on a developed score function to determine objective weights but also designs a combined weighting determination method by integrating the weight function of prospect theory. This combined method capitalizes on the strengths of CRITIC in extracting objective regularities from data while integrating the flexibility of prospect theory in modeling behavioral decision-making, such that the final criterion weights can reflect both the physical disparities among indicators and the psychological cognitive processes under realistic decision contexts, thereby enhancing the robustness and persuasiveness of the decision outcomes. The following details the process of determining criterion weights.

Step 1: Summarize experts' evaluations of each alternative in the form of qROFZN to compile the initial decision matrix.

Step 2: Normalize the decision matrix (9) according to the attributes of criteria, with the rules as follows:

$$Z'_{ij} = \begin{cases} ((\mu_{ij}, \mu_{c_{ij}}), (\nu_{ij}, \nu_{c_{ij}})), & \text{for beneficial criteria;} \\ ((\nu_{ij}, \nu_{c_{ij}}), (\mu_{ij}, \mu_{c_{ij}})), & \text{for cost criteria.} \end{cases} \tag{10}$$

Step 3: Calculate the score value $S_{ij}$ of each element $Z'_{ij}$ by using the developed score function (7).

Step 4: Calculate the standard deviation of each criterion by using the equation

$$\varsigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(S_{ij} - \bar{S}_j)^2}, \tag{11}$$

where $\bar{S}_j = \frac{1}{m}\sum_{i=1}^{m} S_{ij}$.

Step 5: Calculate the correlation coefficients between every two criteria by using the equation

$$r_{jk} = \frac{\frac{1}{m}\sum_{i=1}^{m}(S_{ij} - \bar{S}_j)(S_{ik} - \bar{S}_k)}{\varsigma_j \varsigma_k}. \tag{12}$$

Step 6: Calculate the information content of each criterion by using the equation

$$\Gamma_j = \sigma_j \sum_{k=1}^{n}(1 - r_{jk}). \tag{13}$$

Step 7: Calculate the objective weights of each criterion based on the above-mentioned information content with the help of the equation

$$\varpi_j = \frac{\Gamma_j}{\sum_{k=1}^{n} \Gamma_k}. \tag{14}$$

Step 8: Nxet, calculate the subjective weights, based on the initial weights $\omega = \{\omega_1, \omega_2, ..., \omega_n\}$, with the help of the weighting function (5) in prospect theory:

$$\omega_j' = \begin{cases} \omega_j^+ = \frac{\omega_j^\varepsilon}{(\omega_j^\varepsilon + (1-\omega_j)^\varepsilon)^{1/\varepsilon}}, & \text{for beneficial criteria;} \\ \omega_j^- = \frac{\omega_j^\delta}{(\omega_j^\delta + (1-\omega_j)^\delta)^{1/\delta}}, & \text{for cost criteria.} \end{cases} \tag{15}$$

Step 9: The final weights are obtained by integrating objective and subjective weights via the following formula:

$$\bar{\omega}_j = \frac{\sqrt{\varpi_j \omega_j'}}{\sum_{j=1}^{n} \sqrt{\varpi_j \omega_j'}}. \tag{16}$$

### *5.3. Ranking Method Based on the Developed Similarity Distance*

This section proposes a novel MCDM method for qROFZN based on the developed score function and similarity distance measure and the combined weights above. The detailed summary of the MCDM process is presented in the following steps.

Step 1: Summarize experts' evaluations of each alternative in the form of qROFZN to compile the initial decision matrix.

Step 2: Normalize the decision matrix (9) according to the attributes of criteria, with the rules (10).

Step 3: Determine the combined weights $\bar{\omega}_j$ with the help of (16).

Step 4: Calculate the score values, determine the element with the maximum score value in each column as the ideal value, and then construct the prospect decision matrix based on the value function in the prospect theory and the developed similarity distance measure by

$$V_{ij} = \begin{cases} \Big(D(Z_{ij}, Z_{j_{max}})\Big)^{\alpha} & S_{ij} \geq S_{j_{max}}, \\ -\theta\Big(1 - \big(D(Z_{ij}, Z_{j_{max}})\big)\Big)^{\beta} & S_{ij} < S_{j_{max}}, \end{cases} \tag{17}$$

where $D$ represents the developed similarity distance measure, $S_{j_{max}}$ is the maximum score value in each column. Under gain conditions, a larger value of $D$ corresponds to a closer approximation to the ideal value, thereby yielding a greater positive value. Conversely, under loss conditions, a smaller value of $D$ indicates a larger deviation from the ideal value, which generates a greater negative value. Specifically, under loss conditions, the value is zero when $D = 1$, implying no loss, and equals $-\theta$ when $D = 0$, denoting the maximum possible loss.

Step 5: Construct the regret decision matrix based on the regret–rejoice function in the regret theory by

$$R_{ij} = 1 - e^{-\sigma\left(1 - D(Z_{ij}, Z_{j_{max}})\right)}. \tag{18}$$

Since $D$ is a similarity measure, $1 - D(Z_{ij}, Z_{j_{max}})$ naturally quantifies the degree of deviation of alternative $i$ from the ideal alternative with respect to attribute $j$. On this basis, the function (18) monotonically increases with respect to this deviation. It yields $R_{ij} = 0$ when no deviation occurs (i.e., $D = 1$), and approaches unity as the deviation grows. This formulation aligns with the fundamental tenet of regret

theory, whereby a larger discrepancy between the actual outcome and the ideal state induces a stronger perception of regret.

Step 6: Construct the comprehensive utility value matrix according to (17) and (18), and the specific formula is as follows:

$$U_{ij} = \eta V_{ij} + (1-\eta) R_{ij}, \tag{19}$$

where $\eta \in [0,1]$.

Step 7: Rank the alternatives based on the combined weights (16) and the comprehensive utility value matrix (19) by

$$U_i = \sum_{j=1}^{n} \bar{\omega}_j U_{ij}. \tag{20}$$

In summary, the above constitutes the entirety of the MCDM method proposed in this paper. The decision making algorithm can be summarized in Algorithm 1.

**Algorithm 1** The prospect-regret theory-based MCDM

**Input:** Set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$, set of criteria $C = \{C_1, C_2, \ldots, C_n\}$, initial weight vector $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$, and expert evaluation matrix $Z_{ij}$ in qROFZN form.
**Output:** Ranking of alternatives.
**Step 1: Construct the initial decision matrix**
**for** $i = 1$ to $m$ **do**
  **for** $j = 1$ to $n$ **do**
    $Z_{ij} \leftarrow \left((\mu_{ij}, \mu_{c_{ij}}), (\nu_{ij}, \nu_{c_{ij}})\right)$
**Step 2: Normalize the decision matrix**
**for** $i = 1$ to $m$ **do**
  **for** $j = 1$ to $n$ **do**
    **if** $C_j$ is a benefit criterion **then**
      $Z'_{ij} \leftarrow \left((\mu_{ij}, \mu_{c_{ij}}), (\nu_{ij}, \nu_{c_{ij}})\right)$
    **else**
      $Z'_{ij} \leftarrow \left((\nu_{ij}, \nu_{c_{ij}}), (\mu_{ij}, \mu_{c_{ij}})\right)$
**Step 3: Calculate the combined weights** $\bar{\omega}_j$
Compute objective weights $\varpi_j$ using the CRITIC method (Equations (11)–(14))
Compute subjective weights $\omega'_j$ using the prospect theory weighting function (Equation (15))
Compute combined weights: $\bar{\omega}_j = \frac{\sqrt{\varpi_j \omega'_j}}{\sum_{j=1}^{n} \sqrt{\varpi_j \omega'_j}}$ (Equation (16))
**Step 4: Construct the prospect decision matrix** $V_{ij}$
**for** $j = 1$ to $n$ **do**
  $S_{j_{\max}} \leftarrow \max_i S(Z'_{ij})$, where $S$ is the novel score function (Equation (7))
  $Z'_{j_{\max}} \leftarrow$ the qROFZN with $S_{j_{\max}}$
  **for** $i = 1$ to $m$ **do**
    Calculate similarity distance $d_{ij} \leftarrow D(Z'_{ij}, Z'_{j_{\max}})$ (Equation (8))
    **if** $S(Z'_{ij}) \geq S_{j_{\max}}$ **then**
      $V_{ij} \leftarrow (d_{ij})^{\alpha}$
    **else**
      $V_{ij} \leftarrow -\theta \cdot (1 - d_{ij})^{\beta}$
**Step 5: Construct the regret decision matrix** $R_{ij}$
**for** $i = 1$ to $m$ **do**
  **for** $j = 1$ to $n$ **do**
    $R_{ij} \leftarrow 1 - e^{-\sigma \cdot (1 - d_{ij})}$ (Equation (18))

**Algorithm 1** *Cont.*

**Step 6: Construct the comprehensive utility matrix** $U_{ij}$
Set balance parameter $\eta \in [0, 1]$
**for** $i = 1$ to $m$ **do**
  **for** $j = 1$ to $n$ **do**
    $U_{ij} \leftarrow \eta V_{ij} + (1 - \eta) R_{ij}$ (Equation (19))
**Step 7: Calculate total utility values and rank alternatives**
**for** $i = 1$ to $m$ **do**
  $U_i \leftarrow \sum_{j=1}^{n} \bar{\omega}_j \cdot U_{ij}$ (Equation (20))
Rank alternatives in descending order of $U_i$.
**return** Ranking of alternatives.

## 6. Case Study

To verify the feasibility of the proposed MCDM method, this section applies the proposed method to a medical diagnosis case study adopted from [27]. Additionally, comparative analysis with existing methods is conducted to validate the effectiveness and superiority of the proposed method.

### *6.1. Case Application*

By observing the patient's initial symptoms and manifestations, doctors can preliminarily understand and determine the possible diseases the patient may currently have. Since one symptom may be associated with multiple diseases, less experienced new doctors may struggle to identify the patient's specific cause of illness in a timely manner or may be uncertain about their diagnosis. The MCDM method proposed in this paper can effectively address this issue. Ref. [27] discusses a case involving five possible diseases for a patient: viral fever ($\Phi_1$), malaria ($\Phi_2$), typhoid fever ($\Phi_3$), gastric ulcer ($\Phi_4$), and pneumonia ($\Phi_5$), along with five common symptoms: body temperature, headache, stomachache, cough, and chest pain.

Let $A = \{A_1, A_2, A_3, A_4, A_5\}$ be the set of the possible diseases, $C = \{C_1, C_2, C_3, C_4, C_5\}$ be the set of the symptoms, and $\omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ be the initial weight vector of the symptoms, where $\omega_k \geq 0,\ k = 1, 2, ..., 5, \sum_{k=1}^{5} \omega_k = 1$. The decision-making process is as follows.

Step 1: According to [27], the initial decision matrix of the diseases with respect to the symptoms is given in Table 3.

**Table 3.** The initial decision matrix of the diseases with respect to the symptoms.

| | Body Temperature | Headache | Stomachache | Cough | Chest Pain |
|---|---|---|---|---|---|
| Viral fever | ((0.6,0.6),(0.3,0.4)) | ((0.6,0.5),(0.1,0.7)) | ((0.4,0.4),(0.5,0.3)) | ((0.2,0.7),(0.7,0.2)) | ((0.3,0.5),(0.5,0.6)) |
| Malaria | ((0.7,0.1),(0.1,0.4)) | ((0.3,0.6),(0.6,0.5)) | ((0.1,0.2),(0.6,0.8)) | ((0.5,0.3),(0.4,0.4)) | ((0.1,0.8),(0.8,0.6)) |
| Typhoid fever | ((0.4,0.4),(0.1,0.5)) | ((0.3,0.7),(0.1,0.3)) | ((0.2,0.8),(0.4,0.6)) | ((0.4,0.3),(0.3,0.5)) | ((0.1,0.5),(0.5,0.5)) |
| Gastric ulcer | ((0.1,0.7),(0.7,0.4)) | ((0.2,0.4),(0.4,0.2)) | ((0.6,0.7),(0.2,0.3)) | ((0.2,0.3),(0.7,0.5)) | ((0.2,0.5),(0.7,0.8)) |
| Pneumonia | ((0.1,0.3),(0.8,0.3)) | ((0.2,0.4),(0.5,0.3)) | ((0.3,0.7),(0.5,0.6)) | ((0.4,0.6),(0.5,0.3)) | ((0.5,0.2),(0.1,0.7)) |

Step 2: The entries in Table 3 represent the degree of association between a disease and these observed symptoms. A higher association value indicates a greater likelihood that the corresponding disease is the correct diagnosis. Therefore, in the multi-criteria decision-making framework, these five symptoms should be regarded as benefit-type attributes. Normalize the initial decision matrix according to (10).

Step 3: Calculate the combined weights of each symptom by using the method in Section 5.2, the result is in Table 4, where the initial weights are adopted from [27].

**Table 4.** The weights of each symptom.

| | Body Temperature | Headache | Stomachache | Cough | Chest Pain |
|---|---|---|---|---|---|
| Initial weights | 0.2891 | 0.1797 | 0.1641 | 0.1484 | 0.2188 |
| CRITIC weights | 0.2568 | 0.2321 | 0.2804 | 0.0986 | 0.1321 |
| Subjective weights | 0.3125 | 0.2476 | 0.2370 | 0.2257 | 0.2724 |
| Combined weights | 0.2530 | 0.2141 | 0.2302 | 0.1332 | 0.1694 |

Step 4: Determine the ideal values, and then calculate the prospect decision matrix based on (17), with $\alpha, \beta = 0.88$, $\theta = 2.25$. The prospect decision matrix is in Table 5.

**Table 5.** The prospect decision matrix.

| | Body Temperature | Headache | Stomachache | Cough | Chest Pain |
|---|---|---|---|---|---|
| Viral fever | 0.3237 | −2.1766 | −2.0180 | −1.7360 | −2.1703 |
| Malaria | −1.6410 | −2.2198 | −2.2431 | −1.9838 | −2.1293 |
| Typhoid fever | −1.9863 | 0.0581 | −1.7940 | −2.0798 | 0.0604 |
| Gastric ulcer | −1.9230 | −2.2244 | 0.3840 | −2.2236 | −2.2164 |
| Pneumonia | −2.2158 | −2.2302 | −1.9291 | 0.2627 | −2.0989 |

Step 5: Calculate the regret decision matrix based on (18), with $\sigma = 0.5$, and the result is in Table 6.

**Table 6.** The regret decision matrix.

| | Body Temperature | Headache | Stomachache | Cough | Chest Pain |
|---|---|---|---|---|---|
| Viral fever | 0.3032 | 0.3821 | 0.3571 | 0.3109 | 0.3812 |
| Malaria | 0.2948 | 0.3888 | 0.3924 | 0.3517 | 0.3748 |
| Typhoid fever | 0.3521 | 0.3814 | 0.3206 | 0.3670 | 0.3808 |
| Gastric ulcer | 0.3418 | 0.3895 | 0.2822 | 0.3894 | 0.3883 |
| Pneumonia | 0.3882 | 0.3904 | 0.3428 | 0.3233 | 0.3700 |

Step 6: Calculate the comprehensive utility value matrix according to (19), with $\eta = 0.5$, and the result is in Table 7.

**Table 7.** The comprehensive utility value matrix.

| | Body Temperature | Headache | Stomachache | Cough | Chest Pain |
|---|---|---|---|---|---|
| Viral fever | 0.3134 | −0.8972 | −0.8304 | −0.7125 | −0.8946 |
| Malaria | −0.6731 | −0.9155 | −0.9254 | −0.8161 | −0.8773 |
| Typhoid fever | −0.8171 | 0.2198 | −0.7367 | −0.8564 | 0.2206 |
| Gastric ulcer | −0.7906 | −0.9174 | 0.3331 | −0.9171 | −0.9140 |
| Pneumonia | −0.9138 | −0.9199 | −0.7932 | 0.2930 | −0.8645 |

Step 7: Rank the alternatives based on (20) and the final aggregated values are in Table 8.

**Table 8.** The final aggregated values for each disease.

| | Viral Fever | Malaria | Typhoid Fever | Gastric Ulcer | Pneumonia |
|---|---|---|---|---|---|
| Values | −0.5505 | −0.8367 | −0.4060 | −0.5968 | −0.7182 |

By observing the Table 8, it can be found that $-0.4060 > -0.5505 > -0.5968 > -0.7182 > -0.8367$. So the final ranking for the five diseases is $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$. That is to say, the patient is most likely suffering from $\Phi_3$, i.e., Typhoid fever.

### *6.2. Sensitivity Analysis of Parameters*

In the above experiment, the parameter $q$ is set to 3, the parameter $\eta$ is set to 0.5 and the regret parameter $\sigma$ in (18) is set to 0.5. Moreover, the parameters $\alpha, \beta, \theta$ in (17) are set to $\alpha = \beta = 0.88, \theta = 2.25$ as the optimal values, following Reference [19]. This section investigates the effects of different parameters on the final ranking of alternative solutions. By examining various values of parameters $q$, $\eta$ and $\sigma$, the robustness of the proposed method under parameter uncertainty is analyzed. Through calculating the influence degree of parameter variations on the method's final ranking results, the predictive robustness of the method against parameter uncertainty is evaluated. This process facilitates the identification of key parameters with significant impacts on model performance, enabling prioritization of parameter estimation or optimization.

This section explores the impact of different $q$ values on the aggregated values and ranking results while keeping $\eta, \sigma$ constant, with detailed numerical results shown in Table 9 and Figure 2. As can be observed, these diseases exhibit notably distinct sensitivities to the parameter $q$. Specifically, typhoid fever demonstrates the highest parameter sensitivity, with its aggregated value monotonically decreases from –0.4060 to –0.2085 as $q$ increases from 3 to 15. In contrast, the aggregated values of viral fever and gastric ulcer tend to stabilize at approximately –0.9283 for $q \geq 15$, indicating a relatively low dependence on $q$. The remaining diseases show moderate sensitivity, with their aggregated values generally reaching a minimum around $q = 8$ and then undergoing slight fluctuations. Notably, the ranking results are not invariant with respect to $q$. For $q = 3, 4$, the ranking is $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$. For $q = 5, 8, 10$, the ranking shifts to $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_1 > \Phi_4$. At $q = 15$, the ranking becomes $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_1 = \Phi_4$, where viral fever and gastric ulcer are tied for the lowest position. Consequently, variations in $q$ substantially affect the final ranking: for smaller $q$, viral fever ranks higher, whereas the relative positions of malaria and pneumonia improve as $q$ increases. Nevertheless, typhoid fever consistently ranks first across all cases, indicating that the patient is most likely suffering from typhoid fever.

**Table 9.** The final result for different q-values.

| $q$ | Viral Fever | Malaria | Typhoid Fever | Gastric Ulcer | Pneumonia | Ranking |
|---|---|---|---|---|---|---|
| 3 | −0.5505 | −0.8367 | −0.4060 | −0.5968 | −0.7182 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 4 | −0.6112 | −0.8969 | −0.4697 | −0.6454 | −0.7572 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 5 | −0.8732 | −0.5796 | −0.2319 | −0.9135 | −0.7696 | $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_1 > \Phi_4$ |
| 8 | −0.9250 | −0.6237 | −0.2433 | −0.9271 | −0.7776 | $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_1 > \Phi_4$ |
| 10 | −0.9278 | −0.6332 | −0.2333 | −0.9280 | −0.7893 | $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_1 > \Phi_4$ |
| 15 | −0.9283 | −0.6298 | −0.2085 | −0.9283 | −0.8214 | $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_1 = \Phi_4$ |

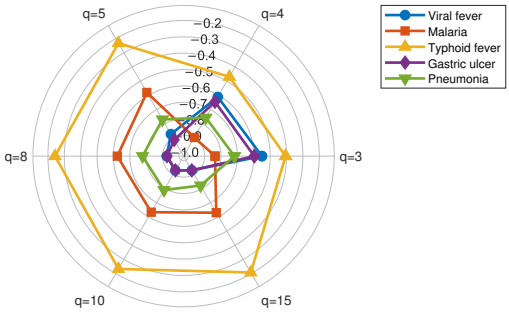



**Figure 2.** The performance of symptom aggregated values under different $q$-values.

Next, the impact of different $\eta$ values on the aggregated values and ranking results while keeping the $q, \sigma$ value constant is explored. As a weighting coefficient between the

prospect value and the regret value, parameter $\eta$ adjusts the relative importance of these two components. The detailed numerical results are shown in Table 10 and Figure 3. As shown in Table 10, as $\eta$ increases gradually from 0.1 to 1, the comprehensive utility values of all five diseases show an approximately linear monotonically decreasing trend, with their absolute values increasing gradually. Remarkably, the ranking order remains invariant across the entire range of $\eta$: $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$. It is particularly noteworthy that when $\eta = 1$, the weight of the regret term becomes zero, and the model degenerates into a decision framework based solely on prospect values. Nevertheless, the ranking order at $\eta = 1$ is exactly identical to that obtained for $\eta < 1$. This implies that the prospect value dominates the overall utility, and the model exhibits strong parameter robustness within this interval. It also indicates that, in the present case, regret-aversion behavior does not reverse the decision boundary. However, the introduction of regret values still affects the magnitude of the comprehensive utility and numerically reflects the decision maker's psychological trade-off. Therefore, incorporating regret aversion into the model retains methodological completeness. The regret term is not redundant, but rather plays a certain role in reshaping the decision boundary when competing alternatives are closely matched.

**Table 10.** The final result for different $\eta$-values.

| $\eta$ | Viral Fever | Malaria | Typhoid Fever | Gastric Ulcer | Pneumonia | Ranking |
|---|---|---|---|---|---|---|
| 0.1 | 0.1673 | 0.1195 | 0.2052 | 0.1626 | 0.1496 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.2 | −0.0121 | −0.1196 | 0.0524 | −0.0272 | −0.0674 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.3 | −0.1916 | −0.3586 | −0.1004 | −0.2171 | −0.2843 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.4 | −0.3710 | −0.5977 | −0.2532 | −0.4070 | −0.5012 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.5 | −0.5505 | −0.8367 | −0.4060 | −0.5968 | −0.7182 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.6 | −0.7299 | −1.0758 | −0.5588 | −0.7867 | −0.9351 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.7 | −0.9093 | −1.3148 | −0.7116 | −0.9766 | −1.1521 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.8 | −1.0888 | −1.5539 | −0.8644 | −1.1665 | −1.3690 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.9 | −1.2682 | −1.7929 | −1.0172 | −1.3563 | −1.5859 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 1 | −1.4476 | −2.0320 | −1.1700 | −1.5462 | −1.8029 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |

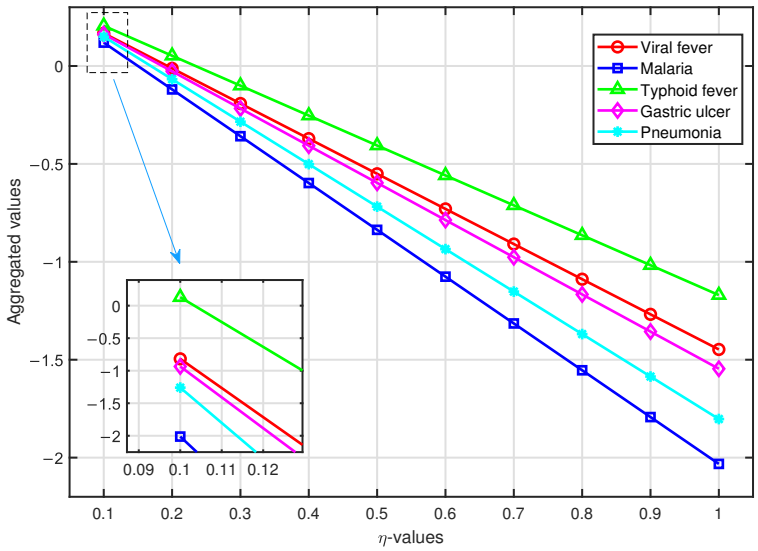



**Figure 3.** The performance of symptom aggregated values under different $\eta$-values.

The parameter $\sigma$ characterizes the degree of sensitivity of decision makers to regret and influences the utility reduction of each alternative arising from possible regret. Adjustment of $\sigma$ enables effective calibration of the regret-avoidance objective within the decision-making framework, thus altering the ultimate perceived utilities of the alternatives. As

indicated in Table 11, when $\sigma$ varies from 0.1 to 1, the aggregated scores of the five diseases exhibit a monotonic increase with $\sigma$, yet their relative ranking remains invariant across the entire range. Consequently, for all values of $\sigma$ examined, the most probable disease is determined to be Typhoid fever. The reason for this ranking invariance is that $\sigma$ only modulates the magnitude of the regret term. As the regret term varies with $\sigma$, the magnitude of adjustment applied to each alternative differs slightly, yet it does not surpass the threshold for ranking reversal, i.e., it fails to alter the order of superiority determined by the prospect utility. In other words, the regret term in this case only fine-tunes the utility values and is insufficient to reverse the priority relationships among the alternatives. As can be seen from Figure 4, the rankings under different $\sigma$ values are completely consistent, with no crossover or fluctuation observed. This indicates that the proposed method exhibits strong robustness with respect to the setting of the regret coefficient $\sigma$, and also suggests that, within the decision context, the prospect utility component plays a dominant role, while the influence of regret on the final choice can be regarded as secondary and stable.

**Table 11.** The final result for different $\sigma$-values.

| $\sigma$ | Viral Fever | Malaria | Typhoid Fever | Gastric Ulcer | Pneumonia | Ranking |
|---|---|---|---|---|---|---|
| 0.1 | −0.6829 | −0.9734 | −0.5425 | −0.7313 | −0.8578 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.2 | −0.6454 | −0.9344 | −0.5037 | −0.6931 | −0.8179 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.3 | −0.6110 | −0.8989 | −0.4682 | −0.6581 | −0.7816 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.4 | −0.5794 | −0.8664 | −0.4357 | −0.6261 | −0.7484 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.5 | −0.5505 | −0.8367 | −0.4060 | −0.5968 | −0.7182 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.6 | −0.5239 | −0.8096 | −0.3788 | −0.5701 | −0.6906 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.7 | −0.4995 | −0.7848 | −0.3540 | −0.5455 | −0.6654 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.8 | −0.4771 | −0.7621 | −0.3312 | −0.5231 | −0.6425 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 0.9 | −0.4566 | −0.7414 | −0.3104 | −0.5025 | −0.6215 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| 1 | −0.4378 | −0.7225 | −0.2914 | −0.4837 | −0.6024 | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |

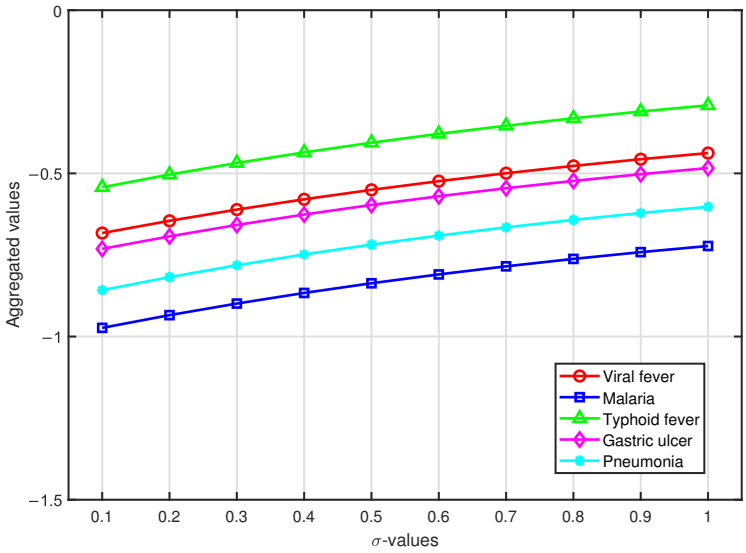



**Figure 4.** The performance of symptom aggregated values under different $\sigma$-values.

### *6.3. Comparative Analysis with Other Decision-Making Methods*

In this subsection, in order to validate the reliability and robustness of the proposed method, the proposed MCDM method is compared with existing decision-making methods, including the Dice similarity distance measure for qROFZN [28], the CRITIC method based on Hamming distance [24], similarity distance combined with BM operator [27], a MCDM based on the fairly aggregation for q-ROFZN [14], AHP-TOPSIS [29], AHP-MABAC [30] and

MABAC for linguistic (p,q)-rung orthopair fuzzy Z-number [31]. These existing approaches typically employ different algorithmic solutions, either relying on established objective methods to derive the final ranking or utilizing entirely subjective methodologies to obtain results. Such limitations can hinder their ability to accurately reflect real-world decision-making environments. Therefore, this paper presents a hybrid method integrating CRITIC weighting with the novel similarity distance measure and prospect regret theory. This method simultaneously incorporates the subjective weights from prospect theory and the objective weights from CRITIC, thereby providing a more comprehensive and reasonable analysis for the composite weights of criteria. Specifically, the MCDM method calculates the ranking results by integrating the similarity between expert-evaluated qROFZN and the ideal qROFZN, combined with the prospect regret theory.

This subsection employs both the proposed decision-making method and the existing methods to solve the same numerical example presented in Section 6.1. The results of the alternative rankings obtained from the proposed and existing methods are presented in Table 12. As shown in Table 12, all methods consistently identify Typhoid fever $\Phi_3$ as the most probable disease for the patient, which is in agreement with the diagnosis reported in Reference [27]. This consensus on the preferred alternative indicates, to a certain extent, the feasibility and effectiveness of the proposed method. However, for the remaining alternatives $(\Phi_1, \Phi_2, \Phi_4, \Phi_5)$, the rankings produced by the various methods differ. These discrepancies are mainly attributable to the different mathematical metrics adopted. The first four comparative methods employ distance-based similarity measures (e.g., Hamming distance and Dice similarity), which have similar mathematical properties (detailed in Table 2). In contrast, the proposed method adopts a Jaccard-based similarity distance measure, which is capable of distinguishing highly similar but non-identical feature sets and capturing the interactions among features, thereby generating reasonable ranking differences for the secondary alternatives. Furthermore, most methods determine weights via either subjective approaches (e.g., expert assignment or AHP) or objective approaches (e.g., CRITIC or the maximizing deviation method). By contrast, the proposed method integrates both subjective and objective dimensions through a combination of CRITIC and prospect theory, thus incorporating a broader range of information. This design offers greater flexibility in reflecting decision makers' risk attitudes and the characteristics of the data.

**Table 12.** Comparison with various decision-making approaches.

| MCDM Methods | Methodology Framework | Mathematical Measure | Weighting Method | Ranking |
|---|---|---|---|---|
| Hu et al. [27] | Bonferroni mean operator-based method | Improved similarity measure | Maximizing deviation method | $\Phi_3 > \Phi_1 > \Phi_5 > \Phi_4 > \Phi_2$ |
| Kumar et al. [28] | TOPSIS | Dice similarity measure | Expert-assigned subjective weights | $\Phi_3 > \Phi_1 > \Phi_5 > \Phi_2 > \Phi_4$ |
| Hameed et al. [14] | Fairly aggregation operator-based method | / | Expert-assigned subjective weights | $\Phi_3 > \Phi_1 > \Phi_5 > \Phi_4 > \Phi_2$ |
| Sharkasi et al. [24] | M-CRITIC-RP method | Hamming distance | CRITIC | $\Phi_3 > \Phi_1 > \Phi_5 > \Phi_2 > \Phi_4$ |
| Liu et al. [29] | TOPSIS | / | AHP | $\Phi_3 > \Phi_2 > \Phi_5 > \Phi_4 > \Phi_1$ |
| Bobar et al. [30] | MABAC | / | AHP | $\Phi_3 > \Phi_1 > \Phi_5 > \Phi_4 > \Phi_2$ |
| Ali et al. [31] | MABAC | / | Expert-assigned subjective weights | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |
| Proposed method | Prospect–regret-based method | Jaccard similarity distance measure | CRITIC and prospect theory | $\Phi_3 > \Phi_1 > \Phi_4 > \Phi_5 > \Phi_2$ |

## 7. Conclusions

In the study of decision-making problems under fuzzy environments, q-rung orthopair fuzzy Z-number demonstrates higher reliability and more flexible degrees of freedom in handling uncertainties compared to ordinary q-rung orthopair fuzzy numbers, thus performing more excellently in fuzzy environments. Given this context, this paper undertakes a series of research studies. A novel score function and an improved Jaccard similarity distance measure are developed for qROFZN, and their theoretical properties are examined. Based on these, a hybrid weight determination method is designed that integrates the objective weighting of the CRITIC method with the subjective weighting of prospect theory, thereby constructing a qROFZN-based MCDM framework that incorporates prospect–regret theory. Finally, the applicability and effectiveness of the proposed method are validated through a case study on medical diagnosis. The results demonstrate that the method can provide more reliable decision support under uncertain environments, enable scientific evaluation and rational ranking of decision alternatives, and take into account both the psychological behavior of decision makers and the inherent uncertainties in real-world decision contexts.

The proposed method also has some limitations. Although its application in medical diagnosis has been validated, its applicability to other decision scenarios requires further investigation. Moreover, in the present case study, the prospect value dominates, while the regret term exerts a relatively minor influence. However, the regret term could theoretically become decisive in altering the ranking when the prospect values of alternatives are very close. Therefore, the proposed method is worth exploring in cases where risk preferences are of particular importance, such as in investment decision-making.

In future research, more in-depth investigations and expansions will be conducted based on the proposed method. Specifically, the novel score function and the improved similarity distance measurement can be further extended to related theoretical systems such as game theory to broaden the method's applicability. Meanwhile, organic integration of this method with several aggregation operators will be attempted, with in-depth model research promoted through optimized integration of the operators to enhance the model's prediction accuracy and result reliability. Additionally, systematic exploration of the method's application in other fields will be carried out, with potential application value mined through empirical analysis. Notably, embedding the model into information system architectures in dynamic decision-making environments is worth exploring, with the practical application scope expanded through dynamic scenario adaptation to provide new methodological support for real-time decision-making in complex systems.

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## Abbreviations

The following abbreviations are used in this manuscript:

| | |
|---|---|
| MCDM | Multi-criteria decision-making |
| qROFZN | q-rung orthopair fuzzy Z-number |
| CRITIC | Criteria Importance Through Intercriteria Correlation |
| IFS | Intuitionistic fuzzy set |
| PFS | Pythagorean fuzzy set |
| FFS | Fermatean fuzzy set |
| q-ROFS | q-rung orthopair fuzzy set |
| p,q-ROFS | p,q-quasirung orthopair fuzzy set |
| WASPAS | Weighted aggregated sum product assessment |
| TOPSIS | Technique for Order Preference by Similarity to Ideal Solution |
| MABAC | Multi-Attributive Border Approximation Area Comparison |
| M-CRITIC-RP | Modified CRITIC with a reference point |

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