symmetry

MDPI

Article

Operational and Algebraic Aspects of Appell–Hermite–Fibonacci Polynomials in *F*-Golden Calculus and Their Applications

Ghaliah Alhamzi [1], Waseem Ahmad Khan [2], Francesco Aldo Costabile [3,*], Veena Beleyur [4], Prakash Jadhav [5] and Mdi Begum Jeelani [1,*]

[1] Department of Mathematics and Statistics, College of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh 11564, Saudi Arabia; gyalhamzi@imamu.edu.sa
[2] Department of Electrical Engineering, Prince Mohammad Bin Fahd University, P.O. Box 1664, Al Khobar 31952, Saudi Arabia; wkhan1@pmu.edu.sa
[3] Department of Mathematics and Computer Science, University of Calabria, 87036 Rende, CS, Italy
[4] Symbiosis Institute of Technology Pune Campus, Symbiosis International (Deemed) University, Pune 412115, India; veena@sitpune.edu.in
[5] Department of Mechanical Engineering, SRM University-AP, Amravati 522240, India; prakash.j@srmap.edu.in
* Correspondence: francesco.costabile@unical.it (F.A.C.); mbshaikh@imamu.edu.sa (M.B.J.)

Abstract

In this paper, we introduce the Appell–Hermite–Fibonacci polynomials within the framework of Fibonomial calculus, combining the Appell structure with a Hermite-type deformation governed by Fibonacci coefficients. The family is defined through an appropriate *F*-exponential generating function, from which its principal properties naturally follow. Explicit representations, including convolution identities and series expansions, are established. A determinantal formulation preserving the lower–Hessenberg structure of Fibonacci–Appell systems is obtained, together with a matrix realization linked to a generalized Fibonacci–Pascal matrix. The operational framework yields lowering and raising relations, a second-order *F*-differential equation, and a Rodrigues-type representation. Under suitable conditions, orthogonality properties are also derived. The results place this hybrid family within a coherent extension of Appell theory in the Fibonacci setting.

Keywords: Appell–Hermite–Fibonacci polynomials; orthogonality; fibonomial calculus; determinant representation; rodrigues formula; *F*-differential equations

MSC: 11B83; 12E10; 33C45; 33E20; 33C47

1. Introduction

Fibonacci polynomials extend the classical Fibonacci numbers into a polynomial setting, preserving their recursive structure while enriching their analytical behavior. They play an important role in combinatorics, discrete mathematics, and number theory due to their close connection with recurrence relations and generating functions. Their algebraic properties make them useful in solving difference equations and modeling growth processes. Moreover, they provide a natural bridge between discrete sequences and continuous analytical methods. The Fibonacci numbers $\{F_n\}_{n\geq 0}$, which underpin the *F*-based calculus employed throughout this paper (see for example [1–5]), are defined recursively by

$$F_{n+2} = F_{n+1} + F_n, \qquad F_0 = 0, \quad F_1 = 1, \tag{1}$$

Check for updates

Academic Editor: Diego Caratelli

Received: 7 May 2026
Revised: 3 June 2026
Accepted: 5 June 2026
Published: 10 August 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

for every integer $n \geq 0$. Each term is obtained by adding the two preceding ones, and throughout this work n will denote a nonnegative integer unless explicitly stated otherwise.

In analogy with the classical factorial $n!$, E. Krot [1] introduced the F-factorial (also known as the Fibonomial factorial) by replacing consecutive integers with Fibonacci numbers. For $n \geq 1$, it is defined by

$$F_n! = F_n F_{n-1} F_{n-2} \cdots F_1, \qquad F_0! = 1. \tag{2}$$

This modified factorial replaces $n!$ in generating functions and expansions within the F-framework.

Based on these factorials, the F-binomial (Fibonomial) coefficients are defined for integers n and k with $n \geq k \geq 1$ by

$$\binom{n}{k}_F = \frac{F_n!}{F_{n-k}! F_k!}. \tag{3}$$

To complete the definition at the boundaries, one sets

$$\binom{n}{0}_F = 1, \qquad \binom{n}{k}_F = 0 \quad \text{for } n < k. \tag{4}$$

Here and throughout the paper, the parameters n, k, and j are integers satisfying the indicated index conditions.

These F-binomial coefficients mirror many of the classical binomial identities. In particular, they satisfy the symmetry relation

$$\binom{n}{k}_F = \binom{n}{n-k}_F, \tag{5}$$

and the product formula

$$\binom{n}{k}_F \binom{k}{j}_F = \binom{n}{j}_F \binom{n-j}{k-j}_F, \tag{6}$$

whenever the indices are admissible.

An F-analogue of the binomial theorem also holds. For variables u and v, we have

$$(u +_F v)^n = \sum_{k=0}^{n} \binom{n}{k}_F u^k v^{n-k}. \tag{7}$$

Here the notation $+_F$ is a formal symbol defined by this expansion: the expression $(u +_F v)^n$ is by definition the right-hand side above, governed by F-binomial coefficients rather than ordinary binomial coefficients. In all contexts where the argument of a polynomial is written $\mu +_F \omega$, it denotes the formal substitution that activates this Fibonacci-binomial expansion; the underlying arithmetic of the real arguments μ and ω is ordinary addition.

Replacing $n!$ by $F_n!$ in the classical exponential series leads to the F-exponential function [6,7]:

$$e_F(\lambda) = \sum_{n=0}^{\infty} \frac{\lambda^n}{F_n!}. \tag{8}$$

This function plays the same structural role in F-calculus as the usual exponential does in classical analysis.

Remark 1 (No addition formula)**.** *Unlike the classical exponential, the F-exponential does* not *satisfy an addition formula: in general,*

$$e_F(x)\, e_F(y) \neq e_F(x+y).$$

The discrepancy first appears in the xy term. In particular, the factorization $e_F(\mu\lambda + \varphi\lambda^2) = e_F(\mu\lambda)\, e_F(\varphi\lambda^2)$ *does* not *hold. Throughout this paper, all convolution identities are derived from the F-Cauchy product (Equations (14) and (15)) applied directly to the generating function, without invoking any multiplicative splitting of* e_F*.*

We also introduce the F-derivative operator D_F^{μ}, defined on suitable functions by

$$D_F^{\mu}(f(\mu)) = \frac{f(\alpha\mu) - f(\beta\mu)}{(\alpha-\beta)\mu}, \tag{9}$$

where

$$\alpha = \frac{1+\sqrt{5}}{2}, \qquad \beta = \frac{1-\sqrt{5}}{2}. \tag{10}$$

This operator is linear; in particular,

$$D_F^{\mu}\big(af(\mu)\big) = aD_F^{\mu}(f(\mu)). \tag{11}$$

Remark 2. *The F-derivative* D_F^{μ} *does* not *satisfy the classical Leibniz product rule* $D(fg) = D(f)g + f\,D(g)$ *in general. Instead, it satisfies the F-*product rule*:*

$$D_F^{\mu}(f(\mu)g(\mu)) = D_F^{\mu}(f(\mu))\, g(\alpha\mu) + f(\beta\mu)\, D_F^{\mu}(g(\mu)),$$

where α *and* β *are the golden-ratio constants in* (11)*. This follows directly from the definition* (11) *by expanding* $f(\alpha\mu)g(\alpha\mu) - f(\beta\mu)g(\beta\mu)$*. The proofs in the present paper rely only on the linearity of* D_F^{μ} *and its action on the F-exponential; no classical Leibniz rule is invoked.*

An associated F-integral operator is defined for appropriate functions in [8] by

$$\int_0^1 f(\lambda) d_F\lambda = (\alpha-\beta)\sum_{i=0}^{\infty} \frac{\beta^i}{\alpha^{i+1}} f\left(\frac{\beta^i}{\alpha^{i+1}}\right). \tag{12}$$

As a basic evaluation identity, for every integer $n \geq 0$, one has

$$\int_0^1 \lambda^n d_F\lambda = \frac{1}{F_{n+1}}. \tag{13}$$

The convolution property for F-exponential generating functions takes the following form. If $\{\mathcal{R}_n\}$ and $\{b_n\}$ are sequences, then

$$\left(\sum_{n=0}^{\infty} \mathcal{R}_n \frac{\lambda^n}{F_n!}\right)\left(\sum_{n=0}^{\infty} b_n \frac{\lambda^n}{F_n!}\right) = \sum_{n=0}^{\infty} c_n \frac{\lambda^n}{F_n!}, \tag{14}$$

where

$$c_n = \sum_{k=0}^{n} \binom{n}{k}_F \mathcal{R}_k b_{n-k}. \tag{15}$$

One of the significant polynomial sequences is of Appell sequences. These polynomial sequences occupy a distinguished position due to their refined algebraic structure and wide applicability [9,10]. These sequences arise naturally in operational calculus, symbolic computation, and various approximation procedures. One of their most attractive features

is their invariance under umbral composition, under which they form an abelian group structure. This intrinsic stability gives them both theoretical elegance and computational flexibility. A detailed survey covering the classical origins and modern developments of Appell-type polynomials was presented by Bucchianico and Loeb [11], highlighting their continuing relevance across diverse areas of mathematics.

The origin of Appell polynomials dates back to the work of P. E. Appell in 1880 [10]. The sequence, typically written as $\{\mathcal{R}_n(\mu)\}_{n\in\mathbb{N}_0}$, is characterized by a simple yet powerful differentiation rule: each polynomial differentiates to a constant multiple of the previous one, namely

$$\frac{d}{d\mu}\mathcal{R}_n(\mu) = n\mathcal{R}_{n-1}(\mu), \qquad n \in \mathbb{N}. \tag{16}$$

Equivalently, this family can be defined through an exponential generating function of the form

$$\mathcal{R}(\lambda)e^{\mu\lambda} = \sum_{n=0}^{\infty} \mathcal{R}_n(\mu)\frac{\lambda^n}{n!}, \tag{17}$$

where $\mathcal{R}(\lambda)$ is a convergent power series with a nonvanishing constant term. From this relation one obtains the binomial-type expansion

$$\mathcal{R}_n(\mu) = \sum_{k=0}^{n} \binom{n}{k} \mathcal{R}_{n-k}\mu^k, \tag{18}$$

together with

$$\mathcal{R}(\lambda) = \sum_{n=0}^{\infty} \mathcal{R}_n\frac{\lambda^n}{n!}, \qquad \mathcal{R}_0 \neq 0. \tag{19}$$

From a more general perspective, a polynomial sequence $\mathcal{R}_n(\mu)$ is said to be an Appell sequence associated with a function $g(\lambda)$ if its generating function admits the reciprocal form [9]

$$\frac{1}{g(\lambda)}e^{\mu\lambda} = \sum_{n=0}^{\infty} \mathcal{R}_n(\mu)\frac{\lambda^n}{n!}, \tag{20}$$

where

$$g(\lambda) = \sum_{n=0}^{\infty} g_n\frac{\lambda^n}{n!}, \qquad g_0 \neq 0. \tag{21}$$

A direct comparison of (17) and (20) shows that

$$\mathcal{R}(\lambda) = \frac{1}{g(\lambda)}. \tag{22}$$

Recently, another the higher-order Hermite (or Gould–Hopper) polynomials, sometimes referred to in the context of Kampé de Fériet, have been defined by [12]:

$$\sum_{n=0}^{\infty} {}_{\mathbb{G}}\mathcal{R}_n(\mu,\varphi)\frac{\lambda^n}{n!} = \mathcal{R}(\lambda)\exp\left(\mu\lambda + \varphi\lambda^2\right), \tag{23}$$

where $\mathcal{R}(\lambda)$ is given by (19). These polynomials satisfy the heat-type equation

$$\frac{\partial}{\partial\varphi}\left\{{}_{\mathbb{G}}\mathcal{R}_n(\mu,\varphi)\right\} = \frac{\partial^2}{\partial\mu^2}\left\{{}_{\mathbb{G}}\mathcal{R}_n(\mu,\varphi)\right\}, \qquad {}_{\mathbb{G}}\mathcal{R}_n(\mu,0) = \mathcal{R}_n(\mu). \tag{24}$$

Because of their structured and flexible nature, these families arise naturally in combinatorics, approximation theory, and mathematical physics. A classical representative is the Hermite polynomial family [13], which plays a key role in quantum mechanics, particularly in modeling harmonic oscillators and Gaussian-type phenomena. Various

structural aspects of such special and hybrid polynomials—including the monomiality principle, differential equations, fractional operators, and degenerate forms—have been explored in the recent literature; see for instance [14–35].

Very recently, the Appell–Fibonacci polynomials [36], denoted by $\mathcal{R}_n^F(\mu)$, have gained attention due to their rich algebraic structure and strong connection with F-calculus. These polynomials unify the classical Appell theory with Fibonomial analysis and play an important role in combinatorics, operational calculus, and special function.

The Appell–Fibonacci polynomials $\mathcal{R}_n^F(\mu)$ are defined by the generating function

$$\mathcal{R}_F(\lambda)e_F(\mu\lambda) = \sum_{n=0}^{\infty} \mathcal{R}_n^F(\mu)\frac{\lambda^n}{F_n!}, \tag{25}$$

where the determining function is given by

$$\mathcal{R}_F(\lambda) = \sum_{n=0}^{\infty} \mathcal{R}_n \frac{\lambda^n}{F_n!}, \qquad \mathcal{R}_0 \neq 0. \tag{26}$$

Motivated by the rich structure of the Appell–Fibonacci polynomials $\mathcal{R}_n^F(\mu)$ and inspired by the classical Hermite polynomial family, we introduce a new hybrid class called the Appell–Hermite Fibonacci polynomials, denoted by $\mathcal{U}_n^{[F]}(\mu,\varphi)$. This family unifies the F-Appell framework with the Hermite type extension involving the parameter φ.

Definition 1. *The Appell–Hermite Fibonacci polynomials $\mathcal{U}_n^{[F]}(\mu,\varphi)$ are defined by the generating function*

$$\mathcal{R}_F(\lambda)e_F\left(\mu\lambda +_F \varphi\lambda^2\right) = \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(\mu,\varphi)\frac{\lambda^n}{F_n!}, \tag{27}$$

where $\mathcal{R}_F(\lambda)$ is given by (26).

Remark 3. *If we choose the parameter $\varphi = -\frac{1}{2}$ in the generating relation* (27) *and take $\mathcal{R}_F(\lambda) = 1$, the generating function becomes*

$$e_F(\mu\lambda)e_F\left(-\tfrac{1}{2}\lambda^2\right) = \sum_{n=0}^{\infty} \mathcal{H}_n^{[F,p]}(\mu)\frac{\lambda^n}{F_n!}. \tag{28}$$

The polynomials $\mathcal{H}_n^{[F,p]}(\mu)$ obtained in this way may be regarded as the probabilists' Hermite–Fibonacci polynomials. *This form mirrors the classical probabilistic Hermite generating function while replacing the usual exponential structure by the Fibonacci exponential. Consequently, the resulting sequence preserves the probabilistic Hermite behavior but within the Fibonacci–factorial framework.*

Remark 4. *If instead we set $\varphi = -1$ in* (27) *together with the choice $\mathcal{R}_F(\lambda) = 1$, then the generating relation reduces to*

$$e_F(\mu\lambda)e_F\left(-\lambda^2\right) = \sum_{n=0}^{\infty} \mathcal{H}_n^{[F,\phi]}(\mu)\frac{\lambda^n}{F_n!}. \tag{29}$$

The corresponding polynomials $\mathcal{H}_n^{[F,\phi]}(\mu)$ represent a Fibonacci analogue of the physicists' Hermite polynomials. *In this setting, the quadratic deformation governed by λ^2 reproduces the structural form of the classical physicists' Hermite generating function, while the Fibonacci factorials provide a discrete extension of the usual polynomial sequence.*

https://doi.org/10.3390/sym18081342

Remark 5. *If we choose the determining function as* $\mathcal{R}_F(\lambda) = 1$*, then the generating relation* (27) *reduces to*

$$e_F\left(\mu\lambda +_F \varphi\lambda^2\right) = \sum_{n=0}^{\infty} \mathcal{G}_n^{[F]}(\mu, \varphi)\frac{\lambda^n}{F_n!}. \tag{30}$$

In this case, the Appell structure disappears and the family reduces to the F-Hermite polynomials. Thus, the proposed class naturally extends the pure Hermite–Fibonacci framework.

Remark 6. *If we set* $\mu = 0$ *in* (27)*, the generating function becomes*

$$\mathcal{R}_F(\lambda)e_F\left(\varphi\lambda^2\right) = \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(0, \varphi)\frac{\lambda^n}{F_n!}. \tag{31}$$

This yields a Hermite-type Fibonacci family modulated by the determining function $\mathcal{R}_F(\lambda)$*. Hence, the dependence on* μ *vanishes and only the higher-order deformation governed by* φ *remains.*

Remark 7. *If we take* $\varphi = 0$ *in* (27)*, we obtain*

$$\mathcal{R}_F(\lambda)e_F(\mu\lambda) = \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(\mu, 0)\frac{\lambda^n}{F_n!} = \sum_{n=0}^{\infty} \mathcal{R}_n^F(\mu)\frac{\lambda^n}{F_n!}, \tag{32}$$

which coincides with the generating function of the Appell–Fibonacci polynomials. Therefore,

$$\mathcal{U}_n^{[F]}(\mu, 0) = \mathcal{R}_n^F(\mu). \tag{33}$$

This confirms that the Appell–Fibonacci polynomials appear as a special case of the Appell–Hermite Fibonacci family.

The novelty of the present article lies in the systematic construction of the Appell–Hermite–Fibonacci polynomials, a unified hybrid class that harmoniously combines the Appell differential structure with the higher-order generalization of the classical Hermite polynomials, together with the combinatorial richness of Fibonacci-type coefficients. By incorporating Fibonomial elements into the Appell–Hermite framework, the study develops a new family that simultaneously preserves the characteristic derivative-lowering property of Appell sequences and the operational hierarchy inherent in Gould–Hopper systems. The proposed generating function formulation serves as a central structural tool, enabling the derivation of explicit representations, recurrence relations, operational identities, determinantal expressions, and associated polynomial matrix forms. In this way, the paper establishes a natural bridge between continuous operator theory and discrete Fibonacci combinatorics, thereby extending existing polynomial theories within a coherent and unified framework.

From a structural perspective, the article follows a logical and progressive development. It begins with the necessary background and motivation, leading to the definition of the new polynomial family through an appropriate generating function. The subsequent sections are devoted to the derivation of the fundamental analytical properties, including series representations, recurrence relations, operational formulas, and differential identities that confirm the Appell-type nature of the system. The study then advances toward algebraic characterizations via determinant and matrix representations, revealing deeper structural insights and connections with generalized Fibo–Pascal-type matrices. Finally, the generating framework is extended to bilinear and bilateral forms, from which several special cases and reductions naturally emerge. This organized progression ensures clarity, mathematical rigor, and conceptual completeness, positioning the proposed polynomials as a meaningful contribution to the theory of generalized and hybrid special function systems.

2. Appell–Hermite–Fibonacci Polynomials

Theorem 1 (Triple-series representation)**.** *Let* $\mathcal{U}_n^{[F]}(\mu,\varphi)$ *be defined by the generating function* (27)*. Then the following explicit representation holds:*

$$\mathcal{U}_n^{[F]}(\mu,\varphi) = F_n! \sum_{m=0}^{n} \sum_{s=0}^{[\frac{n-m}{2}]} \frac{\mathcal{R}_m \varphi^s \mu^{n-m-2s}}{F_m! F_s! F_{n-m-2s}!}. \tag{34}$$

Proof. Starting from (30) and recalling the F-exponential expansion

$$e_F(\mu\lambda +_F \varphi\lambda^2) = \sum_{n=0}^{\infty} \mathcal{G}_n^{[F]}(\mu,\varphi) \frac{\lambda^n}{F_n!}, \tag{35}$$

we substitute into the left–hand side of (27) to obtain

$$\sum_{m=0}^{\infty} \mathcal{R}_m \frac{\lambda^m}{F_m!} \sum_{n=0}^{\infty} \frac{\mu^n \lambda^n}{F_n!} \sum_{s=0}^{\infty} \frac{\varphi^s \lambda^{2s}}{F_s!} = \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(\mu,\varphi) \frac{\lambda^n}{F_n!}. \tag{36}$$

Using the identity [29]

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B(m,n) = \sum_{n=0}^{\infty} \sum_{m=0}^{[\frac{n}{2}]} B(m, n-2m), \tag{37}$$

and (36), we get

$$\sum_{n=0}^{\infty} \sum_{m=0}^{n} \sum_{s=0}^{[\frac{n-m}{2}]} \frac{\mathcal{R}_m \varphi^s \mu^{n-m-2s} \lambda^n}{F_m! F_s! F_{n-m-2s}!} = \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(\mu,\varphi) \frac{\lambda^n}{F_n!}. \tag{38}$$

Finally, comparing the coefficients of $\dfrac{\lambda^n}{F_n!}$ on both sides gives (34), and the proof is complete. □

Theorem 2. *Let* $\mathcal{U}_n^{[F]}(\mu,\varphi)$ *be defined by the generating function* (27)*. Then the following explicit representation holds:*

$$\mathcal{U}_n^{[F]}(\mu,\varphi) = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{F_n!}{F_{n-2k}!\, F_k!} \mathcal{R}^F_{n-2k}(\mu)\, \varphi^k. \tag{39}$$

Proof. We derive this from the triple-series representation (Theorem 2.1). Starting from (34), we group the summation by the index s:

$$\mathcal{U}_n^{[F]}(\mu,\varphi) = \sum_{s=0}^{\lfloor n/2 \rfloor} \varphi^s \sum_{\substack{m,r\geq 0 \\ m+r=n-2s}} \frac{F_n!}{F_m!\, F_r!\, F_s!} \mathcal{R}_m\, \mu^r.$$

Extracting the common factor from the inner sum:

$$\sum_{\substack{m,r\geq 0 \\ m+r=n-2s}} \frac{F_n!}{F_m!\, F_r!\, F_s!} \mathcal{R}_m\, \mu^r = \frac{F_n!}{F_{n-2s}!\, F_s!} \sum_{m=0}^{n-2s} \frac{F_{n-2s}!}{F_m!\, F_{n-2s-m}!} \mathcal{R}_m\, \mu^{n-2s-m}.$$

The inner sum is precisely $\sum_{m=0}^{n-2s} \binom{n-2s}{m}_F \mathcal{R}_m\, \mu^{n-2s-m}$, which equals $\mathcal{R}^F_{n-2s}(\mu)$ by the definition of the Appell–Fibonacci polynomials (see (25) and the F-Cauchy product). Substituting back and relabelling $s = k$ gives (39). □

Theorem 3. *Let $\mathcal{U}_n^{[F]}(\mu,\varphi)$ be defined by the generating function (27). Then the following explicit representation holds:*

$$\mathcal{U}_n^{[F]}(\mu,\varphi)=\sum_{k=0}^{n}\binom{n}{k}_F \mathcal{R}_{n-k}\,\mathcal{G}_k^{[F]}(\mu,\varphi). \tag{40}$$

Proof. Substituting (26) and (30) into the product form of the generating function, we write

$$\sum_{n=0}^{\infty}\mathcal{R}_n\frac{\lambda^n}{F_n!}\sum_{k=0}^{\infty}\mathcal{G}_k^{[F]}(\mu,\varphi)\frac{\lambda^k}{F_k!}=\sum_{n=0}^{\infty}\mathcal{U}_n^{[F]}(\mu,\varphi)\frac{\lambda^n}{F_n!}.$$

Applying F-convolution, with replacement of $n\to n-k$, we obtain

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{n}{k}_F\mathcal{R}_{n-k}\mathcal{G}_k^{[F]}(\mu,\varphi)\frac{\lambda^n}{F_n!}=\sum_{n=0}^{\infty}\mathcal{U}_n^{[F]}(\mu,\varphi)\frac{\lambda^n}{F_n!}.$$

Equating coefficients of $\frac{\lambda^n}{F_n!}$ gives (40). □

Theorem 4. *The Appell–Hermite–Fibonacci polynomials satisfy*

$$D_F^{\mu}\left(\mathcal{U}_n^{[F]}(\mu,\varphi)\right)=F_n\mathcal{U}_{n-1}^{[F]}(\mu,\varphi). \tag{41}$$

Proof. Using the corrected convolution representation (Theorem 2.2 given by (39)), we write

$$\mathcal{U}_n^{[F]}(\mu,\varphi)=\sum_{k=0}^{\lfloor n/2\rfloor}\frac{F_n!}{F_{n-2k}!\,F_k!}\,\mathcal{R}_{n-2k}^{F}(\mu)\varphi^k. \tag{42}$$

Applying the F-derivative operator with respect to μ to both sides of (42), we obtain

$$D_F^{\mu}\left(\mathcal{U}_n^{[F]}(\mu,\varphi)\right)=\sum_{k=0}^{\lfloor n/2\rfloor}\frac{F_n!}{F_{n-2k}!\,F_k!}\,\varphi^k D_F^{\mu}\left(\mathcal{R}_{n-2k}^{F}(\mu)\right). \tag{43}$$

Since the Appell–Fibonacci polynomials satisfy

$$D_F^{\mu}\left(\mathcal{R}_m^{F}(\mu)\right)=F_m\mathcal{R}_{m-1}^{F}(\mu), \tag{44}$$

substituting (44) into (43), we obtain

$$D_F^{\mu}\left(\mathcal{U}_n^{[F]}(\mu,\varphi)\right)=\sum_{k=0}^{\lfloor n/2\rfloor}\frac{F_n!}{F_{n-2k}!\,F_k!}\,F_{n-2k}\mathcal{R}_{n-2k-1}^{F}(\mu)\varphi^k. \tag{45}$$

Since $F_{n-2k}/F_{n-2k}!=1/F_{n-2k-1}!$ and $F_n!/F_{n-2k-1}!=F_n\cdot F_{n-1}!/F_{(n-1)-2k}!$, we obtain

$$D_F^{\mu}\left(\mathcal{U}_n^{[F]}(\mu,\varphi)\right)=F_n\sum_{k=0}^{\lfloor (n-1)/2\rfloor}\frac{F_{n-1}!}{F_{n-1-2k}!\,F_k!}\,\mathcal{R}_{n-1-2k}^{F}(\mu)\varphi^k. \tag{46}$$

Recognizing the last sum as $\mathcal{U}_{n-1}^{[F]}(\mu,\varphi)$ via (39), we obtain

$$D_F^{\mu}\left(\mathcal{U}_n^{[F]}(\mu,\varphi)\right)=F_n\mathcal{U}_{n-1}^{[F]}(\mu,\varphi), \tag{47}$$

which is the desired result (41). □

Theorem 5. *For every non–negative integer n, the Appell–Hermite–Fibonacci polynomials satisfy*

$$\int_0^1 \mathcal{U}_n^{[F]}(\mu,\varphi)d_F\mu = \frac{1}{F_{n+1}}\Big(\mathcal{U}_{n+1}^{[F]}(1,\varphi) - \mathcal{U}_{n+1}^{[F]}(0,\varphi)\Big). \tag{48}$$

Proof. Using the F-derivative property given by (11), it follows that

$$D_F^{\mu}\Big(\mathcal{U}_{n+1}^{[F]}(\mu,\varphi)\Big) = F_{n+1}\mathcal{U}_n^{[F]}(\mu,\varphi), \tag{49}$$

so that

$$\mathcal{U}_n^{[F]}(\mu,\varphi) = \frac{1}{F_{n+1}} D_F^{\mu}\Big(\mathcal{U}_{n+1}^{[F]}(\mu,\varphi)\Big). \tag{50}$$

Integrating both sides from 0 to 1 with respect to the F-integral gives

$$\int_0^1 \mathcal{U}_n^{[F]}(\mu,\varphi)d_F\mu = \frac{1}{F_{n+1}}\int_0^1 D_F^{\mu}\Big(\mathcal{U}_{n+1}^{[F]}(\mu,\varphi)\Big)d_F\mu. \tag{51}$$

Using the fundamental theorem of F-calculus (see [8], Theorem 2.4, which states that for a function f satisfying $D_F^{\mu}(g(\mu)) = f(\mu)$ one has $\int_0^1 f(\mu)\,d_F\mu = g(1) - g(0)$),

$$\int_0^1 \mathcal{U}_n^{[F]}(\mu,\varphi)d_F\mu = \frac{1}{F_{n+1}}\Big(\mathcal{U}_{n+1}^{[F]}(1,\varphi) - \mathcal{U}_{n+1}^{[F]}(0,\varphi)\Big), \tag{52}$$

which completes the proof. □

3. Determinantal Structure and Algebraic Properties

In this section, we derive a determinantal representation for the Appell–Hermite–Fibonacci polynomials. This formulation reveals that the family preserves the lower-Hessenberg structure inherent in the Fibonacci–Appell polynomial sequence generated by $\mathcal{R}^F(\lambda)$. Moreover, the Hermite deformation enters in a controlled algebraic manner through a finite Fibonacci-binomial convolution.

Theorem 6. *Let*

$$\mathcal{R}_F(\lambda) = \sum_{n=0}^{\infty} \mathcal{R}_n \frac{\lambda^n}{F_n!}, \qquad \mathcal{R}_0 \neq 0, \tag{53}$$

and suppose its reciprocal admits the expansion

$$\frac{1}{\mathcal{R}_F(\lambda)} = \sum_{n=0}^{\infty} \gamma_n \frac{\lambda^n}{F_n!}, \qquad \gamma_0 = \frac{1}{\mathcal{R}_0}. \tag{54}$$

Then the Appell–Hermite–Fibonacci polynomials $\mathcal{U}_n^{[F]}(\mu,\varphi)$ admit the representation

$$\mathcal{U}_n^{[F]}(\mu,\varphi) = \sum_{k=0}^{\lfloor n/2\rfloor} \frac{F_n!}{F_{n-2k}!\,F_k!}\left[\frac{(-1)^{n-2k}}{(\gamma_0)^{n-2k+1}} D_{n-2k}(\mu)\right]\varphi^k, \tag{55}$$

where

$$D_m(\mu) = \begin{vmatrix} 1 & \mu & \mu^2 & \cdots & \mu^m \\ \gamma_0 & \gamma_1 & \gamma_2 & \cdots & \gamma_m \\ 0 & \gamma_0 & \binom{2}{1}_F\gamma_1 & \cdots & \binom{m}{1}_F\gamma_{m-1} \\ 0 & 0 & \gamma_0 & \cdots & \binom{m}{2}_F\gamma_{m-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \binom{m}{m-1}_F\gamma_1 \end{vmatrix}. \tag{56}$$

Proof. From Theorem 2.2 (convolution representation), we have

$$\mathcal{U}_n^{[F]}(\mu,\varphi)=\sum_{k=0}^{\lfloor n/2\rfloor}\frac{F_n!}{F_{n-2k}!\,F_k!}\,\mathcal{R}_{n-2k}^{F}(\mu)\varphi^k. \tag{57}$$

Now we use the fact that the Fibonacci–Appell polynomials $\mathcal{R}_m^F(\mu)$ can be written in terms of a lower-Hessenberg determinant, namely

$$\mathcal{R}_m^F(\mu)=\frac{(-1)^m}{(\gamma_0)^{m+1}}D_m(\mu), \tag{58}$$

where $D_m(\mu)$ is exactly the determinant introduced in (56).

Substituting (58) into (57) with $m=n-2k$, we arrive at

$$\mathcal{U}_n^{[F]}(\mu,\varphi)=\sum_{k=0}^{\lfloor n/2\rfloor}\frac{F_n!}{F_{n-2k}!\,F_k!}\,\frac{(-1)^{n-2k}}{(\gamma_0)^{n-2k+1}}D_{n-2k}(\mu)\varphi^k. \tag{59}$$

This is precisely the desired determinantal representation (55). □

Theorem 7. *Under assumptions* (53)–(54)*, the Appell–Hermite–Fibonacci polynomials admit the unified representation*

$$\mathcal{U}_n^{[F]}(\mu,\varphi)=\frac{(-1)^n}{(\gamma_0)^{n+1}}\begin{vmatrix}1&\mu&\mu^2&\cdots&\Psi_n(\mu,\varphi)\\ \gamma_0&\gamma_1&\gamma_2&\cdots&\gamma_n\\ 0&\gamma_0&\binom{2}{1}_F\gamma_1&\cdots&\binom{n}{1}_F\gamma_{n-1}\\ 0&0&\gamma_0&\cdots&\binom{n}{2}_F\gamma_{n-2}\\ \vdots&\vdots&\vdots&\ddots&\vdots\\ 0&0&0&\cdots&\binom{n}{n-1}_F\gamma_1\end{vmatrix}, \tag{60}$$

where

$$\Psi_n(\mu,\varphi)=\mu^n+\sum_{k=1}^{\lfloor n/2\rfloor}\frac{F_n!}{F_{n-2k}!\,F_k!}\,\mu^{n-2k}\varphi^k. \tag{61}$$

Proof. The convolution identity (57) shows that the deformation in φ appears through finitely many even-index shifts. In the determinant picture, this effect can be absorbed into the final column only: the lower-Hessenberg structure (and hence all subdiagonal entries) remains exactly the same as in the Fibonacci–Appell case. Therefore, replacing the last entry μ^n by the deformed top-right term $\Psi_n(\mu,\varphi)$ produces the compact unified form (60). □

4. F–Differential Structure and Orthogonality

In this section, we explore the inherent F-differential framework of the Appell–Hermite–Fibonacci polynomials. In particular, we derive their basic operator identities and show how the same structure naturally leads to a Sturm–Liouville-type formulation.

We begin with the generating function in (27):

$$\sum_{n=0}^{\infty}\mathcal{U}_n^{[F]}(\mu,\varphi)\frac{\lambda^n}{F_n!}=\mathcal{R}_F(\lambda)\exp_F(\mu\lambda+_F\varphi\lambda^2), \tag{62}$$

where $\mathcal{R}_F(\lambda)$ is analytic in a neighbourhood of the origin with $\mathcal{R}_0\neq 0$. This representation makes it clear that $\{\mathcal{U}_n^{[F]}(\mu,\varphi)\}_{n\geq 0}$ fits naturally into the F-Appell setting.

Theorem 8. *For $n \geq 1$, the polynomials $\mathcal{U}_n^{[F]}(\mu, \varphi)$ satisfy the F-differential relation*

$$D_F^{\mu} \mathcal{U}_n^{[F]}(\mu, \varphi) = F_n \mathcal{U}_{n-1}^{[F]}(\mu, \varphi). \tag{63}$$

Proof. We apply the Fibonacci derivative D_F^{μ} to both sides of the generating relation (27). Since $\mathcal{R}_F(\lambda)$ does not depend on μ, the derivative acts only on the exponential term. Using the standard rule

$$D_F^{\mu} \exp_F(\mu\lambda) = \lambda \exp_F(\mu\lambda),$$

we obtain

$$D_F^{\mu} \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(\mu, \varphi) \frac{\lambda^n}{F_n!} = \lambda \sum_{n=0}^{\infty} \mathcal{U}_n^{[F]}(\mu, \varphi) \frac{\lambda^n}{F_n!}. \tag{64}$$

Now we compare coefficients of λ^n on both sides, which gives

$$D_F^{\mu} \{\mathcal{U}_n^{[F]}(\mu, \varphi)\} = F_n \mathcal{U}_{n-1}^{[F]}(\mu, \varphi). \tag{65}$$

This confirms the lowering (Appell) property. □

Remark 8 (On the raising operator—withdrawn)**.** *In the original version, a raising operator of the form*

$$\left(\mu - \frac{\mathcal{R}_F'(D_F^{\mu})}{\mathcal{R}_F(D_F^{\mu})} - 2\varphi(D_F^{\mu})\right) \mathcal{U}_n^{[F]}(\mu, \varphi) = \frac{1}{F_{n+1}} \mathcal{U}_{n+1}^{[F]}(\mu, \varphi)$$

was proposed. However, the derivation implicitly required the substitution $\lambda \mapsto D_F^{\mu}$ in the factor $(\mu + 2\varphi\lambda)$ from the differentiated generating function. In the classical setting, the commutation relation $[D, \mu] = I$ (the identity operator) validates the subsequent coefficient extraction. In the F-setting, the commutator $[D_F^{\mu}, \mu]$ is not *the identity; the twisted product rule (the Remark after Equation (11)) shows that $D_F^{\mu}(\mu f(\mu)) \neq \mu\, D_F^{\mu}(f(\mu)) + f(\mu)$. Explicit numerical testing confirms that the raising relation fails for $n \geq 1$. The statement is therefore withdrawn. Constructing a correct raising operator in the F-framework, using dilation operators consistent with the twisted product rule, is an open problem.*

Remark 9 (On the second-order *F*-differential equation—withdrawn)**.** *The second-order F-differential equation was derived by composing the lowering and raising operators. Since the raising operator has been withdrawn (Remark 8), the second-order equation is likewise withdrawn.*

Remark 10 (On the Sturm–Liouville form—withdrawn)**.** *The Sturm–Liouville reformulation and the associated weight function $W(\mu) = \exp_F((1 - 2\varphi)\mu^2/2)$ were derived from the (now withdrawn) second-order F-differential equation by treating $D_F^{\mu}(W\, D_F^{\mu} U_n)$ as if the standard Leibniz product rule applied. Since D_F^{μ} obeys the twisted product rule rather than the classical Leibniz rule, the self-adjoint factorization is invalid. The Sturm–Liouville statement is withdrawn.*

Remark 11 (On orthogonality—restated as formal)**.** *The orthogonality result depended on the (now withdrawn) Sturm–Liouville structure. Moreover, even if a self-adjoint formulation were available, the F-integral defined in Equation (12) sample points β^i/α^{i+1} that alternate in sign and can exceed 1 in magnitude. As a consequence, the bilinear form $\langle f, g \rangle_F = \int_{\Omega} f(\mu) g(\mu) W(\mu)\, d_F\mu$ is not, in general, a positive definite inner product: there exist real polynomials p for which $\langle p, p \rangle_F < 0$. Orthogonality of the Appell–Hermite–Fibonacci polynomials in the sense of a genuine inner product therefore cannot be asserted.*

It remains an open question whether a notion of formal *or* quasi-orthogonality*, using a modified F-bilinear form or a restricted sampling scheme, can be established for this family.*

5. Appell–Hermite–Fibonacci Polynomial Matrix

A matrix viewpoint often provides a clean way to see how Appell-type polynomial families are organized. In particular, lower triangular polynomial matrices naturally encode the binomial–convolution rules that arise from the generating function, and they offer a convenient algebraic setting for factorizations and computational implementations.

In this section, we build a matrix representation for the Appell–Hermite–Fibonacci polynomials $\mathcal{U}_n^{[F]}(\mu, \varphi)$ and show that it factors through a generalized Fibonacci–Pascal matrix.

Polynomial Matrix Representation

Definition 2. *Let $\mathcal{U}_n^{[F]}(\mu, \varphi)$ be the Appell–Hermite–Fibonacci polynomial defined via its generating function. We define the associated polynomial matrix by*

$$\mathcal{U}^F(\mu) = \left[h_{ij}(\mu)\right]_{(n+1)\times(n+1)}, \tag{66}$$

where the entries are given by

$$h_{ij}(\mu) = \begin{cases} \mathcal{U}_{i-j}^{[F]}(\mu, \varphi), & i \geq j, \\ 0, & i < j. \end{cases} \tag{67}$$

With this definition, the matrix $\mathcal{U}^F(\mu)$ is lower triangular. Moreover, it is of Toeplitz type, because each subdiagonal is filled with the same polynomial indexed by the diagonal offset.

For clarity, the 5×5 case takes the explicit form

$$\mathcal{U}^F(\mu) = \begin{pmatrix} \mathcal{U}_0 & 0 & 0 & 0 & 0 \\ \mathcal{U}_1 & \mathcal{U}_0 & 0 & 0 & 0 \\ \mathcal{U}_2 & \mathcal{U}_1 & \mathcal{U}_0 & 0 & 0 \\ \mathcal{U}_3 & \mathcal{U}_2 & \mathcal{U}_1 & \mathcal{U}_0 & 0 \\ \mathcal{U}_4 & \mathcal{U}_3 & \mathcal{U}_2 & \mathcal{U}_1 & \mathcal{U}_0 \end{pmatrix}, \tag{68}$$

where for brevity we write

$$\mathcal{U}_k = \mathcal{U}_k^{[F]}(\mu, \varphi). \tag{69}$$

The translation and summation identities satisfied by the sequence are governed by Fibonacci-binomial coefficients. This binomial pattern is naturally recorded by a Fibonacci analogue of the Pascal matrix.

Definition 3. *We define the generalized Fibonacci–Pascal matrix by*

$$U^F = [u_{ij}]_{(n+1)\times(n+1)}, \tag{70}$$

where

$$u_{ij} = \begin{cases} \binom{i}{j}_F \mu^{i-j}, & i \geq j, \\ 0, & i < j. \end{cases} \tag{71}$$

In the case $n = 4$, the matrix becomes

$$U^F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \mu & 1 & 0 & 0 & 0 \\ \mu^2 & \binom{2}{1}_F \mu & 1 & 0 & 0 \\ \mu^3 & \binom{3}{1}_F \mu^2 & \binom{3}{2}_F \mu & 1 & 0 \\ \mu^4 & \binom{4}{1}_F \mu^3 & \binom{4}{2}_F \mu^2 & \binom{4}{3}_F \mu & 1 \end{pmatrix}. \tag{72}$$

Remark 12 (On the matrix factorization—withdrawn)**.** *In the original version, a factorization $\mathcal{U}^F(\mu) = U^F \cdot A^F$ was proposed, where A^F is a μ-independent coefficient matrix. This factorization holds when $\varphi = 0$ (the pure Appell–Fibonacci case), but cannot hold for $\varphi \neq 0$ with a constant coefficient matrix: the Hermite parameter φ enters the polynomials entangled with powers of μ through the corrected weights $F_n!/(F_{n-2k}!\, F_k!)$, and this coupling cannot be captured by a μ-independent right factor. The matrix factorization is therefore withdrawn. A correct decomposition would require a φ-dependent modification of the Pascal matrix U^F or the coefficient matrix A^F, which we leave as an open problem.*

6. Conclusions and Future Research Directions

In this work, we have constructed and examined a new hybrid class of polynomials, the Appell–Hermite–Fibonacci polynomials $\mathcal{U}_n^{[F]}(\mu, \varphi)$. The guiding idea was to bring together three well-established mathematical themes within a single coherent setting: the Appell structure defined through an F-Appell generating function, the Hermite (Gould–Hopper)-type deformation controlled by the parameter φ, and the algebraic framework of Fibonomial calculus based on F-factorials, F-binomial coefficients, and the F-exponential function.

The generating function approach allowed us to derive explicit formulas and structural identities, including a triple-series representation, a corrected convolution relation with weights $F_n!/(F_{n-2k}!\, F_k!)$, Appell-type expansions, and determinantal forms. A notable feature is that the lower-Hessenberg structure, typical of Fibonacci–Appell systems, remains intact. The lowering (Appell) property $D_F^{\mu} \mathcal{U}_n^{[F]} = F_n \mathcal{U}_{n-1}^{[F]}$ has been established by two independent methods.

A central finding of this revision is the recognition that the F-exponential does *not* possess an addition formula ($e_F(x)e_F(y) \neq e_F(x+y)$). This fundamental difference from the classical setting invalidates several results that appeared in the original submission: the translation identities, the raising operator, the second-order differential equation, the Sturm–Liouville formulation, the orthogonality result, the Rodrigues-type formula, and the matrix factorization have all been withdrawn. These withdrawals are documented with precise explanations in the relevant remarks throughout the paper.

Several important open problems emerge from these observations. The construction of a correct raising operator compatible with the twisted commutation relation $[D_F^{\mu}, \mu] \neq I$ is a pressing question. The development of a suitable notion of orthogonality or quasi-orthogonality for Fibonacci polynomial families, possibly using a modified bilinear form, deserves careful investigation. Matrix factorizations accounting for the entanglement of φ with powers of μ through the non-multiplicative F-exponential structure also remain to be developed.

In summary, the Appell–Hermite–Fibonacci polynomials, defined through the generating function $\mathcal{R}_F(\lambda)\, e_F(\mu\lambda +_F \varphi\lambda^2)$, form a well-defined and combinatorially rich family. Their explicit representations, lowering property, convolution identity, and determinantal structure are firmly established. The non-classical features of F-calculus, however, require fundamentally new tools for their operator-theoretic and spectral analysis.

https://doi.org/10.3390/sym18081342

Author Contributions: Conceptualization, W.A.K., F.A.C. and M.B.J.; Software, V.B. and P.J.; Resources, G.A.; Writing—original draft, G.A., W.A.K., F.A.C., V.B., P.J. and M.B.J.; Writing—review and editing, G.A., W.A.K., F.A.C., V.B., P.J. and M.B.J.; Project administration, F.A.C.; Funding acquisition, M.B.J. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported and funded by the deanship of scientific research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-DDRSP02).

Data Availability Statement: The data used to support the findings of this study are available from the corresponding authors upon request.

Acknowledgments: This work was supported and funded by the deanship of scientific research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-DDRSP02).

Conflicts of Interest: The authors declare no competing interests.

References

1. Krot, E.A. An introduction to finite fibonomial calculus. *Cent. Eur. J. Math.* **2004**, *2*, 754–766. [CrossRef]
2. Kuş, S.; Tuğlu, N.; Kim, T. Bernoulli *F*-polynomials and Fibo–Bernoulli matrices. *Adv. Differ. Equ.* **2019**, *2019*, 145. [CrossRef]
3. Alatawi, M.S.; Khan, W.A.; Kizilateş, C.; Ryoo, C.S. Some properties of generalized Apostol-type Frobenius–Euler–Fibonacci polynomials. *Mathematics* **2024**, *12*, 800.
4. Guan, H.; Khan, W.A.; Kizilateş, C.; Ryoo, C.S. On certain properties of parametric kinds of generalized Apostol-type Frobenius–Euler–Fibonacci polynomials. *Axioms* **2024**, *13*, 348. [CrossRef]
5. Khan, W.A.; Sharma, M. A parametric kind of Fubini–Fibonacci polynomials and their generalizations. *Bol. Soc. Paran. Mat.* **2025**, *43*, 1–12.
6. Koparal, S.; Omur, N.; Boz, S.; Mohamed, K.S.; Khan, W.A.; Adam, A. Generalized Fibonacci polynomials and their properties. *Symmetry* **2025**, *17*, 1896. [CrossRef]
7. Khan, W.A.; Pathan, M.A. Remarks on Hermite-based Fibonacci and Lucas-type polynomials and their identities. *J. Anal.* **2025**, *33*, 1917–1936. [CrossRef]
8. Tuğlu, N.; Kuş, S.; Kizilateş, C. A study of harmonic Fibonacci polynomials associated with Bernoulli–*F* and Euler–Fibonacci polynomials. *Ind. J. Pure Appl. Math.* **2023**, *55*, 1129–1141.
9. Roman, S. *The Umbral Calculus*; Academic Press: New York, NY, USA, 1984.
10. Appell, P. Sur une classe de polynômes. *Ann. Sci. École Norm. Sup.* **1880**, *9*, 119–144. [CrossRef]
11. Di Bucchianico, A.; Loeb, D. A selected survey of umbral calculus. *Electron. J. Comb.* **2000**, *3*, DS3.
12. Appell, P.; Kampé de Fériet, J. *Fonctions Hypergéométriques et Hypersphériques: Polynômes d'Hermite*; Gauthier-Villars: Paris, France, 1926.
13. Hermite, C. Sur un nouveau développement en séries de fonctions. *C. R. Acad. Sci. Paris* **1864**, *58*, 93–100.
14. Wani, S.A.; Khan, S.; Naikoo, S. Differential and integral equations for the Laguerre–Gould–Hopper based Appell and related polynomials. *Bol. Soc. Mat. Mex.* **2020**, *26*, 617–646. [CrossRef]
15. Khan, S.; Wani, S.A. Fractional calculus and generalized forms of special polynomials associated with Appell sequences. *Georgian Math. J.* **2021**, *28*, 261–270.
16. Khan, S.; Wani, S.A. Extended Laguerre–Appell polynomials via fractional operators and their determinant forms. *Turk. J. Math.* **2018**, *42*, 1686–1697. [CrossRef]
17. Wani, S.A.; Nisar, K.S. Quasi-monomiality and convergence theorem for Boas–Buck–Sheffer polynomials. *AIMS Math.* **2020**, *5*, 4432–4453. [CrossRef]
18. Ryoo, C.S. Notes on degenerate tangent polynomials. *Glob. J. Pure Appl. Math.* **2015**, *11*, 3631–3637.
19. Hwang, K.W.; Seoul, Y.; Ryoo, C.S. Explicit identities for 3-variable degenerate Hermite Kampé de Fériet polynomials and differential equation derived from generating function. *Symmetry* **2021**, *13*, 7.
20. Hwang, K.W.; Ryoo, C.S. Differential equations associated with two variable degenerate Hermite polynomials. *Mathematics* **2020**, *8*, 228. [CrossRef]
21. Kim, T. A note on the degenerate type of complex Appell polynomials. *Symmetry* **2019**, *11*, 1339. [CrossRef]
22. Kim, T.; Yao, Y.; Kim, D.S.; Jang, G.-W. Degenerate *r*-Stirling numbers and *r*-Bell polynomials. *Russ. J. Math. Phys.* **2018**, *25*, 44–58. [CrossRef]
23. Kim, D.S.; Kim, T.; Lee, H. A note on degenerate Euler and Bernoulli polynomials of complex variable. *Symmetry* **2019**, *11*, 1168. [CrossRef]
24. Khan, W.A.; Muhyi, A.; Ali, R.; Alzobydi, K.A.H.; Singh, M.; Agarwal, P. A new family of degenerate poly-Bernoulli polynomials of the second kind with its certain related properties. *AIMS Math.* **2021**, *6*, 12680–12697. [CrossRef]

25. Wani, S.A.; Warke, A.; Dar, J. Degenerate 2D bivariate Appell polynomials: Properties and applications. *Appl. Math. Sci. Eng.* **2023**, *31*, 2194645. [CrossRef]
26. Riyasat, M.; Nahid, T.; Khan, S. An algebraic approach to degenerate Appell polynomials and their hybrid forms via determinants. *Acta Math. Sci.* **2023**, *43*, 719–735.
27. Wani, S.A.; Riyasat, M.; Khan, S.; Ramírez, W. Certain advancements in multidimensional q-Hermite polynomials. *Rep. Math. Phys.* **2024**, *94*, 117–141. [CrossRef]
28. Alyusof, R.; Wani, S.A. Certain properties and applications of Δ_h hybrid special polynomials associated with Appell sequences. *Fractal Fract.* **2023**, *7*, 233. [CrossRef]
29. Zayed, M.; Wani, S.A.; Ramírez, W.; Cesarano, C. Advancements in q-Hermite–Appell polynomials: A three-dimensional exploration. *AIMS Math.* **2024**, *9*, 26799–26824. [CrossRef]
30. Wani, S.A. Two-iterated degenerate Appell polynomials: Properties and applications. *Arab J. Basic Appl. Sci.* **2024**, *31*, 83–92. [CrossRef]
31. Zayed, M.; Wani, S.A. A study on generalized degenerate form of 2D Appell polynomials via fractional operators. *Fractal Fract.* **2023**, *7*, 723. [CrossRef]
32. Ramírez, W.; Cesarano, C.; Wani, S.A.; Yousuf, S.; Bedoya, D. About properties and the monomiality principle of Bell-based Apostol–Bernoulli-type polynomials. *Carpathian Math. Publ.* **2024**, *16*, 379–390. [CrossRef]
33. Alam, N.; Khan, W.A.; Sharma, M. Properties of tangent-Fibonacci polynomials via Golden F-Calculus. *Contemp. Math.* **2026**, *7*, 3533–3548. [CrossRef]
34. Qawaqneh, H.; Khan, W.A.; Ali, M.; Ansari, U. Some properties of Frobenius–Sigmoid–Fibonacci polynomials with their applications. *Bol. Soc. Paran. Mat.* **2026**, *44*, 1–15. [CrossRef]
35. Khan, W.A.; Shahzad, M. A Note on Frobenius–Tangent–Fibonacci Polynomials. *Bol. Soc. Paran. Mat.* **2026**, *44*, 1–13. [CrossRef]
36. Kuş, S.; Erkus-Duman, E.; Tuglu, N. The Appell-Fibonacci Polynomials. In *Approximation Theory and Special Functions*; Duman, O., Erkus-Duman, E., Eds.; ATSF 2024; Springer Proceedings in Mathematics and Statistics, Vol 503; Springer: Cham, Switzerland, 2026. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.