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Even–Odd-Mode Quasi-Static Analysis of Symmetrically Coupled Microstrip Lines with Bi-Isotropic Media

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Abstract

This paper presents a quasi-static analysis of symmetrically coupled microstrip lines on a bi-isotropic Tellegen substrate using the hybrid boundary element method (HBEM). The electromagnetic behaviour of this structure is analyzed under even- and odd-mode excitations in order to determine characteristic parameters. Also, the influence of the degree of bi-isotropy on these parameters is quantified. The bi-isotropic medium, as well as both strips, is modelled by discretizing the interfaces and introducing equivalent electric and fictitious magnetic charges while satisfying the corresponding boundary conditions. Unlike conventional boundary element method formulations, the proposed approach directly forms the system matrix without extensive boundary integrations, leading to reduced computational complexity and computation time. The HBEM results are validated against finite element method solutions obtained using FlexPDE software, showing a close agreement. The presented results demonstrate that the proposed HBEM formulation provides an efficient and accurate approach for quasi-static parameter extraction in complex multiconductor transmission structures.

Keywords: bi-isotropic media; coupled microstrip lines; hybrid boundary element method (HBEM); Tellegen media

1. Introduction

Coupled microstrip transmission lines represent one of the fundamental building blocks in modern microwave and high-frequency circuits, finding widespread application in directional couplers, filters, balanced circuits, and differential signal processing systems [1–3]. Their small size, simple geometry, stability and reliability make them indispensable in highly integrated RF front-ends. Recent advancements have further demonstrated their utility in developing integrated components, such as high-performance filtering switches that combine frequency selectivity with signal switching capabilities [4]. Due to the increasing demand for high-speed wireless communication, symmetrically coupled structures have become essential in designing balanced components for 5G applications, where they ensure phase stability and port isolation [5]. Owing to their planar nature, ease of fabrication, and compatibility with integrated technologies, microstrip structures generally remain an active subject of research, particularly when novel substrate materials and advanced modelling techniques are involved.

In recent years, increasing attention has been given to transmission lines loaded with complex and engineered materials, including anisotropic, chiral, and more generally bi-anisotropic media. Bi-isotropic materials, as a special and practically significant class, are



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characterized by the coupling of electric and magnetic fields through scalar parameters, which may include Tellegen (non-reciprocal) or Pasteur (chiral) effects [6,7]. However, as noted by Mezache and Berka in [8], although chirality and bi-isotropy have reshaped fields such as chemistry and optics over the past century, their full potential in advanced microwave and high-frequency electromagnetic applications remains significantly underexplored. Also, the fundamental physics of these media has revealed unique wave phenomena, such as the omnidirectional excitation of surface waves and super-Klein tunnelling at the interfaces of Tellegen media [9]. Such findings underscore the richness of the electromagnetic interactions in these materials, which further justifies the need for accurate characterization of guided waves in complex substrate configurations.

The significance of these media has been further highlighted by recent findings showing that Tellegen metamaterial substrates can induce phenomena, such as negative input impedance in printed antennas [10]. Recently, the physical realization of Tellegen metamaterials in the microwave region has been demonstrated in [11], showcasing responses that exceed those of natural materials by several orders of magnitude. Furthermore, complex media have also been analyzed in the context of printed antennas to tune their input impedance [12]. Despite these advancements, the majority of existing studies have focused primarily on single-conductor transmission lines or radiation elements. While the quasi-static behaviour and mathematical relations for bi-isotropic media were established in earlier works [7,13–16], more recent investigations have largely remained centred on antenna radiation characteristics.

A quasi-static approximation is commonly used to evaluate the characteristic parameters of the microstrip line under the quasi-TEM propagation mode. This approach was applied in [13] for analysis of the microstrip line with the bi-isotropic substrate using the quasi-static image method. However, as structures become more complex and multilayered, the analytical limitations of the image method have led to the development of more versatile numerical techniques. To address this, various numerical integral methods and non-uniform discretization algorithms have been proposed to accelerate convergence for multilayered planar lines [17]. In addition to these deterministic numerical methods, advanced modelling approaches such as neuro-space mapping (Neuro-SM) have been proposed in [1] to improve the accuracy and efficiency of empirical formulas for coupled microstrip lines. The presented technique avoids the slow trial-and-error process by using neural networks to map geometric and frequency variables, offering a high-speed alternative for standard geometries. Also, time-domain analysis for those coupled structures has been advanced through the development of the BI-FDTD technique, which utilizes wave-field decomposition to uncouple electromagnetic fields in bi-isotropic media [18,19].

In the context of transmission lines, the strong formulation of the finite element method (FEM) has been successfully used in [20] to characterize the effective relative permittivity and characteristic impedance of shielded microstrip lines with Tellegen-type bi-isotropic substrates. However, when coupled microstrip lines are considered, the influence of the Tellegen-type bi-isotropic effect has received little attention, particularly in the context of even and odd modes. Since impedance matching remains a critical aspect of microwave circuit design, a systematic investigation of how bi-isotropic coupling affects the characteristic impedance of symmetrically coupled microstrip lines is both timely and practically relevant.

In this paper, the hybrid boundary element method (HBEM) [21–26] is employed to perform a quasi-static analysis of symmetrically coupled microstrip lines on a bi-isotropic Tellegen substrate. The quasi-static approximation assumes a pure quasi-TEM mode of propagation, which neglects dispersion effects. This assumption is highly accurate and computationally justified as long as the cross-sectional dimensions of the transmission

line remain significantly smaller than the operating electromagnetic wavelength within the substrate [15]. While previous applications of the HBEM were primarily restricted to isotropic media, the fundamental theoretical approach for bi-isotropic Tellegen substrates was established in [25], where the HBEM was successfully applied to evaluate the quasi-static parameters of a single microstrip line. Analysis of a coupled microstrip structure with bi-isotropic substrate using the HBEM is performed here for the first time. To achieve this, the HBEM was extended by incorporating even/odd-mode symmetry directly into the potential formulation, from which the resulting system of linear equations is formed. This allows the algorithm to solve for the unknown free line charges, polarization (bound) line charges and fictitious magnetic line charges at the same time for each propagation mode. Because of this, the matrix size is significantly reduced as well as the calculation time.

The aim of this paper is also to quantify the influence of the degree of bi-isotropy on characteristic impedance and to highlight the modal differences that arise in such media. To ensure the reliability of the proposed approach, a convergence study is performed, and the obtained results are validated versus solutions generated using the software FlexPDE (version 5.0.22) [27]. Although newer commercial versions of this software exist, this version utilizes a strong finite element engine for standard quasi-static partial differential equations. Therefore, it provides a reliable benchmark for validating the proposed HBEM implementation and facilitates reproducibility of the numerical results.

The presented results provide new insight into the role of Tellegen-type bi-isotropic coupling in symmetrically coupled microstrip structures and may serve as a useful reference for the design and optimization of advanced microwave components using engineered substrates.

The paper is organized as follows: after the Introduction, Section 2 provides the theoretical background of bi-isotropic media and details the implementation of the hybrid boundary element method (HBEM) for the analysis of symmetrically coupled microstrip lines. Section 3 presents numerical results, including a convergence study, a comparative analysis with FlexPDE software, and the influence of the bi-isotropic effect on coupled line characteristic parameters. A detailed discussion of the findings is presented in Section 4. Finally, Section 5 concludes the paper with a summary of findings and future research directions.

2. Theoretical Background

2.1. Bi-Isotropic Media Relations

Unlike conventional isotropic media defined by the permittivity ϵ and the permeability μ , bi-isotropic materials require two additional parameters within their constitutive relations to account for magneto-electric coupling [8],

$$\mathbf{D} = \epsilon \mathbf{E} + \xi \mathbf{H} \text{ and} \quad (1)$$

$$\mathbf{B} = \mu \mathbf{H} + \zeta \mathbf{E}, \quad (2)$$

where $\mathbf{D}$ is the electric displacement, $\mathbf{B}$ is the magnetic flux density, and $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field, respectively.

Scalar medium parameters besides permittivity ϵ and permeability μ are magneto-electric coupling coefficients [7,18,28]

$$\xi = (\chi - j\kappa)\sqrt{\epsilon_0\mu_0} \text{ and } \zeta = (\chi + j\kappa)\sqrt{\epsilon_0\mu_0}. \quad (3)$$

The degree of chirality is described with κ , and χ is the Tellegen parameter, a dimensionless quantity for the degree of inherent non-reciprocity. For $\chi \neq 0$ and $\kappa = 0$, the

medium is considered as non-reciprocal and nonchiral, so-called Tellegen material. In that case, $\xi = \zeta$ and Equation (2) obtains the following form:

$$\mathbf{B} = \mu\mathbf{H} + \xi\mathbf{E}. \quad (4)$$

To quantify the strength of the magneto-electric coupling relative to the material's bulk properties, a dimensionless degree of bi-isotropy is introduced as

$$p = \xi^2 / (\epsilon\mu). \quad (5)$$

For passive and physically realizable bi-isotropic media, this parameter is bounded by $0 \leq p < 1$, where $p = 0$ corresponds to a conventional isotropic material. Specifically, a value of $p = 0.9$ signifies a highly pronounced level of bi-isotropy, whereas $p = 0.1$ corresponds to a low level of bi-isotropy.

2.2. Hybrid Boundary Element Method

The hybrid boundary element method (HBEM), developed at the Faculty of Electronic Engineering in Niš, is applied in this paper for characteristic parameter calculation of symmetrically coupled microstrip lines with bi-isotropic substrate. To date, this method has found widespread application across numerous areas of electromagnetics. It has been successfully employed for determining forces between permanent magnets [22], calculating characteristic parameters of structures within isotropic layers [23–25], and analyzing cable terminations and joints [21] as well as grounding configurations [26]. Over recent years, the method has undergone several modifications and advancements [24,25], including its generalization into the boundary element source method [29], and its scope of application continues to expand.

A particular advantage of the proposed HBEM is the capability to analyze multilayered structures, bi-isotropic media, and atypical geometric shapes. Before detailing the specific application of the method to the coupled microstrip configuration, its core formulation will be briefly explained using the generalized case shown in Figure 1a.

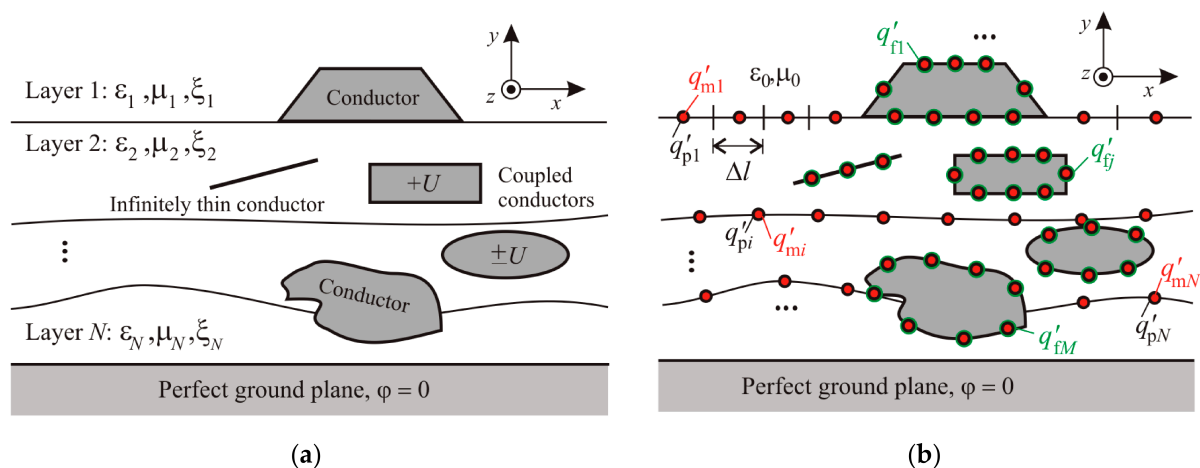


Figure 1. (a) Multiconductor line in a layered bi-isotropic medium with zero potential plane; (b) generalized HBEM model.

In this model, multiple conductors with varying cross-sectional shapes, maintained at different potentials, are embedded within a multilayered bi-isotropic medium. The entire structure is placed above a perfect ground plane at zero potential.

Let the parameters of the i -th layer be ϵ_i, μ_i and ξ_i ($i = 1, \dots, N$), where ϵ_i is the permittivity, μ_i is the permeability and ξ_i is the magneto-electric coupling coefficient.

According to the hybrid boundary element method, developed for the bi-isotropic media and detailed in [25], the strip surface and the material interfaces are segmented. This integral-free calculation procedure replaces each segment Δl with equivalent electrodes placed in free space (air) at the centres of the segments, as shown in Figure 1b. The equivalent electrodes on the strip conductors have radius $a_{e1} = \Delta l/4$ while those on the separating surfaces between the two BI layers have a radius of $a_{e2} = \Delta l/\pi$ [21,23].

For two-dimensional problems, such as the one illustrated in Figure 1a, the equivalent electrodes are modelled as line charges. The equivalent electrodes (EEs) on the strip conductors carry free electric charges per unit length, denoted in Figure 1b as q'_{ij} ($j = 1, \dots, M$). Crucially, due to the bi-isotropic nature of the media, fictitious magnetic charges, q'_{mi} , must be introduced alongside the polarization (bound) electric charges, q'_{pi} ($i = 1, \dots, N$) [25]. These magnetic and bound electric charges are located at the same positions along the interface between the strip and the substrate, as well as at the junction between two layers, as shown in Figure 1b.

Considering that the formed system is placed above a perfect ground plane at zero potential, the electric and magnetic scalar potentials at any arbitrary point in the system are defined after applying the image theorem. By applying the point-matching (collocation) method at the surfaces of the metallic strips with known potentials, as well as for the normal components of the electric and magnetic field at the interface of the two media, a linear system of equations is formed. Solving this system determines the unknown charge values and, consequently, the per-unit-length capacitance of the microstrip line, as well as other characteristic parameters.

2.3. Problem Definition

The cross-section of the analyzed symmetrically coupled microstrip line is shown in Figure 2.

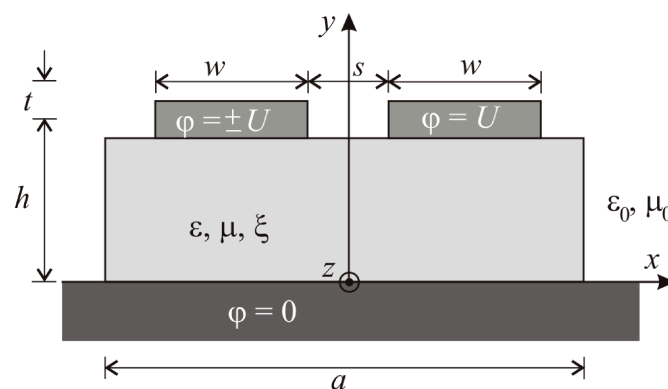


Figure 2. Geometry of a symmetrically coupled microstrip line with a bi-isotropic substrate and zero potential plane.

The configuration consists of two strip conductors, each with a width w and thickness t , separated by a gap s . The underlying Tellegen-type bi-isotropic substrate is characterized by a width a and a height h , with its material properties defined by the permittivity ϵ , the permeability μ and the bi-isotropy parameter ξ . The entire microstrip structure is placed in free space (air) above a perfectly conducting ground plane maintained at zero potential.

To analyze the modal behaviour of the structure, the right conductor is kept at a constant potential of $+U$, while the potential of the left conductor is set to $+U$ for the even propagation mode and $-U$ for the odd propagation mode, following the classical coupled line decomposition framework [30].

In order to determine the characteristic parameters, it is first necessary to determine the electric and magnetic potentials of the system, and to calculate the unknown values of the line charges by satisfying the boundary conditions.

To achieve this, an equivalent HBEM model of the geometry from Figure 2 is formed according to the procedure described in the previous subsection. The entire geometry is discretized and replaced by a system of equivalent electrodes placed in free space (air) as shown in Figure 3. Free line charges, q'_{fj} ($j = 1, \dots, M$), indicated in green, are located on both metallic strips. Bound electric line charges, q'_{pi} (black), and fictitious magnetic line charges, q'_{mi} (red) ($i = 1, \dots, N$), are placed at the air–bi-isotropic interface, as well as along the boundaries between the conductors and the bi-isotropic substrate.

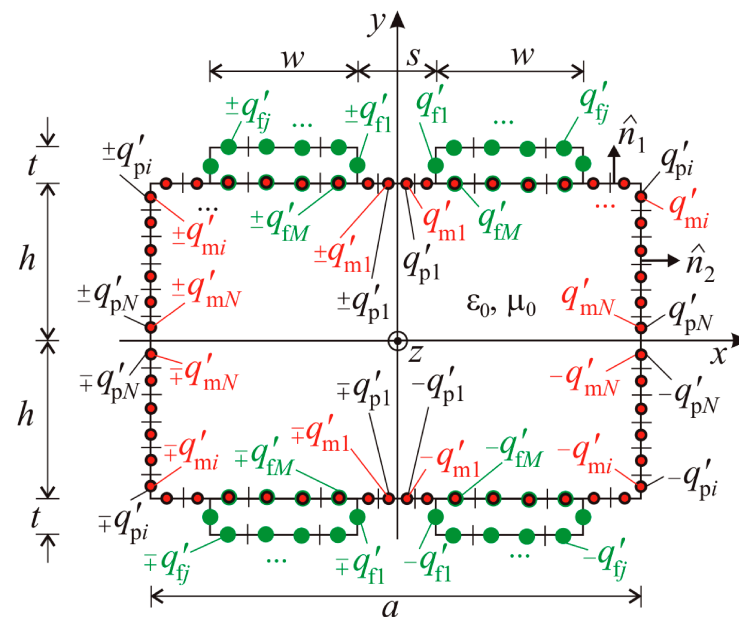


Figure 3. HBEM model.

To reduce the computational cost and exploit the structural symmetry, the image theorem is applied. Due to the presence of the perfectly conducting ground plane along the x -axis, the line charges located in the third and fourth quadrants are mirrored with the opposite sign relative to those in the first and second quadrants. Furthermore, the symmetry with respect to the y -axis (the vertical symmetry plane between the conductors) is governed by the specific propagation mode.

By taking advantage of this two-fold symmetry, the total number of unknown charge values is significantly reduced, as the entire system of equations can be formulated by considering only the boundary elements situated within the first quadrant.

2.4. Determination of Equivalent Electrodes

Based on the equivalent HBEM model illustrated in Figure 3, the electric, ϕ , and magnetic, ϕ_m , scalar potentials at any point (x, y) of the system are given as

$$\begin{aligned} \phi^{(e,o)}(x, y) = & \sum_{j=1}^M \frac{q'_{fj}}{2\pi\epsilon_0} \left[\ln \sqrt{\frac{(x-x_{fj})^2 + (y+y_{fj})^2}{(x-x_{fj})^2 + (y-y_{fj})^2}} \pm \ln \sqrt{\frac{(x+x_{fj})^2 + (y+y_{fj})^2}{(x+x_{fj})^2 + (y-y_{fj})^2}} \right] \\ & + \sum_{i=1}^N \frac{q'_{pi}}{2\pi\epsilon_0} \left[\ln \sqrt{\frac{(x-x_{pi})^2 + (y+y_{pi})^2}{(x-x_{pi})^2 + (y-y_{pi})^2}} \pm \ln \sqrt{\frac{(x+x_{pi})^2 + (y+y_{pi})^2}{(x+x_{pi})^2 + (y-y_{pi})^2}} \right], \end{aligned} \quad (6)$$

$$\varphi_m^{(e,o)}(x,y) = \sum_{i=1}^N \frac{q'_{mi}}{2\pi} \left[\ln \sqrt{\frac{(x-x_{mi})^2 + (y+y_{mi})^2}{(x-x_{mi})^2 + (y-y_{mi})^2}} \pm \ln \sqrt{\frac{(x+x_{mi})^2 + (y+y_{mi})^2}{(x+x_{mi})^2 + (y-y_{mi})^2}} \right], \quad (7)$$

where the sign “+” corresponds to even (e) mode and the sign “−” corresponds to the odd (o) mode. The indices “f”, “p” and “m” serve as labels for free, polarization (bound) and magnetic charges, respectively. Furthermore, M represents the number of free line charges on the right strip conductor, while N denotes the number of polarization as well as magnetic line charges.

The special coordinates (x_{fj}, y_{fj}) , (x_{pi}, y_{pi}) and (x_{mi}, y_{mi}) define the exact positions of the corresponding equivalent electrodes, where $j = 1, \dots, M$ and $i = 1, \dots, N$. The fundamental physical constants, $\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m and $\mu_0 = 4\pi \cdot 10^{-7}$ H/m, represent the free-space permittivity and permeability, respectively.

The linear system of equations is formed by matching expression (6) to the known constant potential value along the right strip conductors. The remaining equations are formulated by enforcing the boundary conditions for the normal components of the electric and magnetic field vectors at the interface between two layers, generally characterized by the parameters ϵ_1, μ_1 and ξ_1 for layer 1 and ϵ_2, μ_2 and ξ_2 for layer 2. These boundary conditions are derived in detail in [27] and take the following mathematical form:

$$\hat{n}E_i^{(0+)} = \frac{\epsilon_2(\mu_1 - \mu_2) - \xi_1\xi_2 + \xi_2^2}{\epsilon_0[(\epsilon_1 - \epsilon_2)(\mu_2 - \mu_1) + (\xi_1 - \xi_2)^2]} \eta_{pi} + \frac{\mu_1\xi_2 - \mu_2\xi_1}{(\epsilon_1 - \epsilon_2)(\mu_2 - \mu_1) + (\xi_1 - \xi_2)^2} \eta_{mi}, \quad (8)$$

$$\hat{n}H_i^{(0+)} = \frac{\epsilon_1\xi_2 - \epsilon_2\xi_1}{\epsilon_0[(\epsilon_1 - \epsilon_2)(\mu_2 - \mu_1) + (\xi_1 - \xi_2)^2]} \eta_{pi} + \frac{\mu_2(\epsilon_1 - \epsilon_2) - \xi_1\xi_2 + \xi_2^2}{(\epsilon_1 - \epsilon_2)(\mu_2 - \mu_1) + (\xi_1 - \xi_2)^2} \eta_{mi}, \quad (9)$$

where $\eta_{pi} = q'_{pi}/\Delta l$ and $\eta_{mi} = q'_{mi}/\Delta l$ ($i = 1, \dots, N$) represent the bound electric and magnetic surface line charges on the i -th segment of the interface. The unit normal vector, denoted as $\hat{n}$, takes the value $\hat{n}_1 = \hat{y}$ for the upper interface and $\hat{n}_2 = \hat{x}$ for the right vertical interface.

The components of the electric and magnetic field strength are determined using the well-known gradient relations $E = -\nabla\varphi$ and $H = -\nabla\varphi_m$, defined respectively as

$$E_x = -\frac{\partial\varphi}{\partial x}, E_y = -\frac{\partial\varphi}{\partial y}, \quad (10)$$

$$H_x = -\frac{\partial\varphi_m}{\partial x} \text{ and } H_y = -\frac{\partial\varphi_m}{\partial y}. \quad (11)$$

In Equations (8) and (9), the notation (0+) indicates that the field matching and evaluation are performed strictly within the air region (layer 1). The remaining parameters correspond to the specific medium configuration given in this paper. For the air region, the parameters are defined as $\epsilon_1 = \epsilon_0$, $\mu_1 = \mu_0$ and $\xi_1 = 0$. For the bi-isotropic (BI) substrate (layer 2), the material parameters are given as $\epsilon_2 = \epsilon$, $\mu_2 = \mu$ and $\xi_2 = \xi = \sqrt{p\epsilon\mu}$, where p represents the dimensionless degree of bi-isotropy.

By expressing the line charges from the boundary equations in terms of the corresponding electric and magnetic field components, the following relations are obtained:

$$q'_{pi} = \frac{\epsilon_0[\mu(\epsilon - \epsilon_0) - \xi^2]}{\epsilon\mu - \xi^2} \Delta l \hat{n}E_i^{(0+)} + \frac{\epsilon_0\xi\mu_0}{\epsilon\mu - \xi^2} \Delta l \hat{n}H_i^{(0+)}, \quad (12)$$

$$q'_{mi} = \frac{\epsilon_0\xi}{\epsilon\mu - \xi^2} \Delta l \hat{n}E_i^{(0+)} + \frac{\epsilon(\mu - \mu_0) - \xi^2}{\epsilon\mu - \xi^2} \Delta l \hat{n}H_i^{(0+)}. \quad (13)$$

By combining potential condition (6) on the right conductor surface with interface boundary conditions (12) and (13), the global HBEM system of linear equations is established in a compact block-matrix form:

$$\begin{bmatrix} A_P \\ A_{B1} \\ A_{B2} \end{bmatrix} \begin{bmatrix} q'_f \\ q'_p \\ q'_m \end{bmatrix} = \begin{bmatrix} U \\ 0 \\ 0 \end{bmatrix}, \quad (14)$$

where the total unknown vector incorporates the free line charges on the conductors, q'_f , alongside the polarization line charges, q'_p , and fictitious magnetic line charges, q'_m , distributed along the dielectric–substrate interfaces. The submatrix A_P enforces the known conductor potentials, while A_{B1} and A_{B2} enforce the continuous magneto-electric field relations at the bi-isotropic interfaces mapping the boundary equations directly to a zero right-hand side.

The even/odd-mode implementation is incorporated directly into potential Equations (6) and (7) using $\pm$ signs. These signs are also used in field Equations (12) and (13). Because of this, there is no need to change the mesh or the matrix format for different modes. The mesh calculation will be done only for the first quadrant. This makes the global matrix smaller, so the code runs much faster and uses less memory.

Based on this formulated system of linear equations, the unknown free, bound, and magnetic line charges are determined. To evaluate the characteristic parameters of the coupled line, it is necessary to first compute the per-unit-length capacitance using

$$C'^{(e,o)} = \sum_{j=1}^M \frac{q'_{fj}}{U}, \quad (15)$$

The corresponding computational code was developed and executed within the Wolfram Mathematica environment. Consequently, the characteristic parameters of interest, which are the effective relative permittivity, ϵ_r^{eff} , and characteristic impedance, Z_c for both modes, are determined using the following expressions [23]:

$$\epsilon_r^{\text{eff}(e,o)} = C'^{(e,o)} / C_0'^{(e,o)} \quad \text{and} \quad (16)$$

$$Z_c^{(e,o)} = Z_{c0}^{(e,o)} / \sqrt{\epsilon_r^{\text{eff}(e,o)}}, \quad (17)$$

where $C'^{(e,o)}$ and $C_0'^{(e,o)}$ represent the per-unit-length capacitance of the coupled microstrip line with and without the substrate, respectively. Similarly, $Z_c^{(e,o)}$ and $Z_{c0}^{(e,o)}$ denote the characteristic impedance of the identical coupled microstrip configuration with the substrate and in the absence of the substrate (in free space), respectively.

It is important to emphasize that for the magnetic substrate configuration, ($\mu_r \neq 1$), the calculated capacitance represents an effective equivalent capacitance C' . In the quasi-static model, the relative permeability μ_r enters the system through the bi-isotropic boundary conditions, given with Equations (8) and (9). Because of this, the coupling between the electric and magnetic field is implicitly included in this effective parameter. This makes the use of standard impedance Equations (16) and (17), mathematically correct and consistent with the exact field solution.

3. Results

Based on the procedure described in the previous section, the numerical results are presented in the following subsections.

3.1. Method Validation, Convergence and Charge Distributions

The convergence of the numerical results, along with the corresponding computation time, is shown in Figure 4. To ensure the general applicability and robustness of the chosen discretization strategy, a dual convergence analysis was performed using two different geometric datasets. The normalized input parameters used for these two cases are: $a/w = 4.0$, $h/w = 2.0$, $s/w = 0.5$, $t/w = 0.1$, $\epsilon_r = 3$, $\mu_r = 2$ and $p = 0.6$ for Figure 4a, and $a/w = 5.0$, $h/w = 1.0$, $s/w = 0.3$, $t/w = 0.05$, $\epsilon_r = 10$, $\mu_r = 1.5$ and $p = 0.9$ for Figure 4b. $N_{\text{tot}} = 2M + 4N$ denotes the total number of unknowns, representing the total number of equivalent electric and magnetic line charges. It should be emphasized that by exploiting the structural symmetry, the actual size of the solved matrix is reduced to one-half of N_{tot} , thereby significantly accelerating the computation.

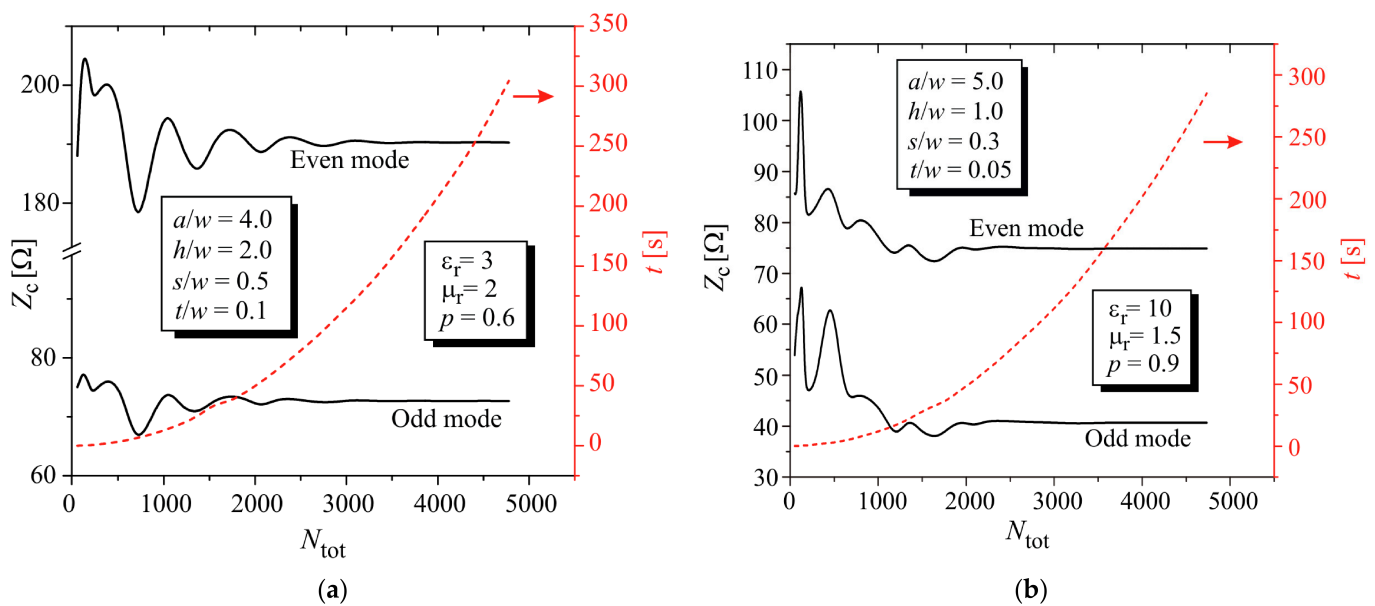


Figure 4. Result convergence and CPU time for two different datasets: (a) $a/w = 4.0$, $h/w = 2.0$, $s/w = 0.5$, $t/w = 0.1$, $\epsilon_r = 3$, $\mu_r = 2$ and $p = 0.6$; (b) $a/w = 5.0$, $h/w = 1.0$, $s/w = 0.3$, $t/w = 0.05$, $\epsilon_r = 10$, $\mu_r = 1.5$ and $p = 0.9$.

The computation time, represented by the red dotted line, includes the CPU time required for the spatial positioning of the equivalent electrodes, solving the linear system of equations, and evaluating the final characteristic parameters. By tracking the convergence curves and the execution time in Figure 4, the results for different geometries become completely stable for both modes with 3500 unknowns. Consequently, this value is used in all subsequent calculations, unless stated otherwise.

To validate the proposed HBEM approach, the FlexPDE software package [27] was applied to model the geometry shown in Figure 2. FlexPDE is a numerical solver based on the finite element method (FEM) for partial differential equation solving. Using the same geometry and material parameters as those for the graphs in Figure 4a, the characteristic impedance values determined by HBEM for both propagation modes are verified with the FlexPDE results for different degrees of bi-isotropy. These quantitative comparisons are summarized in Table 1.

The reference FlexPDE values validate the accuracy of the obtained HBEM results. For the purely isotropic and weakly bi-isotropic cases, the relative deviation is exceptionally low, remaining below 0.5%. As the magneto/electric coupling strengthens to a moderate level, the deviation increases, but remains less than 2%. Even under the extreme bi-isotropic condition, where the field singularities at the strip edges become highly pronounced and

numerically challenging, the maximum relative deviation is below 3%. It should be noted that the FlexPDE values were obtained using a dense mesh of approximately 200,000 nodes, requiring a total simulation time of 25 min. This computational duration accounts for a dual simulation run, which extracts the per-unit-length capacitance of the system first in the absence of the substrate, C'_0 , and subsequently with it, C' . Based on these capacitances, the characteristic impedance and effective relative permittivity are subsequently calculated using expressions (16) and (17). However, it should be noted that the same extraction equations are used in the verification test. Because of this, this close agreement mostly proves that our field solver is accurate under bi-isotropic boundary conditions. It does not independently test the parameter extraction method for magnetic media.

Table 1. Comparison of the characteristic impedance calculated by HBEM and FlexPDE for even and odd modes.

p	Mode	$Z_c[\Omega]$ (HBEM)	$Z_c[\Omega]$ (FlexPDE)	Relative Deviation [%]
0.0 (isotropic)	Even	154.792	154.895	0.07
	Odd	62.185	61.881	0.49
0.1 (low BI)	Even	159.238	159.748	0.32
	Odd	63.590	63.723	0.21
0.5 (medium BI)	Even	182.267	185.780	1.89
	Odd	70.452	71.840	1.93
0.9 (high BI)	Even	222.314	228.368	2.65
	Odd	80.593	83.056	2.96

In contrast, the HBEM approach achieves comparable accuracy with a total computation time of 180 s, demonstrating a significant reduction in computational effort and highlighting the high efficiency of the proposed method.

All computations were performed in double precision on an Intel Core i5 quad-core processor operating at 3.1 GHz with 4 GB of RAM.

The distribution of the calculated normalized bound and magnetic line charges along the interface between the bi-isotropic substrate and the air are presented in Figures 5–8.

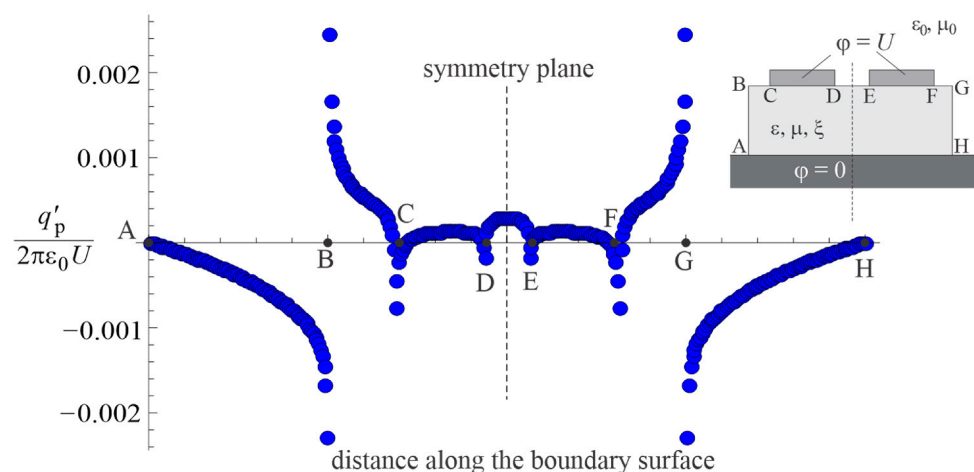


Figure 5. Normalized polarization line charge distribution along the interface (even mode).

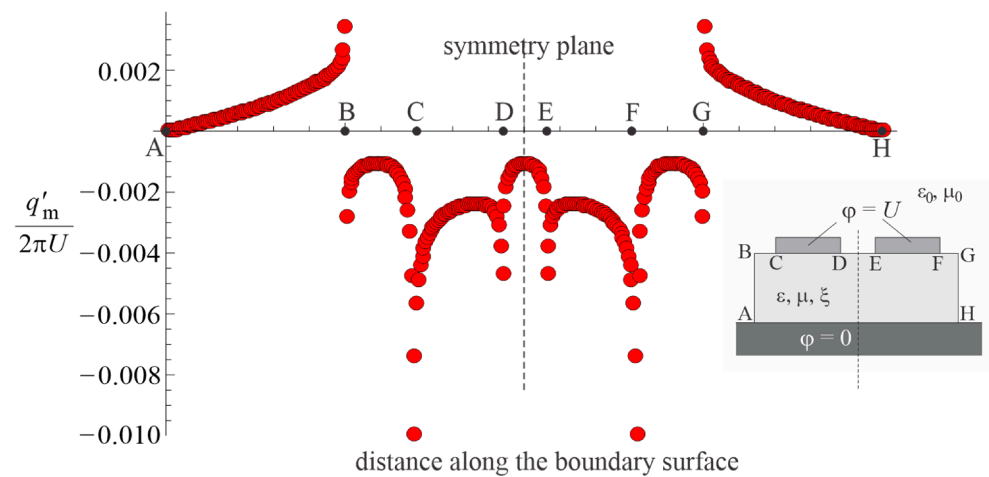


Figure 6. Normalized fictitious magnetic line charge distribution along the interface (even mode).

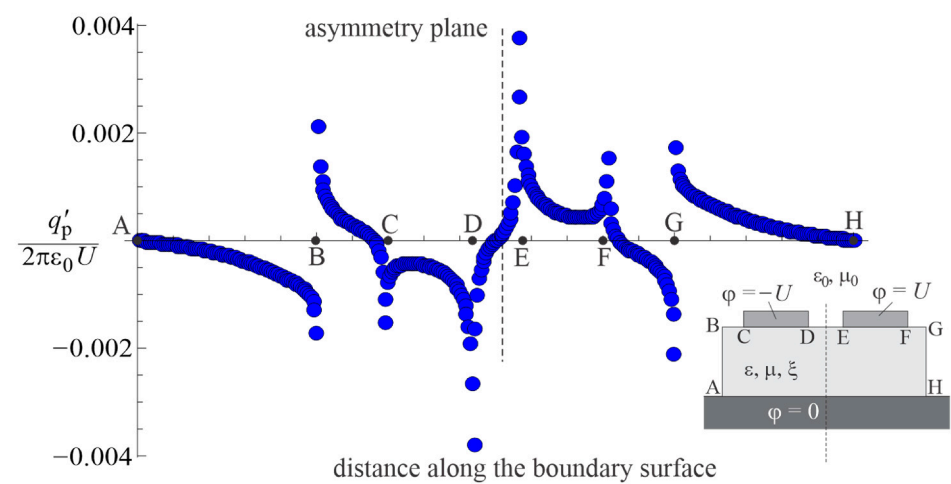


Figure 7. Normalized polarization line charge distribution along the interface (odd mode).

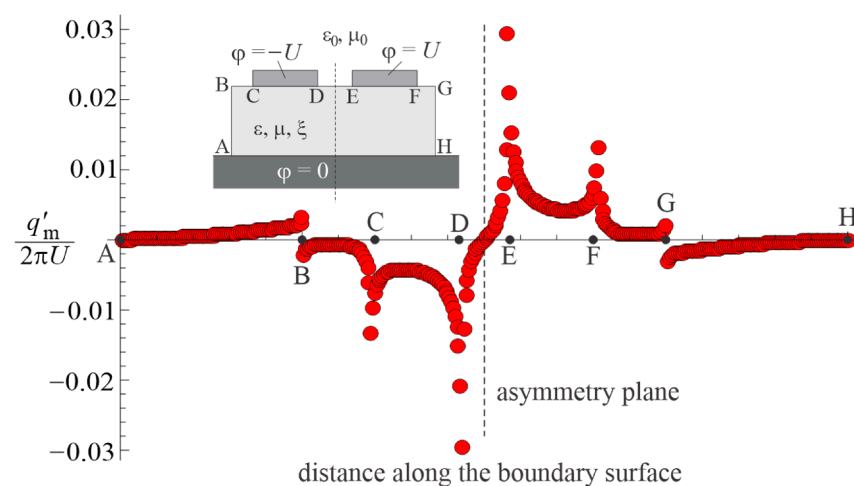


Figure 8. Normalized magnetic line charge distribution along the interface (odd mode).

The path along the interface is defined by the characteristic geometric points from A to H, which are denoted in each figure. These distributions are determined for both propagation modes, utilizing the following normalized input parameters: $a/w = 4$, $h/w = 2$, $s/w = 0.5$, $t/w = 0.1$ and $\epsilon_r = 3$, $\mu_r = 2$, $p = 0.9$, with $N_{\text{tot}} = 1000$.

As can be observed from these figures, a clear physical distinction between the two propagation regimes is demonstrated through the distributions of the equivalent charges. The charge distributions in the even mode (Figures 5 and 6) exhibit perfect symmetry with respect to the central vertical axis of the geometry. The symmetrical distributions are the consequence of the fact that the charges on both strips have the same sign, leading to a specific charge distribution at the substrate interfaces.

Conversely, the odd-mode excitation (Figures 7 and 8) introduces strictly anti-symmetric charge distributions because of the polarity inversion between the left and right conductors.

The abrupt changes and peaks in the charge distributions near the characteristic points corresponding to the strip edges clearly illustrate the edge singularity effect.

The ability of the HBEM to accurately capture these distributions between two layers confirms the physical consistency of the model before following global parametric analysis.

3.2. Parametric Study

To showcase the behaviour of the analyzed structure under different geometric and material conditions, the obtained numerical results are presented within this subsection. The choice of the material parameter ranges for the following study is selected in accordance with the values used in [11,12,31].

Figure 9 presents a comparative analysis for the characteristic impedance as a function of the normalized substrate height as well as the degree of bi-isotropy. The normalized input parameters for this analysis are: $a/w = 4$, $s/w = 0.5$, $t/w = 0.1$, $\epsilon_r = 3.25$ and $\mu_r = 2$.

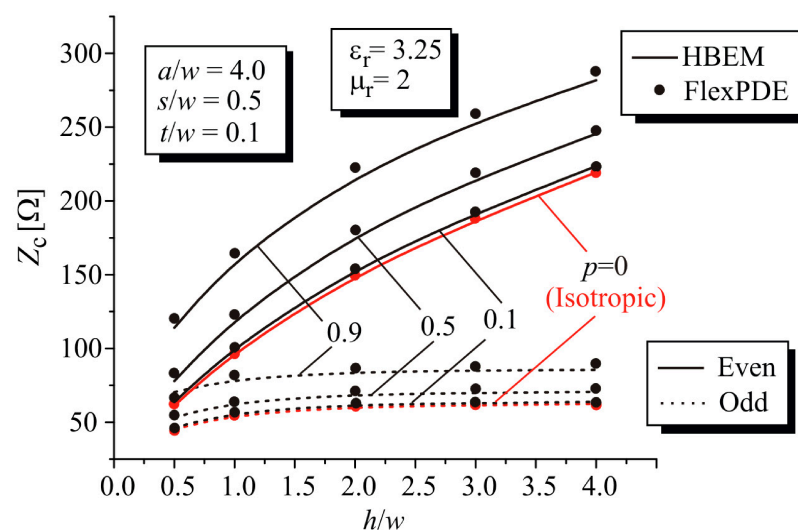


Figure 9. Comparative analysis for characteristic impedance versus substrate height for different values of parameter p .

The solid lines represent the results obtained using the HBEM, while the discrete points indicate the FlexPDE solutions. The relative deviation between the two methods is under 3% across the entire range.

The curves for both modes corresponding to $p = 0$, denoted with red lines, present the specific case of an isotropic substrate.

The characteristic impedance as a function of Tellegen parameter χ and the relative permittivity ϵ_r is illustrated in Figure 10. The normalized input parameters kept constant for this analysis are: $a/w = 4$, $h/w = 2$, $s/w = 0.5$, $t/w = 0.1$ and $\mu_r = 1$. The results are provided for both the even and odd modes.

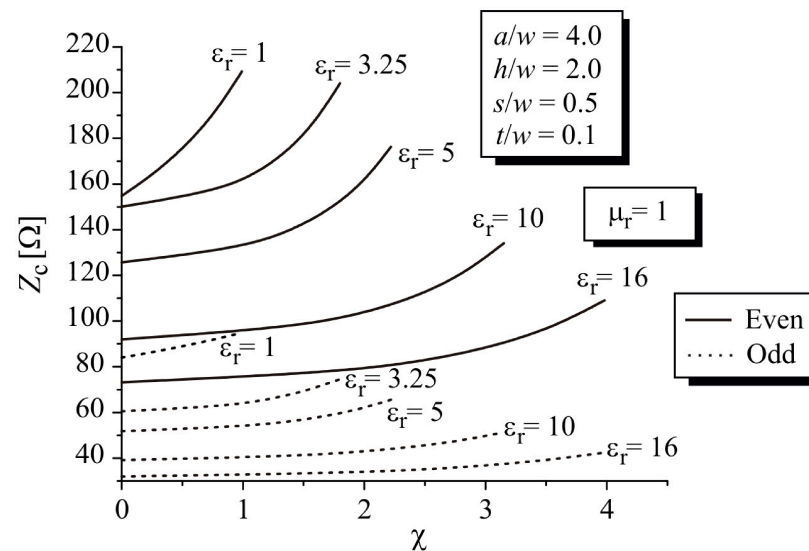


Figure 10. Characteristic impedance versus Tellegen parameter χ for various values of the relative permittivity in the even (solid lines) and odd (dashed lines) modes.

Figure 11 shows the characteristic impedance of the coupled structure as a function of the relative permittivity evaluated for several distinct values of the relative permeability. The normalized input parameters used for this analysis are: $a/w = 4$, $h/w = 2$, $s/w = 0.5$, $t/w = 0.1$ and $p = 0.5$.

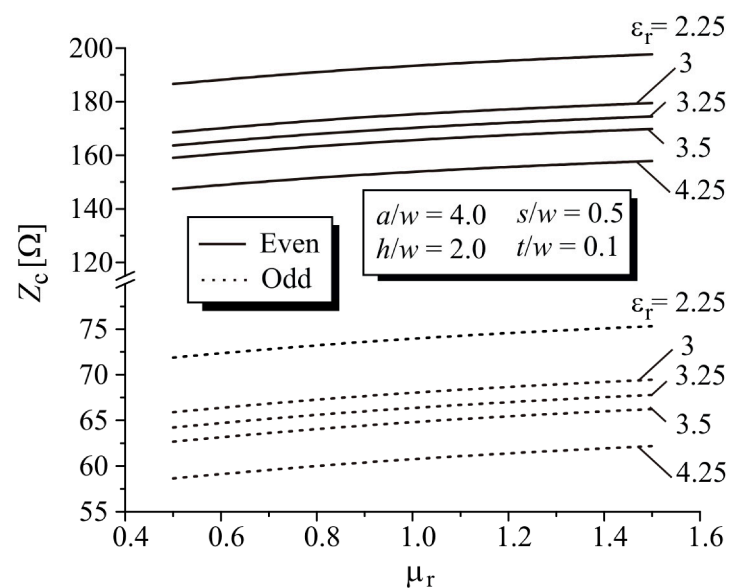


Figure 11. Characteristic impedance versus the relative permeability for various values of the relative permittivity in the even (solid lines) and odd (dashed lines) modes.

Figure 12 illustrates the variation in the characteristic impedance with respect to the gap between the strip conductors. This figure includes curves obtained for two different values of the degree of bi-isotropy.

Additionally, the results for a single microstrip (red lines) are included in the same graph to demonstrate how the modal characteristics asymptotically approach the single-conductor case as the gap expands.

The normalized input parameters used for this analysis are: $a/w = 7$, $h/w = 2$, $t/w = 0.1$ and $\epsilon_r = 3.25$, $\mu_r = 2$.

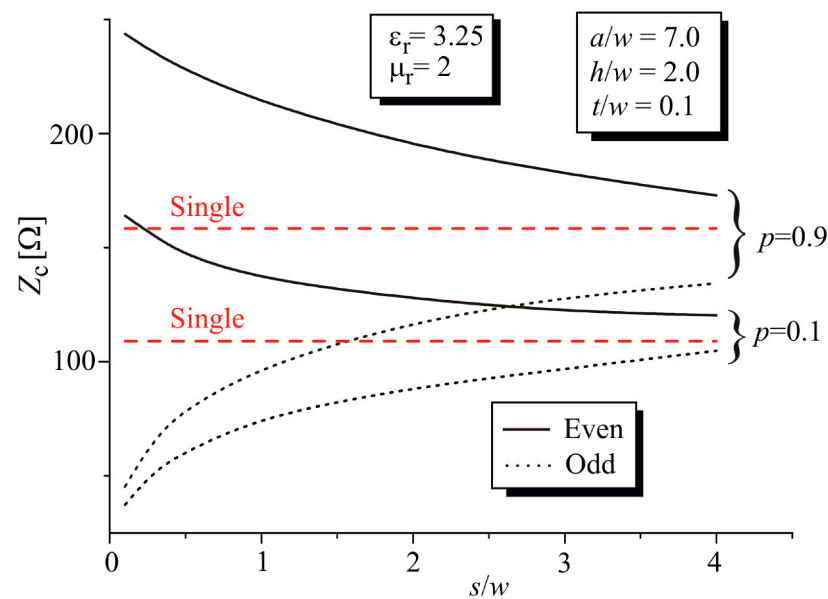


Figure 12. Characteristic impedance versus the strip distance for various values of the degree of bi-isotropy in the even (solid lines) and odd (dashed lines) modes.

4. Discussion

The results presented in Figure 9 indicate a strong correlation between the geometric ratio h/w and the characteristic impedance. As the substrate height increases, the characteristic impedance for both modes increases as well. The reference FlexPDE values validate the accuracy of obtained HBEM results, showing a relative deviation of less than 3%. Varying the bi-isotropy degree from 0 (purely isotropic case) to 0.9 reveals that the presence of magneto-electric coupling increases the characteristic impedance for both the even and odd modes.

The parametric analysis presented in Figure 10 demonstrates the mutual effects of the material parameters on the characteristic impedance. As expected, increasing the relative permittivity ϵ_r leads to a notable decrease in the characteristic impedance due to the enhanced capacitive effect. On the other hand, an increase in the Tellegen parameter introduces a counteracting effect, causing the characteristic impedance to rise for both propagation modes.

Figure 11 introduces the combined impact of ϵ_r and μ_r . The results show that increasing the relative permeability leads to a slight increase in the characteristic impedance. As already concluded from Figure 10, increasing the relative permittivity decreases the characteristic impedance for both modes.

Figure 12 illustrates the decoupling mechanism of the symmetrically coupled microstrip line as a function of the normalized gap s/w . At very small spacing, the maximum divergence between the characteristic impedance for odd and even modes is observed, marking the strongest electromagnetic coupling. As the gap increases, both modal impedances asymptotically converge towards a single, stable curve, which represents the characteristic impedance of a single microstrip line. Crucially, the degree of bi-isotropy affects the rate of convergence: under a higher value of p , the coupling diminishes more slowly. This confirms that bi-isotropy can be utilized as a design degree of freedom to control the isolation or coupling level between adjacent microstrip channels without altering their physical separation.

To understand the underlying physics of how the Tellegen parameter p influences the modal characteristic impedances, it is necessary to consider the electromagnetic field energy redistribution within the bi-isotropic medium. The non-reciprocal magneto-electric

coupling inherently couples the electric and magnetic fields. An increase in the parameter p enhances the presence of equivalent fictitious magnetic charges at the interfaces, which counteracts the primary electric displacement field. This phenomenon effectively reduces the per-unit-length capacitance C' of the coupled structure for both propagation modes. Given that the characteristic impedance is inversely proportional to the square root of the capacitance ($Z_c = \sqrt{L'/C'}$), this capacitive reduction directly leads to the observed rise in both even- and odd-mode impedances.

The values of the characteristic impedances for the even and odd modes, $Z_c^{(e)}$ and $Z_c^{(o)}$, are related through the coupling coefficient, which is determined by applying the expression [30]

$$k = \frac{Z_c^{(e)} - Z_c^{(o)}}{Z_c^{(e)} + Z_c^{(o)}}. \quad (18)$$

The results presented in this paper show that, although increasing the bi-isotropy parameter p increases the impedances in both even and odd modes, it changes them non-uniformly. Observing the mathematical relationship given in (18), it can be concluded that this non-uniform shift directly affects the coupling level. This leads to the possibility to tune the coupling coefficient without modifying the physical dimensions of the strips and the gap spacing, only by adjusting the bi-isotropy of the substrate.

To clearly show why the proposed HBEM is efficient compared to the standard finite element method (FEM) from FlexPDE, its advantages can be summarized in three main points:

- Methods like the FEM need a full-volume dense mesh over the entire space. In contrast, the HBEM only requires discretization along the boundaries and interfaces.
- Because the boundaries are only segmented and the even/odd-mode symmetry is used, the number of unknowns is minimized. As mentioned in Section 3.1, this approach solves the corresponding geometry about 8 times faster than methods that mesh the whole space.
- In the standard FEM, calculating characteristic parameters like per-unit-length capacitance requires complex integration of the energy fields over the whole cross-section. On the other hand, the HBEM directly calculates the unknown line charges, so it is possible to easily find the parameters of interest.

5. Conclusions

In this paper, the hybrid boundary element method, as an efficient semi-numerical approach, was successfully extended to the quasi-static analysis of symmetrically coupled microstrip lines on a bi-isotropic Tellegen-type substrate. By exploiting the symmetry of the system under even- and odd-mode excitations, the computational domain was strategically reduced to a single quadrant. This allows for a significant reduction in the matrix size and overall CPU time.

The developed HBEM model demonstrated numerical stability and convergence of the characteristic impedance results. Verification against the FlexPDE software confirmed close result agreement, with a maximum relative deviation of less than 3%. The presented study successfully proves the capability of the extended HBEM to handle complex coupled electromagnetic structures with bi-isotropic materials.

While the HBEM model presented in this work is very efficient for symmetric structures, its main limitation is that it requires even/odd-mode symmetry to reduce the calculation domain. Future research will focus on extending the proposed HBEM formulation to asymmetric coupled structures. For asymmetric layouts, the lack of geometric symmetry inherently leads to a larger system of linear equations with more variables. However, since

the basic boundary conditions for both isotropic and bi-isotropic media are already derived and well-known, analyzing these structures is only a computational challenge rather than a theoretical one. On the other hand, extending this method to anisotropic media will be a significant theoretical challenge. This is because we will need to derive completely new, complex boundary conditions relations for the HBEM procedure.

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Abbreviations

The following abbreviations are used in this manuscript:

HBEM	hybrid boundary element method
FEM	finite element method
EEs	equivalent electrodes

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