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On the Physical Nature of the Scalar Mode Mass in the Jordan Frame of Metric $f(R)$ Gravity

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Abstract

We analyze the Taylor expansion of the metric $f(R)$ gravity in the Jordan frame around the General Relativity limit, expanding in the small deviation $(\phi - \phi_0)$ with $\phi_0 = 1$. By relating the scalar–tensor representation to the original $f(R)$ formulation, we derive constraints on the expansion parameters from the observed value of the present-day ΛCDM (Λ Cold Dark Matter) deceleration parameter and from cosmological bounds on the variation of Newton's constant. We show that these requirements imply that the scalar degree of freedom must have a mass exceeding the Hubble scale by several orders of magnitude. This result challenges the common assumption that the scalar mode can drive cosmological dynamics with a mass of order of the Hubble constant H_0. We provide a dynamical interpretation of this hierarchy by emphasizing that a proper definition of the scalar mass, in a field-theoretical sense, requires an adiabatic separation between background evolution and perturbations, which naturally leads to a super-Hubble mass scale.

Keywords: $f(R)$ gravity; Jordan frame; scalar field mass; Hubble tension

1. Introduction

A natural extension of General Relativity is provided by the so-called $f(R)$ gravity theories [1–4]. Such models are constructed by a modification of the Einstein–Hilbert action, in which the Ricci scalar R is replaced by a generic function $f(R)$. This procedure leads to a rich phenomenology that is absent in General Relativity [5–9]. It is well known that $f(R)$ gravity theories are equivalent to scalar-tensor theories, in which the scalar field couples either to the metric tensor (Jordan frame) or to the matter sector (Einstein frame). In this work, we will use only metric $f(R)$ gravity in the Jordan frame, in which the self-interacting scalar field is non-minimally coupled to the metric tensor, as described in [1,10].

Chronologically, metric $f(R)$ gravity was first investigated in the context of early universe cosmology, in particular to address the Big Bang singularity [11], and later applied to spherically symmetric black hole spacetimes to regularize central curvature singularities [12].

Recently, after the discovery of the late-Universe acceleration [13,14], $f(R)$ gravity has gained increasing interest, as these theories can effectively reproduce the phenomenology of a negative-pressure fluid through a purely geometric modification of gravity [15]. For the metric-affine formulation see also [16,17].

A simple and well-known example of an $f(R)$ theory is the so-called R^2 gravity [10,18–20], originally introduced in [11,21]. In particular, if we also add a cosmological

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constant term in the R^2 gravity action, we are de facto looking at the first three terms of a Taylor expansion of the generic function $f(R)$ at sufficiently low curvatures. Two fundamental requirements naturally arise when modifying General Relativity:

- Any correction to the standard gravity theory must be in agreement, within experimental uncertainties, with the current tests of GR, the most stringent among which are the Solar System tests [22].
- Any new theory of gravity has to account for the late-time accelerated expansion of the Universe, which takes place at large scales. To do so, the theory introduces corrections that become important at low values of R. See, for example, the Hu–Sawicki model [23].

A further challenge in $f(R)$ gravity is the so-called "Hubble tension", i.e., the 4σ discrepancy in the detections of the Hubble constant value from different sources. On the one hand, the Supernova H0 for the Equation of State (SH_0ES) collaboration [24] that measures H_0 from the supernova data at low redshift z ($z \sim 0.01$–0.1); while on the other hand the Planck satellite collaboration [25] infers H_0 from the CMB measurements at $z \sim 1100$. For a comprehensive review of the various determinations of the Hubble constant, see [26,27]. Many theoretical models have been proposed to alleviate the Hubble tension, with promising results, see, for instance, refs. [5–7,15,28,29]. See also [3,4,30–34].

The capacity of metric $f(R)$ gravity to account for both the Universe acceleration and the Hubble tension does not come without limitations. To satisfy Solar System tests, the mass of the scalar degree of freedom must be sufficiently large to suppress fifth-force effects. Moreover, different experiments [35,36] impose strong local constraints on the value of the Newton constant G, and hence on the coefficient of the term proportional to R in the Taylor series expansion of $f(R)$ at low curvatures R_0. In particular, $|\frac{\Delta G}{G}| \leq 10^{-5}$ [35,36] (ΔG is the maximum variation of the Newton constant according to the local tests). Although we assume G to be constant, hence independent from the place in which it is measured, we prefer to use cosmological constraints on it as a consequence of the fact that we are in cosmological regimes. Hence, for the constraint on the Newtonian constant, we use the results from [37], according to which $\frac{\Delta G}{G} \leq 3\%$. This assumption fixes $|f'(R_0) - 1| \leq 3 \cdot 10^{-2}$. In Table 1 we summarize all the cosmological constraints we use in this manuscript.

Table 1. Cosmological constraints used in this work.

Quantity	Constraint	Reference
Hubble constant H_0	$\sim$70 km s^{-1}Mpc^{-1}	[24,25]
Deceleration parameter q_0	$\simeq -0.55$	[38]
Variation of Newton constant $\Delta G/G$	$\leq$3%	[37]

To simultaneously satisfy local (Solar System) constraints and produce effects at cosmological scales, a possible explanation is that the scalar field undergoes a screening mechanism. Among those proposed, the so-called "chameleon behavior" [39–46] has received considerable attention. In this mechanism, the mass of the scalar mode depends on the local energy density: in other words, the scalar field mass decreases as the spatial scale increases. Consequently, within the high-density environment of the Solar System, the Compton wavelength of the scalar field is short, in such a way that the field becomes effectively non-interacting. On the contrary, in the low-density cosmological environment, the scalar field acquires a mass of order H_0 (i.e., a Compton scale of the Boson field of the order of the Hubble scale), allowing it to drive the accelerated expansion of the Universe.

Although such a formulation could address several theoretical issues, a complete physical and geometrical model that naturally leads to the chameleon mechanism is still lacking (see [47]).

The relation between the scalar field mass, local gravity constraints and cosmological dynamics in $f(R)$ gravity has been widely investigated in the literature. In particular, several studies have analyzed the compatibility between modified gravity models and Solar System experiments by invoking screening mechanisms such as the chameleon effect. For instance, the analysis in [44–46] explores specific functional forms of $f(R)$ (such as Hu–Sawicki or power-law models) and constrains their parameters using a combination of local gravity tests, cosmological evolution and astrophysical observations.

In these works the recovery of General Relativity typically relies on the presence of a screening mechanism that suppresses the scalar interaction in high-density environments. However, the scalar mass itself is not derived from purely cosmological consistency conditions, but is instead determined by the environmental dependence introduced by the screening mechanism.

From the point of view of the underlying theory, however, this mechanism effectively introduces a coupling between the scalar field dynamics and the environmental matter density [47]. Such a coupling weakens the purely geometrical interpretation of the $f(R)$ Lagrangian, since the scalar sector acquires an effective interaction with the matter content of spacetime. In this respect, the result obtained in the present work is related to, but conceptually different from, the chameleon scenario. In fact, our constraint on the scalar field mass is derived entirely from cosmological considerations, namely from the observed value of the deceleration parameter and from bounds on the variation of Newton's constant inferred from cosmological observations. Therefore, the requirement that the scalar mass must largely exceed the Hubble scale emerges independently of Solar System tests and provides a direct consistency condition on the cosmological dynamics of analytic $f(R)$ models.

In this paper, we focus our attention on a related question: is it possible to reconcile the deceleration parameter q_0 of the ΛCDM model with a scalar field mass of order H_0, while simultaneously satisfying the experimental constraints on the variation of Newton's constant ΔG?

Our analysis shows that, in order to satisfy these constraints and remain consistent with the observed value of the deceleration parameter, the scalar mode mass must exceed the Hubble scale by roughly three (five if we consider local measurements of the Newtonian constant instead of cosmological ones) orders of magnitude. Furthermore, we provide a physical explanation for this result that is consistent with the standard interpretation of mass terms in classical field theory [48].

2. Scalar Field Dynamics and Constraints

We begin by recalling that metric $f(R)$ gravity can be written in a scalar–tensor form in the Jordan frame. The action takes the form

$$\mathcal{S} = \frac{1}{2\chi}\int d^4x\,\sqrt{-g}[\phi R - V(\phi)] + S_m, \tag{1}$$

where χ denotes the Einstein constant, g is the determinant of the metric tensor, R is the Ricci scalar, and S_m represents the matter action. The scalar field ϕ corresponds to the additional dynamical degree of freedom introduced by the modification of gravity and is defined through

$$\phi \equiv \frac{df}{dR}. \tag{2}$$

A basic viability requirement for a given $f(R)$ model is that the scalar mode possesses a positive effective mass. In the scalar–tensor representation this mass is defined as

$$m_\phi^2 \equiv \frac{1}{3}\left(\phi\frac{d^2V}{d\phi^2} - \frac{dV}{d\phi}\right), \tag{3}$$

which guarantees the stability of the scalar degree of freedom.

Variation of the action with respect to the metric tensor provides the modified Einstein equations. Taking their trace and using the relation $R = dV/d\phi$, one obtains a Klein–Gordon-like equation for the scalar field. In the presence of matter sources, the trace of the energy–momentum tensor contributes as a source term for the scalar field dynamics.

We consider a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime with a line element [49–52], according to the Planck measurements [53],

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j\,, \tag{4}$$

where $a(t)$ is the cosmic scale factor and t denotes synchronous time.

In this background, the modified Friedmann equations in metric $f(R)$ gravity, as viewed in the so-called "Jordan frame" [1], read as follows:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{\phi}\left(\frac{\chi}{3}\rho_m - H\dot{\phi} + \frac{V(\phi)}{6}\right) \tag{5}$$

and

$$\frac{dV}{d\phi} = 12H^2 + 6\dot{H}\,, \tag{6}$$

where χ is the Einstein constant (we are in $c = 1$ units), the dot indicates differentiation with respect to t, and ϕ denotes the non-minimally coupled scalar field emerging in the Jordan frame, whose self-interaction is described by the potential term $V(\phi)$. Finally, the quantity ρ_m stands for the energy density of the (dark and baryonic) matter component of the Universe.

The first of these equations is the generalized Friedmann equation, while the second one is obtained by varying the gravitational action with respect to the scalar field ϕ.

Now, passing to the time variable $x = \ln(1+z) = \ln(1/a)$ (z denoting the redshift and we set the present-day value of the cosmic scale factor to unity, i.e., $a_0 \equiv a(z=0) = 1$), the two equations above can be rewritten as

$$H^2 = \frac{1}{\phi - \frac{d\phi}{dx}}\left(\frac{\chi}{3}\rho_m(x) + \frac{V(\phi(x))}{6}\right) \tag{7}$$

and

$$\frac{dV}{d\phi} = 12H^2 - 3\frac{dH^2}{dx}\,, \tag{8}$$

respectively. Above, we made use of the relation $\dot{(\dots)} = -Hd(\dots)/dx$.

We now intend to study Equations (7) and (8) very close to us, i.e., for $x \to 0$. To this end, we perform a Taylor expansion of the scalar field dependence for $\phi \simeq \phi_0 \equiv \phi(x=0)$, so that

$$V(\phi) \simeq V(\phi_0) + \left(\frac{dV}{d\phi}\right)_{\phi=\phi_0}(\phi-\phi_0) + \tag{9}$$

$$\frac{1}{2}\left(\frac{d^2V}{d\phi^2}\right)_{\phi=\phi_0}(\phi-\phi_0)^2 + \dots$$

$$= V_0 + \bar{V}(\phi-\phi_0) + \frac{1}{2}V^*(\phi-\phi_0)^2 + \dots \tag{10}$$

The relation between the potential $V(\phi)$ and the underlying $f(R)$ Lagrangian is dictated via the algorithm

$$R = \frac{dV}{d\phi} \quad , f(R) \equiv \phi \frac{dV}{d\phi} - V(\phi)\,, \tag{11}$$

where the function $\phi(R)$ in the right-hand-side of the second equation above is obtained by assuming the invertibility of $dV/d\phi$ in the first one.

By implementing Equation (11) to calculate the $f(R)$ expression, that is associated with the potential term in Equation (10), the first derivative gives

$$\frac{dV}{d\phi} = \bar{V} + V^*(\phi - \phi_0) + \cdots = R\,, \tag{12}$$

which can be inverted at leading order to yield

$$\phi - \phi_0 \simeq \frac{R - \bar{V}}{V^*}. \tag{13}$$

Plugging this into the definition

$$f(R) = \phi R - V(\phi), \tag{14}$$

we obtain the following step by step:

$$f(R) \simeq (\phi_0 + (\phi - \phi_0))R - \left[V_0 + \bar{V}(\phi - \phi_0) + \frac{1}{2}V^*(\phi - \phi_0)^2\right] \tag{15}$$

$$= -V_0 + \left(\phi_0 - \frac{\bar{V}}{V^*}\right)R + \frac{1}{2V^*}R^2 + \frac{\bar{V}^2}{2V^*} + \ldots \tag{16}$$

This result directly follows from the Taylor expansion around the General Relativity limit $\phi_0 = 1$ and illustrates how the parameters V_0, $\bar{V}$, and V^* control the deviation from standard GR.

In order to restore the Einsteinian gravity in the low-curvature limit, we require the following conditions on the expansion parameters:

$$\phi_0 = 1, \quad V_0 \equiv \bar{V} \equiv 0, \quad V^* \gg H_0^2, \tag{17}$$

where $\phi_0 = 1$ ensures that the effective Newton constant matches its standard value, and V_0 and $\bar{V}$ vanish to avoid introducing a cosmological term beyond the one we explicitly include if desired. The condition $V^* \gg H_0^2$ guarantees a sufficiently heavy scalar mode to satisfy local and cosmological constraints. These conditions, applied to Equation (16), restore GR (i.e., $f(R) = R$).

However, we can also retain a cosmological constant term by defining $V_0 \equiv 2\chi\rho_\Lambda$ (ρ_Λ being the vacuum energy density).

Evaluating Equation (8) in $x = 0$ (i.e., $\phi = \phi_0$) according to the expansion in Equation (10), we obtain

$$\bar{U} \equiv \frac{\bar{V}}{6H_0^2} = 2 - \frac{1}{2H_0^2}\left(\frac{dH^2}{dx}\right)_{x=0} = 1 - q_0\,, \tag{18}$$

where we introduced the deceleration parameter q_0 of the Universe, defined as in [38]:

$$q_0 = -1 + \frac{1}{H_0}\left(\frac{dH}{dx}\right)_{x=0}. \tag{19}$$

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Since we have $q_0 \simeq -0.55$ for the ΛCDM model, we should have

$$\bar{V} \simeq 9.3 \times H_0^2 \simeq 10^{-52}\,\mathrm{m}^{-2} \tag{20}$$

at cosmological scales. The situation does not change for models that slightly deviate from the standard cosmological one, such as those in which the Hubble tension [26,54] is addressed through small changes in the matter [55,56] or gravity sector of the theory.

Now, from the measurements of the Newton constant [37], the coefficient multiplying the Ricci scalar in the effective Einstein–Hilbert term cannot deviate significantly from unity. In metric $f(R)$ gravity this coefficient is given by $f'(R)$, which plays the role of an effective gravitational coupling. Cosmological observations constrain its present value to satisfy

$$|f'(R_0) - 1| \lesssim 3 \times 10^{-2}. \tag{21}$$

Using the expansion obtained in Equation (16) and evaluating it around the present epoch $\phi_0 \equiv 1$, we have

$$f'(R_0) = \phi_0 - \frac{\bar{V}}{V^*}. \tag{22}$$

Therefore the observational constraint on the variation of Newton's constant implies

$$\left|\frac{\bar{V}}{V^*}\right| \lesssim 3 \times 10^{-2}. \tag{23}$$

Since Equation (20) gives $\bar{V} \simeq 10^{-52}\,\mathrm{m}^{-2}$, the previous relation immediately yields the bound

$$V^* \gtrsim 10^{-50}\,\mathrm{m}^{-2}. \tag{24}$$

The scalar field mass, as defined in [57], takes, at $x = 0$, the form

$$m_\phi^2 = \frac{1}{3}(V^* - \bar{V}) \simeq \frac{V^*}{3} > 10^{-50}\,\mathrm{m}^{-2} \gg H_0^2. \tag{25}$$

The result obtained above follows from the combination of three independent ingredients:

(i) The observed value of the present-day deceleration parameter $q_0 \simeq -0.55$, which fixes the linear coefficient of the potential expansion through Equation (18), yielding $\bar{V} \sim 10^{-52}\,\mathrm{m}^{-2}$.
(ii) The observational constraint on the variation of the Newton constant, which requires $|f'(R_0) - 1| \lesssim 3 \times 10^{-2}$ and therefore implies $V^* \gtrsim 10^{-50}\,\mathrm{m}^{-2}$ from Equation (16).
(iii) The definition of the scalar-field mass in metric $f(R)$ gravity, $m_\phi^2 = \frac{1}{3}(V^* - \bar{V})$.

Combining these relations leads to

$$m_\phi^2 \gtrsim 10^{-50}\,\mathrm{m}^{-2} \gg H_0^2,$$

which implies that the quadratic scalar field mass is roughly 10^2 times larger than the Hubble scale.

This result is in apparent contradiction with the common expectation that the non-minimally coupled scalar field driving cosmological dynamics should have a Compton wavelength of the order of the Hubble length [39,42,58].

Actually, many metric $f(R)$ models, providing valuable cosmological phenomenology [5–7,23], work only if the value of the scalar field mass is much smaller than required by Solar System tests [10].

However, to address the present discrepancy, we briefly reconsider the derivation of the scalar-field mass in metric $f(R)$ gravity, following the procedure discussed in [57].

On a flat isotropic cosmological background, the Klein–Gordon equation of the non-minimally coupled scalar field emerging in the Jordan frame reads as

$$3\ddot{\phi} + 9H\dot{\phi} + 2V - \phi\frac{dV}{d\phi} = \chi\rho_m. \tag{26}$$

To identify the mass of the scalar degree of freedom, one expands the field around a background cosmological solution $\phi_0(t)$ and introduces a small perturbation $\delta\phi(t)$,

$$\phi(t) = \phi_0(t) + \delta\phi(t), \tag{27}$$

with $|\delta\phi| \ll \phi_0$. Substituting this decomposition into the equation above and expanding to first order in $\delta\phi$, one obtains two equations: a background equation for ϕ_0 and a linearized equation governing the perturbation.

The background dynamics is described by

$$3\ddot{\phi}_0 + 9H\dot{\phi}_0 + 2V(\phi_0) - \phi_0\left(\frac{dV}{d\phi}\right)_{\phi_0} = \chi\rho_m, \tag{28}$$

while the perturbation satisfies

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + m_\phi^2\,\delta\phi = 0, \tag{29}$$

which has the form of a Klein–Gordon equation for a massive scalar mode. The corresponding mass term is therefore

$$m_\phi^2 = \frac{1}{3}\left[\phi_0\left(\frac{d^2V}{d\phi^2}\right)_{\phi_0} - \left(\frac{dV}{d\phi}\right)_{\phi_0}\right]. \tag{30}$$

The expression above, for the expansion in Equation (10) and $\phi_0 = 1$ yields Equation (25).

Another consequence can be obtained taking the second derivative of the potential with respect to the field (from Equation (6)):

$$\frac{d^2V}{d\phi^2} = 24H\frac{dH}{d\phi} - 6\frac{dH}{d\phi} - 6H\frac{d^2H}{d\phi dx} = \frac{6}{\frac{d\phi}{dx}}\left[4H\frac{dH}{dx} - \left(\frac{dH}{dx}\right)^2 - H\frac{d^2H}{dx^2}\right]. \tag{31}$$

Using now the well-known relations

$$\frac{dH}{dz} = \frac{H(1+q)}{1+z}, \tag{32}$$

$$\frac{d^2H}{dz^2} = \frac{H}{(1+z)^2}(J - q^2); \tag{33}$$

and inserting (31) in the inequality of Equation (25),

$$\frac{V^*}{3} \equiv \frac{1}{3}\frac{d^2V}{d\phi^2}|_{x=0} > 10^{-50}\,\mathrm{m}^{-2} \tag{34}$$

produces the following result:

$$\frac{d\phi}{dx}|_{x=0} < 2\cdot 10^{50}(3 + 2q_0 - J_0)H_0^2\,\mathrm{m}^2. \tag{35}$$

Now, using the cosmographic parameters from the ΛCDM model ($q_0 = -0.55$, $J_0 = 1$), the inequality (35) becomes

$$\frac{d\phi}{dx}|_{x=0} < 10^{-2}. \tag{36}$$

This result has a profound consequence on the value of the mass of the scalar field, in particular on its divergence in $z = 0$, as it can be seen in [6].

An important point here is that the expression of the mass in Equation (30) makes sense in Equation (29) only if the perturbation $\delta\phi$ is varying on a much faster time scale with respect to that one of the background variation (see. for reference, [9]). In fact, in this case, the value of the mass, at a given instant, can be regarded as frozen and Equation (29) becomes a real Klein–Gordon equation on an adiabatically varying cosmological background.

In this respect, our estimates, based on cosmological bounds on the variation of Newton's constant, suggest that the time scale over which the massive scalar field formulation evolves is, at least, 10^2 times shorter than the Hubble time and that it seems meaningless to discuss the non-minimally coupled scalar field mass on a much larger scale. In other words, we infer that, at the Hubble scale, ϕ is just a non-trivial scalar mode, coupled to gravity and no real concept of physical mass can be attributed to it. Different would be the story for a fast scalar fluctuation, which, instead, behaves, on a linear approach, as a massive mode. This consideration acquires particular interest in the spirit of the analysis in [59], where the possible decaying process of a fast massive fluctuation of ϕ into dark matter particles has been studied, although the formulation was limited by including a phenomenological decaying rate (see also [28]).

As an illustrative example, let us consider the logarithmic model proposed in [60], defined by the modified $f(R)$ Lagrangian

$$f(R) = -\frac{1}{\beta}\ln(1-\beta R), \tag{37}$$

where β is a free parameter of the model with the dimension of a squared length. Local gravity constraints require the bound $\beta < 6 \times 10^{-6}\,\mathrm{cm}^2$.

Expanding Equation (37) in the regime $\beta R \ll 1$, one obtains

$$f(R) \simeq R + \frac{\beta}{2}R^2 + \mathcal{O}(R^3). \tag{38}$$

Comparing this expansion with the general form obtained from the Taylor expansion of the potential in Equation (16), we immediately obtain the relations

$$\bar{V} \equiv 0, \qquad V_0 = V^* = \frac{1}{\beta}. \tag{39}$$

Using Equation (18), which relates $\bar{V}$ to the present-day value of the deceleration parameter, we see that the condition $\bar{V} = 0$ implies

$$q_0 = 1. \tag{40}$$

This result clearly contradicts the observational value $q_0 \simeq -0.55$. Therefore, the logarithmic model cannot reproduce the present cosmic acceleration within the perturbative expansion around the General Relativity limit discussed above. The accelerated expansion found in [60] must then arise as a genuinely non-perturbative effect, in which contributions from all orders of the expansion become relevant.

The example of a logarithmic Lagrangian, as that one discussed above, is paradigmatic of the applicability of our constraint to extract physical information on the cosmological dynamics predicted by a given model. However, it is worth stressing that this situation does not apply to non-analytic $f(R)$ Lagrangians, i.e., theories that cannot be expanded

in Taylor series around $R = 0$. In such cases the perturbative procedure adopted in the present work is not applicable and the corresponding cosmological dynamics must be studied using different techniques [61–64].

Similarly, the present analysis is not directly relevant for those $f(R)$ models whose structure implies $V^* \equiv 0$ in the scalar–tensor representation, corresponding to a vanishing scalar field mass. This situation occurs, for instance, in models such as the Hu–Sawicki construction [23], where the recovery of General Relativity relies on screening mechanisms rather than on a perturbative expansion around the GR limit.

Furthermore, from the general expression of the scalar field mass given in Equation (30), we obtain

$$m_\phi^2 = \frac{1}{3\beta}. \tag{41}$$

If we require the adiabaticity condition discussed above, namely that the scalar perturbation evolves on a timescale much shorter than the cosmological background evolution, we must impose

$$\beta \ll H_0^{-2}. \tag{42}$$

This condition is fully consistent with the local experimental bound on β and confirms that the scalar mode behaves as a very massive field compared to the Hubble scale.

3. Conclusions

In this work we have analyzed the Taylor expansion of metric $f(R)$ gravity in the Jordan frame around the General Relativity limit at the present cosmological epoch. Assuming that General Relativity must be recovered as the low-curvature limit of the theory, we expanded the scalar potential around $\phi_0 = 1$ and imposed two independent cosmological constraints: the observed value of the present-day deceleration parameter and the cosmological bound on the variation of Newton's constant.

The combination of these requirements leads to a non-trivial consistency condition on the expansion parameters of the theory. In particular, we have shown that the scalar degree of freedom necessarily acquires a mass much larger than the Hubble scale. Even adopting conservative cosmological bounds on $\Delta G/G$, the resulting scalar mass squared exceeds H_0^2 by several orders of magnitude. If one were to impose the more stringent Solar System limits on the variability of Newton's constant, the hierarchy would become even stronger, yielding $m_\phi^2 > 10^5 H_0^2$.

It is important to stress the domain of validity of this analysis. Our result applies to analytic $f(R)$ models that admit General Relativity as an expansion limit around $\phi_0 = 1$. Well-known constructions such as the Hu–Sawicki model or other non-analytic formulations do not reproduce General Relativity through a regular Taylor expansion around this point. In those scenarios, the recovery of the GR limit typically relies on additional dynamical mechanisms, such as chameleon-type screening, which are introduced to ensure compatibility with local gravity tests.

Therefore, our conclusion does not directly constrain such non-analytic models (i.e., models that cannot be Taylor expanded near $\phi - \phi_0$). Rather, it shows that within analytic metric $f(R)$ theories admitting GR as a perturbative limit at the present epoch, observational consistency alone forces a super-Hubble scalar mass scale.

Finally, we have emphasized that the notion of scalar mass in a cosmological background requires an adiabatic separation between background evolution and perturbations. When this condition is properly taken into account, the emergence of a super-Hubble

mass scale appears as a natural consequence of the consistency requirements imposed by observations.

A key novel aspect of our analysis is that, unlike the chameleon mechanism, the requirement for the scalar field to have a large mass emerges solely from cosmological constraints, independent of local Solar System tests. This shows that the mass hierarchy is a consequence of the large-scale cosmological environment and not of local density effects, providing a new perspective on scalar field dynamics. This finding strengthens our results, highlighting that the super-Hubble scalar mass is an intrinsic cosmological feature rather than a consequence of screening mechanisms.

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References

1. Sotiriou, T.P.; Faraoni, V. f(R) Theories of Gravity. *Rev. Mod. Phys.* **2010**, *82*, 451–497. [CrossRef]
2. Capozziello, S.; Cardone, V.F.; Elizalde, E.; Nojiri, S.; Odintsov, S.D. Observational constraints on dark energy with generalized equations of state. *Phys. Rev. D—Part. Fields Gravit. Cosmol.* **2006**, *73*, 043512. [CrossRef]
3. Odintsov, S.D.; Sáez-Chillón Gómez, D.; Sharov, G.S. Analyzing the H_0 tension in $F(R)$ gravity models. *Nucl. Phys. B* **2021**, *966*, 115377. [CrossRef]
4. Odintsov, S.D.; Oikonomou, V.K.; Sharov, G.S. Early dark energy with power-law F(R) gravity. *Phys. Lett. B* **2023**, *843*, 137988. [CrossRef]
5. Montani, G.; De Angelis, M.; Bombacigno, F.; Carlevaro, N. Metric $f(R)$ gravity with dynamical dark energy as a scenario for the Hubble tension. *Mon. Not. R. Astron. Soc. Lett.* **2024**, *527*, L156–L161. [CrossRef]
6. Valletta, A.; Montani, G.; Dainotti, M.G.; Fazzari, E. On the Metric $f(R)$ gravity Viability in Accounting for the Binned Supernovae Data. *arXiv* **2025**, arXiv:2512.19568. [CrossRef]
7. Schiavone, T.; Montani, G.; Bombacigno, F. $f(R)$ gravity in the Jordan frame as a paradigm for the Hubble tension. *Mon. Not. R. Astron. Soc. Lett.* **2023**, *522*, L72–L77. [CrossRef]
8. Montani, G.; Carlevaro, N.; Dainotti, M.G. Running Hubble constant: Evolutionary Dark Energy. *Phys. Dark Universe* **2025**, *48*, 101847. [CrossRef]
9. Olmo, G.J. The Gravity Lagrangian according to solar system experiments. *Phys. Rev. Lett.* **2005**, *95*, 261102. [CrossRef]
10. Capozziello, S.; De Laurentis, M. Extended theories of gravity. *Phys. Rep.* **2011**, *509*, 167–321. [CrossRef]
11. Starobinsky, A.A. A new type of isotropic cosmological models without singularity. *Phys. Lett. B* **1980**, *91*, 99–102. [CrossRef]
12. Elizalde, E.; Nojiri, S.; Odintsov, S.; Sebastiani, L.; Zerbini, S. Nonsingular exponential gravity: A simple theory for early- and late-time accelerated expansion. *Phys. Rev. D—Part. Fields Gravit. Cosmol.* **2011**, *83*, 086006. [CrossRef]
13. Riess, A.G.; Filippenko, A.V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P.M.; Gilliland, R.L.; Hogan, C.J.; Jha, S.; Kirshner, R.P.; et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.* **1998**, *116*, 1009–1038. [CrossRef]
14. Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R.A.; Nugent, P.; Castro, P.G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D.E.; et al. Measurements of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.* **1999**, *517*, 565–586. [CrossRef]
15. Nojiri, S.; Odintsov, S.D. Unified cosmic history in modified gravity: From F(R) theory to Lorentz non-invariant models. *Phys. Rept.* **2011**, *505*, 59–144. [CrossRef]
16. Olmo, G.J. Palatini approach to modified gravity: F (R) theories and beyond. *Int. J. Mod. Phys. D* **2011**, *20*, 413–462. [CrossRef]
17. Bombacigno, F.; Boudet, S.; Olmo, G.J.; Montani, G. Big bounce and future time singularity resolution in Bianchi I cosmologies: The projective invariant Nieh-Yan case. *Phys. Rev. D* **2021**, *103*, 124031. [CrossRef]

18. Hell, A.; Lust, D.; Zoupanos, G. On the degrees of freedom of R^2 gravity in flat spacetime. *J. High Energy Phys.* **2024**, *2*, 39. [CrossRef]
19. Hell, A.; Lust, D. Conformal and pure scale-invariant gravities in d dimensions. *J. High Energy Phys.* **2025**, *9*, 202. [CrossRef]
20. Hell, A.; Ferreira, E.G.M.; Lust, D.; Sasaki, M. The recipe for the degrees of freedom. *arXiv* **2026**, arXiv:2601.10288. [CrossRef]
21. Starobinsky, A.; Schmidt, H.J. On a general vacuum solution of fourth-order gravity. *Class. Quantum Gravity* **1987**, *4*, 695. [CrossRef]
22. Barrow, J.D.; Ottewill, A.C. The stability of general relativistic cosmological theory. *J. Phys. A Math. Gen.* **1983**, *16*, 2757. [CrossRef]
23. Hu, W.; Sawicki, I. Models of $f(R)$ cosmic acceleration that evade solar system tests. *Phys. Rev. D—Part. Fields Gravit. Cosmol.* **2007**, *76*, 064004. [CrossRef]
24. Riess, A.G.; Yuan, W.; Macri, L.M.; Scolnic, D.; Brout, D.; Casertano, S.; Jones, D.O.; Murakami, Y.; Anand, G.S.; Breuval, L.; et al. A comprehensive measurement of the local value of the Hubble constant with 1 km s^{-1} Mpc^{-1} uncertainty from the Hubble Space Telescope and the SH0ES team. *Astrophys. J. Lett.* **2022**, *934*, L7. [CrossRef]
25. Collaboration, P. Planck 2018 results. VI. Cosmological parameters. *Astron. Astrophys.* **2018**, *641*, A6.
26. Di Valentino, E.; Mena, O.; Pan, S.; Visinelli, L.; Yang, W.; Melchiorri, A.; Mota, D.F.; Riess, A.G.; Silk, J. In the realm of the Hubble tension—A review of solutions. *Class. Quantum Gravity* **2021**, *38*, 153001. [CrossRef]
27. Abdalla, E.; Abellán, G.F.; Aboubrahim, A.; Agnello, A.; Akarsu, Ö.; Akrami, Y.; Alestas, G.; Aloni, D.; Amendola, L.; Anchordoqui, L.A.; et al. Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies. *J. High Energy Astrophys.* **2022**, *34*, 49–211. [CrossRef]
28. Montani, G.; De Angelis, M.; Dainotti, M.G. Decay of dark energy into dark matter in a metric $f(R)$ gravity: Effective running Hubble constant. *Phys. Dark Universe* **2025**, *49*, 101969. [CrossRef]
29. Montani, G.; Carlevaro, N.; De Angelis, M. Modified gravity in the presence of matter creation: Scenario for the late Universe. *Entropy* **2024**, *26*, 662. [CrossRef]
30. Dainotti, M.G.; De Simone, B.; Schiavone, T.; Montani, G.; Rinaldi, E.; Lambiase, G. On the Hubble constant tension in the SNe Ia Pantheon sample. *Astrophys. J.* **2021**, *912*, 150. [CrossRef]
31. Dainotti, M.G.; De Simone, B.; Schiavone, T.; Montani, G.; Rinaldi, E.; Lambiase, G.; Bogdan, M.; Ugale, S. Updated analysis of Hubble constant evolution using expanded Pantheon-like samples. *Galaxies* **2022**, *10*, 24. [CrossRef]
32. Nojiri, S.; Odintsov, S.D.; Oikonomou, V.K. Integral F(R) gravity and saddle point condition as a remedy for the H0-tension. *Nucl. Phys. B* **2022**, *980*, 115850. [CrossRef]
33. Simone, B.D.; van Putten, M.H.P.M.; Dainotti, M.G.; Lambiase, G. A doublet of cosmological models to challenge the H0 tension in the Pantheon Supernovae Ia catalog. *arXiv* **2024**, arXiv:2411.05744. [CrossRef]
34. Dainotti, M.G.; Lenart, A.L.; Yengejeh, M.G.; Chakraborty, S.; Fraija, N.; Di Valentino, E.; Montani, G. A new binning method to choose a standard set of Quasars. *Phys. Dark Universe* **2024**, *44*, 101428. [CrossRef]
35. Li, Q.; Xue, C.; Liu, J.P.; Wu, J.F.; Yang, S.Q.; Shao, C.G.; Quan, L.D.; Tan, W.H.; Tu, L.C.; Liu, Q.; et al. Measurements of the gravitational constant using two independent methods. *Nature* **2018**, *560*, 582–588. [CrossRef]
36. Gundlach, J.H.; Merkowitz, S.M. Measurement of Newton's constant using a torsion balance with angular acceleration feedback. *Phys. Rev. Lett.* **2000**, *85*, 2869. [CrossRef]
37. Wang, K.; Chen, L. Constraints on Newton's constant from cosmological observations. *Eur. Phys. J. C* **2020**, *80*, 570. [CrossRef]
38. Weinberg, S. *Cosmology*; OUP Oxford: Oxford, UK, 2008.
39. Khoury, J.; Weltman, A. Chameleon cosmology. *Phys. Rev. D* **2004**, *69*, 044026. [CrossRef]
40. Khoury, J.; Weltman, A. Chameleon fields: Awaiting surprises for tests of gravity in space. *Phys. Rev. Lett.* **2004**, *93*, 171104. [CrossRef]
41. Carroll, S.M. Quintessence and the rest of the world. *arXiv* **1998**, arXiv:astro-ph/9806099. [CrossRef]
42. Quiros, I.; Garcia-Salcedo, R.; Gonzalez, T.; Horta-Rangel, F.A. The Chameleon Effect in the Jordan Frame of the Brans–Dicke Theory. *arXiv* **2015**, arXiv:1506.05420. [CrossRef]
43. Brax, P.; van de Bruck, C.; Davis, A.C.; Shaw, D.J. $f(R)$ gravity and chameleon theories. *Phys. Rev. D—Part. Fields Gravit. Cosmol.* **2008**, *78*, 104021. [CrossRef]
44. Capozziello, S.; Tsujikawa, S. Solar system and equivalence principle constraints on $f(R)$ gravity by the chameleon approach. *Phys. Rev. D* **2008**, *77*, 107501. [CrossRef]
45. Liu, T.; Zhang, X.; Zhao, W. Constraining f(R) gravity in solar system, cosmology and binary pulsar systems. *Phys. Lett. B* **2018**, *777*, 286–293. [CrossRef]
46. Hernández-Aguayo, C.; Baugh, C.M.; Li, B. Marked clustering statistics in f(R) gravity cosmologies. *Mon. Not. R. Astron. Soc.* **2018**, *479*, 4824–4835. [CrossRef]
47. Amendola, L.; Gannouji, R.; Polarski, D.; Tsujikawa, S. Conditions for the cosmological viability of $f(R)$ dark energy models. *Phys. Rev. D—Part. Fields Gravit. Cosmol.* **2007**, *75*, 083504. [CrossRef]
48. Mandl, F.; Shaw, G. *Quantum Field Theory*; John Wiley & Sons: Hoboken, NJ, USA, 2013.

49. Friedman, A. Über die krümmung des raumes. *Z. Phys.* **1922**, *10*, 377–386. [CrossRef]
50. Lemaître, G. Un Univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Ann. Société Sci. Brux.* **1927**, *47*, 49–59.
51. Robertson, H.P. Kinematics and world-structure. *Astrophys. J.* **1935**, *82*, 284. [CrossRef]
52. Walker, A.G. On Milne's theory of world-structure. *Proc. Lond. Math. Soc.* **1937**, *2*, 90–127. [CrossRef]
53. Efstathiou, G.; Gratton, S. The evidence for a spatially flat Universe. *Mon. Not. R. Astron. Soc. Lett.* **2020**, *496*, L91–L95. [CrossRef]
54. Di Valentino, E.; Said, J.L.; Riess, A.; Pollo, A.; Poulin, V.; Gómez-Valent, A.; Weltman, A.; Palmese, A.; Huang, C.D.; van de Bruck, C.; et al. The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics. *Phys. Dark Universe* **2025**, *49*, 101965. [CrossRef]
55. Navonea, I.; Dainotti, M.G.; Fazzari, E.; Montani, G.; Maki, N. Creation of Viscous Dark Energy by the Hubble Flow: Comparison with SNe Ia Master Sample Binned Data. *arXiv* **2025**, arXiv:2511.16130. [CrossRef]
56. Fazzari, E.; Dainotti, M.; Montani, G.; Melchiorri, A. The effective running Hubble constant in SNe Ia as a marker for the dark energy nature. *arXiv* **2025**, arXiv:2506.04162. [CrossRef]
57. Olmo, G.J. Violation of the equivalence principle in modified theories of gravity. *Phys. Rev. Lett.* **2007**, *98*, 061101. [CrossRef]
58. Farajollahi, H.; Salehi, A. Cosmic dynamics in chameleon cosmology. *Int. J. Mod. Phys. D* **2010**, *19*, 621–633. [CrossRef]
59. Montani, G.; Escamilla, L.A.; Carlevaro, N.; Di Valentino, E. Decay of $f(R)$ quintessence into dark matter: Mitigating the Hubble tension? *arXiv* **2025**, arXiv:2512.20193. [CrossRef]
60. Kruglov, S. Logarithmic gravity model. *Int. J. Mod. Phys. D* **2023**, *32*, 2350037. [CrossRef]
61. Lecian, O.M.; Montani, G. Implications of non-analytic gravity at solar system scales. *Class. Quantum Gravity* **2009**, *26*, 045014. [CrossRef]
62. Fiorucci, D.; Lecian, O.M.; Montani, G. Non-analytical power-law correction to the Einstein–Hilbert action: Gravitational wave propagation. *Mod. Phys. Lett. A* **2014**, *29*, 1450178. [CrossRef]
63. Capozziello, S.; Carlevaro, N.; De Laurentis, M.; Lattanzi, M.; Montani, G. Cosmological implications of a viable non-analytical f(R)-gravity model. *arXiv* **2011**, arXiv:1104.2169.
64. Capozziello, S.; Carlevaro, N.; De Laurentis, M.; Lattanzi, M.; Montani, G. Cosmological implications of a viable non-analytical f(R) model. *Eur. Phys. J. Plus* **2013**, *128*, 155. [CrossRef]

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