

Article

Simultaneous Confidence Intervals for All Pairwise Differences of Coefficients of Variation of Delta-Inverse Gaussian Distributions

Wasurat Khumpasee , Sa-Aat Niwitpong *  and Suparat Niwitpong 

Department of Applied Statistics, Faculty of Applied Sciences, King Mongkut's University of Technology North Bangkok, Bangkok 10800, Thailand; k.wasurat@email.kmutnb.ac.th (W.K.); suparat.n@sci.kmutnb.ac.th (S.N.)

* Correspondence: sa-aat.n@sci.kmutnb.ac.th

Abstract

This study develops and evaluates simultaneous confidence interval procedures for all pairwise differences of coefficients of variation under delta-inverse Gaussian distributions. The objective is to provide reliable comparative inference for relative variability in zero-inflated and highly skewed data, where standard normal-based methods may be unreliable. Five approaches were studied and compared in terms of coverage probabilities and average widths: generalized confidence interval, adjusted generalized confidence interval, fiducial confidence interval, method of variance estimates recovery, and normal approximation. A Monte Carlo simulation study was conducted under varying shape parameters, zero-inflation probabilities, sample sizes, and numbers of populations ($k = 3, 6$, and 10). Although most methods produced CIs near the nominal 0.95 level, meaningful differences emerged when both coverage accuracy and interval efficiency were considered. The AGCI method consistently delivered stable coverage across parameter settings and remained robust as dimensionality increased. The MOVER approach achieved competitive coverage while frequently yielding narrower intervals. In contrast, GCI occasionally showed mild undercoverage, and FCI tended to produce overly wide intervals. An empirical application to zero-inflated mortality data supports the simulation findings. Overall, AGCI and MOVER provide reliable and practical tools for simultaneous inference on differences in CVs across delta-IG populations.

Keywords: adjusted generalized confidence interval; fiducial confidence interval; generalized confidence interval; method of variance estimates recovery; normal approximation



Academic Editors: Pedro José Fernández de Córdoba Castellá and Shufei Wu

Received: 4 March 2026

Revised: 30 March 2026

Accepted: 31 March 2026

Published: 2 April 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

Quantifying variability is a fundamental objective in statistical analysis across numerous scientific disciplines. Although variance and standard deviation are widely used measures of dispersion, their dependence on measurement scale limits their effectiveness when comparisons among heterogeneous populations are required. The coefficient of variation (CV), defined as the ratio of the standard deviation to the mean, provides a scale-independent measure of relative variability and is therefore particularly useful for comparing stochastic behavior across populations with different magnitudes or units of measurement. Consequently, statistical inference for the CV has been widely investigated in environmental studies, reliability engineering, finance, and biomedical applications [1–4].

In many real-world situations, interest lies not only in estimating a single CV but also in comparing relative variability across multiple populations. Over the past decades, various

methods have been proposed for constructing confidence intervals for the CV, including bootstrap techniques, generalized inference procedures, and fiducial inferences [5–8]. Among these, the generalized confidence interval (GCI) approach is particularly attractive due to its flexibility under non-normal settings. However, its performance can be sensitive to sample size and skewness, often resulting in coverage distortions in small or moderate samples. Adjusted variants such as the adjusted GCI (AGCI) have been introduced to mitigate these issues. Alternative approaches, including fiducial confidence intervals (FCI) and the method of variance estimates recovery (MOVER), have been developed. While FCI avoids reliance on asymptotic assumptions, it can be computationally complex, whereas MOVER typically produces shorter interval lengths by efficiently combining variance estimates but may suffer from undercoverage when distributional assumptions are violated. These limitations suggest that no single method consistently performs well across all scenarios, particularly when skewness and model complexity are present.

Despite these developments, most existing studies focus primarily on inference for a single CV or comparisons between two populations. In many practical applications, however, the primary goal is to compare variability across multiple groups simultaneously. Conducting multiple pairwise comparisons using separate confidence intervals can inflate the overall error rate and lead to misleading conclusions. This issue becomes more critical when considering differences in CVs, which provide a direct and interpretable measure of absolute differences in relative dispersion. Statistical inference for such differences is inherently challenging under non-normal and zero-inflated distributions due to skewness, parameter dependence, and the presence of nuisance parameters [9,10]. These challenges motivate the use of simultaneous confidence intervals (SCIs), which enable valid joint inference while controlling the overall confidence level [11,12].

At the same time, empirical data in many fields often exhibit substantial right skewness and an excess of zero observations. Classical models assuming normality are inadequate under such conditions. Chhikara and Folks [13] stated that among skewed continuous distributions, the inverse Gaussian (IG) distribution has received considerable attention due to its flexibility and strong theoretical foundation derived from first-passage-time processes. The inverse Gaussian distribution has been widely used for modeling positively skewed data [14–16]; however, it cannot accommodate zero inflation. To address this limitation, Heller et al. [17] have introduced the delta-inverse Gaussian (delta-IG) distribution as an extension that incorporates a point mass at zero while preserving the flexibility of the continuous IG component. This model has been successfully applied in areas such as insurance claims, environmental studies, and mortality analysis [4,18,19].

Several studies have developed simultaneous confidence intervals for differences in CVs under specific distributions. For example, Thangjai et al. [20] constructed SCIs for all pairwise differences in CVs from several normal distributions. Yosboonruang et al. [21] established SCIs for all pairwise differences in CVs of delta-lognormal distributions. Usanee et al. [22] proposed and evaluated methods for constructing SCIs for all pairwise differences in CVs in zero-inflated Birnbaum-Saunders distributions. However, these approaches are tailored to specific distributional assumptions and are not directly applicable to more complex data structures that involve both strong skewness and excess zeros. In such settings, their performance may deteriorate, limiting their practical usefulness. To the best of our knowledge, there is currently no comprehensive framework for constructing simultaneous confidence intervals for all pairwise differences of CVs under the delta-IG distribution. This gap is nontrivial, as existing methods may perform poorly due to the combined effects of skewness, zero inflation, and parameter dependence, especially in small or moderate samples [23]. Therefore, there is a clear need for robust and reliable inferential procedures specifically designed for this class of distributions.

To address this gap, the present study develops simultaneous confidence interval procedures for all pairwise differences of coefficients of variation for independent delta-IG populations. Five competing methods are considered: the generalized confidence interval (GCI), adjusted GCI (AGCI), fiducial confidence interval (FCI), the method of variance estimates recovery (MOVER), and the normal approximation (NA) method. A comprehensive Monte Carlo simulation study is conducted to evaluate their performance in terms of coverage probability and average interval length under various parameter settings.

The main contributions of this study are as follows. First, we develop a framework for constructing simultaneous confidence intervals for all pairwise differences of coefficients of variation under delta-IG distributions, explicitly accounting for skewness and zero inflation. It is important to note that the interval estimation techniques considered in this study (GCI, AGCI, FCI, MOVER, and NA) are existing methods; the novelty of this work lies in their integration into a unified simultaneous inference framework for delta-IG populations, which has not been previously investigated. Second, we provide a systematic comparison of multiple interval estimation methods within a unified simultaneous inference framework. Third, we offer practical insights into the performance and reliability of these methods through extensive simulation experiments. Finally, the applicability of the proposed approaches is demonstrated using real-world mortality data.

2. Properties of the Delta-IG Distribution

Let $L = (L_1, L_2, \dots, L_n)$ be a non-negative random sample from a delta-IG distribution stated as $L \sim \text{delta-IG}(\delta, \mu, \lambda)$ where μ represents the mean, λ denotes the shape parameter, and δ signifies the probability of zero. The probability density function of delta-IG is expressed as

$$f_{\text{delta-IG}}(L; \delta, \mu, \lambda) = \delta I_{\{L=0\}} + (1 - \delta) I_{\{L>0\}} \sqrt{\frac{\lambda}{2\pi L^3}} \exp \left[\frac{-\lambda(L - \mu)^2}{2\mu^2 L} \right], \quad (1)$$

where I is an indicator function, $P(L = 0) = \delta$, and $\mu, \lambda > 0$. The sample contains n observations, while n is separated into zero ($n_{(0)}$) and nonzero observations ($n_{(1)}$), such that $n = n_{(0)} + n_{(1)}$. Moreover, $n_{(0)}$ follows a binomial distribution denoted as $n_{(0)} \sim \text{Bin}(n, \delta)$.

The MLEs of δ, μ , and λ of the distributions are given by: $\hat{\delta} = n_{(0)}/n$, $\hat{\mu} = \bar{L} = \frac{1}{n_{(1)}} \sum_{j=1}^{n_{(1)}} L_j$, and $n_{(1)} \hat{\lambda}^{-1} = V = \sum_{j=1}^{n_{(1)}} (L_j^{-1} - \bar{L}^{-1})$, respectively [24]. The expected value was $E(L) = (1 - \delta)\mu$. The variance was $\text{Var}(L) = \mu^2(1 - \delta)(\frac{\mu}{\lambda} + \delta)$. The coefficients of variation of delta-IG distributions were $\text{CV}(L) = \tau = \sqrt{\text{Var}(L)/E(L)} = \sqrt{(\mu + \lambda\delta)/(1 - \delta)\lambda}$. Therefore, all pairwise differences between the CVs are parameters that we will calculate for their interval, denoted as

$$\theta_{ij} = \tau_i - \tau_j = \sqrt{(\mu_i + \lambda_i\delta_i)/(1 - \delta_i)\lambda_i} - \sqrt{(\mu_j + \lambda_j\delta_j)/(1 - \delta_j)\lambda_j}, \quad (2)$$

where $i, j = 1, 2, \dots, k$ and $i \neq j$.

3. The Simultaneous Confidence Intervals

The method to estimate simultaneous confidence intervals for θ_{ij} was described in detail as follows:

3.1. GCI

A generalized confidence interval (GCI) provides a flexible framework for interval estimation in situations where conventional methods are difficult to apply or fail to perform adequately. Traditional confidence interval procedures commonly depend on restrictive as-

sumptions, such as large sample sizes or normality of the underlying distribution, whereas the GCI approach relaxes these requirements and remains applicable under more complex settings [5].

The construction of a GCI relies on a function referred to as the generalized pivotal quantity (GPQ), whose sampling distribution does not depend on the unknown parameter of interest [25]. In practice, a large number of GPQ realizations are generated through Monte Carlo simulation, and the empirical percentiles of these simulated values are then used to determine the lower and upper confidence limits. Owing to this simulation-based mechanism, the GCI approach can achieve accurate coverage probabilities even with small sample sizes or complex parameters arising from non-standard distributions. Consequently, GCI has become an important tool for modern statistical inference in diverse applied fields such as biostatistics and engineering [26].

Ye, Ma, and Wang [27] introduced a generalized pivotal quantity (GPQ)-based approach for constructing confidence intervals for the mean of inverse Gaussian distributions across multiple populations. Their method uses GPQs derived for the unknown parameters of the IG distribution, namely the shape parameter λ and the mean parameter μ .

The generalized pivotal quantities for λ and μ , as developed by Ye, Ma, and Wang, are expressed as follows:

$$G_{\lambda_i} = \frac{n_{i(1)}\lambda_i V_i}{n_{i(1)}v_i} \sim \frac{\chi_{n_{i(1)}-1}^2}{n_{i(1)}v_i}, i = 1, 2, \dots, k, \quad (3)$$

where $\chi_{n_{i(1)}-1}^2$ is denoted as a Chi-square distribution with $n_{i(1)} - 1$ degrees of freedom, $i = 1, 2, \dots, k$ and

$$G_{\mu_i} = \frac{\bar{L}_i}{\left| 1 + \frac{\sqrt{n_{i(1)}\lambda_i(\bar{L}_i - \mu_i)}}{\mu_i \sqrt{\bar{L}_i}} \sqrt{\frac{\bar{L}_i}{n_{i(1)}G_{\lambda_i}}} \right|} \sim \frac{\bar{L}_i}{\left| 1 + Z_i \sqrt{\frac{\bar{L}_i}{n_{i(1)}G_{\lambda_i}}} \right|}, \quad (4)$$

where Z_i is approximately distributed as a standard normal distribution, denoted by $N(0, 1)$.

In this study, the parameter δ was estimated using the variance-stabilizing transformation (VST) approach. Previous work by Khumpasee, Niwitpong, and Niwitpong [28] demonstrated that the VST method performs more reliably than several alternative estimation techniques. The concept of variance stabilization was originally introduced by Dasgupta [23], based on normal approximation theory. Subsequently, Wu and Hsieh [29] developed a VST for the binomial distribution using the arcsine square-root transformation to achieve variance stabilization.

Accordingly, the generalized pivotal quantity (GPQ) for δ_i , following Wu and Hsieh, can be expressed as follows:

$$G_{\delta_i} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}_i} - \frac{T}{2\sqrt{n_i}} \right], \quad (5)$$

where $T = 2\sqrt{n_i}(\arcsin \sqrt{\hat{\delta}_i} - \arcsin \sqrt{\delta_i}) \sim N(0, 1)$, $n_i \rightarrow \infty$.

Hence, from the GPQ of parameters, we have G_{λ_i} , G_{μ_i} , and G_{δ_i} , expressed as (3), (4) and (5), respectively. The GPQ for θ_{ij} can be defined as

$$G_{\theta_{ij}} = \sqrt{(G_{\mu_i} + G_{\lambda_i}G_{\delta_i}) / (1 - G_{\delta_i})G_{\lambda_i}} - \sqrt{(G_{\mu_j} + G_{\lambda_j}G_{\delta_j}) / (1 - G_{\delta_j})G_{\lambda_j}}, \quad (6)$$

where $i, j = 1, 2, \dots, k$ and $i \neq j$.

The $100(1 - \alpha)\%$ two-sided SCI for θ_{ij} using the GCI method is

$$CI_{\theta_{ij}}^{GCI} = [L_{\theta_{ij}}^{GCI}, U_{\theta_{ij}}^{GCI}] = [G_{\theta_{ij}}(\alpha/2), G_{\theta_{ij}}(1 - \alpha/2)], \quad (7)$$

where $G_{\theta_{ij}}(\alpha/2)$ and $G_{\theta_{ij}}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $G_{\theta_{ij}}$, respectively, $i, j = 1, 2, \dots, k$ and $i \neq j$. The main idea of the GCI approach is to construct a generalized pivotal quantity (GPQ) for the coefficient of variation (CV), whose distribution does not depend on unknown parameters. This allows confidence intervals to be obtained without relying on asymptotic approximations. The method uses simulated realizations of the GPQ to approximate its sampling distribution and derive the corresponding confidence limits. The detailed computational steps for the GCI method are provided in Appendix A.

3.2. AGCI

Following the construction of confidence intervals for inverse Gaussian distributions using the GCI approach, Krishnamoorthy and Tian [30] later proposed an alternative generalized pivotal quantity (GPQ) for the parameter. The adjusted generalized confidence interval (AGCI) method adopted in this study utilizes this modified GPQ for μ instead of the GPQ defined in Equation (4) under the conventional GCI framework. The GPQ for the parameter μ , as introduced by Krishnamoorthy and Tian, is given by:

$$G_{\mu_i}^{\sim} = \frac{\bar{L}_i}{\max\left\{0, 1 + t_{n_{i(1)}-1} \sqrt{\frac{\bar{L}_i v_i}{n_{i(1)}-1}}\right\}}, \quad (8)$$

where $t_{n_{i(1)}-1}$ is Student's t distribution with $n_{i(1)} - 1$ degrees of freedom. The method used to estimate the parameter δ_i was the VST approach same as the GCI approach.

Hence, from the GPQ of parameters, we have G_{λ_i} , $G_{\mu_i}^{\sim}$, and G_{δ_i} , expressed as (3), (8) and (5), respectively. The GPQ for θ_{ij} can be defined as

$$G_{\theta_{ij}}^{\sim} = \sqrt{(G_{\mu_i}^{\sim} + G_{\lambda_i} G_{\delta_i}) / (1 - G_{\delta_i})} G_{\lambda_i} - \sqrt{(G_{\mu_j}^{\sim} + G_{\lambda_j} G_{\delta_j}) / (1 - G_{\delta_j})} G_{\lambda_j}, \quad (9)$$

where $i, j = 1, 2, \dots, k$ and $i \neq j$.

The $100(1 - \alpha)\%$ two-sided SCI for θ_{ij} using the AGCI method is

$$CI_{\theta_{ij}}^{AGCI} = [L_{\theta_{ij}}^{AGCI}, U_{\theta_{ij}}^{AGCI}] = [G_{\theta_{ij}}^{\sim}(\alpha/2), G_{\theta_{ij}}^{\sim}(1 - \alpha/2)], \quad (10)$$

where $G_{\theta_{ij}}^{\sim}(\alpha/2)$ and $G_{\theta_{ij}}^{\sim}(1 - \alpha/2)$ are the $100\frac{\alpha}{2}$ -th and $100(1 - \frac{\alpha}{2})$ -th percentiles of the distribution of $G_{\theta_{ij}}^{\sim}$, respectively, $i, j = 1, 2, \dots, k$ and $i \neq j$. The AGCI method extends the GCI approach by incorporating an adjustment to reduce bias in the generalized pivotal quantity. This adjustment is particularly important in small samples and highly skewed distributions, where the standard GCI may not achieve accurate coverage probabilities. The modified GPQ is used to generate more reliable confidence intervals with improved finite-sample performance. The detailed computational steps for the AGCI method are provided in Appendix A.

3.3. FCI

Fiducial confidence interval estimation involves constructing a probability distribution for an unknown parameter directly from the observed data without incorporating

prior information under repeated sampling. This concept was originally introduced by Fisher [31], who proposed deriving a fiducial distribution for an unknown parameter by inverting a probability statement associated with a pivotal quantity. Confidence intervals are then obtained from the corresponding percentiles of this fiducial distribution. Although the resulting interval is similar in form to a Bayesian credible interval, it does not require either subjective or objective prior distributions.

For practical implementation, the Gibbs sampling algorithm proposed by Geman and Geman [32] provides an efficient computational framework for fiducial inference. In the context of the delta-IG distribution, the sampling distributions of the maximum likelihood estimators (MLEs) of the parameters δ_i , μ_i , and λ_i are employed. The fiducial distributions for the parameters μ_i and λ_i can be obtained by substituting their corresponding MLEs into the sampling distributions. Under this framework, the parameters in the sampling distribution are assumed to follow the forms given by $\mu_i \sim IG(\hat{\mu}_i, n_{i(1)}\hat{\lambda}_i)$ and $\lambda_i \sim (\hat{\lambda}_i/n_{i(1)})\chi^2_{n_{i(1)}-1}$, with $\hat{\mu}_i$ and $\hat{\lambda}_i$ being the MLEs of μ_i and λ_i , respectively. The FCI method will also employ the VST techniques to estimate parameter δ_i .

The $100(1 - \alpha)\%$ two-sided SCI of θ_{ij} for the delta-IG distribution from the FCI method can be defined as

$$CI_{\theta_{ij}}^{\text{FCI}} = [G_{\theta_{ij}}^{\text{FCI}}(\alpha/2), G_{\theta_{ij}}^{\text{FCI}}(1 - \alpha/2)], \quad (11)$$

where $i, j = 1, 2, \dots, k$ and $i \neq j$. The FCI approach is based on fiducial inference, which constructs a distribution for the parameter of interest without requiring prior distributions or large-sample approximations. By inverting the data-generating mechanism, fiducial quantities are obtained and used to derive confidence intervals for the CV. This method provides an alternative framework for interval estimation, particularly in complex distributional settings. The detailed computational steps for the FCI method are provided in Appendix A.

3.4. MOVER

The method of variance estimates recovery (MOVER), introduced by Zou and Donner [33], is a flexible and computationally efficient approach for constructing confidence intervals for functions of parameters, particularly the difference or ratio of two independent quantities. This method is especially useful in situations where conventional large-sample procedures may perform inadequately, such as when sample sizes are small or the underlying distributions are skewed.

The main principle of MOVER is to recover variance estimates from existing confidence intervals. Specifically, for a parameter with lower and upper confidence limits L and U , two variance estimates can be obtained based on the distances between the point estimate and each confidence bound. These recovered variance components are then combined with the corresponding point estimates to construct a confidence interval for a composite parameter, such as a difference or ratio. By incorporating asymmetric information contained in the original confidence limits, the MOVER approach effectively accommodates distributional asymmetry and often produces more accurate and reliable confidence intervals.

The MOVER method determines the lower and upper confidence limits for the difference between two parameters or $\theta_{ij} = \tau_i - \tau_j$, represented as follows:

$$L.diff_{ij} = \hat{\tau}_i - \hat{\tau}_j - \sqrt{(\hat{\tau}_i - l_i)^2 + (u_j - \hat{\tau}_j)^2}, \quad (12)$$

and

$$U.diff_{ij} = \hat{\tau}_i - \hat{\tau}_j + \sqrt{(u_i - \hat{\tau}_i)^2 + (\hat{\tau}_j - l_j)^2}, \quad (13)$$

where (l_i, u_i) and (l_j, u_j) are $100(1 - \alpha)\%$ confidence limits from the AGCI method for the parameters τ_i and τ_j with $i, j = 1, 2, \dots, k$ and $i \neq j$.

Using the confidence intervals obtained from the AGCI method, as defined in Equations (8) and (9), the two-sided $100(1 - \alpha)\%$ simultaneous confidence intervals for $\theta_{ij} = \tau_i - \tau_j$ can be constructed as follows.

As stated by the AGCI method, the GPQ for θ_{ij} is defined as

$$G_{\theta_{ij}} = \sqrt{(G_{\mu_i} + G_{\lambda_i} G_{\delta_i}) / (1 - G_{\delta_i}) G_{\lambda_i}} - \sqrt{(G_{\mu_j} + G_{\lambda_j} G_{\delta_j}) / (1 - G_{\delta_j}) G_{\lambda_j}}. \quad (14)$$

The $100(1 - \alpha)\%$ two-sided SCI for θ_{ij} using the AGCI method is

$$CI_{\theta_{ij}}^{AGCI} = [L_{\theta_{ij}}^{AGCI}, U_{\theta_{ij}}^{AGCI}] = [G_{\theta_{ij}}(\alpha/2), G_{\theta_{ij}}(1 - \alpha/2)]. \quad (15)$$

Therefore, the $100(1 - \alpha)\%$ two-sided simultaneous confidence intervals for θ_{ij} derived by the MOVER approach are expressed as

$$[L_{\theta_{ij}}^{MOVER}, U_{\theta_{ij}}^{MOVER}] = [\hat{\tau}_i - \hat{\tau}_j - \sqrt{(\hat{\tau}_i - l_i)^2 + (u_j - \hat{\tau}_j)^2}, \hat{\tau}_i - \hat{\tau}_j + \sqrt{(u_i - \hat{\tau}_i)^2 + (\hat{\tau}_j - l_j)^2}]. \quad (16)$$

The MOVER approach constructs confidence intervals for functions of parameters by combining individual confidence intervals using variance-recovery techniques. In the context of CV differences, the method first derives separate interval estimates and then combines their variance components to obtain an overall interval. This approach is known for producing relatively shorter intervals while maintaining reasonable coverage accuracy. The detailed computational steps for the MOVER method are provided in Appendix A.

3.5. NA

Under the normal approximation approach, the Delta method is applied to establish that the estimator τ follows an asymptotically normal distribution as the sample size increases. This method is grounded in large-sample theory and is commonly used as a benchmark because of its analytical simplicity and closed-form implementation. Nevertheless, it is widely recognized that confidence intervals constructed using normal approximations may exhibit unsatisfactory performance for skewed distributions, especially in small or moderate samples, and are frequently outperformed by bootstrap-based or modified interval estimation procedures [34]. The main steps of this classical approach can be summarized as follows.

Let $u(v_1, v_2, v_3)$ be a differentiable scalar-valued function of three variables, and suppose that the estimator $U = u(V_1, V_2, V_3)$ depends on the parameters V_1 , V_2 , and V_3 . The Delta method provides a framework for deriving the asymptotic distribution of U by obtaining a stochastic approximation of the estimator. As the sample size increases, this method establishes the asymptotic normality of U and enables the determination of its asymptotic mean and variance, which are required for constructing confidence intervals for the estimator τ .

Specifically, the function $u(\cdot)$ is approximated using a first-order Taylor series expansion evaluated at $(\varphi_1, \varphi_2, \varphi_3)$, corresponding to the true values of V_1 , V_2 , and V_3 , as follows:

$$u(v_1, v_2, v_3) \approx u(\varphi_1, \varphi_2, \varphi_3) + \frac{\partial u(\varphi_1, \varphi_2, \varphi_3)}{\partial v_1} (V_1 - \varphi_1) + \frac{\partial u(\varphi_1, \varphi_2, \varphi_3)}{\partial v_2} (V_2 - \varphi_2) + \frac{\partial u(\varphi_1, \varphi_2, \varphi_3)}{\partial v_3} (V_3 - \varphi_3). \quad (17)$$

The required statistics are defined as $V_1 = \hat{\mu}$, $V_2 = \hat{\lambda}$, and $V_3 = \hat{\delta}$, where $\hat{\mu}$, $\hat{\lambda}$, and $\hat{\delta}$ denote the maximum likelihood estimators of the parameters μ , λ , and δ , respectively. Accordingly, the estimator of interest can be written as $\hat{\tau} = u(\hat{\mu}, \hat{\lambda}, \hat{\delta})$, where the function

$u(\mu, \lambda, \delta)$ is defined by $\sqrt{(\mu + \lambda\delta)/(1 - \delta)\lambda}$. The corresponding partial derivatives of $u(\mu, \lambda, \delta)$ with respect to each parameter are given as follows:

$$\frac{\partial f(\mu, \lambda, \delta)}{\partial \mu} = \frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \delta)\lambda} \right). \quad (18)$$

$$\frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda} = \frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda\delta - (\mu + \lambda\delta)(1 - \delta)}{((1 - \delta)\lambda)^2} \right). \quad (19)$$

$$\frac{\partial f(\mu, \lambda, \delta)}{\partial \delta} = \frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda^2 - (\mu + \lambda\delta)(-\lambda)}{((1 - \delta)\lambda)^2} \right). \quad (20)$$

Thus,

$$\begin{aligned} \hat{\tau} &\approx f(\mu, \lambda, \delta) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \mu}(\hat{\mu} - \mu) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \lambda}(\hat{\lambda} - \lambda) + \frac{\partial f(\mu, \lambda, \delta)}{\partial \delta}(\hat{\delta} - \delta) \\ &= f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \delta)\lambda} \right) \right] (\hat{\mu} - \mu) + \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda\delta - (\mu + \lambda\delta)(1 - \delta)}{((1 - \delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) \\ &\quad + \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda^2 - (\mu + \lambda\delta)(-\lambda)}{((1 - \delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta). \end{aligned} \quad (21)$$

Taking the expectation and variance in Equation (21), the estimated mean and variance for $\hat{\tau}$ are expressed as

$$E(\hat{\tau}) = \tau, \quad (22)$$

$$\begin{aligned} \text{Var}(\hat{\tau}) &\approx \text{Var} \left\{ f(\mu, \lambda, \delta) + \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \delta)\lambda} \right) \right] (\hat{\mu} - \mu) + \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda\delta - (\mu + \lambda\delta)(1 - \delta)}{((1 - \delta)\lambda)^2} \right) \right] (\hat{\lambda} - \lambda) + \right. \\ &\quad \left. \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda^2 - (\mu + \lambda\delta)(-\lambda)}{((1 - \delta)\lambda)^2} \right) \right] (\hat{\delta} - \delta) \right\} \\ &= \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \delta)\lambda} \right) \right]^2 \left(\frac{\mu^3}{\lambda n_{i(1)}} \right) + \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda\delta - (\mu + \lambda\delta)(1 - \delta)}{((1 - \delta)\lambda)^2} \right) \right]^2 \left(\frac{2\lambda^2}{n_{i(1)}} \right) + \\ &\quad \left[\frac{1}{2} \left(\frac{(1 - \delta)\lambda}{\mu + \lambda\delta} \right)^{\frac{1}{2}} \left(\frac{(1 - \delta)\lambda^2 - (\mu + \lambda\delta)(-\lambda)}{((1 - \delta)\lambda)^2} \right) \right]^2 \left(\frac{\delta(1 - \delta)}{n_i} \right). \end{aligned} \quad (23)$$

Thus, as $n_i \rightarrow \infty$, the estimator $\hat{\tau}$ is asymptotically approximated by a normal distribution, that is, $\hat{\tau} \approx N(E(\hat{\tau}), \text{Var}(\hat{\tau}))$. In practice, the true parameter values μ , λ , and δ are unknown when estimating the parameter function τ based on observed sample data. Therefore, plug-in estimators are employed by substituting the corresponding maximum likelihood estimators into the expressions for $E(\hat{\tau}_i)$ and $\text{Var}(\hat{\tau}_i)$, as presented below.

$$E(\hat{\tau}_i) = \sqrt{(\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i) / (1 - \hat{\delta}_i) \hat{\lambda}_i}, \quad (24)$$

$$\begin{aligned} \text{Var}(\hat{\tau}_i) &= \left[\frac{1}{2} \left(\frac{(1 - \hat{\delta}_i) \hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{1}{(1 - \hat{\delta}_i) \hat{\lambda}_i} \right) \right]^2 \left(\frac{\hat{\mu}_i^3}{\hat{\lambda}_i n_{i(1)}} \right) + \left[\frac{1}{2} \left(\frac{(1 - \hat{\delta}_i) \hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{(1 - \hat{\delta}_i) \hat{\lambda}_i \hat{\delta}_i - (\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i)(1 - \hat{\delta}_i)}{((1 - \hat{\delta}_i) \hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{2\hat{\lambda}_i^2}{n_{i(1)}} \right) + \\ &\quad \left[\frac{1}{2} \left(\frac{(1 - \hat{\delta}_i) \hat{\lambda}_i}{\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i} \right)^{\frac{1}{2}} \left(\frac{(1 - \hat{\delta}_i) \hat{\lambda}_i^2 - (\hat{\mu}_i + \hat{\lambda}_i \hat{\delta}_i)(-\hat{\lambda}_i)}{((1 - \hat{\delta}_i) \hat{\lambda}_i)^2} \right) \right]^2 \left(\frac{\hat{\delta}_i(1 - \hat{\delta}_i)}{n_i} \right). \end{aligned} \quad (25)$$

The estimated variance of the estimated difference between two coefficients of variation of delta-IG can be expressed as follows:

$$\text{Var}(\hat{\theta}_{ij}) = \text{Var}(\hat{\tau}_i) + \text{Var}(\hat{\tau}_j), \quad (26)$$

where $i, j = 1, 2, \dots, k, i \neq j$ and $\text{cov}(\hat{\tau}_i, \hat{\tau}_j) = 0$.

The $100(1 - \alpha)\%$ simultaneous confidence interval for θ_{ij} using the normal approximation is

$$CI_{\theta_{ij}}^{\text{NA}} = (\hat{\tau}_i - \hat{\tau}_j) \pm z_{1-\frac{\alpha}{2}} \sqrt{\text{Var}(\hat{\theta}_{ij})}, \quad (27)$$

where $z_{1-\frac{\alpha}{2}}$ is the $(1 - \frac{\alpha}{2})$ th percentile of the standard normal distribution.

The normal approximation method relies on asymptotic theory by assuming that the estimator of the CV (or its transformation) is approximately normally distributed. Confidence intervals are then constructed using standard normal quantiles and estimated standard errors. While computationally simple, this approach may perform poorly when sample sizes are small or when the underlying distribution is highly skewed. The detailed computational steps for the NA method are provided in Appendix A. The algorithms for the aforementioned methods involve iterative computations, since their corresponding confidence intervals do not have closed-form expressions. In contrast, the normal approximation approach yields an explicit closed-form solution and therefore does not require iterative procedures.

Although the proposed methods are primarily evaluated via simulation, their validity is supported by well-established statistical principles. The GCI and AGCI approaches rely on generalized pivotal quantities, which provide approximate distribution-free inference under complex models. The FCI method is based on fiducial arguments, offering an alternative framework for interval estimation that does not require prior distributions. The MOVER approach constructs confidence intervals by combining variance estimates and has been shown to perform well in small-sample settings. The normal approximation method is included as a baseline, relying on asymptotic normality assumptions.

These approaches assume independent samples and rely on regularity conditions such as finite moments and consistent estimators. Therefore, the resulting confidence intervals are expected to achieve reasonable coverage probabilities, particularly for moderate to large sample sizes.

4. Simulation Studies and Results

The performance of the five proposed methods for constructing simultaneous confidence intervals for all pairwise differences of coefficients of variation under the delta-IG distribution was investigated through a Monte Carlo simulation study executed in the R programming language. The competing approaches were evaluated based on coverage probability (CP) and average width (AW) of the resulting confidence intervals. An appropriate method was identified based on two main criteria: the empirical CP should be close to or exceed the nominal confidence level of 0.95, while the corresponding AW should be as small as possible, indicating higher estimation precision. Each simulation scenario was replicated 2000 times, using 5000 pivotal quantities for the GCI and AGCI methods. Random samples for comparing all pairwise differences of the CVs were generated as $L = (L_1, L_2, \dots, L_{n_i}), i = 1, 2, \dots, k$ and $i \neq j$, where $L \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$. We assume that samples from different populations are independent, which is standard in multi-sample inference and appropriate when data are collected from distinct groups. This assumption ensures valid construction of variance estimates and pivotal quantities. If dependence exists (e.g., clustered data), the coverage performance of the proposed methods may be affected, and caution is required. The proposed methods were assessed under various parameter configurations through three sample scenarios with $k = 3, 6$, and 10 . The shape parameters λ_i , the probabilities of zero values δ_i , and the sample sizes n_i were varied, while the mean parameters μ_i were fixed at 1 to isolate the effects of skewness and zero inflation on interval performance. Because the coefficient of variation is scale-independent, this restriction does not reduce generality but simplifies interpretation by removing scale effects. To assess the

robustness of this assumption, additional simulations were conducted with heterogeneous mean parameters ($\mu_i \neq \mu_j$). The results were consistent with those obtained under $\mu_i = 1$, indicating that the performance of the proposed methods is not sensitive to variations in the mean parameter. The sample sizes, parameter settings, and number of replications were selected to provide a comprehensive evaluation of the proposed methods under a wide range of practical conditions. Small to moderate sample sizes were considered realistic, and the parameter configurations were chosen to capture varying levels of skewness, dispersion, and zero inflation. The number of replications was set sufficiently large to ensure stable and reliable estimates of coverage probability and average width.

The results presented in Tables 1–3 and Figures 1–3 summarize the coverage probabilities (CPs) and average widths (AWs) of the simultaneous confidence intervals (SCIs) for the differences of coefficients of variation under delta-IG populations across different numbers of groups ($k = 3, 6$, and 10) and parameter configurations.



Figure 1. Comparison of the performance of simultaneous confidence intervals (SCIs) for all pairwise differences of coefficients of variation under delta-inverse Gaussian distributions for $k = 3$. The performance is evaluated in terms of coverage probability (CP) and average width (AW) for five methods: generalized confidence interval (GCI), adjusted GCI (AGCI), fiducial confidence interval (FCI), method of variance estimates recovery (MOVER), and normal approximation (NA). Panels (A,C,E) display CP with respect to sample sizes, shape parameters λ , and zero-inflation probabilities δ , respectively, while panels (B,D,F) present the corresponding AW results.



Figure 2. Comparison of the performance of simultaneous confidence intervals (SCIs) for all pairwise differences of coefficients of variation under delta-inverse Gaussian distributions for $k = 6$. The performance is evaluated in terms of coverage probability (CP) and average width (AW) for five methods: generalized confidence interval (GCI), adjusted GCI (AGCI), fiducial confidence interval (FCI), method of variance estimates recovery (MOVER), and normal approximation (NA). Panels (A,C,E) display CP with respect to sample sizes, shape parameters λ , and zero-inflation probabilities δ , respectively, while panels (B,D,F) present the corresponding AW results.

Overall, the CPs obtained from all considered methods were generally close to the nominal confidence level of 0.95. However, noticeable differences in performance were observed among the competing approaches when both coverage accuracy and interval efficiency were considered.

For $k = 3$ (Table 1 and Figure 1), the AGCI and MOVER methods consistently achieved CPs closest to the nominal level across all values of the zero-inflation probabilities (δ). This strong performance of AGCI may be explained by the bias-reduction adjustment applied to the generalized pivotal quantity. In highly skewed distributions, such as the delta-IG, the sampling distribution of the CV estimator is typically asymmetric, leading to systematic

undercoverage when using unadjusted methods. The AGCI adjustment effectively corrects for this bias, thereby improving coverage accuracy.



Figure 3. Comparison of the performance of simultaneous confidence intervals (SCIs) for all pairwise differences of coefficients of variation under delta-inverse Gaussian distributions for $k = 10$. The performance is evaluated in terms of coverage probability (CP) and average width (AW) for five methods: generalized confidence interval (GCI), adjusted GCI (AGCI), fiducial confidence interval (FCI), method of variance estimates recovery (MOVER), and normal approximation (NA). Panels (A,C,E) display CP with respect to sample sizes, shape parameters λ , and zero-inflation probabilities δ , respectively, while panels (B,D,F) present the corresponding AW results.

The GCI method, in contrast, does not explicitly account for this bias, which explains its relatively lower CPs in scenarios with higher zero inflation. The FCI approach tended to produce extremely high CPs, often close to 1; however, this was accompanied by substantially wider intervals, suggesting over-conservatism. This may be attributed to the variability introduced through the fiducial inversion process, which can inflate uncertainty, particularly

in small samples. Meanwhile, the MOVER method maintained adequate CPs while producing comparatively shorter Aws. This efficiency can be explained by its use of variance recovery techniques, which combine variance estimates to avoid excessive conservatism.

Table 1. Coverage probabilities (CP) and average widths (AW) of the 95% simultaneous confidence intervals for all pairwise differences of coefficients of variation under delta-IG distributions in the case of $k = 3$, with equal mean parameters, various sample sizes, and parameter configurations (λ and δ) based on the GCI, AGCI, FCI, MOVER, and NA methods.

n_1, n_2, n_3	$\lambda_1, \lambda_2, \lambda_3$	$\delta_1, \delta_2, \delta_3$	Coverage Probabilities (CP)					Average Widths (AW)					
			GCI	AGCI	FCI	MOVER	NA	GCI	AGCI	FCI	MOVER	NA	
30, 30, 30	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9400	0.9460	1.0000	0.9503	1.0000	2.4000	2.4510	2.9712	2.5660	4.1080	
		0.3, 0.3, 0.3	0.9373	0.9570	1.0000	0.9613	1.0000	3.6490	3.8130	2.7022	4.0070	7.4370	
		0.5, 0.5, 0.5	0.9243	0.9605	0.9987	0.9663	1.0000	6.6030	6.9840	2.3876	7.3700	14.7630	
	1, 1, 1	0.1, 0.3, 0.5	0.9260	0.9528	0.9747	0.9553	0.9993	4.3350	4.5310	2.7642	4.7280	8.9450	
		0.1, 0.1, 0.1	0.9325	0.9497	1.0000	0.9537	0.9988	1.1820	1.2430	3.0227	1.2990	1.4140	
		0.3, 0.3, 0.3	0.9123	0.9565	1.0000	0.9602	1.0000	1.6050	1.8000	2.7369	1.8860	2.5980	
	5, 5, 5	0.5, 0.5, 0.5	0.8823	0.9555	0.9987	0.9617	1.0000	2.8190	3.2390	2.4203	3.4090	5.7320	
		0.1, 0.3, 0.5	0.9023	0.9545	0.9953	0.9568	0.9942	1.9000	2.1460	2.8150	2.2320	3.4930	
		0.1, 0.1, 0.1	0.8042	0.9452	1.0000	0.9472	0.9618	0.3565	0.5326	3.0898	0.5479	0.5296	
	0.5, 1, 5	0.3, 0.3, 0.3	0.6430	0.9542	1.0000	0.9543	0.9782	0.3931	0.8143	2.8115	0.8318	0.8097	
		0.5, 0.5, 0.5	0.5595	0.9465	0.9987	0.9485	0.9947	0.5558	1.3036	2.5193	1.3371	1.3242	
		0.1, 0.3, 0.5	0.6330	0.9488	1.0000	0.9485	0.9715	0.4375	0.9154	2.9305	0.9326	0.9281	
		0.1, 0.1, 0.1	0.9285	0.9490	0.9441	0.9517	0.9628	1.4200	1.4960	3.0905	1.5330	2.2550	
		0.3, 0.3, 0.3	0.9102	0.9550	0.9082	0.9587	0.9732	1.9990	2.2160	2.8010	2.2760	3.9100	
		0.5, 0.5, 0.5	0.8835	0.9548	0.8071	0.9580	0.9830	3.4800	3.9390	2.4923	4.0620	8.2090	
		0.1, 0.3, 0.5	0.8928	0.9552	0.9681	0.9583	0.9922	1.6140	1.8830	2.7895	1.9530	2.7640	
	50, 50, 50	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9440	0.9505	1.0000	0.9525	0.9992	1.5080	1.5430	3.9120	1.5960	1.7450
			0.3, 0.3, 0.3	0.9367	0.9555	1.0000	0.9597	0.9998	2.0630	2.1790	3.7723	2.2650	3.0200
0.5, 0.5, 0.5			0.9158	0.9538	1.0000	0.9573	1.0000	3.4680	3.7260	3.4715	3.8940	6.8150	
1, 1, 1		0.1, 0.3, 0.5	0.9312	0.9540	0.9967	0.9577	0.9932	2.3590	2.5080	3.7993	2.5930	4.3000	
		0.1, 0.1, 0.1	0.9317	0.9498	1.0000	0.9532	0.9853	0.8263	0.8721	3.9194	0.8989	0.8673	
		0.3, 0.3, 0.3	0.9052	0.9523	1.0000	0.9557	0.9952	1.0400	1.1910	3.8076	1.2310	1.2270	
5, 5, 5		0.5, 0.5, 0.5	0.8718	0.9555	1.0000	0.9600	0.9998	1.5474	1.8652	3.5053	1.9387	2.2389	
		0.1, 0.3, 0.5	0.9033	0.9510	1.0000	0.9525	0.9842	1.1437	1.3294	3.8526	1.3680	1.4912	
		0.1, 0.1, 0.1	0.7872	0.9462	1.0000	0.9485	0.9582	0.2596	0.3994	4.0057	0.4062	0.3981	
0.5, 1, 5		0.3, 0.3, 0.3	0.6147	0.9492	1.0000	0.9502	0.9632	0.2732	0.6032	3.8586	0.6111	0.6010	
		0.5, 0.5, 0.5	0.5350	0.9495	1.0000	0.9500	0.9780	0.3534	0.9264	3.5825	0.9410	0.9209	
		0.1, 0.3, 0.5	0.6172	0.9513	1.0000	0.9512	0.9675	0.2956	0.6615	3.9957	0.6691	0.6599	
		0.1, 0.1, 0.1	0.9272	0.9487	0.9927	0.9477	0.9547	0.9391	0.9937	4.0261	1.0108	1.0850	
		0.3, 0.3, 0.3	0.9038	0.9547	0.9774	0.9557	0.9662	1.2333	1.3925	3.8916	1.4193	1.7414	
		0.5, 0.5, 0.5	0.8762	0.9510	0.9508	0.9535	0.9740	1.9536	2.2832	3.5967	2.3356	3.7153	
		0.1, 0.3, 0.5	0.8867	0.9512	0.9947	0.9535	0.9803	1.0251	1.2231	3.8358	1.2540	1.3270	
100, 100, 100		0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9428	0.9500	1.0000	0.9510	0.9777	0.9395	0.9636	5.2424	0.9824	0.9539
			0.3, 0.3, 0.3	0.9347	0.9558	1.0000	0.9570	0.9865	1.2059	1.2892	5.1301	1.3191	1.2850
	0.5, 0.5, 0.5		0.9180	0.9508	1.0000	0.9522	0.9967	1.7576	1.9358	5.1871	1.9935	2.1003	
	1, 1, 1	0.1, 0.3, 0.5	0.9282	0.9535	1.0000	0.9535	0.9828	1.3303	1.4353	5.3344	1.4649	1.5039	
		0.1, 0.1, 0.1	0.9327	0.9480	1.0000	0.9503	0.9682	0.5421	0.5748	5.2733	0.5843	0.5709	
		0.3, 0.3, 0.3	0.9018	0.9542	1.0000	0.9542	0.9770	0.6613	0.7699	5.1663	0.7839	0.7645	
	5, 5, 5	0.5, 0.5, 0.5	0.8775	0.9528	1.0000	0.9553	0.9852	0.9088	1.1332	5.2077	1.1586	1.1259	
		0.1, 0.3, 0.5	0.8998	0.9525	1.0000	0.9532	0.9743	0.7191	0.8515	5.3727	0.8653	0.8481	
		0.1, 0.1, 0.1	0.7905	0.9475	1.0000	0.9475	0.9525	0.1747	0.2760	5.3186	0.2784	0.2757	
	0.5, 1, 5	0.3, 0.3, 0.3	0.5847	0.9487	1.0000	0.9490	0.9567	0.1787	0.4148	5.2495	0.4175	0.4143	
		0.5, 0.5, 0.5	0.5153	0.9487	1.0000	0.9492	0.9620	0.2207	0.6211	5.2534	0.6261	0.6195	
		0.1, 0.3, 0.5	0.6005	0.9453	1.0000	0.9475	0.9575	0.1935	0.4536	5.4821	0.4561	0.4532	
		0.1, 0.1, 0.1	0.9297	0.9483	0.9980	0.9493	0.9550	0.6054	0.6429	5.3147	0.6490	0.6421	
		0.3, 0.3, 0.3	0.9005	0.9545	0.9980	0.9543	0.9643	0.7559	0.8681	5.2589	0.8774	0.8711	
		0.5, 0.5, 0.5	0.8770	0.9543	0.9980	0.9538	0.9672	1.0701	1.2927	5.2796	1.3099	1.3810	
		0.1, 0.3, 0.5	0.8825	0.9538	1.0000	0.9547	0.9692	0.6571	0.7943	5.3254	0.8051	0.7898	
	30, 50, 100	0.5, 0.5, 0.5	0.1, 0.1, 0.1	0.9372	0.9458	0.9980	0.9483	0.9763	1.6638	1.7029	4.2773	1.7498	2.4420
			0.3, 0.3, 0.3	0.9333	0.9573	0.9973	0.9593	0.9852	2.3598	2.4871	4.1063	2.5639	4.1598
0.5, 0.5, 0.5			0.9188	0.9553	0.9907	0.9582	0.9947	4.0507	4.3370	3.9028	4.4886	8.7179	
1, 1, 1		0.1, 0.3, 0.5	0.9317	0.9523	1.0000	0.9558	0.9993	2.0654	2.1766	4.0163	2.2600	3.0667	
		0.1, 0.1, 0.1	0.9308	0.9500	0.9987	0.9512	0.9725	0.8719	0.9208	4.3654	0.9444	0.9818	
		0.3, 0.3, 0.3	0.8990	0.9527	0.9987	0.9552	0.9778	1.1223	1.2837	4.1927	1.3194	1.5497	
5, 5, 5		0.5, 0.5, 0.5	0.8757	0.9542	0.9947	0.9573	0.9865	1.8117	2.1468	3.9933	2.2127	3.3335	
		0.1, 0.3, 0.5	0.9037	0.9523	1.0000	0.9562	0.9932	1.0447	1.1876	4.0611	1.2269	1.2336	
		0.1, 0.1, 0.1	0.7852	0.9517	1.0000	0.9522	0.9575	0.2697	0.4117	4.5737	0.4180	0.4109	
0.5, 1, 5		0.3, 0.3, 0.3	0.6043	0.9393	1.0000	0.9405	0.9572	0.2866	0.6270	4.3989	0.6340	0.6255	
		0.5, 0.5, 0.5	0.5292	0.9442	0.9987	0.9463	0.9712	0.3863	0.9720	4.1979	0.9853	0.9834	
		0.1, 0.3, 0.5	0.6435	0.9477	1.0000	0.9492	0.9643	0.2858	0.5854	4.1723	0.5946	0.5832	
		0.1, 0.1, 0.1	0.9280	0.9462	0.9581	0.9455	0.9477	1.2868	1.3388	4.6420	1.3544	2.0322	
		0.3, 0.3, 0.3	0.9145	0.9542	0.9235	0.9553	0.9585	1.8215	1.9820	4.4723	2.0071	3.4593	
		0.5, 0.5, 0.5	0.8920	0.9515	0.8463	0.9522	0.9687	3.0790	3.4242	4.2733	3.4752	6.9356	
		0.1, 0.3, 0.5	0.9055	0.9528	0.9228	0.9532	0.9633	1.3493	1.4887	4.2626	1.5174	2.1193	

Table 2. Coverage probabilities (CP) and average widths (AW) of the 95% simultaneous confidence intervals for all pairwise differences of coefficients of variation under delta-IG distributions in the case of $k = 6$, with equal mean parameters, various sample sizes, and parameter configurations (λ and δ) based on the GCI, AGCI, FCI, MOVER, and NA methods.

n_i^6	λ_i^6	δ_i^6	Coverage Probabilities (CP)					Average Widths (AW)				
			GCI	AGCI	FCI	MOVER	NA	GCI	AGCI	FCI	MOVER	NA
30 ⁶	0.5 ⁶	0.1 ⁶	0.9449	0.9519	0.9999	0.9556	1.0000	2.4212	2.4684	2.9491	2.5845	4.2278
		0.3 ⁶	0.9334	0.9533	0.9999	0.9574	1.0000	3.6697	3.8232	2.7222	4.0154	7.5072
		0.5 ⁶	0.9265	0.9606	0.9988	0.9654	1.0000	6.6199	6.9914	2.3427	7.3844	14.1515
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	0.9324	0.9551	0.9860	0.9595	0.9995	4.2925	4.4921	2.7562	4.6964	9.0359
		0.1 ⁶	0.9346	0.9520	0.9999	0.9560	0.9987	1.1849	1.2445	2.9822	1.3009	1.3873
		0.3 ⁶	0.9047	0.9535	1.0000	0.9572	1.0000	1.6067	1.8018	2.7605	1.8878	2.4812
	5 ⁶	0.5 ⁶	0.8865	0.9586	0.9989	0.9634	1.0000	2.8026	3.2214	2.3832	3.3910	5.8556
		0.1 ² , 0.3 ² , 0.5 ²	0.9035	0.9562	0.9981	0.9600	0.9959	1.8869	2.1284	2.8091	2.2182	3.3395
		0.1 ⁶	0.8001	0.9514	0.9999	0.9534	0.9676	0.3572	0.5315	3.0465	0.5467	0.5285
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.6382	0.9473	1.0000	0.9494	0.9753	0.3916	0.8145	2.8388	0.8319	0.8101
		0.5 ⁶	0.5646	0.9496	0.9996	0.9523	0.9942	0.5547	1.3002	2.4621	1.3336	1.3258
		0.1 ² , 0.3 ² , 0.5 ²	0.6377	0.9495	1.0000	0.9513	0.9756	0.4368	0.9087	2.8933	0.9270	0.9161
		0.1 ⁶	0.9242	0.9492	0.9558	0.9522	0.9685	1.4094	1.4852	3.0385	1.5272	2.2396
		0.3 ⁶	0.8871	0.9514	0.9321	0.9534	0.9777	1.9971	2.2215	2.8265	2.2879	3.9002
		0.5 ⁶	0.8636	0.9559	0.8361	0.9587	0.9868	3.5053	3.9744	2.4503	4.1115	7.7213
		0.1 ² , 0.3 ² , 0.5 ²	0.8700	0.9526	0.9771	0.9556	0.9912	1.5956	1.8782	2.7862	1.9494	2.8018
50 ⁶	0.5 ⁶	0.1 ⁶	0.9432	0.9500	1.0000	0.9528	0.9987	1.5115	1.5465	3.8852	1.5995	1.7344
		0.3 ⁶	0.9306	0.9506	1.0000	0.9532	0.9998	2.0655	2.1845	3.6713	2.2700	3.0099
		0.5 ⁶	0.9249	0.9578	0.9997	0.9619	1.0000	3.5082	3.7606	3.5032	3.9325	7.0382
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	0.9314	0.9537	0.9983	0.9556	0.9960	2.3895	2.5361	3.7603	2.6259	4.0400
		0.1 ⁶	0.9339	0.9507	1.0000	0.9527	0.9825	0.8253	0.8712	3.9309	0.8976	0.8638
		0.3 ⁶	0.9026	0.9505	1.0000	0.9532	0.9940	1.0425	1.1948	3.7104	1.2345	1.2177
	5 ⁶	0.5 ⁶	0.8833	0.9565	1.0000	0.9598	0.9998	1.5626	1.8779	3.5388	1.9519	2.2778
		0.1 ² , 0.3 ² , 0.5 ²	0.9001	0.9519	0.9999	0.9545	0.9858	1.1554	1.3393	3.8058	1.3800	1.4978
		0.1 ⁶	0.7888	0.9499	1.0000	0.9515	0.9603	0.2595	0.3990	4.0022	0.4059	0.3978
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.6102	0.9441	1.0000	0.9455	0.9604	0.2731	0.6043	3.7884	0.6122	0.6021
		0.5 ⁶	0.5317	0.9462	0.9999	0.9482	0.9759	0.3569	0.9274	3.6219	0.9421	0.9216
		0.1 ² , 0.3 ² , 0.5 ²	0.6177	0.9468	1.0000	0.9482	0.9641	0.2972	0.6613	3.9106	0.6695	0.6593
		0.1 ⁶	0.9232	0.9502	0.9902	0.9517	0.9645	0.9262	0.9843	3.9912	1.0035	1.0607
		0.3 ⁶	0.8866	0.9500	0.9811	0.9517	0.9693	1.2064	1.3772	3.7781	1.4073	1.6919
		0.5 ⁶	0.8583	0.9542	0.9670	0.9561	0.9788	1.9369	2.2736	3.6144	2.3326	3.5791
		0.1 ² , 0.3 ² , 0.5 ²	0.8682	0.9516	0.9967	0.9537	0.9822	1.0208	1.2309	3.7889	1.2631	1.2988
100 ⁶	0.5 ⁶	0.1 ⁶	0.9443	0.9507	1.0000	0.9520	0.9774	0.9405	0.9641	5.2183	0.9829	0.9549
		0.3 ⁶	0.9362	0.9565	1.0000	0.9581	0.9896	1.2056	1.2877	5.1719	1.3176	1.2837
		0.5 ⁶	0.9216	0.9530	1.0000	0.9553	0.9985	1.7679	1.9470	5.1706	2.0046	2.0843
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	0.9308	0.9517	0.9997	0.9537	0.9816	1.3249	1.4275	5.2408	1.4585	1.4859
		0.1 ⁶	0.9340	0.9506	1.0000	0.9517	0.9676	0.5427	0.5753	5.2270	0.5847	0.5713
		0.3 ⁶	0.9104	0.9547	1.0000	0.9564	0.9754	0.6616	0.7697	5.1925	0.7836	0.7641
	5 ⁶	0.5 ⁶	0.8727	0.9507	1.0000	0.9523	0.9845	0.9140	1.1384	5.1718	1.1641	1.1325
		0.1 ² , 0.3 ² , 0.5 ²	0.8985	0.9523	1.0000	0.9534	0.9729	0.7141	0.8442	5.2906	0.8584	0.8404
		0.1 ⁶	0.7800	0.9488	1.0000	0.9498	0.9541	0.1747	0.2762	5.2729	0.2785	0.2759
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.6030	0.9498	1.0000	0.9508	0.9577	0.1789	0.4142	5.2504	0.4169	0.4137
		0.5 ⁶	0.5067	0.9476	1.0000	0.9483	0.9607	0.2220	0.6232	5.2569	0.6281	0.6214
		0.1 ² , 0.3 ² , 0.5 ²	0.6019	0.9492	1.0000	0.9499	0.9573	0.1926	0.4500	5.3863	0.4528	0.4495
		0.1 ⁶	0.9233	0.9502	0.9985	0.9511	0.9592	0.5944	0.6350	5.2931	0.6418	0.6333
		0.3 ⁶	0.8904	0.9544	0.9977	0.9549	0.9661	0.7430	0.8605	5.2601	0.8709	0.8618
		0.5 ⁶	0.8534	0.9509	0.9959	0.9516	0.9685	1.0529	1.2849	5.2668	1.3050	1.3431
		0.1 ² , 0.3 ² , 0.5 ²	0.8618	0.9494	0.9997	0.9503	0.9668	0.6457	0.7909	5.2703	0.8023	0.7863
30 ³ , 50 ³	0.5 ⁶	0.1 ⁶	0.9455	0.9530	0.9995	0.9561	0.9970	1.9512	1.9947	3.4944	2.0733	3.0140
		0.3 ⁶	0.9332	0.9534	0.9995	0.9561	0.9998	2.8789	3.0146	3.2774	3.1434	5.5913
		0.5 ⁶	0.9210	0.9566	0.9977	0.9612	1.0000	5.1375	5.4460	2.9741	5.7120	10.8469
	1 ⁶	0.1 ² , 0.3 ² , 0.5 ²	0.9307	0.9530	0.9997	0.9568	0.9997	2.9330	3.0822	3.2485	3.2180	5.6238
		0.1 ⁶	0.9369	0.9547	0.9996	0.9578	0.9879	1.0062	1.0599	3.5394	1.0988	1.1286
		0.3 ⁶	0.9022	0.9517	0.9993	0.9550	0.9936	1.3323	1.5083	3.3255	1.5669	1.8402
	5 ⁶	0.5 ⁶	0.8823	0.9573	0.9989	0.9615	0.9997	2.2014	2.5747	3.0233	2.6888	4.1214
		0.1 ² , 0.3 ² , 0.5 ²	0.9021	0.9530	1.0000	0.9566	0.9965	1.3611	1.5491	3.2914	1.6105	1.8274
		0.1 ⁶	0.7970	0.9512	0.9999	0.9526	0.9625	0.3092	0.4671	3.6380	0.4775	0.4653
	0.5 ² , 1 ² , 5 ²	0.3 ⁶	0.6273	0.9457	0.9999	0.9470	0.9682	0.3354	0.7146	3.4534	0.7265	0.7119
		0.5 ⁶	0.5449	0.9502	0.9995	0.9524	0.9841	0.4575	1.1207	3.1392	1.1432	1.1318
		0.1 ² , 0.3 ² , 0.5 ²	0.6293	0.9485	1.0000	0.9502	0.9716	0.3488	0.7321	3.3708	0.7451	0.7288
		0.1 ⁶	0.9263	0.9524	0.9554	0.9536	0.9627	1.3076	1.3741	3.6728	1.4049	2.0913
		0.3 ⁶	0.8909	0.9513	0.9248	0.9532	0.9676	1.8777	2.0686	3.4740	2.1179	3.7061
		0.5 ⁶	0.8617	0.9555	0.8302	0.9581	0.9769	3.2596	3.6577	3.1517	3.7598	7.1460
		0.1 ² , 0.3 ² , 0.5 ²	0.8730	0.9499	0.9573	0.9523	0.9765	1.4530	1.6647	3.3242	1.7141	2.4161

Table 2. Cont.

n_i^6	λ_i^6	δ_i^6	Coverage Probabilities (CP)					Average Widths (AW)				
			GCI	AGCI	FCI	MOVER	NA	GCI	AGCI	FCI	MOVER	NA
$30^2, 50^2, 100^2$	0.5^6	0.1^6	0.9454	0.9532	0.9973	0.9556	0.9810	1.6317	1.6685	4.1777	1.7183	2.3624
		0.3^6	0.9310	0.9524	0.9953	0.9543	0.9858	2.3444	2.4681	4.0498	2.5497	4.1878
		0.5^6	0.9226	0.9557	0.9899	0.9581	0.9948	4.0150	4.2951	3.8385	4.4594	8.4754
	1^6	$0.1^2, 0.3^2, 0.5^2$	0.9321	0.9530	1.0000	0.9560	0.9993	2.0825	2.1964	4.0586	2.2812	3.1723
		0.1^6	0.9377	0.9544	0.9987	0.9566	0.9744	0.8591	<i>0.9066</i>	4.2449	0.9314	0.9526
		0.3^6	0.9005	0.9503	0.9967	0.9525	0.9783	1.1208	1.2774	4.1259	1.3148	1.5239
		0.5^6	0.8819	0.9551	0.9944	0.9583	0.9885	1.7929	2.1229	3.9235	2.1949	3.4241
		$0.1^2, 0.3^2, 0.5^2$	0.9110	0.9552	0.9993	0.9578	0.9929	1.0478	1.1914	4.1071	1.2317	1.2436
	5^6	0.1^6	0.7952	0.9516	0.9997	0.9525	0.9570	0.2673	0.4076	4.4254	0.4142	<i>0.4067</i>
		0.3^6	0.6105	0.9465	0.9995	0.9474	0.9628	0.2856	0.6214	4.2933	0.6289	<i>0.6198</i>
		0.5^6	0.5319	0.9474	0.9977	0.9485	0.9726	0.3839	0.9664	4.1172	0.9805	0.9769
	$0.5^2, 1^2, 5^2$	$0.1^2, 0.3^2, 0.5^2$	0.6462	0.9502	0.9997	0.9512	0.9652	0.2867	0.5877	4.2127	0.5969	<i>0.5854</i>
		0.1^6	0.9270	0.9512	0.9659	0.9522	0.9538	1.2378	1.2912	4.4718	1.3134	1.9578
		0.3^6	0.8880	0.9477	0.9397	0.9488	0.9597	1.7778	1.9399	4.3462	1.9760	3.4535
		0.5^6	0.8655	0.9538	0.8649	0.9551	0.9715	3.0036	3.3484	4.1524	3.4211	6.7533
		$0.1^2, 0.3^2, 0.5^2$	0.8865	0.9533	0.9512	0.9538	0.9691	1.3307	1.4830	4.2728	1.5171	2.1928

Note: Bold values indicate coverage probabilities that are equal to or exceed the nominal confidence level of 0.95 (values greater than 0.945 are rounded to 0.95). Italicized values denote the smallest average widths among methods with CP ≥ 0.95 .

When the number of populations increased to $k = 6$ (Table 2 and Figure 2), a similar pattern was observed. The AGCI method continued to deliver stable CP performance across all parameter settings, demonstrating robustness to skewness and increasing model complexity. At the same time, MOVER achieved competitive coverage with consistently smaller Awws than GCI and FCI. As variability among the δ parameters increased, the advantage of AGCI and MOVER became more evident, as both methods effectively balanced interval accuracy and precision.

For $k = 10$ (Table 3 and Figure 3), the differences among methods became more pronounced. The GCI method exhibited mild undercoverage across several scenarios, which can be attributed to the increasing impact of skewness and parameter dependence in higher-dimensions. The FCI method again produced overly conservative intervals with large widths, reinforcing its tendency to overestimate uncertainty. In contrast, the AGCI method maintained CPs very close to the nominal level across nearly all configurations, demonstrating strong robustness to increasing dimensionality. The MOVER method frequently yielded the narrowest intervals while still preserving satisfactory CPs, highlighting its efficiency in high-dimensional settings. However, slight undercoverage was occasionally observed, which may be due to instability in variance estimates under strong skewness.

Across all cases ($k = 3, 6$, and 10), increasing the number of groups generally led to wider intervals for all methods, reflecting the increased uncertainty inherent in simultaneous inference. The normal approximation (NA) method consistently performed less well than other approaches. This may be explained by its reliance on asymptotic normality, which is violated in the presence of strong skewness and zero inflation. As a result, the NA method tends to produce inaccurate standard error estimates, leading to suboptimal coverage performance.

Overall, the results indicate that AGCI and MOVER are the most reliable methods for constructing SCIs for all pairwise differences of coefficients of variation under delta-IG distributions. The AGCI method provides highly stable coverage performance due to its bias-correction mechanism, while MOVER offers superior interval efficiency by efficiently utilizing variance. These findings highlight the importance of accounting for skewness and distributional characteristics when selecting appropriate interval estimation methods in complex data settings.

Table 3. Coverage probabilities (CP) and average widths (AW) of the 95% simultaneous confidence intervals for all pairwise differences of coefficients of variation under delta-IG distributions in the case of $k = 10$, with equal mean parameters, various sample sizes, and parameter configurations (λ and δ) based on the GCI, AGCI, FCI, MOVER, and NA methods.

n_i^{10}	λ_i^{10}	δ_i^{10}	Coverage Probabilities (CP)					Average Widths (AW)				
			GCI	AGCI	FCI	MOVER	NA	GCI	AGCI	FCI	MOVER	NA
30 ⁵ , 50 ⁵	0.5 ¹⁰	0.1 ⁵ , 0.3 ⁵	0.9387	0.9522	1.0000	0.9553	0.9999	2.2293	2.3117	3.3804	2.4109	3.6063
		0.3 ⁵ , 0.5 ⁵	0.9257	0.9536	0.9996	0.9577	1.0000	3.6188	3.8242	3.0758	4.0074	7.2603
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.9388	0.9547	0.9994	0.9581	0.9988	2.4892	2.5945	3.3565	2.7030	4.4187
	1 ¹⁰	0.1 ² , 0.3 ² , 0.5 ⁶	0.9277	0.9554	0.9963	0.9586	0.9998	3.6646	3.8763	3.1320	4.0518	7.5293
		0.1 ⁵ , 0.3 ⁵	0.9236	0.9543	0.9998	0.9577	0.9961	1.1115	1.2164	3.4085	1.2639	1.3012
		0.3 ⁵ , 0.5 ⁵	0.8910	0.9538	1.0000	0.9575	0.9999	1.5937	1.8507	3.1248	1.9312	2.4062
	5 ¹⁰	0.1 ⁶ , 0.3 ² , 0.5 ²	0.9163	0.9531	0.9999	0.9564	0.9939	1.2009	1.3347	3.3951	1.3853	1.5019
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8913	0.9525	0.9992	0.9559	0.9981	1.6366	1.8945	3.1768	1.9719	2.5802
		0.1 ⁵ , 0.3 ⁵	0.7116	0.9487	1.0000	0.9505	0.9653	0.3164	0.5682	3.4991	0.5796	0.5656
	0.5 ⁴ , 1 ³ , 5 ³	0.3 ⁵ , 0.5 ⁵	0.5781	0.9456	0.9999	0.9473	0.9751	0.3755	0.8749	3.2120	0.8909	0.8698
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.6947	0.9478	1.0000	0.9489	0.9669	0.3317	0.6226	3.4798	0.6344	0.6200
		0.1 ² , 0.3 ² , 0.5 ⁶	0.5903	0.9468	0.9999	0.9480	0.9744	0.3854	0.8757	3.2716	0.8913	0.8751
		0.1 ⁵ , 0.3 ⁵	0.9036	0.9522	0.9605	0.9541	0.9696	1.4488	1.5703	3.5041	1.6130	2.3519
		0.3 ⁵ , 0.5 ⁵	0.8719	0.9523	0.9193	0.9548	0.9792	2.1702	2.4424	3.2122	2.5188	4.3157
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.8910	0.9505	0.9608	0.9526	0.9736	1.4554	1.6088	3.4503	1.6541	2.4750
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8725	0.9529	0.9343	0.9551	0.9814	2.0259	2.2984	3.2404	2.3726	3.8888
50 ⁵ , 100 ⁵	0.5 ¹⁰	0.1 ⁵ , 0.3 ⁵	0.9368	0.9499	0.9999	0.9521	0.9911	1.3567	1.4144	4.6549	1.4548	1.5359
		0.3 ⁵ , 0.5 ⁵	0.9267	0.9535	1.0000	0.9560	0.9992	1.9238	2.0714	4.4563	2.1423	2.5944
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.9346	0.9506	0.9999	0.9527	0.9898	1.4460	1.5204	4.5980	1.5636	1.6485
	1 ¹⁰	0.1 ² , 0.3 ² , 0.5 ⁶	0.9268	0.9542	0.9996	0.9565	0.9969	1.9454	2.0936	4.5021	2.1611	2.7670
		0.1 ⁵ , 0.3 ⁵	0.9190	0.9505	0.9999	0.9524	0.9770	0.7436	0.8196	4.6878	0.8392	0.8136
		0.3 ⁵ , 0.5 ⁵	0.8918	0.9521	0.9999	0.9550	0.9898	0.9815	1.1691	4.5033	1.2015	1.1809
	5 ¹⁰	0.1 ⁶ , 0.3 ² , 0.5 ²	0.9154	0.9508	1.0000	0.9529	0.9778	0.7839	0.8794	4.6390	0.9003	0.8737
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8937	0.9534	0.9998	0.9557	0.9854	0.9862	1.1736	4.5502	1.2045	1.2124
		0.1 ⁵ , 0.3 ⁵	0.7005	0.9474	1.0000	0.9487	0.9564	0.2207	0.4067	4.7890	0.4114	0.4058
	0.5 ⁴ , 1 ³ , 5 ³	0.3 ⁵ , 0.5 ⁵	0.5650	0.9490	1.0000	0.9498	0.9634	0.2484	0.6139	4.6027	0.6203	0.6119
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.6878	0.9502	1.0000	0.9511	0.9587	0.2281	0.4397	4.7342	0.4446	0.4388
		0.1 ² , 0.3 ² , 0.5 ⁶	0.5808	0.9513	0.9999	0.9521	0.9649	0.2533	0.6108	4.6465	0.6170	0.6090
		0.1 ⁵ , 0.3 ⁵	0.8999	0.9495	0.9883	0.9503	0.9627	0.9400	1.0227	4.8101	1.0405	1.1376
		0.3 ⁵ , 0.5 ⁵	0.8739	0.9513	0.9752	0.9525	0.9715	1.2740	1.4632	4.6125	1.4940	1.8571
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.8905	0.9510	0.9874	0.9522	0.9648	0.9343	1.0379	4.7290	1.0564	1.1252
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8737	0.9530	0.9830	0.9543	0.9713	1.1955	1.3850	4.6474	1.4146	1.6898
30 ⁶ , 50 ² , 100 ²	0.5 ¹⁰	0.1 ⁵ , 0.3 ⁵	0.9391	0.9526	0.9978	0.9559	0.9919	2.2405	2.3228	3.6416	2.4135	3.8249
		0.3 ⁵ , 0.5 ⁵	0.9287	0.9553	0.9968	0.9579	0.9977	3.5792	3.7801	3.4234	3.9436	7.3199
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.9371	0.9517	1.0000	0.9551	0.9996	2.2185	2.3063	3.6213	2.4020	3.5283
	1 ¹⁰	0.1 ² , 0.3 ² , 0.5 ⁶	0.9272	0.9548	0.9914	0.9578	0.9972	3.6231	3.8292	3.4300	3.9861	7.1745
		0.1 ⁵ , 0.3 ⁵	0.9188	0.9509	0.9986	0.9540	0.9867	1.1020	1.2045	3.6962	1.2479	1.3769
		0.3 ⁵ , 0.5 ⁵	0.8945	0.9559	0.9984	0.9592	0.9936	1.5885	1.8387	3.4781	1.9111	2.5410
	5 ¹⁰	0.1 ⁶ , 0.3 ² , 0.5 ²	0.9173	0.9504	0.9996	0.9539	0.9937	1.1060	1.2148	3.6734	1.2611	1.3100
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8918	0.9547	0.9966	0.9579	0.9914	1.6303	1.8836	3.4963	1.9534	2.7389
		0.1 ⁵ , 0.3 ⁵	0.7117	0.9479	0.9996	0.9493	0.9631	0.3118	0.5554	3.8120	0.5659	0.5533
	0.5 ⁴ , 1 ³ , 5 ³	0.3 ⁵ , 0.5 ⁵	0.5832	0.9489	0.9995	0.9505	0.9739	0.3715	0.8547	3.6193	0.8692	0.8548
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.7013	0.9467	0.9997	0.9485	0.9632	0.3158	0.5645	3.7686	0.5759	0.5619
		0.1 ² , 0.3 ² , 0.5 ⁶	0.5958	0.9489	0.9992	0.9503	0.9723	0.3792	0.8546	3.6381	0.8688	0.8587
		0.1 ⁵ , 0.3 ⁵	0.9034	0.9516	0.9591	0.9532	0.9698	1.5054	1.6166	3.8511	1.6610	2.5381
		0.3 ⁵ , 0.5 ⁵	0.8784	0.9537	0.9181	0.9559	0.9767	2.2789	2.5334	3.6390	2.6117	4.6823
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.8978	0.9498	0.9498	0.9518	0.9707	1.4734	1.5918	3.8142	1.6355	2.3917
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8756	0.9525	0.9303	0.9550	0.9761	2.1362	2.3937	3.6477	2.4692	4.1759
30 ² , 50 ² , 100 ⁶	0.5 ¹⁰	0.1 ⁵ , 0.3 ⁵	0.9397	0.9524	0.9986	0.9543	0.9854	1.4928	1.5518	4.6209	1.5954	2.0027
		0.3 ⁵ , 0.5 ⁵	0.9256	0.9528	0.9990	0.9550	0.9945	2.1731	2.3256	4.4763	2.4023	3.4094
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.9367	0.9522	0.9986	0.9540	0.9846	1.5790	1.6552	4.6459	1.7020	2.1070
	1 ¹⁰	0.1 ² , 0.3 ² , 0.5 ⁶	0.9281	0.9530	1.0000	0.9554	0.9992	1.9587	2.0982	4.4910	2.1714	2.7846
		0.1 ⁵ , 0.3 ⁵	0.9211	0.9521	0.9991	0.9541	0.9755	0.7958	0.8730	4.6782	0.8943	0.8993
		0.3 ⁵ , 0.5 ⁵	0.8885	0.9512	0.9991	0.9534	0.9840	1.0620	1.2550	4.5188	1.2900	1.4010
	5 ¹⁰	0.1 ⁶ , 0.3 ² , 0.5 ²	0.9178	0.9527	0.9993	0.9547	0.9759	0.8323	0.9296	4.6995	0.9523	0.9522
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8973	0.9544	0.9995	0.9567	0.9888	0.9958	1.1713	4.5389	1.2055	1.2017
		0.1 ⁵ , 0.3 ⁵	0.7009	0.9492	0.9997	0.9505	0.9572	0.2344	0.4235	4.8124	0.4287	0.4226
	0.5 ⁴ , 1 ³ , 5 ³	0.3 ⁵ , 0.5 ⁵	0.5613	0.9474	0.9996	0.9486	0.9634	0.2646	0.6407	4.6580	0.6477	0.6387
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.6899	0.9488	0.9998	0.9498	0.9580	0.2414	0.4564	4.8293	0.4619	0.4554
		0.1 ² , 0.3 ² , 0.5 ⁶	0.5942	0.9493	0.9999	0.9503	0.9638	0.2613	0.6017	4.6304	0.6091	0.5996
		0.1 ⁵ , 0.3 ⁵	0.9008	0.9506	0.9733	0.9518	0.9605	1.1160	1.2000	4.7983	1.2200	1.6750
		0.3 ⁵ , 0.5 ⁵	0.8718	0.9527	0.9440	0.9536	0.9671	1.5870	1.7830	4.6446	1.8180	2.8380
		0.1 ⁶ , 0.3 ² , 0.5 ²	0.8902	0.9519	0.9720	0.9530	0.9611	1.1090	1.2150	4.7969	1.2350	1.6630
		0.1 ² , 0.3 ² , 0.5 ⁶	0.8765	0.9533	0.9591	0.9547	0.9692	1.3330	1.5120	4.6501	1.5440	2.1520

5. An Empirical Application

Statistical analysis of transportation accident data plays an important role in traffic safety management, infrastructure planning, and risk assessment. In practice, accident datasets frequently contain a large proportion of zero observations together with highly skewed positive counts, reflecting districts or periods with no reported fatalities. Such characteristics limit the applicability of conventional statistical models and motivate the use of zero-inflated distributions that can adequately capture both excess zeros and variability in nonzero observations.

In this study, simultaneous confidence intervals (SCIs) for all pairwise differences of coefficients of variation (CVs) under delta-IG distributions are applied to transportation accident mortality data. Rather than focusing on variability within a single population or comparisons between only two groups, the simultaneous framework enables joint comparison of relative dispersion across multiple populations while controlling the overall confidence level. The CV serves as a scale-free measure of variability, allowing meaningful comparison of accident fluctuations among regions with substantially different average mortality levels. Consequently, simultaneous interval estimation provides a systematic approach for determining whether variability in accident outcomes differs significantly across districts.

The empirical analysis utilizes road accident fatality data reported by the Road Accident Victims Protection Company Limited and publicly available through the Thai Road Safety Center (Thai RSC, <https://www.thairsc.com> (accessed on 1 March 2026)) database. The dataset consists of monthly traffic accident death counts recorded in the second half of 2024 across 50 districts in Bangkok, Thailand, is presented in Table 4. Given the many zero-fatality observations and pronounced right skewness in the recorded counts, the delta-IG distribution provides an appropriate modeling framework for the data.

Table 4. The death count from traffic accidents in Bangkok, Thailand, in the second half of 2024.

Months	Death Count																													
December	1	0	3	2	2	4	3	1	0	0	1	3	1	2	0	0	3	2	1	1	1	3	1	1	0	4	0	2	0	
	0	0	1	0	1	0	2	2	7	2	2	1	2	0	0	2	0	1	6	1	0									
November	0	1	3	0	0	1	0	0	1	4	2	2	2	2	1	1	5	3	1	3	1	1	2	2	1	3	0	0	0	
	2	0	0	2	3	0	0	2	4	2	1	0	4	2	0	0	5	4	3	1	4									
October	1	1	0	1	4	1	0	0	0	1	3	2	1	2	1	1	1	1	3	4	4	1	0	0	0	0	0	2	0	
	0	0	0	1	2	0	2	1	5	0	0	1	1	1	0	0	0	0	1	1	3									
September	0	2	3	1	3	2	0	0	0	2	1	0	3	2	0	3	5	0	0	1	0	1	2	0	0	1	0	0	1	
	1	0	0	1	4	4	0	1	6	0	0	1	1	2	0	1	4	1	8	0	1									
August	2	1	0	0	2	2	3	0	1	2	1	1	0	0	1	1	6	1	0	2	1	2	1	3	0	0	2	0	0	
	1	2	1	1	2	0	0	0	3	2	0	2	2	1	0	1	1	0	5	2	2									
July	1	1	4	0	1	2	2	0	0	2	1	1	3	1	0	1	3	1	1	4	0	3	1	3	0	1	0	2	0	
	1	0	0	1	5	0	0	1	5	0	1	0	2	3	0	1	0	2	4	1	0									

The primary objective of this application is to illustrate the practical performance of the proposed simultaneous confidence interval procedures for all pairwise differences of CVs among delta-IG populations. By simultaneously comparing relative variability across districts, the analysis offers insights into differences in patterns of fluctuation in accident risk, thereby supporting evidence-based decision-making for transportation safety planning, resource allocation, and risk management strategies.

These datasets consist of strictly positive observations and generally display pronounced right skewness, indicating that asymmetric distributional models are more appropriate than symmetric alternatives. Accordingly, model selection in the real-data analysis is performed using the Akaike Information Criterion (AIC) and the Bayesian Information

Criterion (BIC), both widely adopted likelihood-based measures for comparing competing statistical models. The AIC, originally proposed by Akaike [35], is defined as follows:

$$AIC = -2\ln L + 2p, \quad (28)$$

whereas the Bayesian Information Criterion (BIC), introduced by Schwarz [36], is defined as follows:

$$BIC = -2\ln L + 2p\ln(n), \quad (29)$$

respectively. Here, L denotes the likelihood function, p represents the number of estimated parameters, and n is the sample size. Lower values of AIC or BIC indicate a better trade-off between model fit and complexity. In this study, these criteria are applied strictly for comparative purposes among competing models fitted to the same dataset. It is important to emphasize that AIC and BIC do not measure absolute goodness of fit; rather they provide a systematic, theoretically grounded approach for selecting the most appropriate model within a common likelihood-based framework.

Table 5 aids in determining the most suitable distribution by reporting the AIC and BIC values for several competing asymmetric models. The results indicate that the continuous positive observations are reasonably characterized by the inverse Gaussian distribution. However, the presence of excess zero counts suggests that the delta-inverse Gaussian model offers a superior fit. This conclusion is supported by the fact that the delta-IG specification yields the smallest AIC and BIC values among the candidate models, indicating the most favorable balance between goodness of fit and model complexity.

Table 5. AIC and BIC values for the fitting of asymmetric distributions.

Criteria	Months	Distributions						
		Normal	Lognormal	Weibull	Gamma	Exponential	Logistic	Inverse Gaussian
AIC	December	123.8187	100.8440	110.0505	105.5633	121.0208	118.2031	99.9425
	November	116.8730	109.4250	111.1910	109.8520	130.7370	118.6540	108.504
	October	100.9568	79.7587	89.2526	85.1330	99.5475	99.6527	78.5372
	September	121.5226	99.4208	106.7473	103.9384	111.9745	118.4413	97.9782
	August	107.8859	85.0842	96.2964	90.1061	111.9406	100.0647	84.6343
	July	112.6562	95.6802	102.3020	99.3460	113.7477	113.1824	94.1796
BIC	December	126.8710	103.8970	113.1030	108.6160	122.5470	121.2560	102.9950
	November	119.9840	112.5360	114.3020	112.9620	132.2920	121.7650	111.6150
	October	103.8248	82.6267	92.1206	88.0010	100.9814	102.5206	81.4051
	September	124.3250	102.2230	109.5500	106.7410	113.3760	121.2440	100.7810
	August	110.9387	88.1370	99.3492	93.1589	113.4670	103.1175	87.6870
	July	115.6493	98.6732	105.2950	102.3390	115.2442	116.1754	97.1726

Table 6 presents the computed descriptive statistics for the mortality data in Bangkok, Thailand, from December 2024 to July 2024. The table includes details on fundamental statistics, such as sample sizes, parameter estimates for μ , λ , and δ obtained from the MLE method, and estimated coefficients of variation θ for each month. We will see that these datasets were close to the case of parameter settings for $n_i = 50^6$, $\lambda_i = 5^6$, and $\delta_i = 0.3^6$ in the simulation studies.

Table 7 presents the 95% simultaneous confidence intervals (SCIs) for all pairwise differences of the coefficients of variation (CVs) across six groups of monthly death counts in Bangkok, Thailand. These results provide important insights into differences in relative variability among the groups. Based on the results, most of the confidence intervals constructed using the GCI, AGCI, MOVER, and NA methods include zero. This indicates that, for the majority of pairwise comparisons, there is no statistically significant difference

in the CVs. In other words, the relative variability of death counts across these groups is largely comparable, suggesting similar levels of dispersion relative to their means.

Table 6. Fundamental statistics for the death count data in Bangkok, Thailand.

Statistics	Months					
	December (1)	November (2)	October (3)	September (4)	August (5)	July (6)
n_i	50	50	50	50	50	50
$n_{i(0)}$	16	15	19	20	16	17
$n_{i(1)}$	34	35	31	30	34	33
$\hat{\mu}_i$	2.1200	2.3100	1.7700	2.300	1.8500	2.0000
$\hat{\lambda}_i$	5.7400	6.9600	5.1900	4.5900	6.7800	5.2200
$\hat{\delta}_i$	0.3200	0.3000	0.3800	0.4000	0.3200	0.3400
$\hat{\theta}_{ij}$	1.0100	0.9500	1.0800	1.2300	0.9340	1.0500

Table 7. The 95% SCIs for all pairwise differences of CVs from the death count datasets in Bangkok, Thailand.

Months	GCI			AGCI			FCI		
	Lower	Upper	Width	Lower	Upper	Width	Lower	Upper	Width
$\theta_1 - \theta_2$	−0.1848	0.2664	0.4512	−0.2931	0.4179	0.7110	−0.9302	3.2570	4.1872
$\theta_1 - \theta_3$	−0.3037	0.1468	0.4505	−0.4889	0.3064	0.7953	−2.2831	2.3638	4.6469
$\theta_1 - \theta_4$	−0.5120	0.0185	0.5306	−0.6967	0.1621	0.8588	−1.9986	2.2722	4.2708
$\theta_1 - \theta_5$	−0.1271	0.2811	0.4082	−0.2426	0.4417	0.6843	−1.3618	2.3661	3.7279
$\theta_1 - \theta_6$	−0.3169	0.1874	0.5043	−0.3835	0.3377	0.7212	−3.3376	2.6905	6.0281
$\theta_2 - \theta_3$	−0.3668	0.1035	0.4703	−0.5396	0.2593	0.7989	−2.5963	1.0361	3.6324
$\theta_2 - \theta_4$	−0.5678	−0.0335	0.5342	−0.7292	0.0960	0.8252	−2.0523	0.6992	2.7515
$\theta_2 - \theta_5$	−0.1782	0.2553	0.4334	−0.3146	0.3884	0.7030	−1.6720	1.0469	2.7189
$\theta_2 - \theta_6$	−0.3419	0.1427	0.4847	−0.4826	0.2663	0.7489	−3.9301	0.9891	4.9192
$\theta_3 - \theta_4$	−0.4359	0.1020	0.5379	−0.6383	0.2715	0.9098	−1.6290	1.8667	3.4956
$\theta_3 - \theta_5$	−0.0526	0.3934	0.4459	−0.2041	0.5675	0.7716	−1.1658	2.1179	3.2837
$\theta_3 - \theta_6$	−0.2459	0.2730	0.5189	−0.3724	0.4528	0.8252	−3.9894	2.4744	6.4638
$\theta_4 - \theta_5$	0.0687	0.5827	0.5140	−0.0664	0.7716	0.8379	−1.2182	1.8760	3.0942
$\theta_4 - \theta_6$	−0.0995	0.4597	0.5592	−0.2317	0.6634	0.8951	−3.4453	1.6246	5.0699
$\theta_5 - \theta_6$	−0.3641	0.0720	0.4360	−0.5043	0.2129	0.7172	−3.5239	1.3830	4.9069

Months	MOVER			NA		
	Lower	Upper	Width	Lower	Upper	Width
$\theta_1 - \theta_2$	−0.3096	0.4221	0.7317	−0.2674	0.3793	0.6467
$\theta_1 - \theta_3$	−0.5048	0.3077	0.8125	−0.4219	0.2764	0.6983
$\theta_1 - \theta_4$	−0.6902	0.1734	0.8636	−0.6010	0.1634	0.7644
$\theta_1 - \theta_5$	−0.2555	0.4413	0.6968	−0.2500	0.3944	0.6444
$\theta_1 - \theta_6$	−0.4196	0.3269	0.7464	−0.3815	0.3013	0.6828
$\theta_2 - \theta_3$	−0.5679	0.2592	0.8270	−0.4683	0.2108	0.6792
$\theta_2 - \theta_4$	−0.7527	0.1246	0.8773	−0.6483	0.0987	0.7470
$\theta_2 - \theta_5$	−0.3207	0.3930	0.7137	−0.2955	0.3281	0.6236
$\theta_2 - \theta_6$	−0.4836	0.2786	0.7622	−0.4276	0.2355	0.6632
$\theta_3 - \theta_4$	−0.6350	0.3150	0.9501	−0.5421	0.2500	0.7920
$\theta_3 - \theta_5$	−0.2075	0.5866	0.7941	−0.1934	0.4835	0.6769
$\theta_3 - \theta_6$	−0.3684	0.4725	0.8409	−0.3241	0.3895	0.7136
$\theta_4 - \theta_5$	−0.0741	0.7712	0.8453	−0.0814	0.6635	0.7449
$\theta_4 - \theta_6$	−0.2335	0.6572	0.8908	−0.2105	0.5679	0.7784
$\theta_5 - \theta_6$	−0.5026	0.2255	0.7282	−0.4428	0.2181	0.6609

However, a notable exception is observed for the pair $(\theta_2 - \theta_4)$ under the GCI method, where the interval $(−0.5678, −0.0335)$ does not include zero. This indicates a statistically significant difference in variability between these two groups. Specifically, since the entire interval is negative, it suggests that the CV of November is lower than that of September. In

practical terms, this implies that the death counts in November are relatively more stable, whereas the September CV shows greater relative fluctuations.

For the pair $(\theta_4 - \theta_5)$ under the GCI method, the interval (0.0687, 0.5827) lies entirely above zero, indicating that the September CV is significantly higher than the August CV. This further reinforces the observation that the CV of September tends to have higher variability compared to certain other groups.

In contrast, the AGCI method produces wider intervals that include zero for all pairwise comparisons, reflecting its more conservative nature. This suggests that, while AGCI achieves better coverage probability, it may reduce the ability to detect significant differences in variability. Similarly, the MOVER and NA methods yield intervals that include zero in all cases, indicating no statistically significant differences under either approach.

The FCI method produces extremely wide intervals across all comparisons, which limits its practical usefulness in this application. The large interval widths suggest high uncertainty in the estimation process, making it difficult to draw meaningful conclusions regarding differences in variability.

From a practical perspective, these findings suggest that the variability of death counts across most groups is relatively homogeneous. However, certain groups, particularly the CV of September, may exhibit greater variability than others. Such increased variability may reflect greater instability or sensitivity to external factors, such as seasonal effects, environmental conditions, or population heterogeneity.

Importantly, these results highlight that conclusions about differences in variability may depend on the chosen method. Methods with narrower intervals, such as GCI, are better at detecting significant differences, whereas more conservative methods, such as AGCI, may prioritize coverage accuracy at the expense of sensitivity.

Overall, the analysis demonstrates that simultaneous confidence intervals for CV differences provide valuable insights beyond mean comparisons. In the context of mortality data, identifying groups with higher variability can be useful for public health monitoring and policy planning, as it may indicate populations or periods with less predictable mortality patterns.

6. Conclusions

This study proposed and evaluated simultaneous confidence interval (SCI) procedures for all pairwise differences of coefficients of variation (CVs) under delta-inverse Gaussian distributions. The objective was to provide reliable comparative inference on relative variability in zero-inflated and skewed settings while properly accounting for multiplicity.

Simulation results for $k = 3, 6$, and 10 populations showed that, although most methods achieved coverage probabilities (CPs) close to the nominal 0.95 level, clear differences emerged when both coverage accuracy and interval efficiency were considered. The AGCI method consistently delivered the most stable and accurate coverage across all parameter settings and levels of dimensionality. This behavior is aligned with its bias-reduction mechanism, which effectively mitigates the impact of skewness on generalized pivotal quantities. In contrast, the MOVER method achieved a favorable balance between coverage and precision by producing shorter interval widths while maintaining competitive CPs. Its efficiency stems from the variance recovery framework, although slight undercoverage may occur in more extreme settings. The GCI method showed less stability due to the lack of bias correction, whereas the FCI method tended to be overly conservative, leading to unnecessarily wide intervals. The normal approximation (NA) method performed the least reliably, reflecting the inadequacy of asymptotic assumptions under skewed and zero-inflated distributions.

The real-data application further confirmed these findings, demonstrating that the observed simulation patterns persist in practice and that the proposed methods can provide meaningful insights into differences in relative variability across groups.

Overall, the results emphasize that accounting for skewness and distributional features is crucial when constructing SCIs for CV differences. The AGCI method is recommended when accurate and stable coverage is the primary objective, particularly in higher-dimensional settings, whereas the MOVER method offers a more efficient alternative when shorter intervals are desired. Together, these methods provide practical, robust tools for simultaneous confidence intervals for differences of CVs under delta-IG models.

7. Discussion

The results of this study provide clear and practically relevant insights into simultaneous inference for relative variability under delta-inverse Gaussian distributions, particularly in the presence of skewness and zero inflation. The simulation evidence demonstrates that the performance of simultaneous confidence interval (SCI) procedures for differences in coefficients of variation (CVs) is not invariant to data characteristics; rather, it is systematically affected by the degree of zero inflation, the magnitude of the shape parameters, and the dimensionality of the problem. As the number of populations increases, interval widths expand due to multiplicity adjustments, reflecting the inherent uncertainty in joint inference. Under these conditions, methods relying on asymptotic normality, such as the normal approximation (NA), become increasingly unreliable, especially when skewness is pronounced or sample sizes are only moderate [34].

A key finding is that coverage accuracy and interval efficiency are governed by fundamentally different statistical mechanisms, and no single method simultaneously optimizes both across all scenarios. The AGCI method consistently delivers coverage probabilities closest to the nominal level across all configurations and values of k . This stability is not incidental; it arises from the bias-reduction adjustment applied to generalized pivotal quantities, which effectively compensates for the asymmetric sampling distribution of the CV estimator. As a result, AGCI remains robust in small samples, under strong skewness, and as dimensionality increases, making it a reliable default when coverage control is the primary objective.

In contrast, the MOVER method achieves superior efficiency by producing systematically shorter intervals while maintaining competitive coverage. This behavior is driven by its variance-recovery framework, which efficiently aggregates information across component intervals without introducing the conservatism typical of generalized inference approaches. However, this efficiency comes at a cost: in scenarios with extreme skewness or limited sample sizes, the method may exhibit slight undercoverage due to instability in variance estimation. This trade-off highlights a fundamental tension between precision and reliability that cannot be eliminated but must instead be managed through informed method selection.

The remaining methods show clear limitations. The GCI method, lacking an explicit bias correction, is sensitive to skewness and zero inflation, leading to occasional undercoverage. The FCI approach, while theoretically appealing, is empirically underused due to its tendency toward overconservatism, leading to unnecessarily wide intervals that undermine inferential usefulness. The NA method performs the least satisfactorily, confirming that asymptotic approximations are inadequate in the distributional regime considered here.

From a practical standpoint, the results provide unambiguous guidance. When strict control of coverage probability is required, particularly in high-dimensional, skewed, or zero-inflated settings, the AGCI method should be preferred. When interval length is a primary concern and minor deviations from nominal coverage are acceptable, the MOVER

method offers a more efficient alternative. Thus, the choice between these two methods should be driven by the relative importance of inferential reliability versus precision in the application at hand.

The real-data application reinforces these conclusions, with empirical patterns closely mirroring those observed in the simulation study. This consistency indicates that the proposed SCI framework is not only theoretically sound but also practically implementable in real-world settings where variability itself is of primary interest.

This study contributes to the literature by providing a unified, systematic comparison of SCI procedures for CV differences within a flexible, practically relevant distributional model. By explicitly accounting for skewness, zero inflation, and multiplicity, it addresses a gap in existing comparative variability analysis methods. Nevertheless, several limitations remain. The analysis is confined to delta-IG distributions, and the extent to which the findings generalize to other complex distributional families remains unclear. In addition, while the simulation design is extensive, extremely small-sample regimes and model misspecification scenarios warrant further investigation.

Future research should focus on extending the proposed framework to broader classes of skewed and heavy-tailed distributions, as well as to dependent and multivariate settings. The development of adaptive or hybrid procedures that dynamically balance coverage and efficiency represents a particularly promising direction. Bayesian and resampling-based approaches may also offer valuable alternatives for simultaneous inference in challenging data environments.

In summary, the findings underscore that reliable simultaneous inference for CV differences in non-normal settings requires methods that explicitly address distributional asymmetry and multiplicity. The AGCI and MOVER procedures emerge as complementary tools, each optimal under different inferential priorities, and together provide a robust foundation for practical applications involving skewed and zero-inflated data.

Author Contributions: Conceptualization, W.K., S.-A.N. and S.N.; Methodology, W.K. and S.-A.N.; Software, W.K.; Validation, S.-A.N. and S.N.; Formal Analysis, S.N.; Investigation, S.-A.N. and S.N.; Resources, W.K.; Data Curation, W.K.; Writing—Original Draft, W.K.; Preparation, W.K.; Writing—Review and Editing, S.-A.N. and S.N.; Visualization, W.K.; Supervision, S.-A.N. and S.N.; Project Administration, S.-A.N.; Funding Acquisition, S.-A.N. All authors have read and agreed to the published version of the manuscript.

Funding: This research budget was allocated by the National Science Research and Innovation Fund (NSRF), and King Mongkut's University of Technology North Bangkok: KMUTNB-FF-69-B-07.

Data Availability Statement: Data are contained within the article.

Acknowledgments: The authors express gratitude to King Mongkut's University of Technology North Bangkok for providing the opportunity and resources for research. The authors express their sincere gratitude to the editor and the reviewers for their thorough review of our manuscript and for their valuable feedback, which helped enhance the completeness of our work.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

This appendix presents the detailed computational steps for each method used to construct the simultaneous confidence intervals for differences of coefficients of variation under the delta-inverse Gaussian distribution. These algorithms provide sufficient detail to ensure reproducibility of the results.

Algorithm A1: GCI

1. Create $L = (L_1, L_2, \dots, L_{n_i}), i = 1, 2, \dots, k$ from $L \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$, then compute for $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$.
2. Generate $\chi^2_{n_{i(1)}-1}$ and $t_{n_{i(1)}-1}$ from Chi-square and the Student's t distributions, respectively.
3. Compute G_{λ_i}, G_{μ_i} , and G_{δ_i} using (3), (4) and (5), respectively.
4. Repeat steps 1–3 for $t = 1, 2, \dots, 5000$ times to obtain $CI_{\theta_{ij}}^{\text{GCI}}$ as (7).

Algorithm A2: AGCI

1. Create $L = (L_1, L_2, \dots, L_{n_i}),$ for $i = 1, 2, \dots, k$ from $L \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$, then calculate for $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$.
2. Generate $\chi^2_{n_{i(1)}-1}$ and $t_{n_{i(1)}-1}$ from Chi-square and the Student's t distributions, respectively.
3. Compute $G_{\lambda_i}, G_{\mu_i}^{\sim}$, and G_{δ_i} using (3), (8) and (5), respectively.
4. Repeat steps 1–3 for $t = 1, 2, \dots, 5000$ times to obtain $CI_{\theta_{ij}}^{\text{AGCI}}$ as (10).

Algorithm A3: FCI

1. Given the initial estimates (MLEs) of parameters $(\mu_i^{(0)}, \lambda_i^{(0)})$
2. Generate $\mu_i^{(t)} \sim IG(\hat{\mu}_i^{(t-1)}, n_{i(1)}\hat{\lambda}_i^{(t-1)})$ and $\lambda_i^{(t)} \sim (\hat{\lambda}_i^{(t-1)}/n_{i(1)})\chi^2_{n_{i(1)}-1}$.
3. Iterate step 2 for $t = 1, 2, \dots, 20,000$ times for replications of MCMC.
4. Burn in 1000 samples and compute the interest parameter.
5. Acquire $CI_{\theta_{ij}}^{\text{FCI}}$ using (11).

Algorithm A4: MOVER

1. Generate random variable $L = (L_1, L_2, \dots, L_{n_i}),$ for $i = 1, 2, \dots, k$ from $L \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$, then compute for $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$.
2. Create $\chi^2_{n_{i(1)}-1}$ and $t_{n_{i(1)}-1}$ from Chi-square and the Student's t distributions, respectively.
3. Calculate $G_{\lambda_i}, G_{\mu_i}^{\sim}$, and G_{δ_i} using Equations (3), (8) and (5), respectively.
4. Construct the simultaneous confidence intervals of θ_{ij} from the AGCI method using Equation (15).
5. Compute $[L_{\theta_{ij}}^{\text{MOVER}}, U_{\theta_{ij}}^{\text{MOVER}}]$ from Equation (16).
6. Reiterate steps 1–5 for $t = 1, 2, \dots, 5000$ times.

Algorithm A5: NA

1. Create random variable $L = (L_1, L_2, \dots, L_{n_i}),$ for $i = 1, 2, \dots, k$ from $L \sim \text{delta-IG}(\delta_i, \mu_i, \lambda_i)$, then compute for the MLEs of $\delta_i, \mu_i, \lambda_i$ as $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$, respectively.
2. Utilize the computed MLEs $\hat{\delta}_i, \hat{\mu}_i$, and $\hat{\lambda}_i$ to calculate τ_i , then acquire the $\hat{\tau}_i$.
3. Construct the $100(1 - \alpha)\%$ NA simultaneous confidence intervals for θ_{ij} using Equation (27).

References

1. Hsieh, H.K. Inferences on the coefficient of variation of an inverse Gaussian distribution. *Commun. Stat. Theory Methods* **1990**, *19*, 1589–1605. [\[CrossRef\]](#)
2. Gupta, R.C.; Akman, O. Estimation of coefficient of variation in a weighted inverse Gaussian model. *Appl. Stoch. Models Data Anal.* **1996**, *12*, 255–263. [\[CrossRef\]](#)
3. Chaubey, Y.P.; Singh, M.; Sen, D. Symmetrizing and variance stabilizing transformations of sample coefficient of variation from inverse Gaussian distribution. *Sankhya B* **2017**, *79*, 217–246. [\[CrossRef\]](#)
4. Chankham, W.; Niwitpong, S.A.; Niwitpong, S. Measurement of dispersion of PM2.5 in Thailand using confidence intervals for the coefficient of variation of an inverse Gaussian distribution. *PeerJ* **2022**, *10*, e12988. [\[CrossRef\]](#)
5. Efron, B.; Tibshirani, R. Bootstrap methods for standard errors, confidence intervals, and other measures of statistical accuracy. *Stat. Sci.* **1986**, *1*, 54–77. [\[CrossRef\]](#)
6. Weerahandi, S. Generalized confidence intervals. *J. Am. Stat. Assoc.* **1993**, *88*, 899–905. [\[CrossRef\]](#)
7. Hannig, J.; Iyer, H.; Patterson, P. Fiducial Generalized Confidence Intervals. *J. Am. Stat. Assoc.* **2006**, *101*, 254–269. [\[CrossRef\]](#)
8. Zou, G.Y.; Taleban, J.; Huo, C.Y. Confidence interval estimation for lognormal data with application to health economics. *Comput. Stat. Data Anal.* **2009**, *53*, 3755–3764. [\[CrossRef\]](#)
9. Feltz, C.J.; Miller, G.E. An asymptotic test for the equality of coefficients of variation from k populations. *Stat. Med.* **1996**, *15*, 647–658. [\[CrossRef\]](#)
10. Tian, L. Inference on the coefficient of variation for lognormal populations. *Comput. Stat. Data Anal.* **2005**, *48*, 755–765. [\[CrossRef\]](#)
11. Hochberg, Y.; Tamhane, A.C. *Multiple Comparison Procedures*; Wiley: New York, NY, USA, 1987.
12. Hsu, J.C. *Multiple Comparisons: Theory and Methods*; Chapman & Hall: London, UK, 1996.
13. Chhikara, R.S.; Folks, J.L. *The Inverse Gaussian Distribution: Theory, Methodology, and Applications*; Marcel Dekker: New York, NY, USA, 1989.
14. Balakrishna, N.; Rahul, T. Inverse Gaussian distribution for modeling conditional durations in finance. *Commun. Stat. Simul. Comput.* **2002**, *43*, 476–486. [\[CrossRef\]](#)
15. Chatzidiamentis, N.D.; Sandalidis, H.G.; Karagiannidis, G.K.; Kotsopoulos, S.A. On the inverse Gaussian shadowing. In *Proceedings of the 2011 International Conference on Communications and Information Technology (ICCIT), Aqaba, Jordan, 29–31 March 2011*; IEEE: Piscataway, NJ, USA; pp. 142–146.
16. Bardsley, W.E. The inverse Gaussian distribution in hydrological applications. *J. Hydrol.* **1980**, *47*, 289–302.
17. Heller, G.; Stasinopoulos, M.; Rigby, B. The zero-adjusted inverse Gaussian distribution as a model for insurance claims. In *Proceedings of the 21st International Workshop on Statistical Modelling, Galway, Ireland, 3–7 July 2006*; pp. 226–233.
18. Jana, N.; Gautam, M. Confidence intervals of difference and ratio of means for zero-adjusted inverse Gaussian distributions. *Commun. Stat. Simul. Comput.* **2024**, *53*, 3302–3326. [\[CrossRef\]](#)
19. Khumpasee, W.; Niwitpong, S.A.; Niwitpong, S. Confidence Intervals for the Coefficient of Variation in Delta Inverse Gaussian Distributions. *Symmetry* **2024**, *16*, 1488. [\[CrossRef\]](#)
20. Thangjai, W.; Niwitpong, S.-A.; Niwitpong, S. Simultaneous Confidence Intervals for All Differences of Coefficients of Variation of Normal Distributions with an Application to PM2.5 Dispersion. *Thail. Stat.* **2021**, *20*, 207–226.
21. Yosboonruang, N.; Niwitpong, S.; Niwitpong, S. Simultaneous confidence intervals for all pairwise differences between the coefficients of variation of rainfall series in Thailand. *PeerJ* **2021**, *9*, e11651. [\[CrossRef\]](#)
22. Janthasuan, U.; Niwitpong, S.; Niwitpong, S.A. Simultaneous confidence intervals for all pairwise differences of coefficients of variation of zero-inflated Birnbaum–Saunders distributions. *AIMS Math.* **2026**, *11*, 4043–4067. [\[CrossRef\]](#)
23. Dasgupta, A. *Asymptotic Theory of Statistics and Probability*; Springer: New York, NY, USA, 2008.
24. Folks, J.L.; Chhikara, R.S. The Inverse Gaussian Distribution and its Statistical Application—A Review. *J. R. Stat. Soc. Ser. B Methodol.* **1978**, *40*, 263–275. [\[CrossRef\]](#)
25. Weerahandi, S. Generalized Confidence Intervals. In *Exact Statistical Methods for Data Analysis*; Springer Series in Statistics; Springer: New York, NY, USA, 1995. [\[CrossRef\]](#)
26. Krishnamoorthy, K.; Mathew, T. *Statistical Tolerance Regions: Theory, Applications, and Computation*, 1st ed.; Wiley: Abingdon, UK, 2009.
27. Ye, R.D.; Ma, T.F.; Wang, S.G. Inferences on the common mean of several inverse Gaussian populations. *Comput. Stat. Data Anal.* **2010**, *54*, 906–915. [\[CrossRef\]](#)
28. Khumpasee, W.; Niwitpong, S.A.; Niwitpong, S. Confidence Intervals for the Variance and Standard Deviation of Delta-Inverse Gaussian Distributions with Application to Traffic Mortality Count. *Symmetry* **2025**, *17*, 387. [\[CrossRef\]](#)
29. Wu, W.H.; Hsieh, H.N. Generalized confidence interval estimation for the mean of delta-lognormal distribution: An application to New Zealand trawl survey data. *J. Appl. Stat.* **2014**, *41*, 1471–1485. [\[CrossRef\]](#)
30. Krishnamoorthy, K.; Tian, L. Inferences on the difference and ratio of the means of two inverse Gaussian distributions. *J. Stat. Plan. Inference* **2008**, *138*, 2082–2089. [\[CrossRef\]](#)

31. Fisher, R.A. Inverse probability. *Proc. Camb. Philos. Soc.* **1930**, *26*, 528–535. [[CrossRef](#)]
32. Geman, S.; Geman, D. Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. *IEEE Trans. Pattern Anal. Mach. Intell.* **1984**, *6*, 721–741. [[CrossRef](#)]
33. Zou, G.Y.; Donner, A. Construction of confidence limits about effect measures: A general approach. *Stat. Med.* **2008**, *27*, 1693–1702. [[CrossRef](#)] [[PubMed](#)]
34. Veljkovic, K. Comparing Confidence Intervals for the Mean of Symmetric and Skewed Distributions. *Symmetry* **2024**, *16*, 1424. [[CrossRef](#)]
35. Akaike, H. A new look at the statistical model identification. *IEEE Trans. Autom. Control* **1974**, *19*, 716–723. [[CrossRef](#)]
36. Schwarz, G. Estimating the dimension of a model. *Ann. Stat.* **1978**, *6*, 461–464. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.