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An Uncertain Financial Fragility Modeling Framework Based on Uncertainty Theory

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Abstract

Financial fragility is a forward-looking indicator that characterizes the potential financial deterioration risks of economic entities under adverse shocks. Household financial fragility assessment is often subject to epistemic uncertainty induced by limited information and subjective expert judgement, which cannot be properly captured by classical probabilistic frameworks. This paper develops an uncertain logistic regression framework for household financial fragility modeling rooted in uncertainty theory. We first construct two measurement indicators, namely financial margin and liquidity capacity, to quantify household financial fragility. Then we derive the uncertain maximum likelihood estimator and design a numerical algorithm to implement parameter estimation. Using micro household data from the China Family Panel Studies (CFPS), we carry out empirical analysis for rural labor-transfer households. The empirical results show that the proposed uncertain logistic regression model delivers stable and economically interpretable estimates under epistemic uncertainty with unstable event frequencies. Compared with classical probabilistic logistic regression, the uncertain framework exhibits better applicability for such small-sample scenarios dominated by cognitive uncertainty.

Keywords: uncertainty theory; uncertain financial fragility modeling framework; uncertain regression analysis; parameter estimation; household financial fragility

1. Introduction

The concept of fragility originated in the field of natural disaster research, and has been widely spread to many disciplines such as disaster science, ecology, sociology and economics. According to Gabor and Griffith [1], fragility is regarded as the possibility of being damaged in the event of an adverse event, and this interpretation emphasizes exploring potential adverse effects and their probability of occurrence from a "risk" perspective. On the other hand, Timmermann [2] defined fragility as the degree of damage when adverse events occurred, and this view paid more attention to the specific impact brought by adverse events, and Kates [3] defined it from the perspective of "resilience", emphasizing the ability to cope with adverse events. In the field of economics, the discussion on fragility began in the 1980s, when the United Nations Development Program (UNDP) introduced the concept of "financial fragility", which is mainly used to assess the sustainable economic development ability of countries and regions, focusing on the analysis from a macro perspective. Financial fragility generally refers to the susceptibility of economic agents to adverse financial shocks and the likelihood that their financial positions will deteriorate in the

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future. It is a forward-looking risk concept that characterizes the potential propensity of an economic agent to fall into financial distress when faced with adverse shocks. Unlike static financial deficit indicators, this concept focuses on the inherent weaknesses in an agent's financial structure that render it susceptible to external shocks. Depending on the subject of analysis, financial fragility can manifest at various levels, including households, enterprises, financial institutions, and other economic entities. At the microeconomic level, financial fragility is closely linked to an agent's ability to maintain financial stability when facing income fluctuations, employment changes, unexpected expenses, or other adverse shocks.

Among the various forms of financial fragility at the micro level, household financial fragility is receiving increasing attention. Household financial fragility is a specific manifestation of financial fragility at the micro-household level, representing the risk that a household may struggle to continue covering daily expenses or meeting debt obligations following a shock. This paper quantifies this fragility using two dimensions: financial margin and liquidity surplus. Research on household financial fragility has focused on declines or fluctuations in household consumption levels, levels of household debt, and households' ability to raise funds. The key point of these research perspectives is the characterization and measurement of household financial fragility, which covers multiple dimensions and reflects the economic stability of households in the face of uncertain financial shocks. Debt-related indicators are the traditional way to assess households' financial fragility. For instance, Jappelli et al. [4] focused on the proportion of debt in income and believed that the increase of this proportion increased the financial fragility of households. Similarly, Brown and Taylor [5] studied debt-to-asset ratios, linking high debt burdens to households' financial fragility. In addition to debt levels, the concept of financial spreads is also widely used to assess the financial fragility of households, Johansson and Persson [6] defined financial spread as disposable income minus the necessary cost of living and the principal and interest of the loan that should be repaid, and a negative financial spread was regarded as a sign of financial fragility. Ampudia et al. [7] further defined it as financially vulnerable households where financial spreads were negative and liquid assets could not cover debts during a specific period of time. Anderloni et al. [8] provided a more comprehensive definition of household financial fragility, including consumption expenditure fragility, income and savings fragility, and loan responsibility fragility, and conducted a survey through questionnaire design. This definition covers not only excessive debt, but also the ability to pay rent and buy necessities. He and Shi [9] analyzed the impact of changes in health status on family economic activities. In general, the measure of household financial fragility involves multiple aspects, including debt levels, financial spreads, socio-economic characteristics, and health risks. Different scholars have put forward methods and indicators to assess the family financial fragility from different angles, reflecting the complexity and multi-dimensional nature of this concept. With the deepening of research, the understanding and measurement methods of financial fragility will be more refined and diversified.

However, the above characterization and modeling studies on family financial fragility are all carried out under the conditions of certainty or randomness, while in practice, the judgment and measurement of family financial fragility are carried out according to the experience and cognition of specific experts. This process contains a large number of cognitive uncertainties, and it is inappropriate to describe them as deterministic factors or random factors. At this time, we should choose the uncertainty theory to describe the corresponding cognitive uncertainty, and develop a financial fragility modeling framework based on uncertainty and apply it to household financial fragility. Uncertain financial fragility is distinct from the concept of financial fragility as defined within a probabilistic framework. It specifically highlights the cognitive uncertainty inherent in the process of assessing and measuring household risk. In this paper, we employ uncertainty theory and

uncertain logistic regression to characterize this type of cognitive uncertainty during the modeling and measurement phases. The core theoretical advantage of the uncertainty theory lies in its ability to depict the cognitive uncertainty arising from limited information, expert judgment, and subjective cognition, which is precisely the main source of uncertainty in the assessment of family financial fragility. Probability theory is more suitable for inherent statistical uncertainties with repeatable experiments and stable frequency characteristics. When the research cannot conduct repeated sampling and the uncertainty stems from expert subjective judgment rather than random phenomena based on frequency, the modeling framework based on uncertainty theory can avoid imposing probability assumptions on limited and subjective information forcibly. Regarding the theoretical advantages of the uncertainty theory, one can refer to the empirical analysis studies in the works such as Yang and Liu [10], Liu et al. [11], and Chen [12]. The core contribution of this paper is to construct a modeling framework for uncertain financial fragility in financial scenarios based on uncertainty theory. Uncertain logistic regression is applied to financial indicator modeling, and corresponding research on uncertain maximum likelihood estimation is conducted. This study expands the application scenarios of existing uncertain regression analysis, and there are essential differences in the methodological system compared to classical logistic regression.

Specifically, we use uncertain regression model to describe and model the family financial fragility. In view of this, this paper aims to present an uncertain family financial fragility model based on the uncertain regression analysis theory by characterizing the cognitive uncertainty in the characterization of family financial fragility as uncertain variables and symmetrical uncertain disturbance terms, and carries out statistical inference research on the proposed model. Finally, the main works of this paper will be illustrated by combining the data of the China Family Panel Survey (CFPS), which is implemented by the China Social Science Survey Center of Peking University.

2. Literature Review on Uncertain Regression Analysis

Uncertainty theory was first developed by Liu [13] and further refined by Liu [14] in order to reasonably characterize the possibility of events involving a large amount of cognitive uncertainty. Up to now, uncertainty theory has developed into an axiomatic mathematical system for the study of belief degrees of experts and cognitive uncertainty.

Among the many branches of uncertainty theory, uncertain regression analysis, as a collection of methods using uncertainty theory to explore the relationship between explanatory variables and response variables, has attracted the attentions of many scholars. The study of uncertain regression analysis began with the work of Yao and Liu [15], which proposed an uncertain regression model by assuming that the disturbance terms of the regression model are independent and equally distributed uncertain variables, and then depicted the causal relationship under the uncertain environments. Subsequently, many scholars began to devote themselves to the study of different uncertain regression models. For example, for the scenario where both response and explanatory variables are multidimensional data, Ye and Liu [16] studied the multivariate uncertain regression model to characterize the relationship between multiple groups of response variables and explanatory variables simultaneously. For scenarios where effective information in disturbance terms cannot be fully extracted, uncertain regression models with autoregressive time series errors and moving average time series errors were proposed by Chen [17] and Chen [18] to further mine the effective information in disturbance terms corresponding to regression models. Meanwhile, for panel data with multiple dimensions (such as time dimension and space dimension), Jiang and Ye [19] explored uncertain panel regression model to reveal the interaction between panel data in complex scenes. In addition, in the face of more complex

scenarios where explicit regression relationships cannot be obtained, Ding and Zhang [20] studied non-parametric regression models in uncertain environments, which can effectively identify regression equations in the case of unknown functional relationships.

Another research focus of uncertain regression analysis is its statistical inference research, which mainly includes two tasks, one is to estimate the unknown parameters of the uncertain regression model, and the other is to estimate its disturbance term. In order to estimate unknown parameters in an uncertain regression model based on observed data, Yao and Liu [15] first investigated least squares estimates by minimizing the sum of squares of the deviation between the predicted and observed values. However, this form of objective function may amplify the effect caused by outliers, and to avoid this problem, Liu and Yang [21] presented a least absolute deviations estimator by replacing the sum of the squares of the deviations with the sum of the absolute values of the deviations. In addition, the idea of maximum likelihood was also applied to uncertain regression analysis by Lio and Liu [22] and the uncertain maximum likelihood estimation was proposed. Recently, Liu and Qin [23] revised the uncertain maximum likelihood estimate from the perspective of removing outliers from the process of estimating unknown parameters. In addition, there are also many estimation methods such as Tukey's biweight estimation (Chen [24]), moment estimation (Liu [25]), least squares estimation based on the perspective of minimizing the deviation between the empirical distribution function and the population distribution function (Wang et al. [26]) have been studied by other scholars. On the other hand, in order to estimate the disturbance term in the uncertain regression model, Lio and Liu [27] first proposed the moment estimation of the disturbance term. Subsequently, Lio and Liu [22] estimated the disturbance term based on a maximum likelihood perspective. Similarly, the work of Lio and Liu [22] was modified by Liu and Liu [28] to consider that the influence of outliers should be excluded when estimating disturbance term. At the same time, Liu and Liu [29] also presented a least squares estimate based on the perspective of minimizing the sum of squares of the deviation between the empirical distribution function of the observation data of the disturbance term and its population distribution function.

3. An Uncertain Financial Fragility Modeling Framework

In this section, we propose a modeling framework for financial fragility in uncertain environments based on uncertain logistic regression model, and apply this framework to modeling household financial fragility and analyzing its influencing factors.

3.1. General Model Formulation

Financial fragility can occur in various microeconomic scenarios, such as household finance, personal credit conditions, and the financial stability of small economic entities. Although the specific influencing factors vary in different application scenarios, these issues share a common modeling objective: to explore the correlation between a set of explanatory variables and the likelihood of economic entities falling into an unfavorable financial state.

In practical applications, the information used to assess financial fragility may be incomplete, inaccurate, or insufficient to support reliable probability descriptions. For such cases, uncertainty theory provides an alternative mathematical framework for expressing uncertainty and constructing logistic regression models. This section establishes a modeling framework based on the theory of uncertainty, aiming to depict the relationship between binary financial fragility indicators and related explanatory variables.

Let Y be a binary indicator of financial fragility, where $Y = 1$ indicates being in a vulnerable financial state, and $Y = 0$ indicates not being in such a state. Let $X_1, X_2, \cdots, X_p$ be a set of explanatory variables, which reflect the financial situation of the economic entity under examination, and can cover economic characteristics, income status, employment

status, financial burden, or other factors related to specific applications. In order to model the relationship between the response variable and the explanatory variables, we adopt the logistic form within the framework of uncertainty theory. Specifically, the response level is expressed as the logistic function of the linear predictor. That is, the belief degree of the response variable with a value of 1 can be characterized as

$$\mathrm{M}\{Y = 1|X_1, X_2, \cdots, X_p\} = \frac{\exp\left(\beta_0 + \sum_{j=1}^{p} \beta_j X_j\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_j X_j\right)},$$

where $\beta_0, \beta_1, \cdots, \beta_p$ are unknown parameters. The intercept β_0 represents the baseline level of the linear prediction term, while each coefficient β_i with $1 \leq i \leq p$ characterizes the contribution of the corresponding explanatory variable to the prediction term under the given model. The logistic transformation maps the linear prediction term to the interval $(0, 1)$, making it suitable for describing a binary response. On the other hand, the belief degree of the response variable with a value of 0 can also be characterized as

$$\mathrm{M}\{Y = 0|X_1, X_2, \cdots, X_p\} = 1 - \frac{\exp\left(\beta_0 + \sum_{j=1}^{p} \beta_j X_j\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_j X_j\right)} = \frac{1}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_j X_j\right)}.$$

3.2. Uncertain Maximum Likelihood Estimation

The fundamental difference between the uncertain financial fragility modeling framework and the classical stochastic framework lies in that the former uses the operational rules of uncertainty theory to conduct statistical inferences on the constructed financial fragility model. Since the above uncertain financial fragility model predicts the belief degree of the response variable taking a certain value, it is most convenient to determine the unknown parameters in the above uncertain financial fragility model based on uncertain maximum likelihood estimation.

Suppose that there is a set of observations

$$(x_{i1}, x_{i2}, \cdots, x_{ip}, y_i),\ i = 1, 2, \cdots, n$$

regarding the explanatory variables and the response variable. Before conducting the uncertain maximum likelihood estimation of the model parameters, we first assume that the corresponding uncertain financial fragility model satisfies the following assumptions.

Assumption 1. *The observational data*

$$(x_{i1}, x_{i2}, \cdots, x_{ip}, y_i),\ i = 1, 2, \cdots, n$$

are independent.

Assumption 2. *Suppose the parameter vector* $(\beta_0, \beta_1, \cdots, \beta_p)$ *lies within a compact set* Θ*, that is,*

$$\|(\beta_0, \beta_1, \cdots, \beta_p)\| \leq M$$

for some sufficiently large constant $M > 0$*.*

In combination with the economic research background of this paper, the explanatory variables (such as income, debt ratio) in the uncertain financial fragility modeling

framework, after being standardized processing, the corresponding regression coefficients should not tend to infinity. An excessively large coefficient value indicates that the model has a serious overfitting problem, which will weaken the generalization performance of the model. Imposing constraints on the parameter space is precisely the embodiment of the regularization idea. Thus Assumption 2 is reasonable.

Under the above assumptions, for each index i with $1 \leq i \leq n$, if y_i takes value of 1, then the belief degree predicted by uncertain financial fragility model is

$$\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, \cdots, x_{ip}\} = \frac{\exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}.$$

If y_i takes value of 0, then the belief degree predicted by uncertain household financial fragility model is

$$\mathrm{M}\{y_i = 0 \mid x_{i1}, x_{i2}, \cdots, x_{ip}\} = \frac{1}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}.$$

Thus the predicted belief degree can be written as

$$\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, \cdots, x_{ip}\}^{y_i} \times \mathrm{M}\{y_i = 0 \mid x_{i1}, x_{i2}, \cdots, x_{ip}\}^{1-y_i},$$

which is equivalent to

$$\left(\frac{\exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}\right)^{y_i} \times \left(\frac{1}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}\right)^{1-y_i}$$

$$= \frac{\exp\left(y_i \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}.$$

Then it follows from the independence of the observational data and the method of uncertain maximum likelihood proposed by Lio and Liu [22] that the likelihood function is

$$L(x_{i1}, x_{i2}, \cdots, x_{ip}, y_i, i = 1, 2, \cdots, n) = \bigwedge_{i=1}^{n} \frac{\exp\left(y_i \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}, \tag{1}$$

and the corresponding maximum likelihood estimation solves the following maximization problem,

$$\max_{\beta_0, \beta_1, \beta_2, \cdots, \beta_p} \bigwedge_{i=1}^{n} \frac{\exp\left(y_i \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)}. \tag{2}$$

It is noted that for each i with $1 \leq i \leq n$, we have

$$\frac{\exp\left(y_i \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)} = \frac{1}{1 + \exp\left((1 - 2y_i) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)}$$

since y_i can only take the value 1 or 0. Then it follows the monotonicity of the expression on the right-hand side of the above equation that

$$\begin{aligned}
&\max_{\beta_0,\beta_1,\beta_2,\cdots,\beta_p} \bigwedge_{i=1}^{n} \frac{1}{1 + \exp\left((1 - 2y_i) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)} \\
&= \max_{\beta_0,\beta_1,\beta_2,\cdots,\beta_p} \frac{1}{1 + \exp\left(\bigvee_{i=1}^{n} \left((1 - 2y_i) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)\right)} \\
&= \max_{\beta_0,\beta_1,\beta_2,\cdots,\beta_p} \frac{1}{1 + \exp\left(\bigwedge_{i=1}^{n} \left((2y_i - 1) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)\right)} \\
&= \frac{1}{1 + \exp\left(\min\limits_{\beta_0,\beta_1,\beta_2,\cdots,\beta_p} \bigwedge_{i=1}^{n} \left((2y_i - 1) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right)\right)}.
\end{aligned}$$

Thus, the maximization problem (2) is equivalent to the following minimization problem

$$\min_{\beta_0,\beta_1,\beta_2,\cdots,\beta_p} \bigwedge_{i=1}^{n} \left((2y_i - 1) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)\right). \tag{3}$$

That is, the maximum likelihood estimation $(\beta_0^*, \beta_1^*, \cdots, \beta_p^*)$ solves the minimization problem (3).

Remark 1. *It is easy to see that*

$$(2y_i - 1) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij}\right)$$

is continuous with respect to the unknown parameters $\beta_0, \beta_1, \cdots, \beta_p$ for each index i with $1 \leq i \leq n$. Thus the objective function of the minimization problem (3) is also continuous with respect to the unknown parameters $\beta_0, \beta_1, \cdots, \beta_p$. Then according to the extreme value theorem for continuous function, it can be concluded that the optimal solution of the minimization problem (3) must exist within its compact parameter domain. That is, the maximum likelihood estimation $(\beta_0^*, \beta_1^*, \cdots, \beta_p^*)$ must exist within a compact parameter domain Θ.

Since the above minimization problem is usually difficult to solve, we design a numerical solution to solve the maximum likelihood estimation. Please refer to Algorithm 1 for details.

Algorithm 1 Numerical algorithm for maximum likelihood estimation

1: **Step 0:** Input observational data $\{(x_{i1}, x_{i2}, \cdots, x_{ip}, y_i),\ i = 1, 2, \cdots, n\}$, feasible region Θ its dimension $p + 1$, a sufficiently large positive number M, stopping tolerances `TolFun`$= 10^{-4}$, `TolX` $= 10^{-4}$, `MaxFunEvals` $= 200(p + 1)$, N_{init} initial points.
2: **Step 1:** Define the objective function

$$E(\beta_0, \beta_1, \cdots, \beta_p) = \bigwedge_{i=1}^{n} \left((2y_i - 1) \times \left(\beta_0 + \sum_{j=1}^{p} \beta_i x_{ij} \right) \right).$$

3: **Step 2:** Set $E_{\text{best}} \leftarrow M$.
4: **for** $k = 1, 2, \cdots, N_{\text{init}}$ **do**
5: **Step 3:** Sample initial guess $(\beta_0^{(k)}, \ldots, \beta_p^{(k)}) \in \Theta$.
6: **Step 4:** Minimize E using MATLAB (version number '9.13.0.2049777 (R2022b)') `fminsearch`, stop by pre-set tolerances.
7: **Step 5:** Record local minimizer $\boldsymbol{\beta}^{(k)}$ and objective value $E^{(k)}$.
8: **if** $E^{(k)} < E_{\text{best}}$ **then**
9: Update E_{best} and $\boldsymbol{\beta}_{\text{best}}$.
10: **end if**
11: **end for**
12: **Step 6:** Output the final estimates $\boldsymbol{\beta}^* = \boldsymbol{\beta}_{\text{best}}$.

4. Uncertain Financial Fragility Model from the Household Financial Perspective

In this section, we will model household financial fragility based on uncertain regression analysis.

4.1. Characterization of Household Financial Fragility

Since most literature does not provide a unified standard for "household financial fragility", and different research perspectives will also lead to differences in definitions and measurement methods. Therefore, we need to first provide a unified standard for describing "household financial fragility". Here we draw on the work of Ampudia et al. [7] and other researchers, and based on the characteristics of labor transfer families, we construct a financial fragility index from the dual perspectives of a family's financial margin and the ability to liquidate assets when facing risks. It mainly includes two definition criteria: the ability of income to bear expected expenses and current liabilities and the liquidity level of assets when the family is hit.

From the financial planning perspective of the transfer household, we consider constructing the first defining indicator, namely whether the income is sufficient to meet the expected expenses and current liabilities. Let Y, EC, and DP represent the income, living cost, and debt expenditure of a household in a certain period, respectively. Then we can use $Y - EC - DP$ to characterize the financial margin (denoted as FM) of the household after meeting its expected expenditure and debt commitment in the current period, that is, we can assume that

$$FM = Y - EC - DP. \tag{4}$$

This indicator essentially refers to the difference between total household income and total consumer expenditure and annual mortgage, which can be used to measure the financial constraints faced by the household after the increase in living costs such as food, housing, and transportation and the increase in debt repayment pressure.

Taking into account the uncertainty risks of transfer households and the financing barriers they face, in order to characterize the liquidity level of assets when households are hit by shocks, we next construct a second indicator: whether the liquid assets held

by households are sufficient to cope with potential unexpected expenses. Let LA and UC represent the liquid assets and unexpected expenditures of a household in a certain period, respectively. Then we can use $LA - UC$ to characterize the liquidity surplus (denoted as LC) of the household in the current period, that is, we can assume that

$$LC = LA - UC. \tag{5}$$

In particular, in order to meet the uncertainty of expenditure, the unexpected expenditure here is generally selected as those expenditure items that cannot be predicted in advance, such as medical expenses.

Based on the above analysis, we choose to define the household financial fragility as

$$\text{Fragility} = \mathbf{I}(FM < 0 \text{ or } LC < 0), \tag{6}$$

where $\mathbf{I}$ is the indicative function. This indicator states that as long as one of the financial margin and liquidity surplus is negative, the family's finances are considered fragile. It comprehensively reflects the family's daily livelihood and financial distress in the face of uncertainty, covering the fragility in both general and sudden situations.

4.2. Characterization of Household Income Quality

Unlike most existing studies that focus on how labor mobility promotes income (unit: ten thousand yuan) growth and thus affects household consumption or wealth, we will construct household income quality indicator for migrant workers from four dimensions: income adequacy, structure, knowledge, and volatility, so as to evaluate the impact of income quality rather than just income quantity on household financial fragility from multiple dimensions.

Let $IQ_1, IQ_2, IQ_3, IQ_4, IQ$ denote the household income adequacy, structure, knowledge, volatility and quality, respectively. At first, we discretize household income and use the stratified household income to reflect the adequacy characteristics of income. Specifically, the calculation formula for household income adequacy is

$$IQ_1 = \begin{cases} 1, & \text{if household income is less than 0.5} \\ 2, & \text{if household income is between 0.5 and 1.5} \\ 3, & \text{if household income is between 1.5 and 3} \\ 4, & \text{if household income is between 3 and 5} \\ 5, & \text{if household income is more than 5.} \end{cases}$$

The increase in the channels of household income sources can effectively disperse risks and reflect the diversified structural characteristics of household income. Therefore, we classify household income channels into agricultural production income, wage income, non-agricultural business income, property income, transfer income and other income, and add up the number of household income channels to represent the structural characteristics of household income (IQ_2). In addition, we use the average years of education of each household (the ratio of the total years of education of household members to the number of household members) to characterize the knowledge characteristics of household income (IQ_3). Finally, we use per capita household income as the dependent variable, and use the average age of household members, average years of education, and gender, employment status, and political status of the head of the household as independent variables to perform regression to obtain the residual values. The square of the residual values is used as a measure of the income volatility characteristics (IQ_4). The larger the residual value, the greater the income volatility faced by the household.

Next, we will construct household income quality indicator IQ based on the obtained indicators IQ_1, IQ_2, IQ_3, IQ_4 of the four dimensions. Assume that there are m household in total, and let z_{ij} denote the household income indicator IQ_i of household j for $i = 1, 2, 3, 4$ and $j = 1, 2, \cdots, m$. Then we normalize z_{ij} by

$$\hat{z}_{ij} = \frac{z_{ij} - \bigwedge_{j=1}^{m} z_{ij}}{\bigvee_{j=1}^{m} z_{ij} - \bigwedge_{j=1}^{m} z_{ij}},$$

and calculate its proportion under household income indicator IQ_i as

$$wl_{ij} = \frac{\hat{z}_{ij}}{\sum_{j=1}^{m} \hat{z}_{ij}}.$$

Based on the above results, the information entropy e_i and redundancy rate d_i of household income indicator IQ_i can be obtained as

$$e_i = -\frac{1}{\ln m} \sum_{j=1}^{m} wl_{ij} \ln wl_{ij}, \; d_i = 1 - e_i,$$

and then the weight of IQ_i can be calculated by

$$w_i = \frac{d_i}{d_1 + d_2 + d_3 + d_4},$$

immediately. Thus we can use

$$\sum_{i=1}^{4} wl_{ij} w_i$$

to represent the household income quality indicator IQ of household j.

4.3. Construction of Uncertain Household Financial Fragility Model

Because the reasons affecting the family financial fragility are complex, this paper only considers the impact of family labor flow on the family financial fragility. Therefore, the corresponding explanatory variable is whether labor mobility occurs (denoted x_1), the number of labor mobility (denoted x_2), household income quality (denoted x_3), and logarithm of household savings (denoted x_4), and the response variable is household financial fragility (denoted y).

It follows from the definition (6) that y always takes the value of 0 or 1. Thus we should use uncertain logistic regression model to model the relationship between y and x_1, x_2, x_3, x_4.

Let M denote the uncertain measure, and let x_{i1}, x_{i2}, x_{i3}, x_{i4}, y_i, $i = 1, 2, \cdots, n$ represents the set of observed values of the explanatory variables and response variable. Then $\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\}$ can be used to denote the belief degree that the response variable is $y_i = 1$ when the explanatory variables are x_{i1}, x_{i2}, x_{i3}, and x_{i2}, and it takes value on the interval $[0, 1]$. Thus

$$\ln \frac{\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\}}{1 - \mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\}}$$

takes the values over the set of real numbers R, which can be modeled by a linear regression model based on the idea of uncertain logistic regression. That is, we can assume that

$$\ln \frac{\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\}}{1 - \mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\}} = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4}.$$

Rearranging the above formula, we can get

$$\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} = \frac{\exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}{1 + \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}. \tag{7}$$

On the other hand, we have

$$\begin{aligned} \mathrm{M}\{y_i = 0 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} &= 1 - \mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} \\ &= 1 - \frac{\exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}{1 + \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})} \\ &= \frac{1}{1 + \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}. \end{aligned} \tag{8}$$

The above Equations (7) and (8) are called household financial fragility model. Because the above predicted likelihood is characterized here as belief degree (uncertain measure), and subsequent statistical inference studies are all based on uncertainty theory, it is also called uncertain household financial fragility model.

5. Empirical Study

In this section, we will conduct an empirical study with the data of China Family Panel Survey (CFPS) released by Peking University's China Center for Social Science Surveys in https://www.isss.pku.edu.cn/sjsj/cfpsxm/index.htm (accessed on 14 March 2026) to illustrate the model and parameter estimation proposed above.

5.1. Data Set

Based on the data of CFPS 2022, we extracted the data of Shandong Province separately, and sorted out the household financial xfragility of each family according to Formulas (4)–(6). Here Y, EC and DP are calculated according to the corresponding data of household total income, total consumption expenditure, mortgage expenditure in CFPS2022 data, respectively, LA and UC are calculated according to the corresponding data of household deposits, medical care expenditure in CFPS2022 data, respectively. The household income quality data is calculated according to the formulas in Section 4.2, and we also take the logarithm of the savings deposit held by each household to obtain the logarithm of household savings data. In a word, after deleting invalid data, a total of 161 sets of data are obtained, as shown in Table 1.

Table 1. The data of household financial fragility, whether labor mobility occurs, the number of labor mobility, household income quality, and logarithm of household savings belonging to Shandong Province in CFPS 2022 data.

y	x_1	x_2	x_3	x_4	y	x_1	x_2	x_3	x_4
0	0	0	0.4831	8.5174	0	0	0	0.6707	12.2061
1	1	1	0.3377	11.0021	1	1	1	0.4986	0
1	1	2	0.4383	0	1	0	0	0.4182	10.5321
1	0	0	0.5389	9.9035	0	0	0	0.4793	11.7753
0	0	0	0.534	11.5129	0	0	0	0.4525	11.2898
0	0	0	0.4621	11.2898	0	1	1	0.5785	11.1563
0	1	1	0.6021	7.6014	1	0	0	0.6342	11.0021
0	0	0	0.4027	9.2104	0	0	0	0.5225	12.2061
1	0	0	0.4219	9.2104	0	0	0	0.594	11.9184
0	0	0	0.3817	0	0	0	0	0.486	10.309
0	0	0	0.1711	8.5174	0	1	2	0.4391	0
0	0	0	0.3587	10.8198	1	0	0	0.4161	0
0	1	1	0.3662	9.3927	1	0	0	0.5426	0

Table 1. *Cont.*

y	x_1	x_2	x_3	x_4	y	x_1	x_2	x_3	x_4
0	1	2	0.4123	11.2898	0	0	0	0.4584	11.9184
1	0	0	0.215	9.2104	1	1	2	0.4756	0
0	1	2	0.4391	11.5129	0	1	1	0.4734	10.8198
0	0	0	0.4316	7.6014	0	0	0	0.3469	8.0067
1	0	0	0.407	0	1	0	0	0.5627	0
0	0	0	0.5501	12.6115	1	1	1	0.4621	11.0021
0	0	0	0.5292	10.309	0	1	4	0.4383	9.9035
1	0	0	0.2322	8.6997	1	0	0	0.5003	9.9035
0	0	0	0.4429	9.9035	0	1	1	0.2515	10.8198
1	1	1	0.6976	9.7982	1	0	0	0.2343	0
1	1	1	0.4584	10.309	0	1	1	0.4316	11.5129
1	1	2	0.2365	10.8198	0	1	1	0.4391	12.8992
0	1	1	0.4219	11.2898	0	0	0	0.4525	11.2898
1	1	1	0.4048	0	0	1	3	0.5627	10.8198
1	0	0	0.2322	9.2104	0	0	0	0.4756	12.6115
0	0	0	0.3394	10.8198	0	0	0	0.3705	10.8198
0	1	1	0.5367	11.9184	0	0	0	0.482	9.9035
0	0	0	0.4621	11.5129	1	0	0	0.4793	5.425
0	1	2	0.4621	12.6115	1	0	0	0.3952	9.9035
1	0	0	0.3051	0	1	0	0	0.4593	11.5129
1	0	0	0.259	9.2104	1	0	0	0.5903	9.2104
1	0	0	0.4182	5.8608	1	0	0	0.3683	12.2061
0	1	1	0.4621	0	1	0	0	0.3185	0
0	1	1	0.6506	12.5425	0	1	1	0.4018	10.8198
0	0	0	0.3834	10.309	0	0	0	0.4085	11.6082
0	0	0	0.5717	11.8494	0	0	0	0.4927	11.0021
0	0	0	0.5098	11.5129	0	0	0	0.4734	11.5129
0	0	0	0.5024	10.8198	1	1	1	0.4756	0
0	0	0	0.5538	12.6115	1	0	0	0.4852	0
0	1	1	0.5259	10.4631	0	1	1	0.4144	11.2253
0	0	0	0.4236	9.9035	1	0	0	0.5731	0
1	0	0	0.1957	0	0	0	0	0.5225	9.2104
0	0	0	0.5694	10.5967	0	1	1	0.5292	12.2061
0	1	3	0.5463	12.7657	0	1	1	0.4064	11.0021
0	0	0	0.4927	10.5321	0	0	0	0.3432	11.9184
0	1	1	0.5098	11.9184	1	1	1	0.4659	0
0	1	2	0.5806	9.9035	1	1	2	0.3588	0
1	1	1	0.442	0	1	1	1	0.4458	10.8198
0	0	0	0.4027	11.9184	0	0	0	0.4273	10.9151
1	0	0	0.5635	12.2061	0	1	1	0.6067	11.2898
0	0	0	0.5098	11.5129	0	1	2	0.4353	11.5129
0	0	0	0.5329	12.1007	0	1	1	0.3874	10.309
0	0	0	0.4927	9.2104	1	0	0	0.5302	0
0	0	0	0.3796	10.309	0	1	1	0.4889	9.9035
0	0	0	0.4391	12.8992	0	0	0	0.7802	11.5129
1	0	0	0.5509	0	0	1	1	0.5262	9.2104
0	1	1	0.4458	10.7144	0	0	0	0.378	8.5174
0	0	0	0.6439	12.2061	0	1	1	0.4756	10.8198
1	0	0	0.5788	12.6115	0	0	0	0.3952	9.2104
1	1	2	0.463	11.5129	1	0	0	0.5024	0
1	0	0	0.53	8.2943	0	1	1	0.332	9.2104
0	0	0	0.4236	10.309	0	0	0	0.5828	13.1224

Table 1. *Cont.*

y	x_1	x_2	x_3	x_4	y	x_1	x_2	x_3	x_4
1	1	1	0.4429	8.5174	1	1	1	0.4391	9.9035
0	0	0	0.4048	10.8198	0	0	0	0.378	11.5129
0	0	0	0.4659	6.9088	1	0	0	0.4354	7.6014
1	0	0	0.5078	9.2104	1	0	0	0.4823	8.8538
0	1	3	0.542	11.5129	1	1	1	0.2418	8.9873
0	1	2	0.6506	10.309	0	1	1	0.4584	0
0	1	1	0.4466	9.9035	1	0	0	0.512	12.2061
1	1	1	0.3855	0	0	0	0	0.4584	7.6967
0	0	0	0.5003	9.9035	0	0	0	0.5098	11.6953
0	0	0	0.2754	11.0021	0	1	1	0.4756	11.5129
1	1	1	0.5624	0	0	1	4	0.3435	12.2061
1	0	0	0.2716	0	1	1	1	0.6193	9.9035
0	0	0	0.3952	8.0067	1	0	0	0.332	10.1267
1	0	0	0.5731	11.2898	1	0	0	0.3788	9.2104
0	0	0	0.4852	11.9829	0	1	1	0.4085	9.9035
0	0	0	0.5195	11.2898					

5.2. Uncertain Household Financial Fragility Model

Let us denote the collected data in Table 1 by

$$x_{i1}, x_{i2}, x_{i3}, x_{i4}, y_i, i = 1, 2, \cdots, 161, \tag{9}$$

and use uncertain household financial fragility model (7) and (8) to model it. By substituting the above data (9) into Equation (2), we can obtain that the maximum likelihood estimations β_0^*, β_1^*, β_2^*, β_3^* and β_4^* is the optimal solutions to the following maximization problem

$$\min_{\beta_0,\beta_1,\beta_2,\beta_3,\beta_4} \bigwedge_{i=1}^{161} \frac{\exp\left(y_i \times \left(\beta_0 + \sum_{j=1}^{4} \beta_j x_{ij}\right)\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{4} \beta_j x_{ij}\right)}.$$

Solving the above maximization problem by MATLAB, we obtain

$$\beta_0^* = 0.0651,\ \beta_1^* = 0.0125,\ \beta_2^* = -0.0101,\ \beta_3^* = -0.0950,\ \beta_4^* = -0.0031.$$

Thus the estimated uncertain household financial fragility model is obtained as

$$\begin{aligned} &\mathrm{M}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} \\ =&\frac{\exp(0.0651 + 0.0125x_{i1} - 0.0101x_{i2} - 0.0950x_{i3} - 0.0031x_{i4})}{1 + \exp(0.0651 + 0.0125x_{i1} - 0.0101x_{i2} - 0.0950x_{i3} - 0.0031x_{i4})}, \end{aligned} \tag{10}$$

and

$$\begin{aligned} &\mathrm{M}\{y_i = 0 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} \\ =&\frac{1}{1 + \exp(0.0651 + 0.0125x_{i1} - 0.0101x_{i2} - 0.0950x_{i3} - 0.0031x_{i4})}. \end{aligned} \tag{11}$$

5.3. Discussion

From the above results, we can see that the regression coefficient of the explanatory variable "whether labor force flows" (x_1) is 0.0125, which means that there is a positive correlation between labor force flow and family financial fragility. This is because families

with fragile finances usually choose to work outside, which in turn shows the mobility of family labor. On the other hand, the regression coefficient of the explanatory variable "the number of labor flow" (x_2) is -0.0101, which means that there is a negative correlation between the number of labor flow and family financial fragility. This implies that the greater the number of labor flow, the more it can alleviate the fragility of family finances to a certain extent. In addition, the regression coefficient of the explanatory variable "household income quality" (x_3) and "logarithm of household savings" are -0.0950 and -0.0031, respectively, which means that the higher the household income quality and the household savings, the less likely the family finances are to show their fragility.

6. Comparison with Stochastic Regression Analysis

In this subsection, we will apply stochastic regression analysis to model the data in Table 1 and explain why this type of data is more suitable for modeling using uncertain regression analysis.

Similarly, we can use the stochastic linear regression equation to characterize the probability that the response variable takes the value y_i for each index i with $1 \leq i \leq 161$. Thus we can obtain

$$\Pr\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} = \frac{\exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}{1 + \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}, \tag{12}$$

and

$$\Pr\{y_i = 0 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} = \frac{1}{1 + \exp(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4})}, \tag{13}$$

where Pr is probability measure. According to the operational rules of probability theory, the corresponding stochastic maximum likelihood estimation solves the following maximization problem,

$$\min_{\beta_0,\beta_1,\beta_2,\beta_3,\beta_4} \prod_{i=1}^{161} \frac{\exp\left(y_i \times \left(\beta_0 + \sum_{j=1}^{4} \beta_j x_{ij}\right)\right)}{1 + \exp\left(\beta_0 + \sum_{j=1}^{4} \beta_j x_{ij}\right)}. \tag{14}$$

By substituting the above data (9) and solving the above maximization problem via MATLAB, we have

$$\beta_0^* = 2.7618,\ \beta_1^* = 0.3448,\ \beta_2^* = -0.5570,\ \beta_3^* = -1.3687,\ \beta_4^* = -0.2902.$$

Thus the estimated stochastic household financial fragility model is obtained as

$$\begin{aligned} &\Pr\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} \\ =&\frac{\exp(2.7618 + 0.3448 x_{i1} - 0.5570 x_{i2} - 1.3687 x_{i3} - 0.2902 x_{i4})}{1 + \exp(2.7618 + 0.3448 x_{i1} - 0.5570 x_{i2} - 1.3687 x_{i3} - 0.2902 x_{i4})}, \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\Pr\{y_i = 0 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\} \\ =&\frac{1}{1 + \exp(2.7618 + 0.3448 x_{i1} - 0.5570 x_{i2} - 1.3687 x_{i3} - 0.2902 x_{i4})}. \end{aligned} \tag{16}$$

At this point, we focus on the probability that the estimated stochastic household financial fragility model predicts that the response variable takes the value of 1. For the actual observed data, the probability that the response variable takes the value of 1 is y_i

for $i = 1, 2, \cdots, 161$. The probability that the response variable predicted by the estimated stochastic household financial fragility model takes the value 1 is

$$\hat{\Pr}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\}$$

$$= \frac{\exp(2.7618 + 0.3448x_{i1} - 0.5570x_{i2} - 1.3687x_{i3} - 0.2902x_{i4})}{1 + \exp(2.7618 + 0.3448x_{i1} - 0.5570x_{i2} - 1.3687x_{i3} - 0.2902x_{i4})},$$

for $i = 1, 2, \cdots, 161$. By using

$$\varepsilon_i = y_i - \hat{\Pr}\{y_i = 1 \mid x_{i1}, x_{i2}, x_{i3}, x_{i4}\},\ i = 1, 2, \cdots, 161, \tag{17}$$

we can obtain 161 residuals of the estimated stochastic household financial fragility model as shown in Table 2 and Figure 1.

Table 2. Residuals of the estimated stochastic household financial fragility model.

−0.4082	0.7514	0.1990	0.7006	−0.2124	−0.2410	−0.3821	−0.3864	0.6198
−0.9037	−0.5139	−0.2954	−0.3368	−0.1360	0.5512	−0.1246	−0.4912	0.0993
−0.1609	−0.2780	0.5202	−0.3277	0.7771	0.7446	0.8133	−0.2134	0.1197
0.5571	−0.3009	−0.1619	−0.2293	−0.0911	0.0875	0.5661	0.3803	−0.8718
−0.1212	−0.3197	−0.1885	−0.2180	−0.2561	−0.1602	−0.2302	−0.3335	0.0763
−0.2510	−0.0467	−0.2750	−0.1669	−0.1576	0.1251	−0.2229	0.8252	−0.2180
−0.1855	−0.3576	−0.3209	−0.1704	0.1184	−0.2368	−0.1595	0.8443	0.8790
0.5917	−0.3079	0.6292	−0.2824	−0.5297	0.6471	−0.0661	−0.1312	−0.2817
0.1169	−0.3106	−0.3082	0.1443	0.0839	−0.4743	0.7857	−0.2010	−0.2269
−0.1546	0.1339	0.7042	−0.2122	−0.2434	−0.1854	0.7857	−0.1830	−0.1809
−0.2900	−0.8008	0.1004	0.1172	−0.2100	0.2072	−0.2247	−0.4908	0.1201
0.7818	−0.0694	0.6894	−0.2819	0.0801	−0.2006	−0.1424	−0.2434	−0.0776
−0.1752	−0.2920	−0.3160	0.3702	0.6578	0.7700	0.6725	0.7833	0.0890
−0.2422	−0.2375	−0.2487	−0.2266	0.1303	0.1093	−0.2183	0.1216	−0.3483
−0.1522	−0.2315	−0.2374	0.1288	0.1822	0.7687	−0.2707	−0.1740	−0.1251
−0.2743	0.1155	−0.2701	−0.1614	−0.3007	−0.4433	−0.2241	−0.3888	0.1116
−0.3594	−0.1365	0.7162	−0.2503	0.5101	0.6149	0.5962	−0.8724	0.8148
−0.4751	−0.2091	−0.1911	−0.0417	0.7636	0.6529	0.6058	−0.2924	

For each household i, since the behaviors of observing the values of the explanatory variables and the response variable are independent, no matter what the population distribution of the residual ε is, as long as it obeys a certain probability distribution function and all the calculation processes obey the operation rules of probability theory, the obtained residuals sequence $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_{161}$ must be a random sequence in the sense of probability theory. If these 161 residuals are indeed a random sequence in the sense of probability theory, then their graph should be very similar to a random graph. We can also see from Figure 1 that if the 161 residuals $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_{161}$ are classified as positive or negative, their continuous ranks are not random at all. In particular, we picked out two consecutive parts from the residuals sequence,

$$\varepsilon_{36}, \varepsilon_{37}, \cdots, \varepsilon_{50}$$

and

$$\varepsilon_{62}, \varepsilon_{63}, \cdots, \varepsilon_{79},$$

and applied the "kstest2" function in MTALAB to test these two subsequences. The p-value of the test is 0.0210, and the original hypothesis that the two subsequences came from the same probability population is rejected at the 0.05 significance level, immediately. Therefore, the original sequence does not come from the same probability population, let alone a random sequence in the sense of probability theory. Therefore, for the data in Table 1,

its frequency (even the residuals sequence obtained) is far from stable, and it should be processed using uncertain regression analysis rather than stochastic regression analysis.

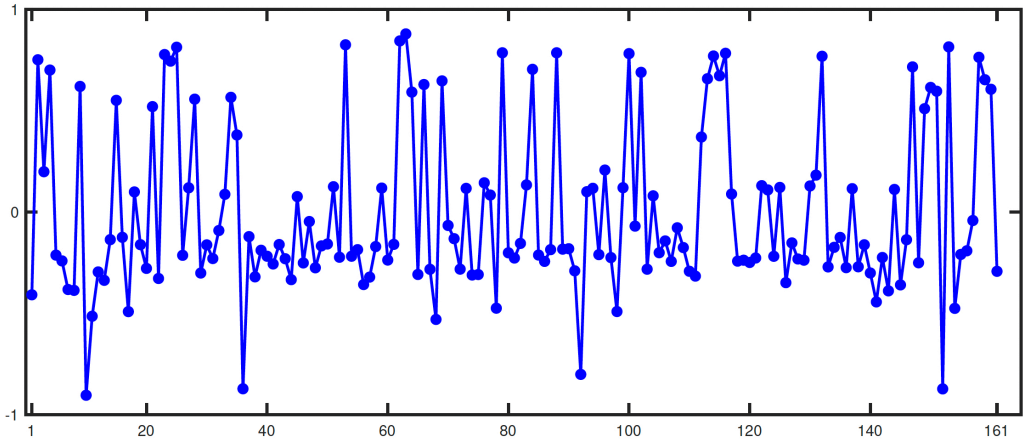


Figure 1. Residuals of the estimated stochastic household financial fragility model.

7. Conclusions

This paper constructed an uncertain logistic regression modeling framework for household financial fragility in uncertain environments based on uncertainty theory, derived the uncertain maximum likelihood estimation for the unknown parameters of the model and designed the corresponding numerical solution algorithm. This framework is suitable for modeling scenarios with scarce information and subjective judgment components in the data, and compensates for the reliance of traditional probability models on frequency assumptions. Relying on transfer household micro-research data, this paper empirically examined the correlation between labor mobility and household financial fragility. The results show that in scenarios dominated by cognitive uncertainty and unstable event frequencies, the uncertain model constructed in this paper can obtain stable parameter estimates with economic explanations, and has better adaptability compared to traditional stochastic models.

This study provides a new analytical tool for assessing family financial risks under information constraints. Regarding the relevant policy practices for managing household risks, when there are insufficient repeatable observable samples, the uncertain modeling method can serve as an additional analysis means to complement the traditional probability model. At the same time, this study also has several limitations: Firstly, the empirical sample comes only from a single province, and the generalizability of the conclusion is constrained to some extent. Secondly, the advantages of this method are concentrated in the cognitive uncertainty scenario. When the research object has a stable frequency of random characteristics, the probability framework model is still the preferred choice. Future research can be expanded to panel-type uncertain time series models, incorporating data from multiple provinces on family surveys, to further examine the dynamic evolution patterns of family financial risks.

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