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Review

Review on the Theoretical and Practical Applications of Symmetry in Thermal Sciences, Fluid Dynamics, and Energy

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Abstract

This literature review explores the role of symmetry in thermal sciences, fluid dynamics, and energy applications, emphasizing the theoretical and practical implications. Symmetry is a fundamental tool for simplifying complex problems, enhancing computational efficiency, and improving system design across multiple engineering and physics domains. Thermal and fluid processes are applied in several modern energy use technologies, essentially involving the complex, multidimensional interaction of fluid mechanics and thermodynamics, such as renewable energy applications, combustion diagnostics, or Computational Fluid Dynamics (CFD)-based optimization, where symmetry is highly considered to simplify geometric parameters. Indeed, the interconnection between experimental analysis and the numerical simulation of processes is an important field. Symmetry operates as a unifying principle, its presence determining everything from the stability of turbulent flows to the efficiency of nuclear reactors, revealing hidden patterns that transcend scales and disciplines. Rotational invariance in pipelines; rotors of hydraulic, thermal, and wind turbines, and in many other cases, for instance, not only lowers computational cost but also guarantees that solutions validated in the laboratory can be effectively scaled up to industrial applications, demonstrating its crucial role in bridging theoretical concepts and real-world implementation. Thus, a wide range of symmetry solutions is exhibited in this research area, the results of which contribute to the development of science and fast information for decision making in industry. In this review, essential findings from prominent research were synthesized, highlighting how symmetry has been conceptualized and applied in these contexts.

Keywords: thermal and fluid science; energy production and use; fuels and combustion; diagnostic techniques of measurement; analysis for decision making in industry

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1. Introduction

The exploration of symmetry within thermal, fluid, and energy applications has garnered significant attention across various scientific domains, particularly in physics and engineering. In this literature review, we will delve into the critical insights provided by notable contributions that have shaped our understanding of symmetry and its implications in these themes-related contexts.

The interdependence between symmetry and computational cost plays a crucial role in modern engineering design, dimensioning, and production optimization. Hence, this topic has been receiving increasing attention from academia and the general public. A comprehensive and in-depth understanding of the nexus between symmetry applications in engineering is essential to achieve efficient resource management. The intersections

between theoretical and practical challenges partly overlap and reflect the transversal nature of symmetry: the same principle found in Navier–Stokes equation analysis (mathematics), heat exchanger optimization (materials science), and, e.g., rotational invariance, applies simultaneously to turbine design (engineering). Thus, there is a gap in the existing literature on which the motivation for the review was based; that is, it intends to address and connect the interdependence, interconnections, and intersections between theory and practice, considering symmetry, as discussed in this review.

Quantifying symmetry's impact on computational efficiency remains a critical challenge, as most studies focus either on its theoretical benefits (e.g., reduced degrees of freedom) or computational costs without systematic metrics to evaluate trade-offs, such as whether enforcing symmetry in CFD simulations (e.g., Navier–Stokes) reduces runtime more than the overhead of symmetry-aware meshing. Furthermore, while symmetry techniques are applied across domains like turbine design (rotational invariance) and heat exchangers (periodic boundaries), few studies compare optimization gains or explore cross-disciplinary generalization, such as adapting materials science's dimensionality reduction methods to civil engineering. The literature also heavily emphasizes static symmetry, leaving dynamic systems lacking frameworks to balance symmetry-breaking adaptations against computational savings. Additionally, while engineers often manually identify symmetries, machine learning automation lacks benchmarks for cost-effective detection. Finally, the environmental implications of symmetry-driven computations, such as energy use in HPC clusters or lifecycle carbon footprints, are rarely studied, representing a significant gap in sustainable design practices.

Symmetry plays a crucial role in reducing computational costs across various scientific and engineering domains by leveraging inherent structural properties to simplify complex problems. In quantum chemistry, symmetry-adapted basis functions and block-diagonalized Hamiltonians reduce matrix dimensions by a factor proportional to the symmetry group size $|G|$, significantly cutting down on computational effort [1,2]. Similarly, in optimization, symmetric problems such as convex programs can be solved more efficiently, with complexity dropping from $O(n^k)$ to $O(n^{\{k-1\}})$ when symmetry is exploited [3,4]. Machine learning benefits from symmetry through architectures like group-equivariant, which reduce parameter counts by a factor of $|G|$, accelerating training and inference [5]. Computational methods for partial differential equations and finite element analysis also see gains, as symmetry-aware meshes decrease degrees of freedom by $O(1/n)$, where (n) is the number of symmetry operations [6]. Even in graph algorithms, exploiting automorphism groups transforms intractable problems like graph isomorphism from factorial $O(n!)$ complexity to quasipolynomial time [7]. These quantitative improvements demonstrate that symmetry not only enhances theoretical elegance but also delivers concrete computational savings, making it a powerful tool for efficiency-driven disciplines.

In thermal sciences and fluid dynamics, symmetry mainly helps describe the behavior of physical systems under different conditions, facilitating measurements, modeling, and analysis. Symmetry is a principle that permeates science and engineering, providing a powerful tool for simplifying and solving complex problems [8]. Symmetry frequently appears in various scheduling problems, such as when assigning jobs to multiple identical machines or creating production schedules for uniform equipment [9]. The most commonly used symmetry-breaking methods are reinforced by the symmetry group special structure in scheduling problems. The symmetry-breaking efficiency of the scheduling methods problems was examined in a modified version of a powerful symmetry-breaking procedure [10]. Computational results were produced in order to compare different methods of symmetry breaking and discuss the cases in which it could be used in practice [11].

The physical analyses consider the simplest possible natural phenomena models in order to relate the most important features to formulate predictions due to the limitations of the mathematical apparatus [12]. Thus, describing multicomponent systems, sometimes complicated, as a simple result led to the success of statics in machine design and construction, allowing us to understand the interactions between these elements. On the other hand, the subsequent success of Newton's gravity theory and Maxwell's electromagnetism theory allows us to detail distinct phenomena. Such behaviors that are apparently very different and unrelated intuitively can be described by elementary interactions through simple equations. As a consequence of these achievements, this reduction employing symmetry has become an important tool in science [13]. This article reviews the fundamental concepts of symmetry in these fields and explores its practical implications in energy applications.

Following the framework of the knowledge structure, this article systematically reviews the evolution and progress of information, methodology, knowledge, and wisdom that have emerged in this area over the last 55 years, but emphasizing more the studies published in the last 20 years. The search was based on the following databases of articles: MDPI, sciencedirect, scopus, web of science, and scholargoogle. The terms used for the search were symmetry, energy, and thermofluid dynamics. Analysis of the abstracts allowed the identification of three major themes by similarity analysis demonstrating the main connections between the terms: (i) thermal and fluid science; (ii) energy production and use; and (iii) fuels, combustion, and renewable sources. A total of more than 1000 articles were retrieved, of which 113 were used as references, resulted from eliminating similar articles, and applying the following inclusion/exclusion criteria: articles relating to renewable energy applications, combustion diagnostics, or Computational Fluid Dynamics (CFD)-based optimization; a search period of 20 years; and a systematic approach based on the use of symmetry in theoretical or practical engineering applications. By synthesizing previous works, this article highlights existing knowledge gaps as well as practices and challenges for prospective research.

A schematic summary of this review highlights key energy technologies driven by thermal and fluid processes in complex systems integrating thermodynamics and fluid dynamics. From CFD modeling to renewable energy and combustion studies, symmetry emerges as a critical tool for geometric simplification, performance optimization, and computational efficiency.

2. Analysis of the Application of Symmetry in Thermal Sciences

The application of symmetry in thermal sciences has garnered significant attention, reflecting its profound implications across various subfields, including mechanical systems, turbulence modeling, plasma physics, and nanotechnology. This literature review synthesizes key contributions from seminal articles that elucidate the role of symmetry in enhancing our understanding and capabilities within the thermal sciences.

2.1. Heat Transfer Modeling

The exploration of variational principles in mechanical systems with symmetry was performed, highlighting how symmetry facilitates the reduction in complex variational principles and informs the development of accurate integration algorithms [14]. Also, this work emphasizes the importance of preserving structures during discretization, which is crucial for accurately simulating mechanical systems, including thermal dynamics. Building on the foundation of symmetry in fluid dynamics, the consequences of symmetries in turbulence modeling were examined [15]. Also, it was demonstrated that symmetries not only lead to conservation laws via Noether's theorem but also influence the derivation of Navier–Stokes equations and their solutions. The findings of this work underscore the

necessity of maintaining symmetry in turbulence models to accurately represent physical phenomena, particularly in the context of heat transfer.

Figure 1 shows the mean velocity profiles simulated using the invariant model in a flow within a ventilated room (Nielsen's cavity), compared with those provided by the Smagorinsky model and the dynamic model, which shows good agreement at medium values and poor agreement at extremes. At $x_1/L = 2/3$ and $x_3/W = 0.5$, the invariant model's velocity profile matches experimental data more closely than the Smagorinsky and dynamic models, even without test filtering or a wall model. The result, except near the floor, shows good agreement with experimental data.

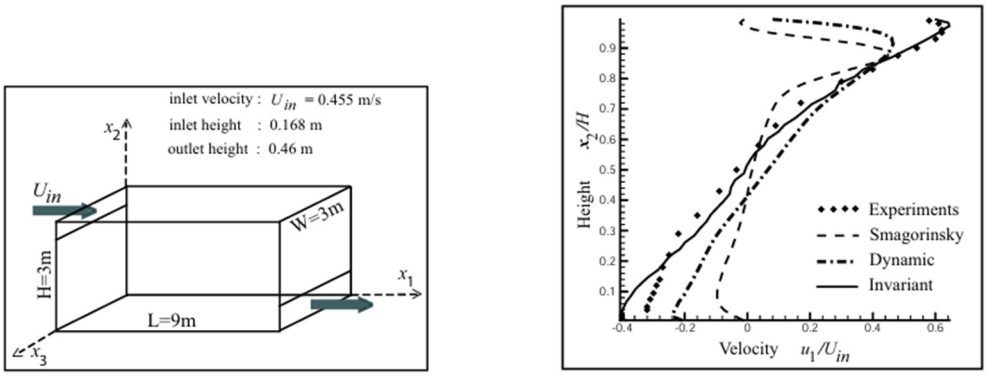

Figure 1. Comparing results of normalized height and velocity measures and several models. Adapted from [16].

Expanding the discussion to plasma physics, symmetry considerations are pivotal in addressing nonlinear problems [17]. Identifying symmetries in integrate-differential equations of kinetic plasma theory illustrates how these principles can simplify the analysis of complex plasma behaviors, which is relevant to thermal processes in energetic environments.

Further, contributing to the discourse by connecting symmetry principles to thermodynamic theory, it is posited that symmetry is foundational to understanding the macroscopic properties of matter [18]. This perspective aligns with the notion that symmetry informs the conservation laws governing thermal processes, thereby reinforcing its significance in the thermal sciences.

The evolving concept of symmetry within theoretical physics was synthesized [19], identifying its multifaceted roles in model construction and classification. This conceptual framework is vital for understanding how symmetry influences thermal phenomena, particularly in the context of emerging technologies and materials.

2.2. Nanoscale Thermal Transport

A conceptual framework can be applied to classify didactically the phenomena by scale (nano/micro/macro) and symmetry type (geometric/statistical/dynamical), clarifying interrelationships. In this way, studies considering the following three main themes are identified: how nanoscale symmetry breaks asymmetric nanoparticle arrays and influences macroscopic heat transfer; whether statistical symmetries in turbulence, Kolmogorov's scaling, can inform scalar field detection algorithms; highlighting methodological bridges that connect nanoscale fluctuations to continuum-scale symmetries.

A comprehensive review of advancements in nanoscale thermal transport was performed, emphasizing how symmetry principles underpin the behavior of materials at the atomic level [20]. Their analysis highlights the challenges and opportunities presented by nanoscale effects, which necessitate a refined understanding of thermal conduction and related phenomena.

An innovative method for detecting symmetry in scalar fields was introduced, emphasizing the importance of symmetry detection in data analysis and visualization [21]. This work suggests that understanding symmetry can enhance our ability to analyze thermal fields, potentially leading to improved thermal management strategies.

The implications of statistical scaling symmetries in incompressible Navier–Stokes turbulence were critically examined [22], cautioning against misinterpretations of symmetries, which could lead to erroneous conclusions in thermal modeling and highlighting the need for rigorous analysis in thermal sciences.

2.3. Thermal Sciences

The work [23] illustrates the practical implications of symmetry in optimizing thermal systems, particularly in engineering contexts, focused on symmetry detection in 3D models and proposing algorithms that leverage symmetry for applications in thermal management.

Revisiting the symmetry properties of linear parabolic equations provides a unified framework for understanding transformation and symmetry group properties [24]. This theoretical grounding is essential for advancing thermal analysis methodologies. The authors discuss emergent symmetries in various physical systems, emphasizing their relevance in low-energy regimes. These insights contribute to the understanding of how symmetry influences thermal phenomena in complex systems.

The significance of identifying symmetries in heat exchanger network design was highlighted [25], demonstrating how symmetry can streamline optimization processes. Their findings illustrate the practical benefits of symmetry in enhancing thermal system efficiency.

The interplay between symmetry and symmetry breaking in elliptic PDEs reveals how these concepts are fundamental to understanding thermal phenomena, particularly in phase transitions and stability analysis [26]. Thermal convection and its relationship with symmetry in thermal metamaterials were examined, proposing a new perspective on heat transfer that integrates symmetry considerations into the design of thermal systems [27].

A review on heat transfer analysis in various cavity geometries, underscoring the diverse applications of symmetry in optimizing thermal performance across different configurations, was conducted [28]. Focused on symmetry breaking in thermal photonics, the work [29] illustrates how manipulating symmetries can enhance control over thermal radiation. The work reflects the innovative applications of symmetry in modern thermal engineering. The non-equilibrium thermodynamics of heat transport in advanced materials was addressed, emphasizing the role of symmetry in understanding and tailoring thermal properties at the nanoscale [30].

However, symmetry is not merely an aesthetic consideration in thermal sciences but a critical principle that informs theoretical frameworks, enhances modeling accuracy, and drives innovation in thermal management technologies. Also, symmetry in thermal sciences refers to the invariance of the physical properties of a system under specific transformations such as rotations or reflections. This property is essential for the formulation and solution of heat transfer equations, allowing significant simplifications. The implicit solution of the second-order nonlinear ordinary differential equation that governs heat transfer in a rectangular fin was achieved by reduction methods based on symmetry [31–33].

The influence of the Reynolds number (Re) on velocity profiles is illustrated in [31], where an increase in Re leads to a corresponding rise in velocity (Figure 2). The accuracy of the proposed method improves with additional convergence-control parameters, enabling new explicit analytical solutions for annular axisymmetric stagnation flow and heat transfer on a moving cylinder of finite radius. The approximate solutions for the similarity function f—derived using convergence-control parameters—show strong agreement

with numerical results obtained via a fourth-order Runge–Kutta method combined with a shooting approach.

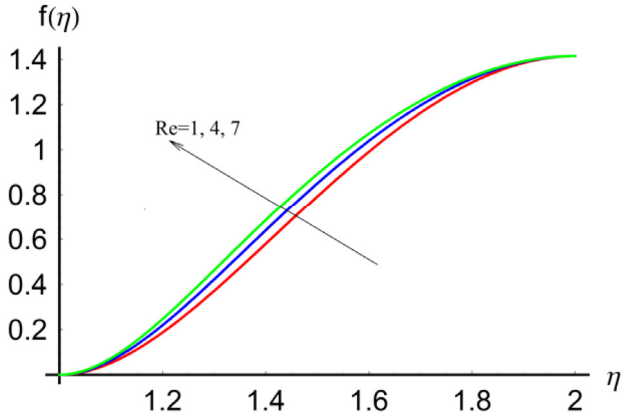

Figure 2. The variation in velocity profiles for three different values of Re. Adapted from [31].

Describing heat transfer in nonlinear domains and discontinuities, as happens at propagation fronts between different phases, is very complex, which makes the solution very computationally expensive. Conservation laws are expressed as partial differential equations (PDEs), typically solved using discretization methods like the finite element method. An alternative approach employs nonlocal formulations. In practical engineering applications, these problems are often simplified by assuming axial or spherical symmetry, which reduces computational complexity and processing time.

In addition, optimizing the nonlocal approach for these situations reduces the required particles by orders of magnitude, leading to a massive decrease in runtime. This allows analysis with higher resolution and larger domains, as best suited to demand [34]. So, symmetry in heat transfer due to geometry can be derived in fundamental parts: Spherical symmetry is used in the analysis of heat transfer in spheres such as particles or droplets. Spherical symmetry simplifies the general heat conduction equation by reducing it to one radial dimension, which facilitates analytical and numerical solutions. Cylindrical symmetry is common in problems involving tubes and cylinders, such as heat exchangers. Cylindrical symmetry allows the analysis of problems in cylindrical coordinates, which is particularly useful for heat exchange systems between fluids moving inside and outside tubes. Plane symmetry is relevant to heat transfer in flat walls and surfaces. Plane symmetry is often used in heat conduction problems in two-dimensional geometries such as plates and thin films.

The main topics of symmetry in thermodynamics (and sometimes encompassing thermo-fluid dynamics) involve fundamental and applied concepts that help us understand how thermal and fluid systems behave under different conditions. Here are some of the most important topics. Noether's theorem and conservation of energy: Noether's theorem relates continuous symmetries of physical systems to conservation laws. In thermodynamics, this can manifest itself as conservation of energy, momentum, and other physical quantities [35,36]. Symmetry and equations of state: the study of how symmetries impact the equations of state that describe the properties of pure substances and mixtures, such as pressure, volume, and temperature [37,38]. Heat transfer and symmetry: analysis of how the symmetry of a system can simplify the solution of problems of conduction, convection, and thermal radiation. Examples include heat conduction in spheres (spherical symmetry) and in tubes (cylindrical symmetry) [39,40]. The instantaneous temperature fields shown in Figure 3 allow us to observe that for lower Prandtl numbers, the heat diffusion increases compared with eddy transport near the wall.

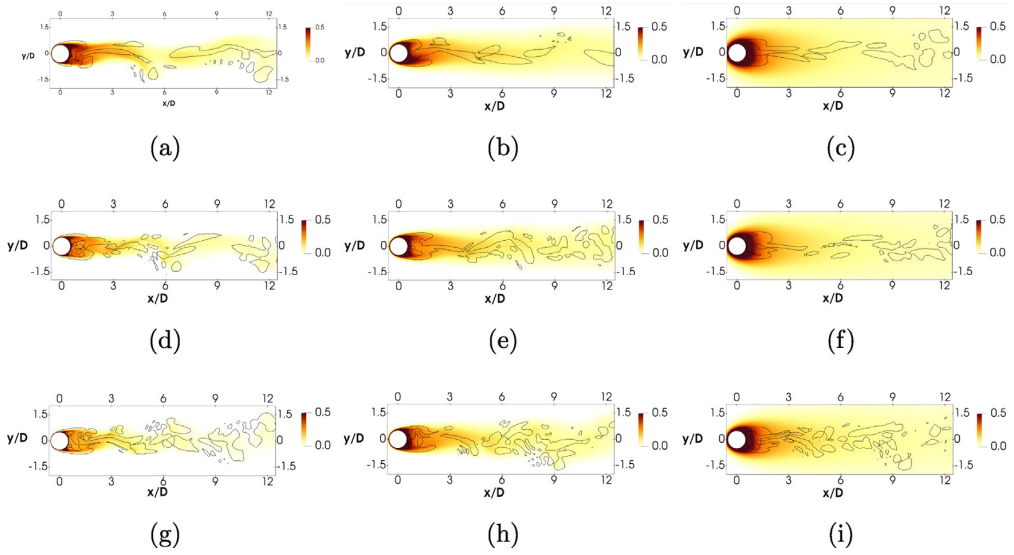

Figure 3. Prandtl number effect on the instantaneous fields of temperature. The temperature field superimposed with means of Q-iso-contours (black) identifies the vortical structures. (**a**) Re = 500, Pr = 0.7; (**b**) Re = 500, Pr = 0.1; (**c**) Re = 500, Pr = 0.01; (**d**) Re = 750, $P\,r$ = 0.7; (**e**) Re = 750, Pr = 0.1; (**f**) Re = 750, Pr = 0.01; (**g**) Re = 1000, Pr = 0.7; (**h**) Re = 1000, Pr = 0.1; (**i**) Re = 1000, Pr = 0.01. Adapted from [39].

The enhanced thermal diffusivity attenuates temperature fluctuations and erodes small-scale thermal gradients. Observations from Figure 3c,f,i demonstrate that for Pr = 0.01, small-scale temperature variations are fully damped, causing the temperature field to deviate from the underlying vortical flow structures. This behavior stems from the overwhelming molecular thermal diffusivity at low Prandtl numbers, with turbulent thermal diffusivity dominating only under high-Reynolds-number conditions, for Pr = 0.025 at Reynolds number $Re > 6 \times 10^4$). Indeed, for Re relative to Pr = 0.01, the flow thermal diffusivity dominates the heat transport.

2.4. Symmetry in Irreversible Processes

In the investigation of how symmetry applies to irreversible thermodynamic processes, such as entropy generation and energy dissipation, the analysis of relaxation processes and equilibrium in nonlinear systems often involves dynamical symmetries [41–43]. Fluid thermodynamics and symmetry breaking involve the exploration of phenomena in fluids where symmetry is broken, such as in the transition from laminar to turbulent flow, where complex patterns emerge due to instabilities [44–46]. Symmetry in thermal materials involves the study of materials with specific thermal properties that exhibit particular symmetries, such as thermoelectric crystals and materials with thermal anisotropy, which have different thermal conductivities in different directions [47,48].

2.5. Thermal Fluid Dynamics

The analysis of fluid systems under thermal gradients such as bottom-up heated convection cells, which often display hexagonal or other geometric symmetries [49,50], was conducted in a rectangular cavity (aspect ratio AR = 4) filled with a liquid at a low Prandtl number (Pr = 0.015). A temperature difference was imposed between the vertical walls, while the horizontal walls remained adiabatic. The bifurcation diagram in Figure 4 summarizes the computational results, illustrating the transition from a stationary flow regime to a time-dependent one. At low Grashof numbers, the flow consists of a unique convective cell.

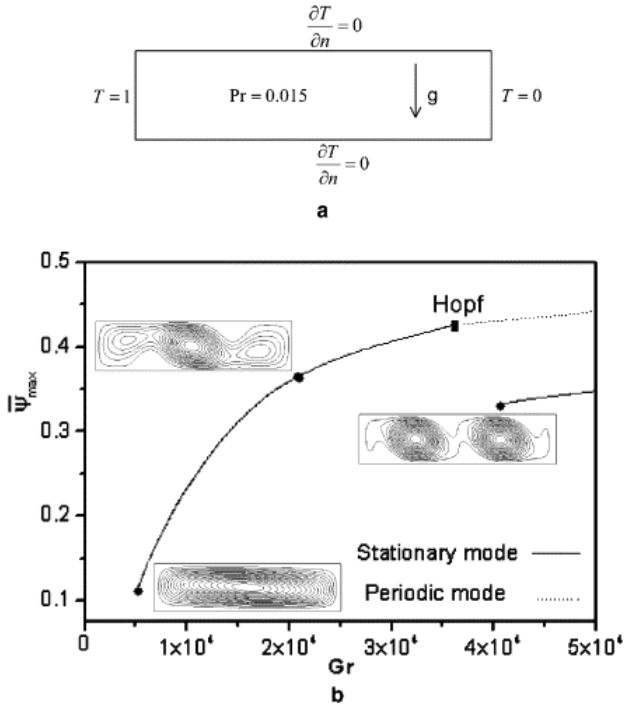

Figure 4. Cases: (**a**) configuration of the geometry, (**b**) diagram of bifurcation: rigid adiabatic upper and lower boundaries, 5000 < *Gr* < 50,000. Adapted from [42].

A comprehensive validation of the proposed numerical code was conducted to accurately model complex melt dynamics, including cases with and without phase transition. The simulations focused on a rectangular enclosure (Figure 4a, AR = 4) containing a low-Prandtl-number fluid (*Pr* = 0.015) subjected to a vertical temperature gradient under adiabatic horizontal boundaries.

The computational results are shown in the bifurcation diagram, Figure 4b, illustrating the transition from steady to unsteady flow; for a low Grashof number (*Gr*), the flow consists of a single convective cell. As the Rayleigh number (*Ra*, where *Gr* = *Ra*/*Pr*) increases, this structure evolves into three counter-rotating cells. Beyond a critical threshold (*Gr*_c = 35,500), the flow becomes time-dependent and highly influenced by *Gr*. This behavior agrees well with previous studies, validating the accuracy of the ultimate scheme for capturing the mono-to-multicellular transition. All simulations used a 64 × 64 mesh, with grid independence confirmed by tests ranging from 32 × 32 to 128 × 128 (differences < 0.2%). Convergence was achieved when the maximum relative residual between iterations fell below 10^{-6}.

2.6. Symmetry in Renewable Energy Systems

Symmetry have applications in renewable energy system optimization and design, such as solar panels and wind turbines, where symmetry can influence efficiency and performance [51–53]. In mathematical modeling and computational simulations, the use of symmetry techniques to simplify governing equations in thermal and fluid systems facilitates more efficient analytical and computational solutions [54,55]. Symmetry in combustion processes: in the study of symmetry in flames and combustion processes, the temperature distribution and the formation of combustion products can exhibit symmetry under certain conditions [56–59].

The Fourier equations for heat conduction, together with the appropriate boundary conditions, can be significantly simplified in systems with symmetry. For example, the

heat conduction equation in spherical or cylindrical coordinates reduces the mathematical complexity involved, allowing the obtainment of analytical or semi-analytical solutions [60].

3. Symmetry in Fluid Dynamics

The exploration of symmetry in fluid dynamics is a multifaceted endeavor that has garnered significant attention in the academic literature. From the foundational principles of hydrodynamic stability to the intricate applications of symmetry in turbulence modeling, researchers have sought to elucidate the critical role that symmetry plays in understanding fluid behavior.

The stage was set by reviewing the relationship between hydrodynamic stability and turbulent flows [61], establishing a rigorous mathematical framework that connects nonlinear energy stability criteria to global transport bounds. This foundational work highlights how stability considerations, particularly in thermal convection, can yield insights into turbulence dynamics, emphasizing the importance of laminar boundary layers in controlling transport phenomena. The reconstructed velocity field in Figure 5 clearly reveals a burst–sweep cycle, demonstrating that even this severely truncated five-mode model qualitatively reproduces key turbulent boundary layer dynamics, including vortex lateral shifting. The cycle suggests a heteroclinic orbit in phase space, where bursting is driven by excitation of unstable modes with spanwise wave numbers 1, 3, and 5. Key observations include the following: (1) a globally stable equilibrium (within the 2, 4 subspace) losing stability; (2) mixed-mode solutions emerging, with two distinct heteroclinic cycle windows; and (3) traveling wave solutions. The specific cycles occur within the second window. At lower values, complex interactions between modulated traveling waves and heteroclinic cycles are found, with solutions exhibiting chaotic characteristics. Further reduction yields simpler spanwise traveling and modulated traveling waves. Notably, as demonstrated in the bifurcation diagram, modes become unstable by the trivial equilibrium state in the sequence 2, 1, 3, 4, 5, which matches the Fourier space energy distribution.

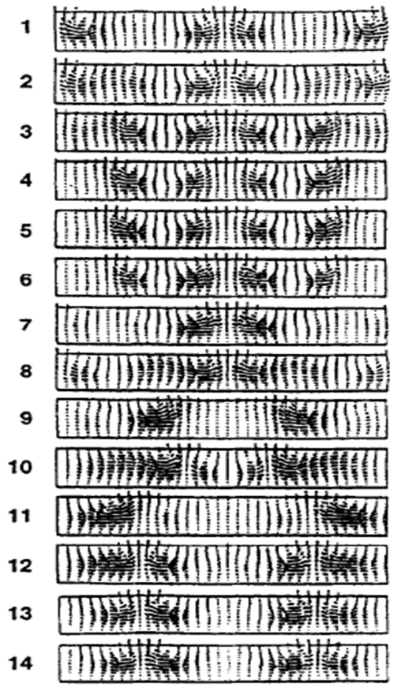

Figure 5. Velocity field components in the cross-stream direction (~2, ~3), obtained through reconstruction from the time-dependent modal coefficients, characterize an individual heteroclinic trajectory. Adapted from [61].

3.1. Noether's Theorem Links Symmetry to Conservation Laws

Building on this foundation [62] delves into the Lagrangian actions of symmetries in ideal compressible fluid dynamics and magnetohydrodynamics (MHD). This work, rooted in Noether's theorem, reveals how these symmetries lead to conservation laws, providing a deeper understanding of the physical underpinnings of fluid dynamics. The introduction of relabeling symmetries and their implications for conservation laws in fluid systems further enriches the discourse on symmetry in fluid dynamics.

3.2. Symmetry Simplifies the Navier–Stokes Equations, Particularly in Axial, Radial, and Translational Flows

The Navier–Stokes equations can be simplified under certain symmetry assumptions, such as (*i*) axisymmetry (invariance under rotation about an axis) or (*ii*) translational symmetry (invariance under translation along a direction). The reduced forms of the incompressible Navier–Stokes equations under these symmetries are as follows:

For axisymmetric flow (cylindrical coordinates), we assume the following:

- No swirl ($u_\theta = 0$);
- All derivatives with respect to θ vanish ($\partial/\partial\theta = 0$);
- Flow is described in cylindrical coordinates (r,θ,z).

The incompressible Navier–Stokes equations reduce to the following:

Continuity Equation:

$$\partial u_r/\partial r + u_r/r + \partial u_z/\partial z = 0 \quad (1)$$

(where u_r and u_z are the radial and axial velocities, respectively).

Momentum Equations:

Radial (r) direction:

$$\partial u_r/\partial t + u_r\partial u_r/\partial r + u_z\partial u_r/\partial z = -1/\rho\ \partial p/\partial r + \nu(\partial^2 u_r/\partial r^2 + 1/r\ \partial u_r/\partial r - u_r/r^2 + \partial^2 u_r/\partial z^2) \quad (2)$$

Axial (z) direction:

$$\partial u_z/\partial t + u_r\partial u_z/\partial r + u_z\partial u_z/\partial z = -1/\rho\ \partial p/\partial z + \nu(\partial^2 u_z/\partial r^2 + 1/r.\partial u_z/\partial r + \partial^2 u_z/\partial z^2) \quad (3)$$

Key Features:

- The θ momentum equation vanishes if $u_\theta = 0$.
- The term u_r/r^2 in the radial momentum equation arises from curvature effects.

For translational symmetry (2D or quasi-2D flow), flows invariant along a direction (e.g., z-direction in Cartesian or cylindrical coordinates), the equations simplify to a 2D form:

Cartesian Coordinates (2D Flow):

Assume $\partial/\partial z = 0$ and $u_z = 0$ (or constant).

- Continuity:

$$\partial u_x/\partial x + \partial u_y/\partial y = 0 \quad (4)$$

- Momentum Equations:

$$\partial u_x/\partial t + u_x\partial u_x/\partial x + u_y\partial u_x/\partial y = -1/\rho\ \partial p/\partial x + \nu(\partial^2 u_x\partial x^2 + \partial^2 u_x\partial y^2) \quad (5)$$

$$\partial u_y/\partial t + u_x\partial u_y/\partial x + u_y\partial u_y/\partial y = -1/\rho\ \partial p/\partial y + \nu(\partial^2 u_y/\partial x^2 + \partial^2 u_y/\partial y^2) \quad (6)$$

Cylindrical Coordinates (Translational Symmetry along z):

Assume $\partial/\partial z = 0$, but u_z may exist (e.g., pipe flow).

- Continuity:

$$1/r.\ \partial(ru_r)/\partial r + 1/r\ \partial u_\theta/\partial\theta = 0 \quad (7)$$

- Momentum Equations:

$$\partial u_r/\partial t + u_r\partial u_r/\partial r + u_\theta/r\ \partial u_r/\partial\theta - {u_\theta}^2/r = -1/\rho\ \partial p/\partial r + \nu(\nabla^2 u_r - u_r/r^2 - 2/r^2\ \partial u_\theta/\partial\theta) \quad (8)$$

$$\partial u_\theta/\partial t + u_r\partial u_\theta/\partial r + u_\theta/r\ \partial u_\theta/\partial\theta + u_r u_\theta/r = -1/\rho r\ \partial p/\partial\theta + \nu(\nabla^2 u_\theta - u_\theta/r^2 + 2/r^2\ \partial u_r/\partial\theta) \quad (9)$$

$$\partial u_z/\partial t + u_r\partial u_z/\partial r + u_\theta/r\ \partial u_z/\partial\theta = \nu(\nabla^2 u_z) \quad (10)$$

where $\nabla^2 = \partial^2/\partial r_2 + 1/r\ \partial/\partial r + 1/r^2\ \partial^2/\partial\theta^2$.

These simplifications are useful for analytical solutions or numerical simulations of flows with high symmetry (e.g., pipe flow, jets, boundary layers): Axisymmetry eliminates θ-dependence, reduces to (r,z) equations. Translational symmetry eliminates z-dependence, reduces to 2D (x,y) or (r,θ) equations. For swirling flows, if $u_\theta \neq 0$ (e.g., rotating flows), additional terms appear in the momentum equations.

In a more applied context, ref. [63] discusses the implications of symmetries on turbulence modeling. They argue that many existing turbulence models neglect the Navier–Stokes equation's symmetry group, which can lead to the loss of essential physical properties. Their work emphasizes the necessity of incorporating symmetries into turbulence models to preserve the underlying physics of fluid flows.

Expanding the discussion to symmetry breaking in free surface flows, ref. [64] employs bifurcation theory to analyze the stability of falling film flows. This work illustrates how instabilities can arise from symmetry breaking and how these phenomena can be beneficial or detrimental depending on the specific context, such as in industrial applications.

Shifting the focus to atmospheric sciences, symmetry methods are employed to derive group-invariant solutions to the barotropic vorticity equation [65]. This application underscores the significance of symmetry in understanding complex weather patterns and suggests future directions for applying symmetry techniques to more sophisticated geophysical models. Continuing this trajectory, ref. [66] apply the Lie group symmetry theory to model non-isothermal turbulent flows. Their work emphasizes how symmetry groups can guide the development of turbulence models that respect the physical properties encoded in the governing equations. In the realm of turbulence quantification, ref. [67] explores the application of Lie group symmetry to the Navier–Stokes equations. They highlight the importance of symmetry in understanding the invariance of solutions under transformation, which can lead to new insights into the complex dynamics of turbulent flows. Figure 6 shows that, for high-quality data from direct numerical simulations (DNSs), the centerline contains the outer edge. The analysis excluded regions within $r < 0.1$ of the centerline to eliminate measurements with insignificant velocity defects.

These results show excellent agreement with previous values derived from compensated stress–length function analysis. The methodological consistency is further verified through the linear relationship between the scaling function f and the mean velocity defect, as demonstrated in Figure 6c. This linear correlation (slope = 0.452) persists throughout the outer region ($y^+ = 150$ to $y^+ = Re\tau$). Figure 6d confirms the theoretical mean-velocity profile (MVP) matches DNS data with remarkable precision, exhibiting relative errors consistently below 0.1%. Based on this robust validation, we select $\kappa \approx 0.45$ and $r_core \approx 0.27$ as representative parameters for the current DNS dataset.

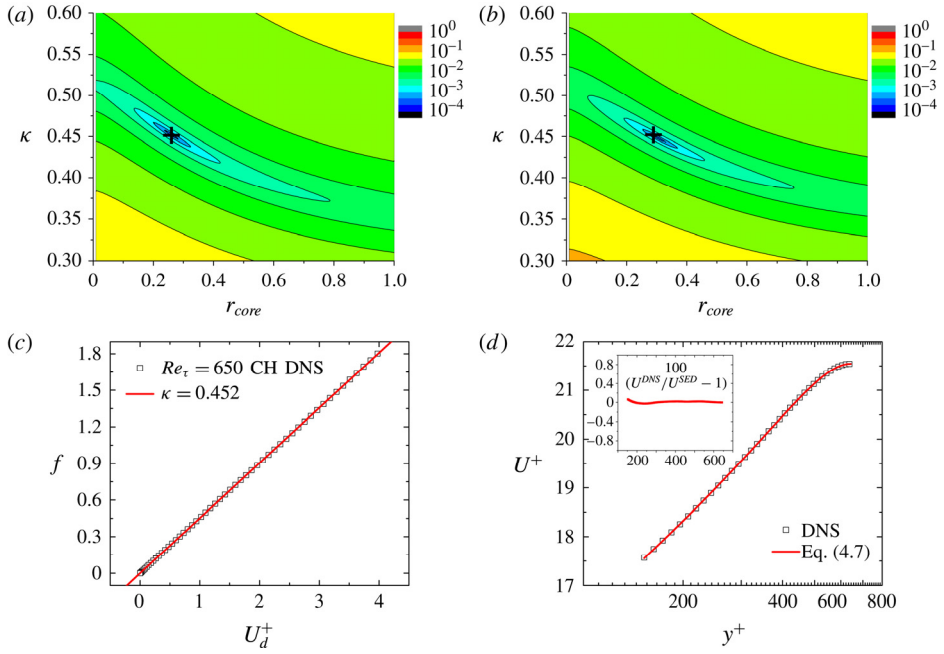

Figure 6. Error contours for DNS channel flow simulations using the theoretical model, with color gradients representing error magnitude. (**a**,**b**) Error distributions for Reτ = 650 and Reτ = 940, respectively, computed using the theoretical mean-velocity profile (MVP). Crosses mark optimal parameters: (κOpt ≈ 0.452, $r_{coreOpt}$ ≈ 0.26) for Reτ = 650 and (κ_{Opt} ≈ 0.447, $r_{coreOpt}$ ≈ 0.31) for Reτ = 940. (**c**) Validation at Reτ = 650 showing linear correlation between f(r,0.26) and Ud+DNS; the slope (κ = 0.452) corresponds to the von Kármán constant. (**d**) Theoretical MVP demonstrates excellent agreement with DNS data, with inset revealing relative errors < 0.1%. Adapted from [67].

The authors caution against the potential pitfalls of interpreting symmetries in fluid dynamics, particularly in the context of scaling symmetries in the incompressible Navier–Stokes equations [22]. Their analysis reveals how misinterpretations of symmetry can lead to inconsistencies in physical modeling. They investigate symmetry breaking in azimuthal thermoacoustic modes, illustrating how changes in symmetry can impact flow stability and lead to significant changes in flow dynamics. This research points to the critical role that symmetry plays not only in fluid mechanics but also in related fields such as combustion dynamics. Also, the study contributes to the understanding of stability theory in the Euler ideal fluid equations, elucidating how shear flows can transition from stability to instability. This work highlights the mathematical intricacies involved in predicting turbulence and emphasizes the importance of stability analysis in fluid dynamics.

A historical perspective on invariant solutions in turbulence emphasizes the challenges of studying fluid mechanics analytically and the necessity of numerical methods in exploring complex flow dynamics. The role of physical symmetry in discrete velocity models, such as those used in the Lattice Boltzmann Method (LBM), is fundamental to ensuring the correct recovery of macroscopic equations, such as the Navier–Stokes equations, while maintaining computational efficiency. Works such as those by [68,69] highlight two essential aspects: the importance of symmetry for macroscopic accuracy and the impact of this symmetry on computational optimization.

Preserving physical symmetry in discrete velocity sets is crucial to correctly recovering the Navier–Stokes equations [68]. The authors emphasized that Galilean invariance, isotropy, and consistency with continuum mechanics are fundamental requirements. Without these properties, numerical artifacts such as incorrect viscosities or spurious anisotropies arise, compromising the results. For example, if the velocity set is asymmetric

or insufficient, higher-order tensors will not adequately reproduce the isotropic behavior of the fluid, leading to errors in modeling viscous forces.

The intelligent use of symmetry can lead to significant performance gains, such as a reduction of approximately 50% in simulation time in methods such as Dissipative Particle Dynamics (DPD) and its many-body variant (MDPD) [69]. Although their study did not focus on LBM, the principles are analogous: symmetry allows for the optimization of collision operators, reduction in redundant calculations, and improvement of efficiency in parallel architectures. This is because symmetric velocity sets, such as those used in LBM (D2Q9, D3Q19, D3Q27), facilitate standardized memory access and allow for the vectorization of operations.

In the context of LBM, the choice of velocity set is a balance between accuracy and computational cost. Lattices like D3Q19 are widely used because they offer a good compromise between isotropic symmetry and complexity. On the other hand, faster schemes like D3Q27 improve accuracy but increase computational cost. Furthermore, collision operators like MRT (Multiple Relaxation Time) can be optimized by exploiting the system's symmetry, reducing memory bandwidth and speeding up calculations.

In short, symmetry plays a dual role: ensuring the physical validity of the model and enabling computational optimizations. Future work can further explore this relationship by applying symmetry principles to adaptive meshes, multiphase models, or GPU implementations. The combination of physical rigor and numerical efficiency is essential for advances in complex fluid simulations, making LBM and related methods increasingly powerful tools for computational fluid mechanics.

3.3. Symmetry Breaking Explains Transitions

Computational gains are achieved via symmetry in CFD and turbulence modeling. Focusing on symmetry-breaking phenomena in turbulent flow through porous media reveals how flow instabilities can lead to significant deviations from expected flow patterns. A representative application demonstrates how hairpin vortex orientation depends on microscale spatial variations in turbulent dissipation arising from distinct primary and secondary flow regions [70]. Figure 7 displays micro-vortex core lines at Rep = 300, revealing their concentration within secondary flow regions. These regions are demarcated from the primary flows by intense shear layers, evident in the vorticity contours. The micro-vortices exhibit localized swirling motions exclusively within secondary flow zones.

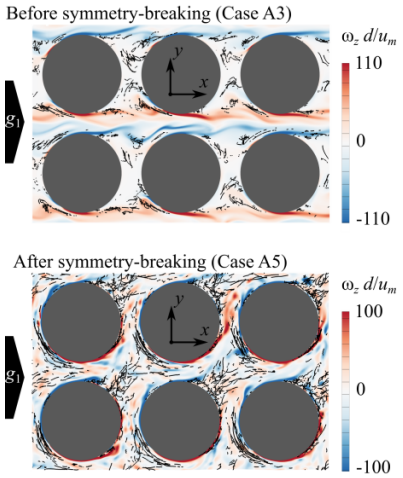

Figure 7. Vortex core lines (pre- and post-symmetry breaking) projected on the z = 0 plane, overlaid with corresponding z-vorticity contours. Their numerical simulations highlight the complex interplay between symmetry and turbulence in porous structures. Adapted from [70].

The instantaneous flow streamlines, plotted at two distinct time steps (separated by nondimensional time units), reveal oscillatory flow instabilities that drive alternating vortex shedding above and below the solid obstacles. Under turbulent forcing, the micro-vortex distorts and elongates in the z-direction. During this stretching, it interacts concurrently with both the swirling, low-velocity secondary flow (which sustains the vortex via rotational motion) and the highly dissipative primary flow (characterized by strong strain rates and pressure gradients). This dichotomy leads to vortex breakup in the primary flow region, while the secondary flow preserves the vortex structure, forming a reverse hairpin vortex with its head anchored in the secondary flow and its tail penetrating the primary flow. Over time, energy dissipation diminishes the vortex head's size. Phase differences in micro-vortex dynamics emerge clearly when comparing vortex head scales behind distinct obstacles.

Also, rounding out this body of work by discussing the role of continuous symmetries in solving the 3D Euler fluid equations is [71]. The contributions emphasize the practical applications of symmetry in understanding fluid behavior and the potential for further exploration in related fields.

These articles underscore the centrality of symmetry in fluid dynamics, providing a rich tapestry of insights that illuminate both theoretical and practical aspects of the theme. The literature reveals a dynamic interplay between stability, turbulence, and symmetry, offering a comprehensive understanding of how these elements interact to shape fluid behavior across various contexts. In fluid dynamics, symmetry helps describe the motion of fluids and the formation of flow patterns. The Navier–Stokes equations, fundamental to fluid dynamics, often exploit symmetries to facilitate their solutions [72,73]. Symmetry can be axial, radial, or translational, depending on the geometry and conditions of the problem [74,75]. Axial symmetry is common in flows around rotating objects, such as turbines and propellers. Axial symmetry simplifies the analysis of flows around cylindrical bodies and reduces the dimension of the problem. Radial symmetry is observed in convergent and divergent flows such as in nozzles and diffusers. Radial symmetry is essential to the analysis of flows in radial geometries, such as in centrifugal pumps and injectors. Translational symmetry is useful in the analysis of flows in channels and ducts with repetitive geometries. Translational symmetry allows the application of periodic solution methods, simplifying the analysis of continuous flow systems in repetitive geometries.

The Navier–Stokes equations, when applied to problems with symmetry, can be simplified to reduce computational complexity. For instance, in cylindrical coordinates, the continuity and momentum equations can be simplified into more tractable forms to enable analytical solutions [76]. This mathematical simplification proves particularly useful when examining phenomena such as the polydisperse sphere suspension effects on relative viscosity, as demonstrated in Figure 8 [77]. The relative viscosity of sphere suspensions is a complex function of the volume fraction of solids, particle size distribution, and other factors. While the Einstein equation provides a starting point for dilute suspensions, more advanced models are needed to accurately describe the viscosity of concentrated and bimodal suspensions. Polydispersity, the presence of particles of varying sizes, significantly impacts the relative viscosity of sphere suspensions. It generally decreases the relative viscosity compared with a monodisperse system at a given particle volume fraction, because larger particles can create a more open structure, allowing for easier flow compared with a dense packing of smaller particles; also, the proportionality of these mentioned terms is due to the geometric symmetry. Indeed, polydispersity can break the symmetry of a monodisperse system, leading to a more complex and potentially lower viscosity.

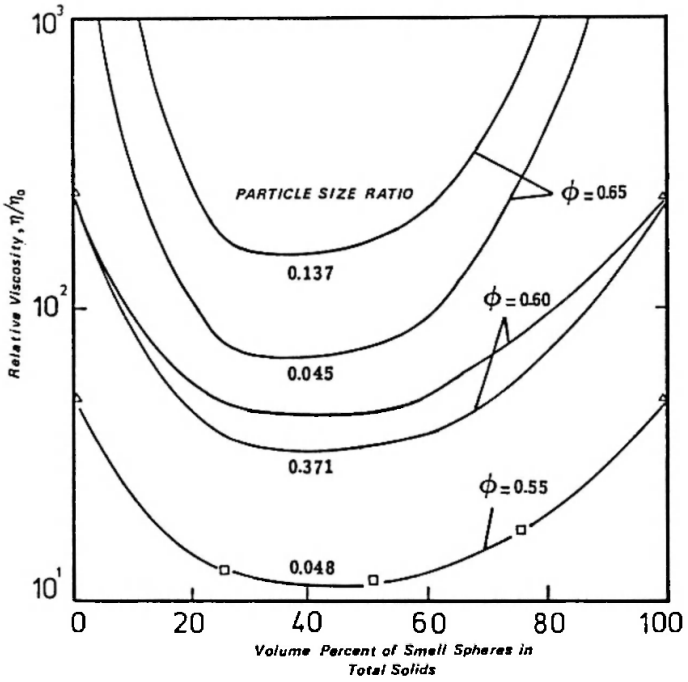

Figure 8. Relative viscosity of sphere suspensions in a function of the volume percent of small spheres in total solids, in which Φ is the total solids concentration. Adapted from [77].

The main topics on symmetry in fluid dynamics address a variety of phenomena and concepts that help us understand the behavior of fluids. The most important topics are as follows. Rotational and Axial Symmetry: systems of fluids that exhibit rotational or axial symmetry, such as flows around cylinders or spheres and flows in circular pipes, are often easier to analyze due to their simplified geometry [72]. Symmetry Breaking: Phenomena where symmetry is broken, such as hydrodynamic instabilities, lead to complex patterns and phase transitions in fluids. Examples include the transition to turbulence in fluid flows and the formation of vortices [76]. Non-Newtonian Fluids: The behavior of fluids whose rheological properties do not follow Newton's law of viscosity, and how their symmetries affect flow, including fluids such as blood, mud, and certain polymers [78]. Symmetric Boundary Conditions: The study of how different boundary conditions can impose or break symmetry in a fluid system is critical in applications such as the design of piping systems and flow channels [79]. Symmetry in Micro- and Nanofluids: Investigation of how symmetry manifests itself at the microscopic and nanoscopic scales, affecting the behavior of fluids in microchannels and lab-on-a-chip devices [80]. Mathematical Modeling and Computational Simulations: The use of symmetries to simplify Navier–Stokes and other governing equations facilitates more efficient analytical and computational solutions. This includes the application of Lie symmetry methods and other advanced techniques [55,81]. Multiphase and Multicomponent Fluids: The study of systems where more than one phase (solid, liquid, gas) or component is present and how symmetries affect the interactions between phases or components [82–84]. Symmetry in Natural Phenomena: Observation of symmetries in natural phenomena, such as the patterns formed by ocean currents, planetary atmospheres, and magmatic flows [85,86]. Droplet and Bubble Dynamics: Analysis of how symmetries affect the formation, coalescence, and breakup of drops and bubbles in different media and under different flow conditions [87–89]. Symmetry in Turbulence: Study of the transition from symmetric laminar flows to asymmetric turbulent states, including the analysis of coherent structures within turbulent flows [90,91].

4. Symmetry in Energy Applications

The role of symmetry in energy applications has been widely studied in multiple scientific domains, with significant emphasis on physics and engineering. This section of the review highlights crucial discoveries from key publications that have deepened our knowledge of symmetry's implications in energy contexts.

4.1. Heat Exchanger Designs

Ref. [92] underscores the pivotal role of symmetry as a methodological theme in 20th-century physics, positing that its significance is likely to persist into the 21st century. This work differentiates between the symmetries of crystals and gauge symmetries, emphasizing that while these distinctions may appear subtle, they are essential for grasping the nuances of symmetry in physical systems. This review not only elucidates various notions of symmetry but also highlights their applications, setting a foundational framework for understanding how symmetry can influence energy systems.

Expanding on this notion, ref. [93] reflects on the evolution of the concept of symmetry throughout the 20th century, particularly in light of advancements in quantum physics and relativity. Four critical facets of symmetry are identified—transformation, comprehension, invariance, and projection—arguing that these elements are interrelated and collectively enhance the theoretical framework within contemporary physics. Furthermore, ref. [21] illustrates how symmetry has transitioned from an aesthetic criterion to a powerful theoretical tool, exemplified by significant developments such as the unification of forces in general relativity and the interactions described in the Standard Model. This conceptual evolution underscores the broader applications of symmetry beyond mere classification, suggesting its role in rational thinking and modeling within scientific discourse.

The search for new electrode materials that offer lower costs, enhanced safety, and high energy density has become crucial to meet the increasing demands of energy storage systems. Recently, symmetric electrodes have gained significant attention due to their unique design, using the same active material for both the cathode and anode within a single system. This approach not only lowers manufacturing costs and simplifies production but also improves safety, extends lifespan, and enables bidirectional charging. In this way, a comprehensive review of this topic considering various symmetric electrodes indicates a particular focus on their applications across different energy storage systems. In a more applied context, ref. [94] investigates the practical implications of symmetry in the design of networks.

The importance of identifying symmetries in mathematical optimization is highlighted, particularly in expediting computational algorithms. The layered lithium trivanadate (LiV_3O_8) cathode material has garnered significant interest due to its low cost, high specific capacity (~200 mAh g^{-1}), and excellent safety features. However, its commercial viability in aqueous electrolytes is hindered by poor cycling stability, attributed to structural collapse and vanadium dissolution during charge/discharge cycles. To mitigate these issues, replacing Li^+ with Na^+ in LiV_3O_8 forms sodium vanadate ($Na_{1.16}V_3O_8$, NVO), which exhibits greater structural stability during lithium intercalation/deintercalation and reduced vanadium dissolution.

Given NVO's wide operating potential window, researchers have explored its use in symmetric lithium-ion batteries (LIBs), where the same material serves as both cathode and anode. Madhavi's group pioneered this approach, demonstrating a functional aqueous rechargeable LIB using NVO electrodes. Electrochemical performance was evaluated for NVO samples synthesized at different annealing temperatures (200 °C, 300 °C, and 400 °C). Notably, NVO-400 delivered the highest initial capacity (>150 mAh g^{-1}) at high current rates and retained ~75% capacity after 100 cycles, showcasing superior rate capability

and cyclability. Although symmetric NVO-based LIBs exhibit lower energy densities than conventional LIBs due to a narrower voltage window, their enhanced safety, cost-effectiveness, and simple design make them promising for specialized applications where these factors outweigh energy density limitations.

Figure 9 shows an example of employing group theory to analyze the transshipment model. The authors reveal several types of symmetry inherent in the problem, demonstrating how recognizing these symmetries can streamline the design process and enhance efficiency in energy applications. Figure 9a shows cyclic voltammograms (CVs) of NVO-400 measured in a three-electrode configuration (vs. SCE): (i) full cell (−1.0 to 0 V), (ii) anode (0 to 2.0 V), and (iii) cathode (0 to 1.0 V) responses at 5 mV/s scan rate with corresponding galvanostatic charge/discharge profiles for NVO-200, NVO-300, and NVO-400. (b) Anode CV (−1 to 0 V vs. SCE). (c) Cathode CV (0 to 1 V vs. SCE) measured in 4 M LiCl aqueous solution at 5 A/g. (d) Cycling performance showing specific capacity and Coulombic efficiency for symmetric NVO-200/NVO-200, NVO-300/NVO-300, and NVO-400/NVO-400 cells (0–1.9 V, 5 A/g).

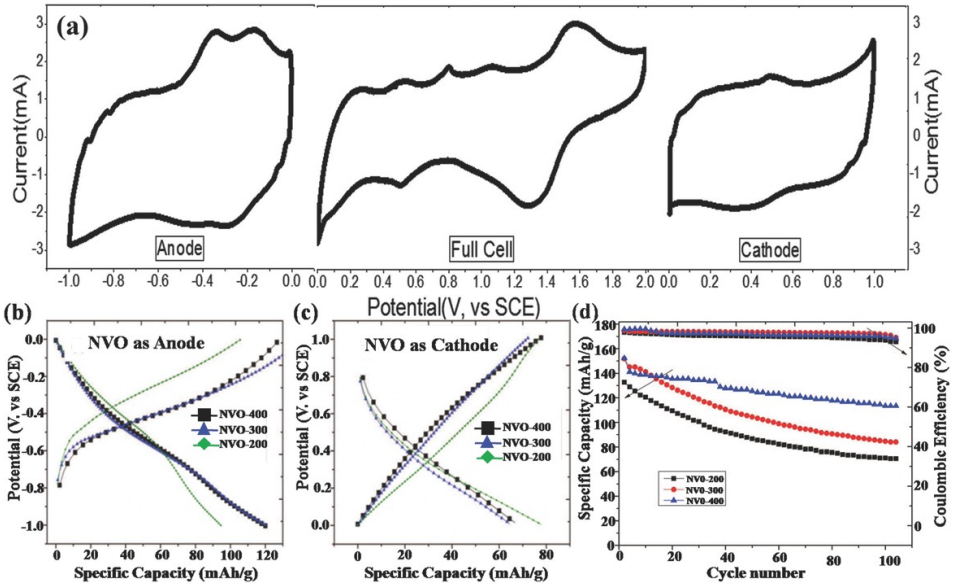

Figure 9. (**a**) Cyclic voltammograms (CVs), (**b**) anode electrode configuration, (**c**) cathode electrode configuration, and (**d**) specific capacity versus cycle number and Coulombic efficiency. Adapted from [94].

While these symmetric LIBs exhibit lower energy densities than commercial lithium-ion batteries due to their narrower operating potential window, they offer compelling advantages for specialized applications. Their enhanced safety profile, combined with advantages in cost-effectiveness and simplified construction, makes them particularly suitable for use cases where safety and operational reliability are prioritized over maximum energy density [94].

4.2. Applied Thermal Sciences for Energy

Through these articles, the literature illustrates a multifaceted understanding of symmetry, from its theoretical underpinnings in physics to its practical applications in engineering [95–97]. Each contribution builds upon the last, creating a comprehensive narrative that underscores the critical importance of symmetry in energy applications and its potential to inform future research and development in the field of symmetry use in thermal system designs [98–101]. Following are some specific cases of symmetry applications in the thermo-energy industry. Heat Exchanger Design with Shell and Tube Heat Exchangers [102,103]:

In shell and tube heat exchangers, cylindrical symmetry allows a uniform distribution of fluid flow through the tubes, improving heat transfer and minimizing hot spots that can cause equipment failures. Turbines and Compressors in Gas Turbines [104]: The blades of gas turbines are arranged symmetrically around the rotational axis. This ensures that centrifugal forces are balanced, minimizing vibrations and increasing the useful life of the turbines. Solar Energy Systems with Solar Concentrators: Symmetrical parabolic reflectors concentrate sunlight at a focal point, where a receiver absorbs the solar energy and converts it into heat. This heat is then used to generate electricity.

Nuclear fusion reactors use symmetry in thermal system designs [105]. The tokamak uses symmetrical magnetic fields to confine the plasma in a toroidal shape. This symmetry is crucial to maintaining plasma stability and sustaining fusion reactions. A technical discussion of how symmetry enables stability and computational savings involves the tokamak's axisymmetric (toroidally symmetric) magnetic field structure, which simplifies equilibrium and stability analysis. The primary magnetic field components are the toroidal field, which is generated by external toroidal field coils, or the poloidal field, which is induced by plasma current (Ohmic heating or auxiliary systems). The stability benefits are as follows: (i) Reduced ripples and perturbations: a perfectly axisymmetric tokamak avoids neoclassical transport and magnetic islands that arise from 3D perturbations; (ii) Avoidance of resonances: symmetry minimizes field-line resonances that drive instabilities (e.g., tearing modes); (iii) Axisymmetry helps maintain Kruskal–Shafranov stability (safety factor $q > 1$) and suppresses kink modes; (iv) Divertor function: The up–down symmetric poloidal field allows for an X-point divertor, which improves particle exhaust and reduces impurity accumulation [105].

Batteries and energy storage systems use symmetry in the thermal system design [106] in constructing a symmetrical air-cooled system. This concept is primarily employed to enhance heat dissipation in battery systems. We developed a thermal model for a liquid-cooled pouch battery pack to evaluate different design configurations using a fin-cooled pack as the reference. The analysis focused on two key metrics: (i) cooling efficiency and (ii) temperature uniformity across the pack [107]. Numerical simulations revealed two critical limitations: (i) Inefficient thermal conduction between the cell stack's base and the cooling plate, which restricts heat dissipation; (ii) temperature non-uniformity caused by asymmetric fin–cell arrangements. Conversely, symmetric electrode designs in lithium-ion batteries (LIBs) promote uniform current distribution during cycling, improving both efficiency and longevity [108].

Symmetry in Computational Fluid Dynamics (CFD) models in thermal sciences, commonly applied to turbomachinery and its various components, allows for simplifying simulations and reducing the computing time required to obtain accurate results on fluid flow and heat transfer [109]. Axial symmetry in fusion systems provides significant computational gains, allowing the use of 2D codes for magnetohydrodynamic equilibria rather than complex 3D simulations. In gyrokinetic simulations, symmetry reduces the computational domain to a single field period, while the Fourier decomposition into the toroidal angle limits the required modes, saving memory and processing. In addition, 1.5D transport codes further simplify the problem by modeling only the radial evolution and averaging over the poloidal and toroidal directions. However, symmetry breaks such as coil misalignments introduce error fields that can trigger locked or disruptive modes, highlighting the importance of precise symmetry control in these systems [110,111].

Symmetry in combustion processes is often addressed in the simplification of models and even in experimental measurements performed. The simulation and measurement objects range from fuel droplets or flames with axial symmetry to engines, especially rotary engines such as aircraft engines; the symmetry of the combustion chambers ensures uniform

combustion of the fuel, increasing efficiency and reducing pollutant emissions. From the geometric symmetry of flames produced by gaseous fuels, especially considering the combustion of liquids, which are generally atomized in droplets [112], and the mitigation of particle emissions such as soot, to flames stabilized inside porous materials [113,114], the most frequent applications are in energy, steam, and heating production, and also in commercial and military aviation, as the following work shows.

The study [113] addressed this research gap by revisiting the problem through a coupled analysis of the Navier–Stokes equations with species and energy transport equations. Figure 10 presents key snapshots of temperature distributions (color contours) and reaction rate isolines (black curves) in a turbulent combustion simulation. The initial condition featured a single hot spot on the symmetry axis positioned sufficiently far from the porous plug. After a transient phase, the system evolved toward a flame state resembling case (e). All simulations used the following dimensionless parameters: injection gas velocity (*m* = 10); Damköhler number (Da = 1685); activation energy (N = 72); Prandtl number (Pr = 0.72). Notably, time-dependent cases incorporated a small but finite Mach number. The results reveal that hot spots closer to the symmetry axis yield long-term symmetric states with dual-edge flames, whereas off-axis positions lead to asymmetry. This trend persisted even in the constant-density model, confirming the existence of a critical vertical threshold for hot spot placement that dictates solution symmetry under fixed parameters.

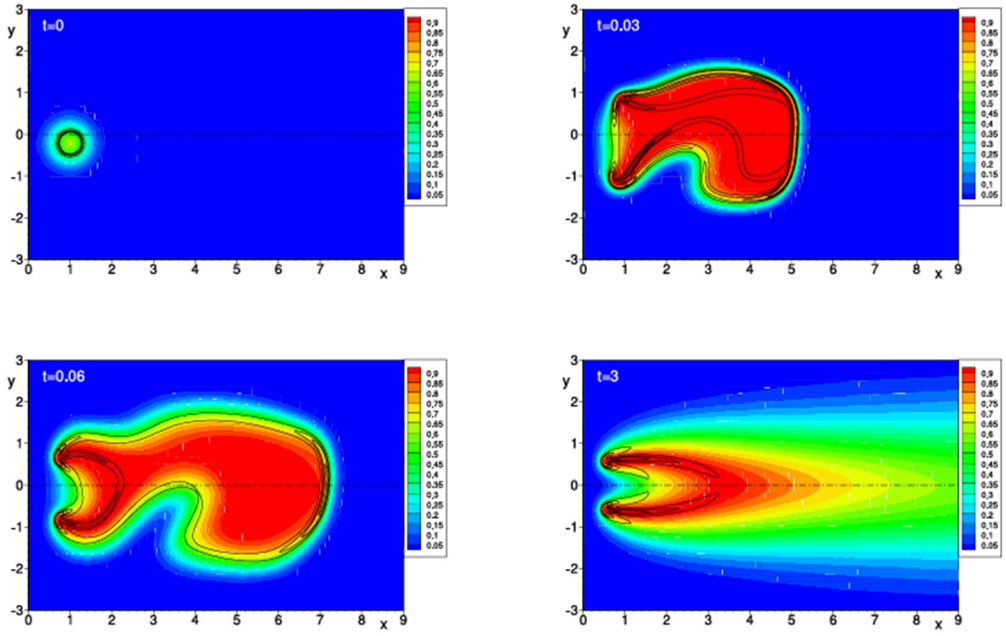

Figure 10. The temporal evolution of flame structures initiated from a single hot spot located at (x = 4, y = 0), showing snapshots at t = 0 (initial condition), t = 0.03, t = 0.06, and t = 3. The long-term flame structure (t = 3) demonstrates excellent agreement with computational predictions, confirming the validity of the numerical model. Adapted from [113].

Time-dependent simulations require significantly greater computational resources than steady-state iterative solutions. To mitigate these costs, multiple studies have employed geometric symmetry to reduce computational domain size and accelerate calculations while maintaining solution accuracy.

A deeper critical analysis indeed strengthens the discussion, especially for practitioners who must weigh efficiency against accuracy. The discussion on symmetry would benefit from a deeper critical analysis that goes beyond its utility to address its limitations and physical trade-offs. While symmetry reduces computational costs in CFD, such as in the simulations of wings or symmetric bodies, improving efficiency and enabling validation

against theoretical cases, it can suppress key phenomena like vortex shedding, turbulent transitions, or flow bifurcations, leading to non-physical results at high Reynolds numbers or in geometries with asymmetric separation. For instance, it is advantageous in low Reynolds (Re) microchannels but misleading in automotive wake flows, where asymmetry dictates drag. Concrete examples of how symmetry aids versus distorts analysis are found in initial symmetric simulations, which may be useful, but full-domain validation is crucial for capturing critical physics. A comparison table of "safe" (controlled laminar flows) versus "risky" (unstable or transitional flows) applications, along with cautionary notes on interpreting results, guide practitioners in balancing efficiency and accuracy, avoiding pitfalls like artificial suppression of instabilities or unaccounted-for real-world asymmetries from manufacturing tolerances. References to classic benchmarks (e.g., cylinders at high Re) strengthen the discussion by highlighting cases where symmetry obscures true flow behavior. The following examples underscore the need for practitioners to justify symmetry use case-by-case, ensuring it does not mask critical flow physics.

Cases where symmetry is beneficial include low-Re microchannel flow, in which symmetric simulations accurately model laminar, fully developed flows where no instabilities arise [79]. Examples include the following: (i) In aerodynamic initial design [12,13], half-car or half-aircraft simulations reduce computational cost in early design stages while preserving key flow features (e.g., pressure distribution on a symmetric fuselage) [4]. (ii) In heat exchanger analysis [25], periodic symmetry simplifies modeling repeating fin structures, assuming uniform flow conditions [37,38].

Cases where symmetry misleads or fails include cylinders at high Reynolds numbers, in which imposing symmetry suppresses the von Kármán vortex street (Re > 47), leading to incorrect drag and lift predictions [113]. Examples include the following: (i) With airfoils at a high angle of attack, symmetric meshes may enforce unphysical flow attachment, masking stall behavior caused by asymmetric separation [61]. (ii) In automotive aerodynamics, real-world vehicle wakes are inherently asymmetric; symmetry artificially suppresses the cross-flow vortices critical for drag and cooling flow predictions [76]. (iii) In turbomachinery blade interactions, symmetry assumptions in rotor–stator simulations can miss secondary flow instabilities or rotating stall precursors [102–104].

5. Conclusions

While symmetry serves as a cornerstone for optimizing energy systems, its application faces shared theoretical and practical challenges that cut across disciplines. This section synthesizes these limitations, analyzes unresolved research gaps, and proposes actionable future directions. The main topics on symmetry in energy cover several areas of physics and engineering, reflecting how symmetry influences the behavior and efficiency of energy systems.

Common problems in symmetry applications are about over-idealization and exhibiting real-world complexity. Symmetric models often assume perfect geometries, isotropic materials, or axisymmetric flows, ignoring manufacturing defects, material heterogeneity, or operational instabilities. Also, scale-transition failures can occur. Symmetries valid at one scale (e.g., nanoscale crystal periodicity) often break at larger scales, such as polycrystalline thermoelectrics with grain boundaries. And there are computational trade-offs, where symmetry reduces computational costs but may obscure critical asymmetrical phenomena, as in vortex shedding in symmetric CFD simulations.

The critical research gaps are based on the disconnects between theory and experiment, where mathematical symmetry tools are rarely validated against real-world asymmetric conditions such as corroded heat exchangers, which can integrate scalar field methods as probabilistic symmetry detection with experimental fluid dynamics to quantify uncertainty.

Also lacking is asymmetry as a design tool; the studies considered in this revision focus on preserving symmetry rather than exploiting controlled asymmetry. Moreover, there are cross-disciplinary inconsistencies; no unified framework links symmetry principles across fields, such as how statistical symmetries in turbulence (physics) could inform fault-tolerant power grids.

Unifying all this, in this work the themes related to symmetry and its applications in industry were identified. Symmetry in Renewable Energy Systems: Symmetry in photovoltaic cell arrangements can optimize sunlight capture and energy conversion efficiency. The symmetrical design of wind turbine blades maximizes wind energy capture, improving performance and reducing structural fatigue. Symmetry in Thermoelectric Materials: Materials with symmetrical crystal structures can have improved thermoelectric properties, which are essential for the efficient conversion of heat into electricity. Symmetry in Combustion Processes: Symmetry in combustion chambers can lead to more uniform fuel combustion, increasing energy efficiency and reducing pollutant emissions. Symmetry in Power Distribution Networks: The symmetrical design of power distribution networks can improve system stability and resilience, facilitating the management of power flows and minimizing losses. Symmetry in Energy Storage Systems: Batteries and supercapacitors with structural symmetry can enhance the charge and discharge properties, as well as provide longer service life and efficiency. Symmetry in Nuclear Fusion Reactors: In fusion reactors, magnetic symmetry is crucial to confine the plasma and sustain fusion reactions efficiently and stably. Symmetry in photonic devices: Devices that manipulate light to perform non-intrusive measurements [115] or for energy conversion and transmission, such as solar cells and LEDs, can benefit from symmetrical structures to optimize photon flux and quantum efficiency. Symmetry in thermodynamics and heat transfer: Symmetry in heat transfer systems such as heat exchangers can result in more uniform temperature distribution and improved thermal efficiency and is exhibited in a wide range of applications [116,117]. Symmetry and conservation laws: Noether's theorem relates symmetries to fundamental conservation laws, such as the conservation of energy, which are essential for understanding the principles underlying energy systems. Symmetry in fluid dynamics: In systems where fluids are used for energy transfer (such as in hydroelectric or refrigeration systems), symmetry can facilitate mathematical modeling and optimization of fluid flow.

A wide range of studies have been related in more general research [118] and several other important works on specific topics, which are not included in this article due to the limitations of its nature, but which can be found from the topics and works mentioned in this review. Symmetry's true potential lies in strategic adaptation, balancing idealizations with real-world constraints. By systematizing these challenges and fostering cross-domain collaboration, researchers can transform symmetry from a simplifying assumption into a dynamic tool for energy innovation.

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