

Article

Optimization of Actuator Arrangement of Cable–Strut Tension Structures Based on Multi-Population Genetic Algorithm

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Abstract: This study addresses the optimization of actuator arrangements in adaptive cable–strut tension structures to enhance structural controllability and performance. Two novel optimization criteria are proposed: (1) a weighted sensitivity criterion that integrates nodal displacements and internal force increments, and (2) a system strain energy criterion reflecting overall structural stiffness. Nonlinear optimization models are formulated for these criteria, with actuator positions as design variables, and solved using a robust multi-population genetic algorithm. The weighted sensitivity criterion prioritizes targeted control of specific nodes and members, while the strain energy criterion ensures balanced global response. Numerical validation is conducted on a Geiger cable dome and a four-layer tensegrity structure. Results demonstrate that both criteria yield actuator arrangements satisfying geometric symmetry while achieving high sensitivity in displacement and internal force control. The proposed framework offers practical insights for optimizing adaptive structures under static control requirements, and advances the field by bridging localized and global response optimization, enabling smarter, more resilient tension structures.

Keywords: actuator arrangement optimization; cable–strut tension structures; multi-population genetic algorithm; weighted sensitivity criterion; system strain energy; structural controllability



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1. Introduction

Cable–strut tension structures are spatial systems that derive their stiffness from prestressing, with cables and struts serving as their fundamental components [1]. These lightweight systems are widely used in large-span roofs, deployable aerospace structures, and adaptive architectural frameworks due to their high strength-to-weight ratio and geometric flexibility. The cables, typically made of steel strands or synthetic fibers, resist tension forces, while struts—often hollow steel or composite tubes—provide compression resistance, creating a stable equilibrium through carefully calibrated prestress levels. This type of structure aligns well with the concept of smart structures, as its members can be designed to simultaneously fulfill specific requirements for strengthening, sensing, actuation, and control [2]. By adjusting member lengths, the structural morphology, stiffness, and prestress distribution can be actively controlled, offering advantages such as high resilience, adaptability, and energy efficiency. For instance, in deployable aerospace mechanisms, cable–strut systems enable compact stowage and precise deployment [3], while in civil engineering, they mitigate structural responses to wind or thermomechanical loads through

real-time actuator adjustments [4]. Active control can be effectively achieved using smart actuators based on shape-memory alloys, piezoelectric materials, or clustered elements [5].

In the field of active control of cable–strut tension structures, researchers have explored various optimization criteria and methods. For instance, Zhang and Feng [6] conducted a detailed study on the self-stress mode analysis and optimal prestress design of cable–strut tension structures, providing a theoretical basis for the morphological control of such structures. Zhang et al. [7] investigated the mechanical properties and shape-control abilities of cable dome structures under asymmetrical loads, which demonstrated the potential of using advanced computational methods to optimize the performance of cable domes under complex loading conditions. Raja and Narayanan [8] compared the effectiveness of different active control strategies in the vibration control of tensegrity structures, offering experimental evidence for actuator arrangement optimization. Their findings underscored the need for a systematic approach to evaluate the performance of various control strategies. Hrabaka et al. [9] presented a novel methodology for actuation planning and vibration control of active tensegrity structures, combining dynamic relaxation-based optimization for cable rest-length adjustments with an H_2 controller to suppress structural oscillations. These studies highlight the importance of understanding the underlying mechanics of tension structures to develop effective control strategies.

The control accuracy and sensitivity of active components in adaptive structures pose significant challenges to meeting the control requirements in different working states, which seriously limits their adaptability to various external environments. To address this issue, Li et al. [10] studied the response sensitivity of adaptive cable–strut tension structures to changes in member length, providing theoretical support for the weighted sensitivity criterion proposed in this paper. Their work highlighted the importance of understanding how changes in actuator positions affect structural responses. Goyal et al. [11] introduced a unified approach for jointly optimizing structural design, sensor/actuator precision, and control parameters in tensegrity systems using Linear Matrix Inequalities (LMIs) and convexification techniques, demonstrating effective integration for enhanced system performance. Zou et al. [12] presented a double rhombic-strut adaptive beam string structure that significantly enhances control accuracy and sensitivity through a rhombic amplification mechanism, validated by experimental and numerical analyses.

Additionally, Lu et al. [13] presented a machine learning-based active control framework for lightweight cable–strut antennas, integrating the force density method for shape sensing and a nested genetic algorithm for actuator optimization, achieving high deformation accuracy and rapid response under wind loads. Sui and Shao [14] proposed a strength control method for statically indeterminate truss structures, offering new insights for the optimal design of adaptive structures. Veuve et al. [15] presented a near full-scale deployable tensegrity footbridge with a focus on mid-span connection control, comparing symmetric and uniform topologies. It demonstrated the potential of using computational algorithms to realize real-time control in complex cable–strut systems. Feng et al. [16] proposed a fast model predictive control strategy combined with the artificial fish swarm algorithm for active vibration control in tensegrity structures, addressing nonlinearity and actuator arrangement. Validated on tensegrity beams and a tower, the method effectively mitigated vibrations, offering a promising solution for complex engineering challenges. Kripakaran and Smith [17] investigated the configuration and enhancement of measurement systems for damage identification, providing a new perspective for actuator arrangement optimization. Their work emphasized the importance of considering sensor characteristics during the design of measurement systems to ensure accurate structural identification.

The controllability of intelligent structures is intrinsically linked to the layout of their actuating devices, making the optimization of actuator arrangement a critical task in the

design and implementation of such systems. A key challenge in this field lies in defining appropriate optimization criteria that can effectively guide the placement of actuators to achieve desired structural responses. Researchers have proposed various criteria to address this challenge, including observability/controllability metrics [17,18], minimum system energy principles [8,19], and reliability-based approaches [20,21]. Each of these criteria offers unique insights into the optimal placement of actuators, but there remains a need for a more comprehensive approach that can balance multiple structural responses, such as nodal displacements and internal forces.

This study proposes two controllability criteria for the optimal arrangement of actuators in structural static control: (1) a weighted sensitivity criterion that integrates nodal displacements and internal force increments, and (2) a system strain energy criterion reflecting global structural stiffness. It offers a new framework for optimizing the placement of actuators in complex structures, with the goal of enhancing structural performance and controllability. Unlike existing methods that focus on isolated controllability metrics, the weighted sensitivity criterion reconciles multi-scale responses and introduces variance constraints to prevent actuator over-localization, while the strain energy criterion ensures balanced energy distribution across the structure. Based on these criteria, two nonlinear optimization models are established, with actuator arrangement as the design variable under specific constraints. To solve these models, a multi-population genetic algorithm (MPGA) is employed, enhancing exploration–exploitation balance and avoiding premature convergence—a common limitation in traditional genetic algorithms (GAs) and particle swarm optimization (PSO). The proposed framework is rigorously tested on both Geiger cable dome and multi-layer tensegrity structure, demonstrating its scalability, robustness, and potential for practical applications in structural engineering. Table 1 illustrates the main contributions over existing techniques from four aspects. Following this introduction, Section 2 details the formulation of optimization criteria and the MPGA framework. Section 3 validates the method through numerical examples, while Section 4 discusses conclusions and future directions.

Table 1. Comparison of existing techniques vs. proposed approach.

Existing Techniques	Proposed Approach
Single controllability metrics	Dual-criteria framework: Combine weighted sensitivity (local control) and system strain energy (global stiffness balance)
Risk of actuator over-concentration	Variance-constrained sensitivity: Avoid actuator over-concentration through displacement/internal force variance thresholds
Standard GA/PSO algorithms	MPGA algorithm: Adaptive crossover/mutation probabilities and migration operators enhance exploration–exploitation balance
Limited validation scope	Comprehensive validation: Tested on Geiger cable dome and multi-layer tensegrity structure, demonstrating scalability and robustness

2. Materials and Methods

2.1. Structural Controllability Optimization via Weighted Sensitivity

As a spatial flexible system deriving stiffness from prestress, the cable–strut tension structures exhibit strong geometric nonlinearity due to the high interdependence between their shape and internal forces. These kinds of structures demonstrate pronounced sensitivity to external loads, where even minor external disturbances can induce significant load effects within the structure, potentially compromising its serviceability and safety [22]. To address these challenges, active cable–strut tension structures are designed with embedded control elements such as actuators. When external perturbations occur, these systems adaptively adjust member lengths to regulate structural responses—including internal forces and nodal displacements—thereby maintaining optimal operational states under dy-

namic conditions. Clearly, the effectiveness of the control system is inherently linked to the spatial distribution of actuators, making sensitivity analysis—the calculation of structural response gradients with respect to design variables—a cornerstone of the optimization process. This section establishes a weighted sensitivity criterion to evaluate actuator layout effectiveness, followed by the formulation of a constrained optimization model tailored for static control scenarios.

2.1.1. Fundamental Assumptions

The static control process investigated in this study prohibits material plasticity and cable slackness. Prior to establishing the optimization model, the following fundamental assumptions are adopted:

- (1) All nodes are idealized as frictionless hinged nodes.
- (2) Cable elements sustain axial tension only while strut elements sustain axial compression only.
- (3) The structure is subjected exclusively to nodal loads.
- (4) The cross-sectional area of each member remains constant throughout the calculation.
- (5) Cable and strut materials are idealized elastic bodies, and their constitutive relationships obey Hooke's Law.
- (6) For active members embedded with actuators (as shown in Figure 1), the axial stiffness k is determined as follows.

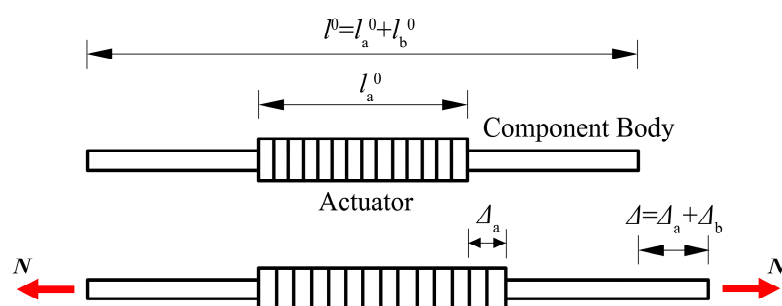


Figure 1. Schematic diagram of actuator working principle.

Let k_a and l_a^0 denote the actuator's axial stiffness and original length, respectively. Within the elastic range, the axial stiffness k_b of the component body is given by the following:

$$k_b = E_b A_b / l_b^0 \quad (1)$$

where E_b , A_b and l_b^0 represent Young's modulus, the cross-sectional area, and the rest-length of the component, respectively. When the active member is subjected to a tensile force N , the total axial deformation Δ comprises two parts: the actuator deformation Δ_a and the component deformation Δ_b :

$$\Delta = \Delta_a + \Delta_b = \frac{N}{k_a} + \frac{N}{k_b} = N / \left(\frac{k_a k_b}{k_a + k_b} \right) \quad (2)$$

Thus, the axial stiffness of the active member is derived as follows:

$$k = \frac{k_a k_b}{(k_a + k_b)} = \frac{E_b A_b}{E_b A_b / k_a + l_b^0} \quad (3)$$

where Equation (1) is substituted into.

2.1.2. Weighted Sensitivity Index Formulation

To analyze the influence of different actuator arrangements on nodal displacements and internal forces in cable–strut tension structures, this study proposes a weighted sensitivity criterion for structural response. For a given actuator layout scheme, a unit regulation amount applied to the actuator induces the displacements of controlled degrees of freedom $(1, 2, 3, \dots, b_1)$, denoted by a b_1 -dimensional vector $\mathbf{U}$, and changes in the internal forces of controlled members $(1, 2, 3, \dots, b_2)$, denoted by a b_2 -dimensional vector $\mathbf{T}$. The mean of the absolute values of elements in arrays $\mathbf{U}$ and $\mathbf{T}$ are represented by U_m and T_m , respectively, and their variances by U_v and T_v . The weighted sensitivity H of the structural configuration is defined as follows:

$$H = (\alpha U_m + \beta) + T_m \quad (4)$$

where α and β are weight coefficients used to reconcile the difference in magnitude between U_m and T_m .

Then, the following optimization model is constructed with the objective of maximizing the weighted sensitivity H and with the actuator layout $\mathbf{e}$ as the decision variable:

$$\begin{aligned} \text{Find } & \mathbf{e} = [e_1, e_2, \dots, e_q]^T \\ \text{Max } & S(\mathbf{e}) \\ \text{s.t. } & U_v(\mathbf{e}) \leq U_v^*, \quad T_v(\mathbf{e}) \leq T_v^* \\ & e_i \in \{0, 1\}, \quad \sum e_i = q_a \end{aligned} \quad (5)$$

Here, $\mathbf{e}$ is a q -dimensional vector where q denotes the total number of structural members in the system. For the i -th member: $e_i = 1$ if it is an active member, indicating an actuator-induced length adjustment of 1 mm; $e_i = 0$ otherwise. The sum of all entries in $\mathbf{e}$ equals q_a , representing the total number of active members in the system. Consequently, distinct vectors $\mathbf{e}$ correspond to unique actuator arrangement schemes. The variance thresholds U_v^* and T_v^* ensure actuator layouts avoid excessive concentration on specific regions leading to localized over-control, thereby preserving global structural controllability.

2.1.3. Governing Equation Derivation and Actuator Quantity Analysis

Assume the structural system has d nodal degrees of freedom and q members totally. The self-equilibrium equations at nodes and the kinematic compatibility equations relating nodal displacements to bar elongations can be respectively given as follows [23]:

$$\mathbf{A}\mathbf{n} = \mathbf{f} \quad (6)$$

$$\mathbf{A}^T \delta \mathbf{x} = \delta \mathbf{l} \quad (7)$$

where the matrix $\mathbf{A}$ of order $d \times q$ is the so-called equilibrium matrix containing the orientations of members of the structure, and $\mathbf{A}^T$ denotes the transpose of matrix $\mathbf{A}$; $\mathbf{n}$ is the $n \times 1$ vector consisting of the tension in each member; $\mathbf{f}$ is the $d \times 1$ vector composed of concentrated forces acting on the nodes; $\delta \mathbf{x}$ is the d -dimensional vector of incremental coordinates, describing infinitesimal movements of nodes; $\delta \mathbf{l}$ is the q -dimensional elongation vector, containing infinitesimal extension of each member.

The elongation δl for each member consists of two parts: (i) an inelastic part due to the imposed length change, and (ii) a linear-elastic part caused by the change of axial force. Thus, the following relation holds the following:

$$\delta \mathbf{l} = \mathbf{e} + \mathbf{F} \cdot \delta \mathbf{n} \quad (8)$$

where δn is the q -dimensional vector of axial force change in each member; the q -dimensional vector e contains the imposed elongation of each member; the $q \times q$ diagonal matrix F has diagonal elements representing the axial flexibility of each member. Specifically, each diagonal entry is calculated as the ratio of the element length to the product of Young's modulus and cross-sectional area ($l/(EA)$).

Substituting Equation (7) into Equation (8) and tidying up gives the following:

$$\delta n = F^{-1}(A^T \delta x - e) \quad (9)$$

On the other hand, based on the finite element theory, the relationship between the nodal load increment vector δf , nodal displacement vector δx , and member elongation vector e can be expressed by the following equation [19]:

$$K \cdot \delta x = \delta f + AF^{-1}e \quad (10)$$

where K is defined as the tangent stiffness matrix of the structure, with its full derivation provided in [1].

During the control process, the nodal loads remain unchanged, i.e., $\delta f = 0$. Considering that K may be either invertible (for common cable–strut tension structures) or singular (for free standing tensegrity structures), the Moore–Penrose generalized inverse of K , denoted as K^+ , is employed to solve δx from Equation (10):

$$\delta x = K^+ AF^{-1}e \quad (11)$$

By substituting Equation (11) into Equation (9), the relationship between δn and e is obtained as follows:

$$\delta n = F^{-1}(A^T K^+ AF^{-1} - I)e \quad (12)$$

where I is $q \times q$ identity matrix.

In summary, the relationship among the nodal displacements, internal force increments, and actuator adjustments are derived by combining Equations (11) and (12):

$$\begin{bmatrix} \delta x \\ \delta n \end{bmatrix} = \begin{bmatrix} K^+ AF^{-1} \\ F^{-1}(A^T K^+ AF^{-1} - I) \end{bmatrix} e \quad (13)$$

For convenience, the above equation can be abbreviated as:

$$\Phi = He \quad (14)$$

Assuming the number of actuators in the structural system is q_a , the vector e is partitioned into the subvector associated with active members e_c (of size $q_a \times 1$) and the one associated with passive members e_u (of size $(q - q_a) \times 1$), with $e_u = 0$. Let b_c denote the total number of control parameters. Then, Φ is partitioned into the controlled subvector Φ_c (of size $b_c \times 1$) and the uncontrolled subvector Φ_u (of size $(d + q - b_c) \times 1$). Correspondingly, by partitioning the coefficient matrix H , Equation (14) can be rewritten as follows:

$$\begin{bmatrix} \Phi_c \\ \Phi_u \end{bmatrix} = \begin{bmatrix} H_{11}^{(b_c \times q_a)} & H_{12}^{(b_c \times (q - q_a))} \\ H_{21}^{((d + q - b_c) \times q_a)} & H_{22}^{((d + q - b_c) \times (q - q_a))} \end{bmatrix} \begin{bmatrix} e_c \\ 0 \end{bmatrix} \quad (15)$$

Then, the governing equation characterizing the relationship between the actuator adjustments and the control parameters of the structure is obtained as follows:

$$\Phi_c = H_{11}e_c \quad (16)$$

The necessary and sufficient condition for Equation (16) to have a solution is that the rank r of $\mathbf{H}_{11}$ is equal to the rank r' of the augmented matrix $[\mathbf{H}_{11} \ \Phi_c]$, i.e., $r = r' \leq q_a$. When the structural configuration, control objectives, and control schemes are unknown, the values of r and r' cannot be undetermined. Computational practice shows that in most cases, $\mathbf{H}_{11}$ satisfies the row full-rank condition approximately, i.e., $r \approx b_c \leq q_a$. Therefore, the number of actuators q_a should, in principle, be no less than the number of control parameters b_c . From an economic perspective, it is generally appropriate to set $q_a = b_c$.

On the other hand, the governing Equation (16) is also utilized in this study to determine the weight coefficient α in the objective function (see Equation (4)). The specific procedure is as follows: for a given actuator layout scheme, $\mathbf{H}_{11}$ can be determined according to Equations (13) and (15). By defining $\mathbf{e}_c$ as a q_a -dimensional vector with all elements equal to 1, Equation (16) allows linear calculation of displacement control parameters and internal force control parameters, from which their mean values U_m and T_m can be derived. By randomly generating i groups (typically $i \geq 50$) of actuator layout schemes, i pairs of associated datasets $[U_m, T_m]$ are obtained. These datasets are then subjected to linear fitting via the least squares method to establish the linear correlation between U_m and T_m , thereby determining the weight coefficient α to reconcile their magnitude differences.

2.1.4. Nonlinear Iterative Refinement of Sensitivity Indices

For precise computation of U_m and T_m , let us revisit Equation (10). As previously stated, during the control process, $\delta f = \mathbf{0}$ holds. Therefore, Equation (10) can be rewritten as follows:

$$\mathbf{K} \cdot \delta \mathbf{x} = \mathbf{A} \mathbf{F}^{-1} \mathbf{e} \quad (17)$$

The right-hand side of it can be regarded as the driving force caused by $\mathbf{e}$ in control process. Here, we denote it as $\delta f'$ since it is fully equivalent to the nodal force increment δf , and $\delta \mathbf{x}$ is also solved through Equation (17), i.e., $\delta \mathbf{x} = \mathbf{K}^+ \delta f'$.

Then the nodal coordinates can be updated as follows:

$$\mathbf{x}^2 = \mathbf{x}^1 + \delta \mathbf{x} \quad (18)$$

where the nodal coordinate vectors $\mathbf{x}^1$ and $\mathbf{x}^2$ are associated with initial configuration and deformed configuration, respectively. Correspondingly, the member length vector $\mathbf{l}$ and equilibrium matrix $\mathbf{A}$ should also be updated in terms of the new location $\mathbf{x}^2$.

Noting that the actuator works as if the rest-length of the member would be changed, the current member tension vector $\mathbf{n}$ can be computed as follows:

$$\mathbf{n} = \mathbf{F}^{-1}(\mathbf{l} - \mathbf{l}^0 - \mathbf{e}) \quad (19)$$

where $\mathbf{l}^0$ is the vector containing rest-length l^0 of each member before the imposed contraction is acted upon.

Obviously, for the current location, the equilibrium Equation (6) usually does not hold precisely because the derivation of $\mathbf{K}$ is based upon linear-elasticity hypothesis and the out-of-balance forces are given as follows:

$$\delta f' = \mathbf{f} - \mathbf{A} \mathbf{n} \quad (20)$$

Then, the Newton–Raphson method is employed to eliminate $\delta f'$. The structural location will be updated iteratively, unless the 2-norm of $\delta f'$ is less than an allowable error ξ . Thus, the regulated structural configuration is obtained, thereby enabling precise computation of U_m and T_m in Equation (4). This nonlinear program can be summarized by the flowchart shown in Figure 2.

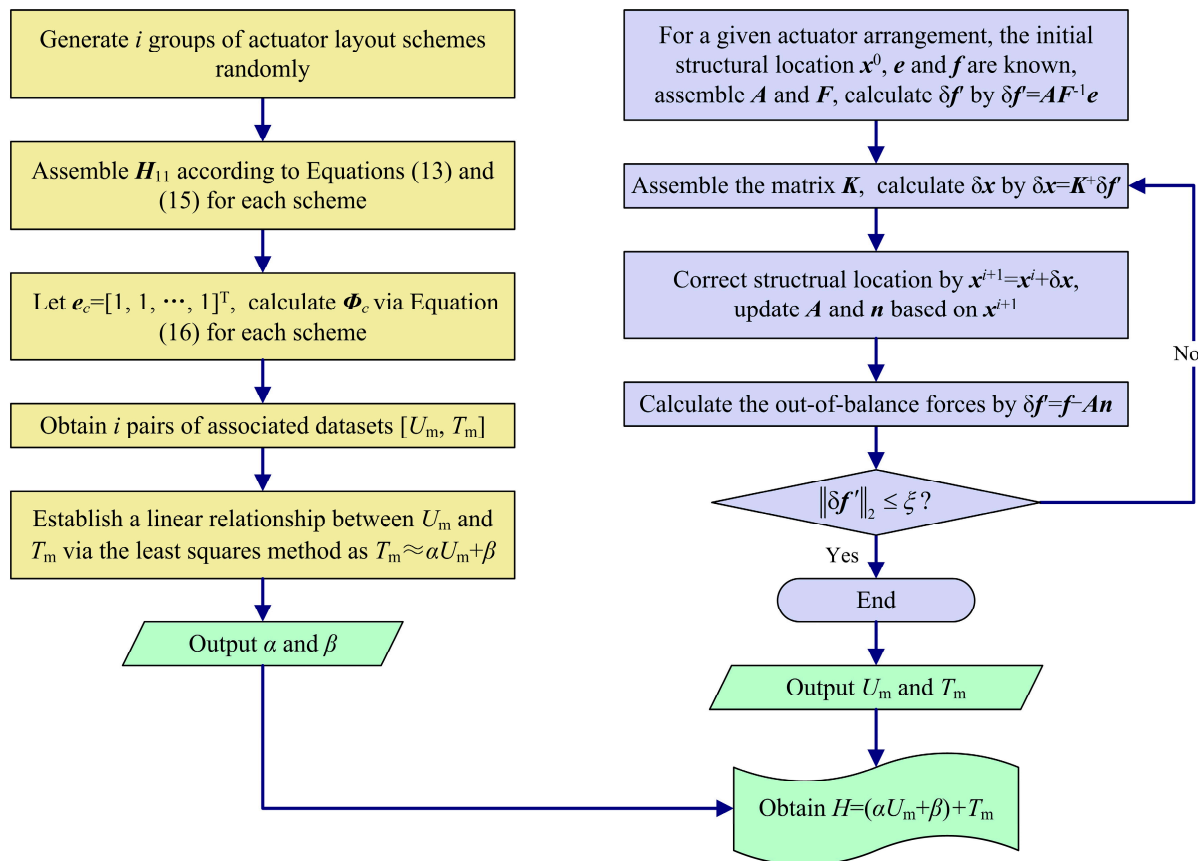


Figure 2. Flowchart of nonlinear iterative program to calculate structural response for a certain actuator layout.

2.2. Structural Controllability Optimization via System Strain Energy Criterion

The work performed by actuators during the control process is stored within the structure as strain energy. For a given actuator arrangement scheme, the strain energy generated by activating all actuators simultaneously with a unit quantity of imposed adjustment can holistically characterize the control efficiency of that scheme. In other words, the greater the strain energy stored in the system, the higher the regulation efficiency of the scheme. The expression of strain energy for i -th member in a tension structure is the following:

$$W_i = \int_L n_{0i} \frac{\Delta n_i}{E_i A_i} dl_i + \frac{1}{2} \int_L \Delta n_i \frac{\Delta n_i}{E_i A_i} dl_i \quad (21)$$

where n_{0i} and Δn_i represent the initial prestress and the internal force increment in the i -th member, respectively.

Assembling the strain energy expression of each member into the matrix form, the total strain energy W stored in the system can be obtained as follows:

$$W = n_0 \cdot e + \frac{1}{2} \Delta n \cdot e \quad (22)$$

where both n_0 and Δn are q -dimensional vectors, consisting of n_{0i} and Δn_i in each member, respectively. For a given actuator layout e , Δn can be precisely calculated by using the iterative program provided in Section 2.1.4, thereby obtaining the value of W through Equation (22).

Then, the following optimization model is constructed with the objective of maximizing the strain energy W with the actuator layout e as the decision variable:

$$\begin{aligned} &\text{Find } e = [e_1, e_2, \dots, e_q]^T \\ &\text{Max } W(e) \\ &\text{s.t. } U_v(e) \leq U_v^*, \quad T_v(e) \leq T_v^* \\ &\quad e_i \in \{0, 1\}, \quad \sum e_i = q_a \end{aligned} \quad (23)$$

For clarity, the optimization models formulated based on the weighted sensitivity criterion (defined in Equation (5)) and the strain energy criterion (defined in Equation (23)) will be termed Model 1 and Model 2, respectively, in the following sections of this paper.

2.3. Multiple Population Genetic Algorithms

In this paper, the multi-population genetic algorithm (MPGA) is employed to solve the aforementioned optimization models. MPGA enhances traditional genetic algorithms by maintaining multiple subpopulations that evolve independently, periodically exchanging high-quality individuals through migration operators. This framework prevents premature convergence and local optima traps by preserving population diversity and balancing exploration–exploitation dynamics. Adaptive crossover/mutation probabilities and elitism further stabilize global search, ensuring consistent and reproducible optimal solutions even in complex, multi-modal design spaces. The specific steps are as follows.

(1) Encoding: Binary encoding is used. e is a q -dimensional vector arranged in the order of member numbers, where members with actuators are set to 1, and the rest are set to 0.

(2) Initial Population Generation: s subpopulations are initialized to explore different regions of the solution space, with each subpopulation containing m individuals. In each individual of the population, the number of ‘1’ entries must equal the predetermined number of actuators.

(3) Fitness Evaluation: The objective functions defined in Equations (4) and (22) can be transformed into fitness functions through the following normalization procedure. First, assume all components are active members, and all actuators are simultaneously shortened by 1 mm (i.e., let $e = [-1, -1, \dots, -1]^T \in \mathbb{R}^{q \times 1}$). The values of the objective functions $S(e)$ and $W(e)$ are then precisely calculated using the nonlinear program described in Section 2.1.4. Clearly, these values are greater than those corresponding to any individual with q_a ($< q$) actuators, hence denoted as $S_{\max}$ and $W_{\max}$, respectively. The fitness function is defined as the ratio of the objective function of a given individual to $S_{\max}$ or $W_{\max}$, expressed as follows:

$$F(e) = \begin{cases} S(e)/S_{\max} & \text{For Model 1} \\ W(e)/W_{\max} & \text{For Model 2} \end{cases} \quad (24)$$

When an individual violates the constraints, the penalty function method is applied to appropriately reduce its fitness, with a penalty factor of 0.6.

(4) Selection: The roulette wheel selection strategy is adopted to ensure that individuals with higher fitness have a greater probability of being selected for the next generation.

(5) Crossover: An adaptive crossover probability $P_c(i)$ for i -th individual is defined as follows:

$$P_c(i) = \sqrt{i/N} \cdot P_a \quad (25)$$

where the baseline crossover probability P_a is a constant, ranging from 0.5 to 1. This mechanism is designed such that individuals with higher fitness (assigned smaller indices) are given lower crossover probabilities to protect high-quality solutions, while those with lower fitness (assigned larger indices) receive higher crossover probabilities to promote

population diversity. Combined with the elitism strategy, this strategy ensures that superior solutions are preserved without loss. After identifying the individuals selected for crossover operations, the single-point crossover method is employed for gene exchange.

(6) Mutation: Bit-flip mutation is used. In the early stage of evolution, to prevent the population from prematurely converging to local optima, the mutation probability for low-fitness individuals is adaptively increased. During the later evolutionary phase, the overall mutation probability of the population is constrained to remain below 10%, thereby avoiding degradation of the algorithm into purely random search. Thus, the mutation probability function is designed as follows:

$$P_m = \begin{cases} \frac{\exp(i/N)-1}{\exp(i/N)+1} & \text{Early stage} \\ 0.05 \sim 0.1 & \text{Later stage} \end{cases} \quad (26)$$

where an adaptive mechanism governs mutation probability in the initial phase, and a constant value is adopted in the later stage.

(7) Migration Operator and Elitism Strategy: The migration operator replaces the worst individuals in the target population with the fittest individuals from the source population, facilitating information exchange between independently evolving subpopulations. Simultaneously, the fittest individuals from each subpopulation are preserved in an elitist pool.

(8) Termination Condition: The minimum retained generations of the optimal individual in the elitist pool is adopted as the convergence criterion. Specifically, the algorithm is considered converged when the number of generations for which the optimal individual remains unchanged exceeds a predefined threshold. This criterion fully leverages the knowledge accumulated during the evolutionary process, offering a more rational approach compared to the conventional maximum generation criterion.

In summary, the flowchart of the proposed MPGA algorithm is illustrated in Figure 3, which can be implemented in Matlab 7.0 (R14) software.

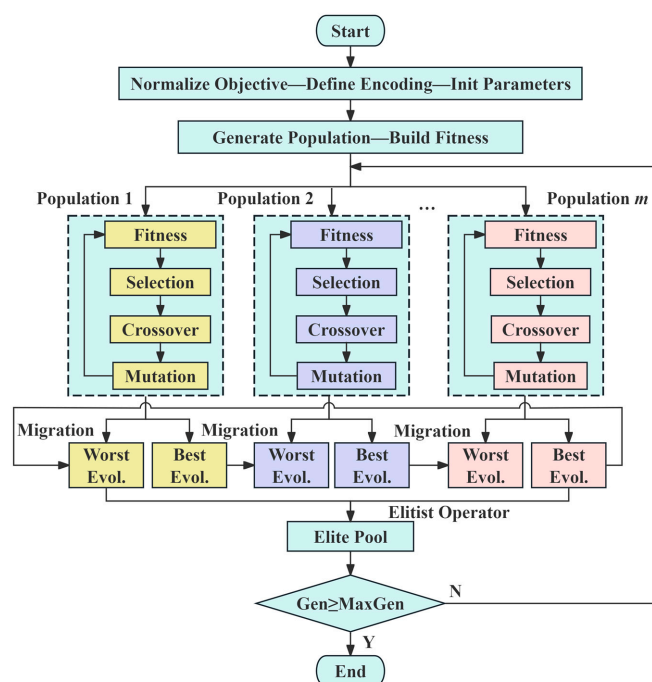


Figure 3. Flowchart of the proposed MPGA algorithm.

3. Results and Examples

The numerical examples in this section follow a progressive principle from simplicity to complexity to validate the proposed methodology comprehensively. The first example employs a Geiger cable dome, a canonical and simple structural form widely adopted in tension structure research due to its symmetric geometry and straightforward prestress distribution. This example serves as a foundational validation of the optimization framework. The second example adopts a tensegrity structure derived from the form-finding process in [24], which features a more intricate topological configuration. The selected tensegrity structure forms a horizontal working platform, enabling us to explicitly define the optimization objective of surface flatness (i.e., controlling vertical displacements of upper nodes). This progression from basic to complex structures ensures rigorous testing of the method's adaptability and scalability across varying geometric and mechanical challenges.

3.1. Geiger Cable Dome

A three-layer, eight-ribbed Geiger cable dome is used as an example, and the section and plan of the structure are shown in Figure 4. Its radius is 5 m; the radii of the inner and outer ring cables are 1.667 m and 3.334 m, respectively; the heights of the upper, middle, and lower struts are 0.308 m, 0.959 m, and 1.736 m, respectively. The system has 11 groups of 81 members. The cables adopt $\phi 12$ high-strength steel strands (the elastic modulus is 185 GPa, and the allowable stress is 1850 MPa). The struts employ $\phi 45 \times 3$ hollow steel rods (the elastic modulus is 206 GPa, and the allowable stress is 210 MPa). Assume that the actuators are of the same specification—the original length is 100 mm, the axial stiffness $Ka = 100$ kN/mm, and the allowable axial force is 100 kN. The nodes located around the outer ring, numbered from 18 to 25, are pin-jointed. The prestress applied to the structure can be determined via the equilibrium matrix theory [25], with results summarized in Table 2.

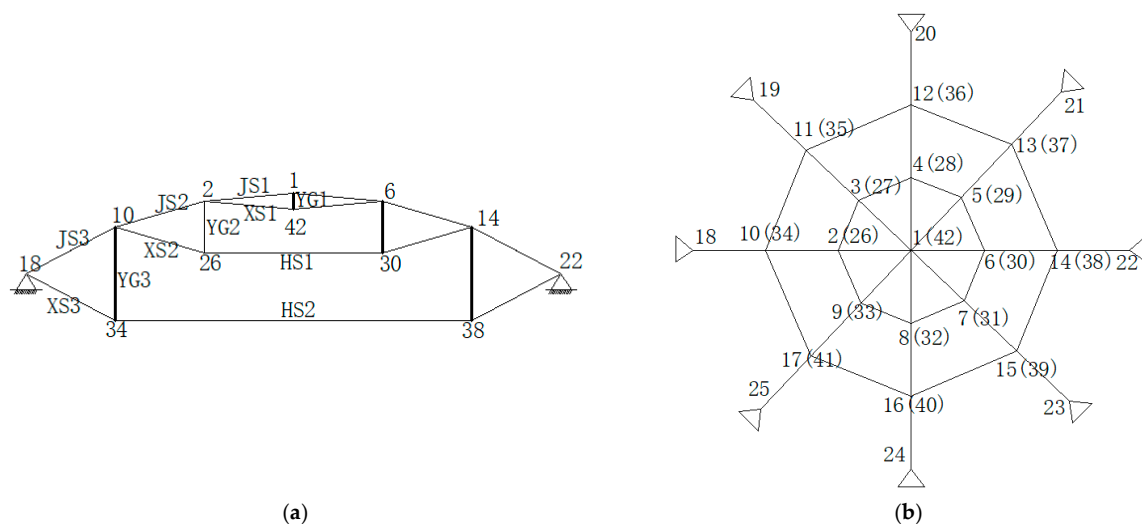


Figure 4. Geiger cable dome including labels of nodes and members: (a) Section view; (b) Plan view.

Table 2. Prestress distribution of Geiger dome (kN).

Member Group	Prestress	Member Group	Prestress	Member Group	Prestress
JS1	9.3752	XS1	9.3752	YG1	−5.8994
JS2	19.4234	XS2	19.3115	YG2	−5.3538
JS3	41.9695	XS3	41.9695	YG3	−19.3832
HS1	24.2426	HS2	48.6374		

Considering that the surface of the Geiger cable dome needs to be covered with membrane material, to prevent the tensioned membrane from wrinkling or slackening, it is necessary to control the vertical displacements of the 17 nodes ($u_{1z} \sim u_{17z}$) located on the upper surface. On the other hand, as shown in Table 2, the internal forces of the eight members included in the JS1 group (inner ridge cables) are smaller than those of other types of members under the prestressed state. To prevent these members from ceasing to function, which would affect the stability and stiffness of the structure, it is necessary to control their internal forces. As can be seen from Section 2.1.3, the number of actuators must satisfy $q_a \geq r'$ to ensure the existence of feasible solutions in the search space. Given $r' = 25$ ($b_1 = 17$, $b_2 = 8$), and considering cost-effectiveness, the number of actuators q_a is set to 25 in this case study.

As discussed earlier, the governing Equation (16) is utilized to determine the weight coefficient α in Model 1. For a given actuator layout scheme, Equation (16) enables linear computation of controlled displacements and internal force increments resulting from unit-imposed adjustments (i.e., by setting the control vector $e_c = [1, 1, \dots, 1]^T \in \mathbb{R}^{q_a \times 1}$). Then, the mean absolute values U_m and T_m can be derived. By randomly generating 50 distinct actuator layout schemes, 50 paired datasets $[U_m, T_m]$ are obtained, as illustrated in Figure 5. After linear fitting via the least squares method, the correlation between U_m and T_m is established, ultimately yielding $\alpha = 1.53$ and $\beta = 0$. The nonlinear iterative framework presented in Section 2.1.4 enables precise computation of S and W , the objective functions of Optimization Models 1 and 2.

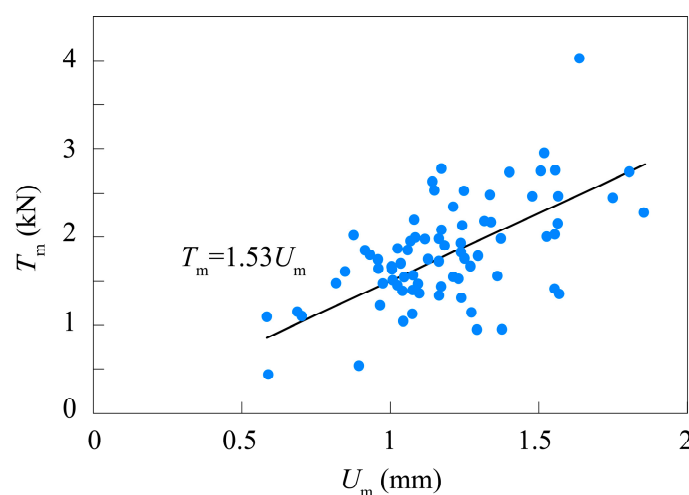


Figure 5. Fitted linear relationship between U_m and T_m in Example 1.

The variance thresholds are set as $U_v^* = 2$ and $T_v^* = 1$, and the key control parameters adopted by the MPGA algorithm are listed in Table 3. These values are determined through a trial-and-error process to assure stable convergence and acceptable time consumption of the algorithm. For Optimization Model 1, we executed the MPGA algorithm provided in Section 2.3 for 100 consecutive runs and obtained an optimal solution with a fitness value of 0.83. The corresponding actuator arrangement is illustrated in Figure 6a. Among the 100 runs, this optimal solution was achieved 72 times, with an average convergence generation of 68 and a fastest convergence generation of 53. Figure 7a depicts the fastest convergence process of the maximum fitness for Model 1. For Optimization Model 2, we similarly conducted 100 consecutive runs of the MPGA algorithm, yielding an optimal solution with a fitness value of 0.71. The corresponding actuator arrangement is shown in Figure 6b. This optimal solution was attained 78 times out of 100 runs, with an average convergence generation of 151 and a fastest convergence generation of 132. Figure 7b

illustrates the fastest convergence process of the maximum fitness for Model 2. As observed in Figure 6, the optimal actuator distributions for both two models exhibit rotational symmetry, which is rational and consistent with theoretical expectations.

Table 3. Parameter settings for MPGA in Example 1.

Parameter Name	Parameter Value	Parameter Name	Parameter Value
Number of populations (s)	4	Size of Single Population (m)	40
Minimum Retained Generations	15	Baseline crossover probability (P_a)	0.9
Mutation Probability in later stage (P_m)	0.1	Penalty factor	0.6

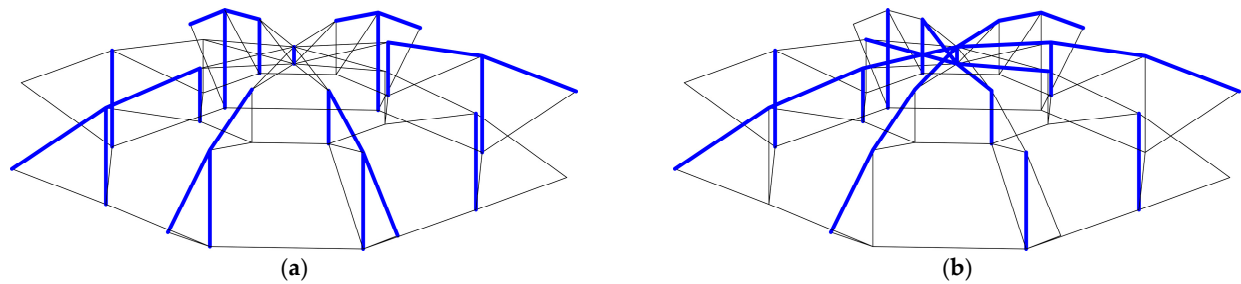


Figure 6. Optimal actuator layouts in Example 1 (Blue thick lines represent active components): (a) Based on Optimization Model 1; (b) Based on Optimization Model 2.

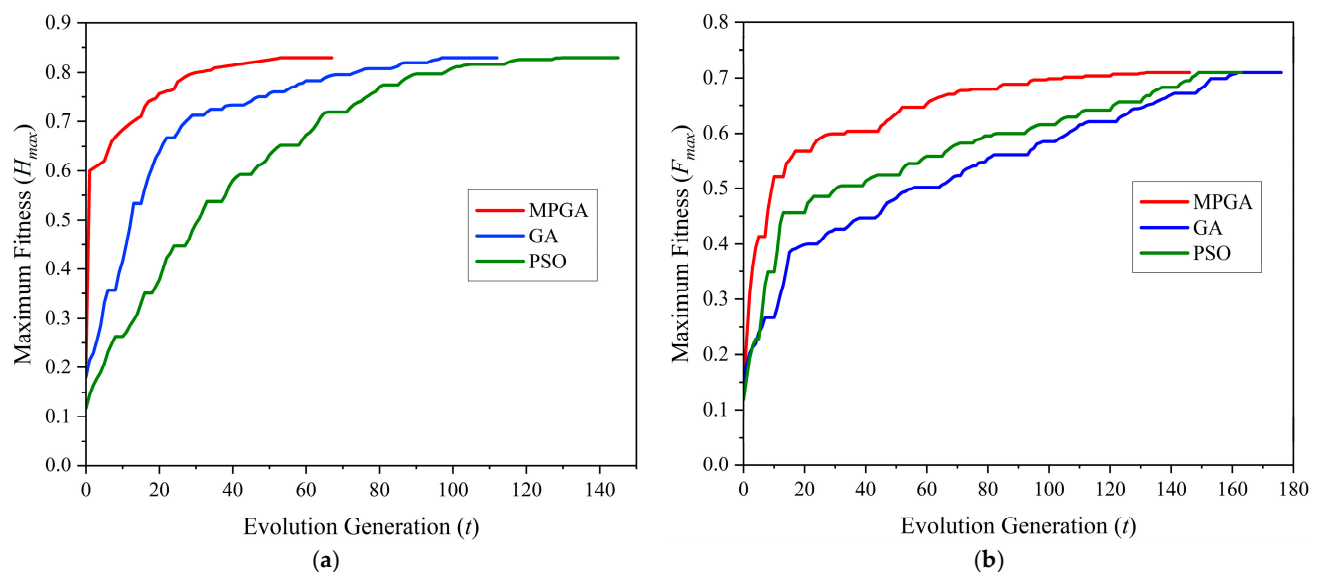


Figure 7. Fastest convergence curves of MPGA, standard GA, and PSO algorithms in Example 1: (a) Based on Optimization Model 1; (b) Based on Optimization Model 2.

For comparative purposes, two benchmark algorithms—the standard genetic algorithm (GA) and the particle swarm optimization (PSO) algorithm—are applied to address the same problem. In the GA, parameters including population size, encoding method, fitness function design, constraint handling, selection operator, crossover and mutation strategies, elitism strategy, and termination criteria are entirely consistent with those in the MPGA algorithm. The crossover rate and mutation rate are empirically set to constant values of 0.75 and 0.05, respectively. Following the methodology in [26], the key control parameters of the PSO algorithm are configured as follows: Population size: 50; Inertia weight: Linearly decreases from 0.9 to 0.4; Acceleration coefficients: The individual learning factor (c_1) and social learning factor (c_2) are both set to 2.0 (i.e., $c_1 = c_2 = 2.0$); Velocity clamping: Limited to 10% of the solution space range for each dimension; Constraint

handling: almost identical to those in the MPGA method, where a penalty is imposed on particles violating constraints by reducing their fitness values, thereby guiding the particles to move toward feasible regions; Termination criteria: Consistent with the MPGA algorithm, where evolution stops if the optimal individual shows no significant improvement over 15 consecutive generations. Both the GA and PSO algorithms were executed 100 times, yielding the optimal solutions shown in Figure 6. The fastest convergence curves of the two algorithms are shown in Figure 7a (for Optimization Model 1) and Figure 7b (for Optimization Model 2). A detailed comparison of the computational performance for the three algorithms is provided in Table 4. The experimental findings conclusively show that the MPGA algorithm achieves markedly higher convergence probabilities and faster convergence rates than both standard GA and PSO, as evidenced by its robust performance across multiple optimization models.

Table 4. Comparative performance of MPGA, GA, and PSO algorithms in Example 1.

Model	Algorithm	Convergence Count/100 Runs	Average Convergence Generation	Fastest Convergence Generation
Model 1	MPGA	72	68	53
	GA	37	131	97
	PSO	18	172	130
Model 2	MPGA	78	151	132
	GA	28	186	162
	PSO	18	176	149

To verify the reliability of the results, the control quantities of all actuators in the system were set to 1 mm, and the sensitivity of the nodal displacements and member internal forces are calculated. The results are shown in Tables 5 and 6. Due to the large number of structural members, Table 6 only presents the internal forces of a portion of members.

Table 5. Nodal displacement sensitivity of u_z to be controlled in Example 1 (unit: mm).

Model Number	1	2	3	4	5	6	7	8	9
Model 1	3.77	4.00	1.37	4.00	3.71	4.00	1.37	4.00	3.71
Model 2	3.57	2.10	−1.75	−4.16	4.52	2.10	−1.75	−4.16	4.52
Model Number	10	11	12	13	14	15	16	17	
Model 1	2.53	−1.27	2.53	3.07	2.53	−1.27	2.53	3.07	
Model 2	5.12	−3.52	1.95	2.44	5.12	−3.52	1.95	2.44	

The calculation results indicate that both the displacement of the controlled nodes and the internal forces of the controlled members exhibit high sensitivity. Additionally, the node displacement and internal forces of the active members where the actuators are located also show high sensitivity. Among the controlled members, the internal force sensitivity of the cables is significantly higher than that of the struts. By further comparing the results of Model 1 and Model 2, it is evident that the sensitivity of the control indicators in the former is mostly higher than that in the latter; however, the sensitivity of the non-control indicators in the latter is mostly higher than that in the former, demonstrating better balance. This is because the system strain energy indicator adopted by Model 2 comprehensively reflects the overall response of the structure, whereas Model 1 focuses more on the response of specific nodes and members.

Table 6. Internal force sensitivity of a portion of members in Example 1 (unit: kN).

Model Number	JS1 Group							
	1–2	1–3	1–4	1–5	1–6	1–7	1–8	1–9
Model 1	−5.72	6.09	−5.72	−5.16	−5.72	6.09	−5.72	−5.16
Model 2	−3.04	−3.23	4.01	3.54	−3.04	−3.23	4.01	3.54
Model Number	XS1 Group							
	2–42	3–42	4–42	5–42	6–42	7–42	8–42	9–42
Model 1	0.22	−1.75	0.22	−0.01	0.22	−1.75	0.22	−0.01
Model 2	−1.47	2.65	−1.47	−1.04	−1.47	2.65	−1.47	−1.04
Model Number	HS1 Group							
	26–27	27–28	28–29	29–30	30–31	31–32	32–33	33–26
Model 1	−0.72	−0.72	−0.73	−0.73	−0.72	−0.72	−0.73	−0.73
Model 2	1.74	2.70	0.86	2.70	1.74	2.70	0.86	2.70
Model Number	YG2 Group							
	2–26	3–27	4–28	5–29	6–30	7–31	8–32	9–33
Model 1	0.17	0.14	0.17	0.49	0.17	0.14	0.17	0.49
Model 2	−0.49	2.56	−1.32	−2.33	−0.49	2.56	−1.32	−2.33

‘1–2’ represents the member with node numbers 1 and 2 at ends, and so on.

3.2. Tensegrity Structure

This section utilizes the classic four-prism tensegrity as the fundamental unit to construct complex multi-layer tensegrity structures. Taking a double-layer four-prism tensegrity structure as an example (see Figure 8), the construction principles will be briefly introduced as follows.

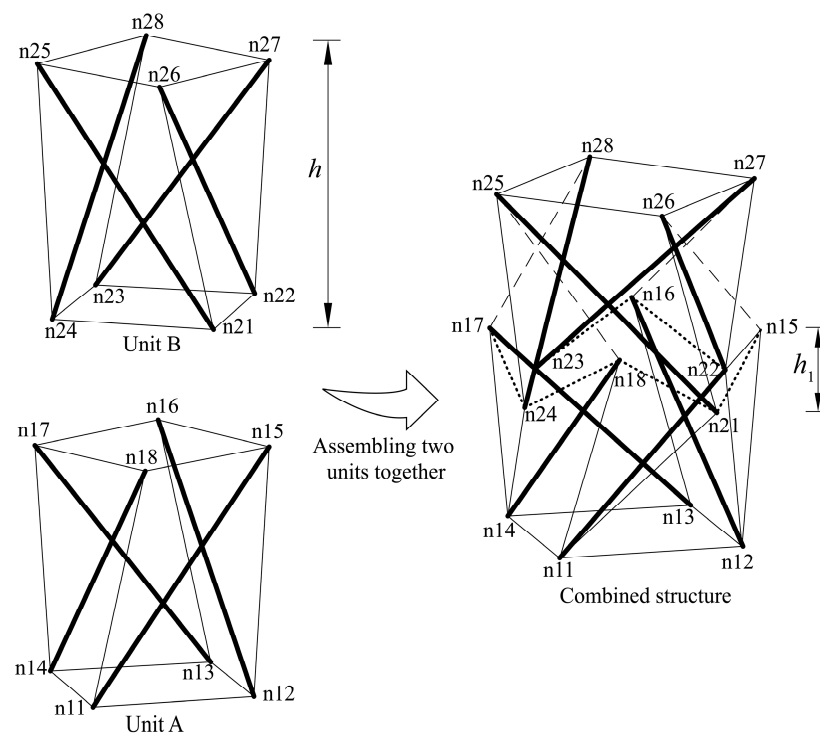


Figure 8. Schematic diagram of the construction of a double-layer tensegrity structure (Thick lines are struts, while thin lines are cables).

In Figure 8, Unit A and Unit B are four-prism tensegrity units with the same dimensions but different rotation directions. Unit A is a right-handed unit, while Unit B is a left-handed unit. First, Unit B is placed directly above Unit A and rotated by a certain angle θ , which is called the torsion angle. Then, the upper horizontal cables of Unit A and the lower horizontal cables of Unit B are removed, and new cables, called saddle cables (shown by the dashed lines in Figure 8), are used to connect the corresponding nodes of the two units. Additionally, to maintain the stability of the structure, auxiliary cables (shown by the dotted lines in Figure 8) are added. The height of the basic unit is denoted by ' h ', while the overlapping height of Unit A and Unit B is denoted by ' h_1 '. Then, $\mu (=h_1/h)$ is defined as the overlap ratio of adjacent units. The nodes are denoted as n_{ij} ($i = 1, 2; j = 1, 2, 3, 4, 5, 6, 7, 8$), where i represents the layer and j represents the nodal number.

Based on the above combination method, a four-layer tensegrity structure shown in Figure 9 can be constructed with the main geometric parameters as $h = 0.565$ m and $\beta = 0.3$.

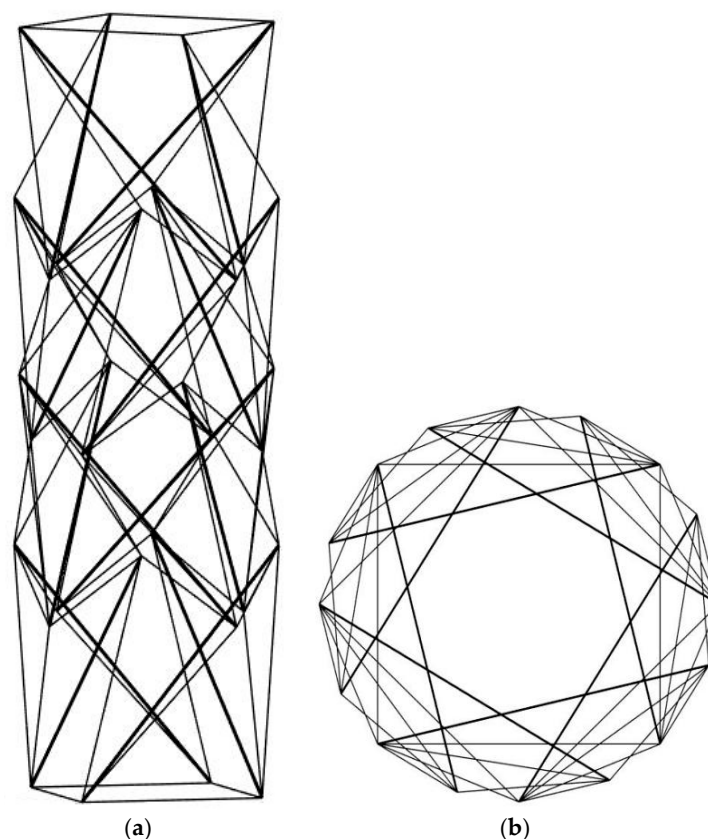


Figure 9. Four-layer prismatic tensegrity structure: (a) Perspective View; (b) Plan View.

The node numbering principle of this structure is consistent with the double-layer structure shown in Figure 8. From bottom to top, the units are Unit 1 with node numbers n_{11} to n_{18} , Unit 2 with node numbers n_{21} to n_{28} , Unit 3 with node numbers n_{31} to n_{38} , and Unit 4 with node numbers n_{41} to n_{48} . To accurately describe the types of members and their spatial positions, the structure is unfolded into a plan view as shown in Figure 10, along nodes n_{11} , n_{24} , n_{31} , n_{44} , and n_{45} .

In Figure 10, the thick lines represent struts, and the struts from the bottom to the top layers are denoted as YG1, YG2, YG3, and YG4, respectively. The entire structure consists of 24 types of members, totaling 96 members. In addition to struts YG1~YG4, there are saddle cables AS1~AS6, auxiliary cables FZS1~FZS6, vertical cables SS1~SS6, and horizontal cables SPS1~SPS2. Each type of member has four elements, and the material properties are the same as in Example 1. Nodes n_{11} , n_{12} , n_{13} , and n_{14} are pin-jointed.

According to the equilibrium matrix theory [25], the structure includes 12 independent self-stress modes, without mechanism displacement mode, indicating that the structure is geometrically stable. The initial prestress applied to the structure is shown in Table 7.

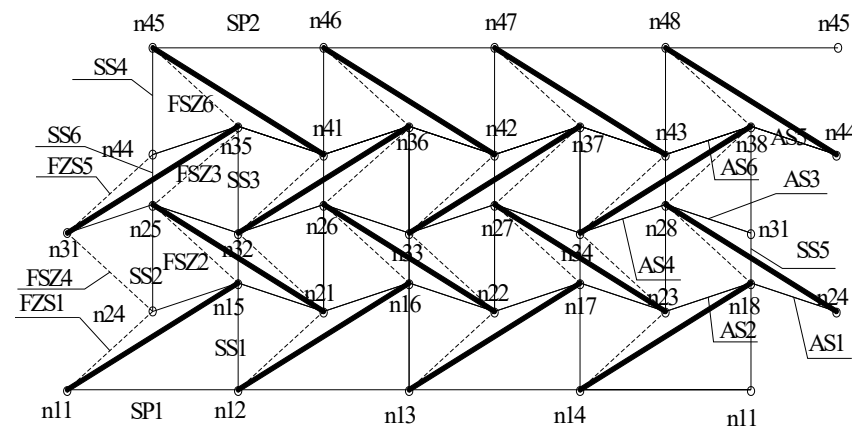


Figure 10. Planar unfolded diagram of four-layer prismatic tensegrity structure.

Table 7. Prestress distribution of four-layer prismatic tensegrity structure (unit: kN).

Member Group	Prestress	Member Group	Prestress	Member Group	Prestress
YG1, YG4	−212.55	AS5, AS6	159.94	SS1, SS4	73.81
YG2, YG3	−253.61	FZS1, FZS6	101.75	SS1, SS4	62.35
AS1, AS2	181.53	FZS2, FZS5	1.73	SS1, SS4	128.15
AS3, AS4	209.47	FZS3, FZS4	4.15	SPS2	62.49

Assuming the structure is used as a support system for a work platform, it is necessary to ensure that the upper plane is as level as possible. Therefore, vertical displacement control must be applied to the four nodes on the upper plane, numbered as n45~n48. As shown in Table 7, the prestress levels of the auxiliary cable elements (FZS2~FZS5) are significantly lower than those of other types of cable elements. Thus, their internal forces need to be controlled to prevent the cables from becoming inactive. Since the number of control targets is equal to 20 (4 nodal displacements and 16 internal forces of elements), the number of actuators is set to 20 as well. Following the same procedure outlined in Example 1, 50 paired datasets $[U_m, T_m]$ were generated and visualized in Figure 11. Linear fitting using the least squares method established the relationship between T_m and U_m as $T_m = 0.30U_m + 5.60$. Then, the nonlinear iterative framework given in Section 2.1.4 can be employed to determine the objective functions H and W for Optimization Models 1 and 2. The variance thresholds are empirically set as $U_v^* = 1$ and $T_v^* = 5$.

The control parameters of the MPGA algorithm, determined through parameter sensitivity analysis, are listed in Table 8. For Optimization Model 1, the MPGA algorithm was independently executed 100 times, yielding an optimal solution with a fitness value of 0.57. This optimal solution was achieved 64 times out of 100 runs, with an average convergence generation of 136 and a fastest convergence generation of 103 across these successful runs. The fastest convergence process of the fitness function for Model 1 is illustrated in Figure 12a. For Optimization Model 2, the MPGA algorithm was similarly executed 100 times, producing an optimal solution with a fitness value of 0.67. This solution was also attained 64 times out of 100 runs, with an average convergence generation of 146 and a fastest convergence generation of 112. The fastest convergence process for Model 2 is depicted in Figure 12b.

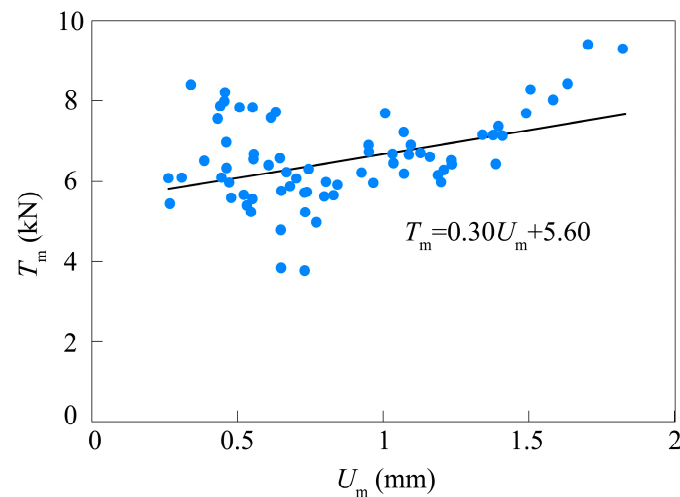


Figure 11. Fitted linear relationship between U_m and T_m in Example 2.

Table 8. Parameter settings for MPGA in Example 2.

Parameter Name	Parameter Value	Parameter Name	Parameter Value
Number of populations (s)	4	Size of Single Population (m)	60
Minimum Retained Generations	15	Baseline crossover probability (P_a)	0.8
Mutation Probability in later stage (P_m)	0.08	Penalty factor	0.6

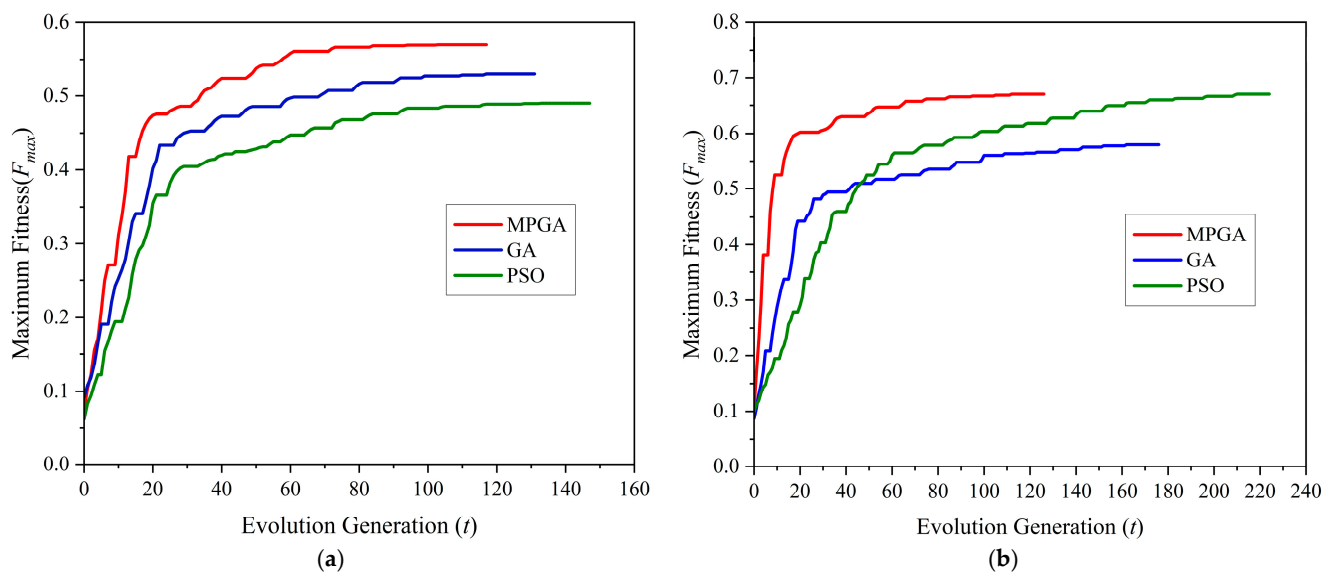


Figure 12. Fastest convergence curves of MPGA, standard GA, and PSO algorithms in Example 2: (a) Based on Optimization Model 1; (b) Based on optimization Model 2.

The optimal solutions obtained by the two models are illustrated in Table 9. The adopted structure and the indices to be controlled exhibit geometric rotational symmetry [27]. The optimal actuator arrangement schemes derived from both models also satisfy the rotational symmetry condition, which aligns with the anticipated outcome.

For comparative analysis, the standard genetic algorithm (GA) and particle swarm optimization (PSO) were also applied to address the aforementioned problem. GA shares parameters with MPGA (population size, encoding, fitness function, constraints, selection, crossover/mutation strategies, elitism, termination criteria), while the crossover and muta-

tion rates are specifically set to 0.75 and 0.05 based on empirical tuning. Given the increased complexity of Example 2 compared to Example 1, the population size of the particle swarm optimization (PSO) algorithm is expanded to 80, while all other control parameters retain the same configurations as in Example 1.

Table 9. Optimal actuator layouts in Example 2.

Model Number		Actuator Arrangement			
Model 1	YG3	SS1	SS4	SS5	SS6
Model 2	YG3	YG4	AS1	AS3	SS6

For Optimization Model 1, both the standard genetic algorithm (GA) and particle swarm optimization (PSO) were executed 100 times, yielding optimal solutions with fitness values of 0.53 and 0.49, respectively. These results are inferior to the MPGA-derived optimal solution ($F = 0.57$), suggesting potential convergence to local optima. For Optimization Model 2, GA and PSO were similarly tested over 100 runs. The GA achieved a suboptimal fitness value of 0.58, again underperforming MPGA's optimal solution ($F = 0.67$), likely trapped in local optima. While PSO occasionally matched MPGA's result ($F = 0.67$), it succeeded in only 9/100 runs—far fewer than MPGA's 64/100 success rate—and required 210 generations for its fastest convergence, significantly slower than MPGA's 112 generations. Figure 12 illustrates the fastest convergence curves of GA and PSO for both models, highlighting their slower and less stable performance compared to MPGA. A detailed comparison of the computational performance for the three algorithms is provided in Table 10. It demonstrates that MPGA's multi-population framework mitigates premature convergence and enhances solution diversity, ensuring higher success rates. Meanwhile, the adaptive crossover/mutation probabilities and elitism accelerate global search while preserving high-quality solutions.

Table 10. Comparative performance of MPGA, GA, and PSO algorithms in Example 2.

Model	Algorithm	Convergence Count/100 Runs	Maximum Fitness	Fastest Convergence Generation
Model 1	MPGA	64	0.57	103
	GA	/	0.53	/
	PSO	/	0.49	/
Model 2	MPGA	64	0.67	112
	GA	/	0.58	162
	PSO	9	0.67	210

Finally, the sensitivity of the nodal displacements and member internal forces are calculated, the results of which are shown in Tables 11 and 12.

Table 11. Nodal displacement sensitivity of u_z in Example 2 (unit: mm).

Model Number	15 to 18	21 to 24	25 to 28	31 to 34	35 to 38	41 to 44	45 to 48
Model 1	−0.55	−1.27	−0.97	−0.13	1.71	2.25	<u>−7.26</u>
Model 2	−1.95	−2.84	−3.29	−3.99	−4.37	−4.12	<u>5.18</u>

Underlined numbers correspond to displacement control targets.

Table 11 reveals that the optimal actuator arrangement schemes obtained through the two optimization models both ensure that the displacement sensitivity of the controlled

nodes (n45–n48) is higher than that of the non-controlled nodes. The difference lies in the fact that in Model 1, the displacement sensitivity of the controlled nodes is higher than in Model 2, whereas in Model 2, the displacement sensitivity of the non-controlled nodes is higher than in Model 1. In other words, the scheme provided by Model 1 is superior for the specific optimization objective, while the results of Model 2 are more balanced, which is essentially similar to Example 1.

Table 12. Internal force sensitivity in Example 2 (unit: kN).

Model Number	YG1	YG2	YG3	YG4	AS1	AS2	AS3	AS4
Model 1	−9.79	−1.41	8.65	0.93	11.10	7.45	−1.28	4.32
Model 2	−13.53	−11.24	4.28	13.05	8.98	−6.10	−6.91	7.82
Model Number	AS5	AS6	FZS1	FZS2	FZS3	FZS4	FZS5	FZS6
Model 1	4.04	5.86	16.24	<u>18.73</u>	<u>−15.53</u>	<u>−13.41</u>	<u>15.66</u>	−2.59
Model 2	−2.26	−16.83	10.12	<u>14.32</u>	11.59	<u>−9.62</u>	<u>11.33</u>	−3.42
Model Number	SS1	SS2	SS3	SS4	SS5	SS6	SP1	
Model 1	5.23	0.84	−3.22	1.23	12.34	16.86	0.26	
Model 2	6.37	2.45	−1.48	−6.80	−3.35	−20.59	−4.37	

Underlined numbers correspond to internal force control targets.

In Table 12, the underlined numbers correspond to the internal force sensitivity of the controlled members, while the bold numbers correspond to the internal force sensitivity of the active members (i.e., the members where actuators are located). As can be seen from this table, the internal force sensitivity of these two types of members is higher than that of other members, which once again proves the effectiveness of the method proposed in this paper. Similar to the displacement sensitivity, the placement scheme provided by Model 1 yields better control effects for the internal forces of specific members, whereas the placement scheme provided by Model 2 offers more balanced control effects, maintaining the internal force sensitivity of most non-controlled members at a certain level.

4. Conclusions

This study addresses the challenge of optimizing actuator arrangement in adaptive cable–strut tension structures by proposing a weighted sensitivity criterion that incorporates both nodal displacements and internal force increments, and a system strain energy criterion reflecting global structural stiffness. The variance constraints are introduced to balance actuator distribution, preventing excessive focus on local regions. The optimization models, based on the two proposed criteria, were solved using a multi-population genetic algorithm.

The case study results demonstrate that the proposed optimization criteria and genetic algorithm effectively identify optimal actuator arrangements that enhance structural controllability and performance. The weighted sensitivity criterion (Model 1) focuses on specific nodal displacements and internal forces, providing higher sensitivity for targeted control indicators. In contrast, the system strain energy criterion (Model 2) offers a more balanced approach, reflecting the overall structural response and ensuring better control over non-targeted indicators.

Both models successfully identified actuator arrangements that satisfy geometric symmetry and achieve high sensitivity in controlling nodal displacements and internal forces. Through comparative analysis with standard genetic algorithm and particle swarm optimization, the multi-population genetic algorithm is proven to be a robust tool for solving the complex optimization problems associated with actuator arrangement, demonstrat-

ing strong exploration capability, high convergence reliability, rapid convergence speed, superior robustness across diverse scenarios, and inherent symmetry preservation in actuator layouts.

Future research could explore the application of these optimization criteria to other types of adaptive structures and investigate the integration of real-time control systems to further enhance the performance and adaptability of tension structures. Additionally, the impact of different loading conditions and the robustness of the proposed methods under varying environmental factors (e.g., wind, seismic activity, and temperature variations) could be examined to comprehensively assess their practical applicability in real-world engineering scenarios.

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