

Article

Hybrid Nanofluid Flow and Heat Transfer in Inclined Porous Cylinders: A Coupled ANN and Numerical Investigation of MHD and Radiation Effects

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Abstract

This study investigates the thermal characteristics of two hybrid nanofluids, single-walled carbon nanotubes with titanium dioxide ($SWCNT - TiO_2$) and multi-walled carbon nanotubes with copper ($MWCNT - Cu$), as they flow over an inclined, porous, and longitudinally stretched cylindrical surface with kerosene as the base fluid. The model takes into consideration all of the consequences of magnetohydrodynamic (MHD) effects, thermal radiation, and Arrhenius-like energy of activation. The outcomes of this investigation hold practical significance for energy storage systems, nuclear reactor heat exchangers, electronic cooling devices, biomedical hyperthermia treatments, oil and gas transport processes, and aerospace thermal protection technologies. The proposed hybrid ANN-numerical framework provides an effective strategy for optimizing the thermal performance of hybrid nanofluids in advanced thermal management and energy systems. A set of coupled ordinary differential equations is created by applying similarity transformations to the governing nonlinear partial differential equations that reflect conservation of mass, momentum, energy, and species concentration. The boundary value problem solver *bvp4c*, which is based in MATLAB (R2020b), is used to solve these equations numerically. The findings demonstrate that, in comparison to the $MWCNT - Cu$ /kerosene nanofluid, the $SWCNT - TiO_2$ /kerosene hybrid nanofluid improves the heat transfer rate (Nusselt number) by up to 23.6%. When a magnetic field is applied, velocity magnitudes are reduced by almost 15%, and the temperature field is enhanced by around 12% when thermal radiation is applied. The impact of important dimensionless variables, such as the cylindrical surface's inclination angle, the medium's porosity, the magnetic field's strength, the thermal radiation parameter, the curvature ratio, the activation energy, and the volume fraction of nanoparticles, is investigated in detail using a parametric study. According to the comparison findings, at the same flow and thermal boundary conditions, the $SWCNT - TiO_2$ /kerosene hybrid nanofluid performs better thermally than its $MWCNT - Cu$ /kerosene counterpart. These results offer important new information for maximizing heat transfer in engineering systems with hybrid nanofluids and inclined porous geometries under intricate physical conditions. With its high degree of agreement with numerical results, the ANN model provides a computationally effective stand-in for real-time thermal system optimization.

Keywords: hybrid nanofluid; Prandtl fluid model; thermal radiation; activation energy; stretching cylinder; artificial neural networks



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1. Introduction

The inclusion of nanoparticles in standard heat transfer fluids can increase convective heat transmission. In 1995, Choi and Eastman [1] created the concept of nanofluids. Nanofluids have the potential to be used as heat-transferring fluids due to their improved durability and surprising increase in thermal conductivity, independent of the small volume fractions of scattered NPs. Due to their improved heat transfer qualities over traditional working fluids, nanofluids have attracted interest in a variety of scientific industries during the past ten years [2–6]. Based on a previous study, the behavior of nanofluid may be strongly influenced by numerous aspects such as the technique of manufacture, the concentration of suspended nanoparticles, and the base fluid employed. Due to the goal of achieving a more favorable nanofluid, the dispersion of various nanoparticles into the base fluid has resulted in a unique product known as a hybrid nanofluid. Former researchers produced and examined many forms of hybrid nanofluids that complied with the criteria necessary for their investigations. Many experiments have been conducted to investigate the characteristics of these nanofluids. The combined use of oxide and metallic nanoparticles or oxide and carbon nanotubes (CNTs) was employed as a dispersing nanoparticle in the base fluid in these investigations. Because of their complementary qualities, the *SWCNT – TiO₂ MWCNT – Cu* pairings were chosen for this investigation. *SWCNT* offers excellent heat conductivity and aspect ratio, whereas *TiO₂* nanoparticles are chemically stable and cost-effective. When combined with *Cu* nanoparticles, which have greater thermal conductivity, the *MWCNTs* become more physically resilient and economically feasible. Kerosene was chosen as the basis fluid because of its widespread industrial use, excellent thermal stability, and adaptability for high-temperature applications such as electronic cooling, jet fuel systems, and heat exchangers. Its intermediate viscosity and low heat conductivity make it an ideal choice for demonstrating the benefits of nanoparticle addition. Furthermore, when suitably sonicated or treated with surfactants, kerosene-based nanofluids have been shown to display stable dispersion of metallic and oxide nanoparticles, indicating their suitability for hybrid nanofluidic systems. The choice of an inclined porous cylinder reflects conditions found in practical applications such as heat exchangers, packed-bed reactors, and electronic cooling systems. Curvature alters boundary-layer formation, inclination affects buoyancy and mixed convection, and porosity introduces flow resistance and fluid penetration. Studying these combined effects helps explain how geometric and porous features influence overall heat- and mass-transfer performance. The different nanoparticles with heat flux were discussed by [7,8]. Nadeem et al. [9] investigated the flow of hybrid nanofluid across a porous exponentially stretched tunnel. An investigation into the impact of hybrid nanofluid *CuO–TiO₂* on radiator performance was conducted by Sudarmadji et al. [10]. Rehman et al. [11] studied the mathematical responses to coupled nonlinear equations, including bioconvection in MHD Casson nanofluid flow. Ref. [12] analyzed boiling heat transfer for porous structures. Waqas et al. [13] investigated the effects of nonlinear temperature radiation on hybrid nanoparticles suspended in water-based nanofluid flow over a spinning disc. Bi et al. [14] studied a new passive residual heat removal system's design and transient behavior. Chabani et al. [15] investigated heat transmission via a hybrid nanofluid within a porosity ring among a wandering triangle and cylinder using the Darcy–Brinkman–Forchheimer model. Tetik et al. [16] assessed an experimental investigation on the heat transfer performance of a radiator utilizing a hybrid nanofluid comprising *MWCNT* and *SiO₂*. Manimaran et al. [17] conducted a critical assessment of the thermal conductivity and longevity of water-based hybrid nanofluids for heat transfer purposes. Abdollahi et al. [18] investigated the heat transmission associated with a hybrid nanofluid flow combining the oxide of graphene and particles of copper in water in a spinning device.

Magnetohydrodynamics (MHD) is an investigation of how the magnetic field and the velocity field interact when a fluid has electrical conductivity. The magnetic field may produce electricity in a flowing fluid, resulting in forces operating on the fluid and causing the magnetic field to change. The whole phenomenon of MHD is described by a set of differential equations that includes the Navier–Stokes equations and Maxwell’s equations. The mass-based hybrid nanofluid framework for thermal radiation evaluation of MHD flow across a wedge incorporated in a porous medium was investigated by Choudhary et al. [19]. Shoaib et al. [20] investigated the mass and heat transfer in 3-D MHD convective flow based on a water hybrid nanofluid over an expanding sheet using the effectiveness of numerical computers. Huang et al. [21] analyzed how droplet size affects the performance of a magnesium water ramjet engine. Yaseen et al. [22] investigated the aiding and opposed flow impacts of hybrid nanofluid flow through a moving surface when thermal radiation and velocity slip were present. Wahid et al. [23] investigated the MHD radiation flow of a hybrid nanofluid ($Al - Cu/H_2O$) via a porous horizontal surface with mixed convection. Mahesh et al. [24] examined how radiation affected the MHD stress distribution in a hybrid nanofluid across a permeable sheet with poration. Raisee et al. [25] used a generalized slip model to forecast gas flow across short and long 2-D micro and nanochannels. Furthermore, studies into mixed convection flow in many types of fluids are becoming increasingly important among scientists due to their numerous applications in industries and technology, such as smartphones, nuclear reactors, and pipeline transit. Convection is a heat-exchange mechanism that involves the movement of fluid from a warmer to a cooler medium. Furthermore, free convection is a mixed motion caused by a difference in density, whereas forced convection is a mixing motion caused by an outside force. Shah et al. [26] investigated the dynamics of methanol conveying mono and hybrid nanoparticles to optimization of heat transfer across stretching cylinder. Islam et al. [27] examined the thermal impact of a Maxwell nanofluid rotating movement created by a spinning stretched cylinder in a mixed convection flow. Patil and Kulkarni [28] explored MHD combined convection in an $Ag - TiO_2$ hybrid nanofluid flow across a thin cylinder. The bioconvection flow of Sutterby nanofluid with Darcy–Forchheimer porous media across an inclined stretched cylinder was numerically examined by Ahmad et al. [29]. Jiang et al. [30] studied the mixed convection process involving heat and mass transfer of an MHD hybrid nanofluid within a porous cubic cavity that contains oscillating walls and rotating cylinders. The study focused on understanding how these structural and magnetic factors affect the flow dynamics and thermal efficiency of the system. The researchers discussed different numerical methods for heat transfer [31,32].

In 1889, Svante Arrhenius described activation energy as the minimum energy needed to start a chemical reaction. This concept is widely applied in chemical engineering, nuclear reactor optimization, and heating oil recovery. The impacts of activation energy and multiple buoyant forces on MHD Prandtl fluid as it traverses a stretchable cylinder under convective boundaries were studied by Shah et al. [33]. Awan et al. [34] investigated the implications of biological convection on the Williamson nanofluid flow, including nonlinear heat radiation, activation energy, and chemical response on a stretched sheet. Zafar et al. [35] examined the cumulative impact of mixed convection, activation energy, chemical reactions, and motile microorganisms when examining the magnetohydrodynamic (MHD) flow of a Prandtl nanofluid.

Artificial neural networks (ANNs) use sophisticated data processing techniques to simulate how the human brain operates. The results of the numerical simulation are replicated with great fidelity and at a lower computational cost using an artificial neural network (ANN). Three layers make up the feed-forward architecture of the ANN: an output layer, a hidden layer with 20 neurons, and an input layer. Because of its quick convergence, the training algo-

rithm uses the Levenberg–Marquardt (LMA) optimization. Ul-Haq et al. [36] used numerical simulations combined with an artificial neural network (ANN) to investigate the non-similar solution of MHD mixed convection transport of viscous nanofluids across a curved stretching surface. Das and Mamun [37] used an ANN model and numerical analysis to study the MHD mixed convection phenomena in a round container with a revolving cylinder and different nanofluids and hybrid nanofluids. Abbas et al. [38] employed artificial neural network (ANN) modeling to analyze Darcy–Forchheimer nanofluid flow over a porous Riga plate, focusing on how Brownian motion, thermal radiation, and activation energy influence heat transfer behavior. Rameshkhar et al. [39] used machine learning to study the flow of Casson hybrid nanofluid across a heated stretched surface. Reddy et al. [40] used an artificial neural network (ANN) to incorporate the Thompson and Troian slip boundary condition into their analysis of the two-phase flow of a hybrid dusty Eyring–Powell nanofluid across a porosity tube. The combined impacts of MHD on non-Newtonian Carreau ternary nanofluid flow, including heat and mass transport, via a vertically expanding cylinder while taking chemical reactions and radiation factors into account, were examined by Mehta, R., and Senthilvadiru [41] using an ANN. By analyzing the heating capacity of a Prandtl hybrid nanofluid flowing in an inclined cylinder, the current work makes a unique addition to the field of heat management. This research addresses the impacts of inclined geometry, mixed convection, thermal radiation, and activation energy on the thermal distribution features of hybrid nanofluid, which has received little attention in the literature. This work contributes to a better understanding of the thermal stability of hybrid nanofluids across an inclined cylinder, which is an important milestone in the field of heat management. To improve forecast accuracy and system optimization, a unique ANN model is created utilizing the acquired simulation data in addition to numerical simulations. ANN enables the prediction and optimization of heat transfer rates by analyzing flow properties, enhancing the efficiency of thermal systems. A system of partial differential equations is modeled using boundary layer assumptions. The nonlinear PDE system is transformed into comparable equations by implementing a suitable transformation. The resulting flow problems are numerically solved to investigate the effect of the physical restrictions present by employing the MATLAB (R2020b) bvp4c function. The ANN showed high regression accuracy ($R = 1$) across all outcomes, with MSE values as low as 10^{10} . The prediction error for ANN and bvp4c solutions was less than 1.5%, indicating strong adaptability and minimal overfitting. The results of the MATLAB (R2020b) neural network training technique are the gradient values shown in Table 1. With regard to the network weights and biases, the Levenberg–Marquardt approach determines the gradient of the mean squared error (MSE) during optimization. In addition to showing the rate of convergence, these values provide an approximation of the error surface’s slope. The training process has nearly reached an optimal solution when the gradient values are lower.

Table 1. ANN-LMA training summary.

Epoch	MSE	Gradient	mu	
332	2.3568×10^{-9}	9.9841×10^{-8}	1×10^{-8}	λ_p
282	1.2298×10^{-5}	9.9741×10^{-8}	1×10^{-8}	Rd
688	3.3141×10^{-10}	9.98×10^{-8}	1×10^{-9}	Ea

2. Novelty

The present study offers a combined methodological and physical advancement in the modeling of hybrid nanofluid flow over inclined porous cylinders. While earlier studies have separately investigated magnetohydrodynamic (MHD) and porous effects on nanofluid transport, the current work integrates Darcy–Forchheimer drag, thermal radiation, activation-type thermal sensitivity, and Prandtl non-Newtonian rheology

into a single formulation. The novelty, therefore, lies in the comprehensive coupling of these effects rather than in the use of any single parameter. The comparison between *SWCNT – TiO₂/kerosene* and *MWCNT – Cu/kerosene* nanofluids provides fresh insights into their relative heat transfer efficiency under complex boundary conditions. On the methodological side, the integration of ANN with the *bvp4c* solver represents a novel hybrid framework, offering over 90% reduction in computational cost while preserving high accuracy $R \approx 1.0$, $MSE \sim 10^{-9}$ – 10^{-10} . This approach establishes ANN as a reliable surrogate for real-time prediction and optimization in thermal system design. Together, these contributions place the present work at the frontier of AI-driven thermal analysis of hybrid nanofluids in complex geometrical configurations.

3. Research Gap

The majority of current research is limited to basic geometries like flat plates, channels, or disks, despite the fact that hybrid nanofluids have been thoroughly examined for their improved thermal properties. In inclined porous stretched cylindrical surfaces, where curvature and inclination significantly impact flow and heat transport, very little research has been devoted to these surfaces. Similarly, although the effects of thermal radiation, activation energy, and magnetohydrodynamics (MHD) have all been studied separately, nothing is known about how they interact in a non-Newtonian Prandtl fluid framework.

- Experimental validation and AI-based modeling of hybrid nanofluids, especially *SWCNT – TiO₂* and *MWCNT – Cu* systems, are still limited.
- Few studies integrate MHD, radiation, porous media, and activation energy effects within a single framework.
- The activation energy term has rarely been justified or applied to non-reactive fluids like kerosene.
- Existing AI approaches focus mainly on ANN models without benchmarking against other surrogates such as Gaussian Processes or Physics-Informed Neural Networks (PINNs).

4. Mathematical Formulation

In this work, a Prandtl-type non-Newtonian hybrid nanofluid implanted in a Darcy–Forchheimer porous medium has a constant, incompressible, mixed convection flow along an inclined, stretched cylindrical surface. Both *SWCNT – TiO₂/kerosene* and *MWCNT – Cu/kerosene* are hybrid nanofluids that are being examined. Arrhenius-type activation energy, thermal radiation, and magnetic field effects are all included. The velocity components u_1 v_1 in the x - and r -directions characterize the flow. The cylinder with radius ‘ a ’ expands at a velocity termed the stretched velocity, defined by $\check{u}_w(\check{x}_1) = \frac{\check{u}_0 \check{x}_1}{l}$, where the steady u_0 is the characteristic speed and l is the cylindrical length. In the radial direction, an inclined magnetic field B_0 is applied. Figure 1 depicts the flow geometry and coordinate system.

The governing equations of continuity, momentum, energy, and concentration are defined as [42–44]:

$$\frac{\partial(ru_1)}{\partial x_1} + \frac{\partial(rv_1)}{\partial r_1} = 0, \quad (1)$$

$$u_1 \frac{\partial u_1}{\partial x_1} + v_1 \frac{\partial u_1}{\partial r_1} = \frac{A}{\rho_{hnf} c} \left(1 + \frac{1}{r_1} \frac{\partial u_1}{\partial r_1} + \frac{1}{2c^2} \left(\frac{\partial u_1}{\partial r_1} \right)^2 \right) \frac{\partial^2 u_1}{\partial r_1^2} + \frac{A}{r_1 \rho_{hnf} c} \frac{\partial u_1}{\partial r_1} + \frac{A}{6r_1 \rho_{hnf} c^3} \left(\frac{\partial u_1}{\partial r_1} \right)^3 - \frac{\sigma_{hnf} B_0^2 u_1}{\rho_{hnf}} \sin^2(\beta) - v_{hnf} \frac{u_1}{k^*} - \frac{C_b}{\sqrt{k^*}} u_1^2, \quad (2)$$

$$u_1 \frac{\partial T}{\partial x_1} + v_1 \frac{\partial T}{\partial r_1} = \frac{\alpha_{hnf}}{r_1} \frac{\partial}{\partial r_1} \left(r_1 \frac{\partial T}{\partial r_1} \right) - \frac{1}{(\rho C_p)_{hnf}} \frac{\partial}{\partial r_1} (r_1 q_r), \quad (3)$$

$$u_1 \frac{\partial C}{\partial x_1} + v_1 \frac{\partial C}{\partial r_1} = \frac{1}{r_1} \frac{\partial}{\partial r_1} \left(r_1 \frac{\partial C}{\partial r_1} \right) - K_r^2 \left(\frac{T}{T_\infty} \right)^n \exp \left(-\frac{E}{kT} \right) (C - C_\infty). \quad (4)$$

Here, A represents the Prandtl fluid consistency index, c denotes the material time constant, k^* is the permeability of the porous medium, and u_0 is the reference stretching velocity. Although kerosene is a non-reactive base fluid, the inclusion of the activation energy term E serves as a phenomenological representation of thermally activated diffusion processes. Similar formulations have been used in hybrid nanofluid literature (Shah et al. [33]; Awan et al. [34]; Zafar et al. [35]) to model temperature-dependent nanoparticle mobility rather than chemical reactions. This interpretation maintains physical consistency with the non-reactive nature of kerosene while allowing a broader analysis of temperature-dependent transport effects with the following associated boundary conditions [45]:

$$\begin{cases} u_1 = u_w, & v_1 = 0, & k_{hnf} \frac{\partial T}{\partial r_1} = h_2^* (T - T_w), & C = C_w, & \text{as } r_1 = R, \\ u_1 \rightarrow 0, & C \rightarrow C_\infty, & T \rightarrow T_\infty, & & \text{as } r_1 \rightarrow \infty. \end{cases} \quad (5)$$

Temperature-dependent features of the hybrid nanofluid are listed in Table 2. The continuity equation establishes the foundation for understanding nanofluid motion by preserving mass. Newton's second law, which is impacted by changes in pressure, viscous strains, and magnetic forces, is projected by the momentum equation. Applications such as the delivery of medication are impacted by the Lorentz force, which affects fluid behavior. The energy equation incorporates various heat transfer mechanisms within the nanofluid, including convection, Brownian motion, and thermophoresis, which collectively influence the temperature distribution. The concentration equation explains how NPs spread, with chemical interactions influencing concentration and Brownian motion and thermophoresis aiding in NP dispersion. These related equations concurrently resolve to offer a thorough understanding of how to improve hybrid nanofluids' thermal efficiency. The thermophysical characteristics of the nanoparticles and base fluid evaluated at 293 K [35] are provided in Table 3.

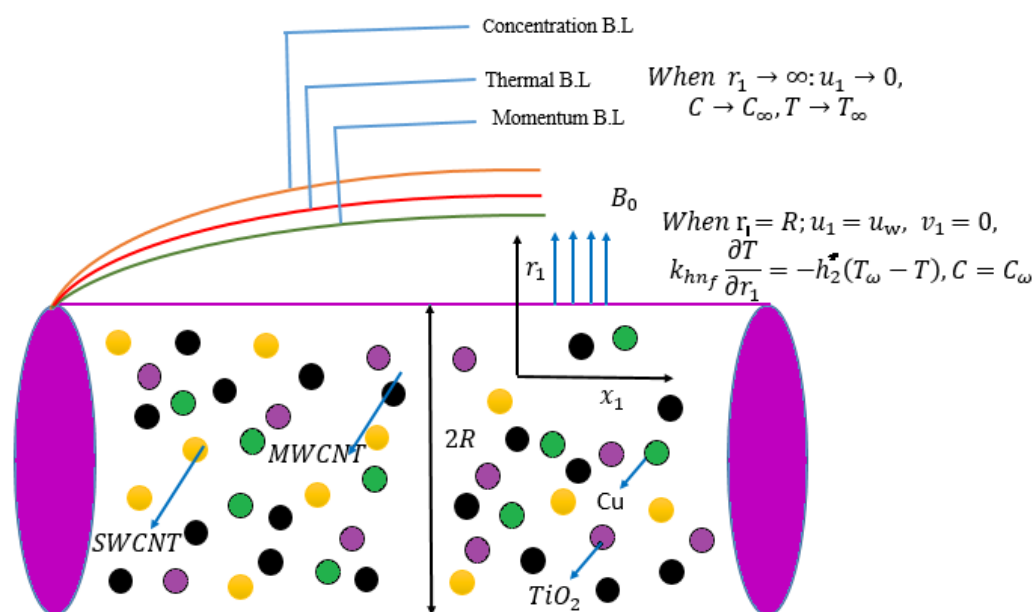


Figure 1. Geometry of the problem.

Table 2. Temperature-dependent characteristics of hybrid nanofluids [46].

Properties	Hybrid Nanofluid
Viscosity	$\mu_{hnf} = \frac{\mu_f}{(1-\phi_2)^{2.5}(1-\phi_1)^{2.5}}$
Density	$\rho_{hnf} = \phi_2 \rho_{s2} + \rho_f \left((1-\phi_1) + \phi_1 \left(\frac{\rho_{s1}}{\rho_f} \right) \right) (1-\phi_2)$
Heat Capacity	$(\rho C_p)_{hnf} = (\rho C_p)_f \left((1-\phi_2)(1-\phi_1) + \phi_1 \left(\frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right) + \phi_2 \frac{(\rho C_p)_{s2}}{(\rho C_p)_f} \right)$
Thermal Conductivity	$\frac{k_{hnf}}{k_f} = \frac{k_{s2} + (m-1)k_{bf} - (m-1)\phi_2(k_{bf} - k_{s2})}{k_{s2} + k_{bf}(m-1) + (k_{bf} - k_{s2})\phi_2} \frac{k_{s1} + (m-1)k_f - (m-1)\phi_1(k_f - k_{s1})}{k_{s1} + (m-1)k_f + (k_f - k_{s1})\phi_1}$
Electrical Conductivity	$\frac{\sigma_{hnf}}{\sigma_{nf}} = \frac{\sigma_2 + 2\sigma_{nf} - 2\phi_2(\sigma_{nf} - \sigma_2)}{\sigma_2 + 2\sigma_{nf} + \phi_2(\sigma_{nf} - \sigma_2)}$

where s , f , and nf stand for nanomaterials, base fluid, and nanofluid, respectively.

Table 3. Material features of kerosene and other nanoparticles at the temperature 293 K.

Properties	Kerosene	SWCNT	MWCNT5	TiO ₂	Cu
ρ (kg/m ³)	783	2600	1600	4230	8933
C_p (J/kg ¹ ·K ¹)	2090	425	796	692	385
k (W/m ¹ ·K ¹)	0.15	6600	3000	8.4	401

Introducing the following similarity transformations:

$$\sqrt{\frac{a}{v}} = \eta, v_1 = -f(\eta)\sqrt{av}, u_1 = f'(\eta)ax_1, \theta(\eta) = \frac{-T_\infty + T}{-T_\infty + T_w}, \phi(\eta) = \frac{-C_\infty + C}{-C_\infty + C_w}, \quad (6)$$

into Equations (2)–(4), we have

$$\alpha_1(1 + 2\gamma\eta)f'''' + \alpha_2(1 + 2\gamma\eta)^2(f'')^2 f'''' + \frac{4}{3}\gamma\alpha_2(1 + 2\gamma\eta)(f'')^3 - \phi_2 M f' \sin(\beta) + \phi_2(f f'' - f'^2 - Fr f'^2) - \lambda_p f' = 0, \quad (7)$$

$$\left(\phi_4 + \frac{4}{3}Rd\right)(1 + 2\gamma\eta)\theta'' + 2\left(\phi_4 + \frac{4}{3}Rd\right)\gamma\theta' + Pr\phi_3 f\theta' = 0, \quad (8)$$

$$(1 + 2\gamma\eta)\phi'' + 2\gamma\phi' + Sc\left(f\phi' - \sigma_1 \exp\left(-\frac{E}{1 + \delta\theta}\right)(1 + \delta\theta)^n \phi\right) = 0, \quad (9)$$

and the corresponding boundary conditions (4) are transformed as:

$$\begin{cases} f' = 1, & f = 0, & \frac{\partial\theta}{\partial\eta} = Bi(\theta - 1), & \phi = 1, & \text{at } \eta = 0, \\ \frac{\partial f}{\partial\eta} \rightarrow \infty, & \theta \rightarrow \infty, & \phi \rightarrow \infty, & & \text{as } \eta \rightarrow \infty, \end{cases} \quad (10)$$

where $Rd = \frac{4\sigma T_\infty^3}{k_{hnf}k^*}$ is the radiation parameter, $Pr = \frac{v_f(\rho C_p)_{hnf}}{k_{hnf}}$ is the Prandtl number, $Bi = \frac{h_1}{k_{hnf}}\sqrt{\frac{v}{a}}$ is the heat mass transfer Biot parameter, $Ea = \frac{E}{KT_\infty}$ is the activation energy, $\sigma_1 = \frac{k_r^2}{\alpha}$ is the chemical reaction rate, $\alpha_1 = \frac{\check{A}}{c}$ and $\alpha_2 = \frac{a^3 x^2 \check{A}}{2c^3 v_{hnf}}$ are the Prandtl fluid parameters, $Fr = \frac{Cb}{\sqrt{k^*}}$ is the Forchheimer number, $\gamma = \frac{1}{R}\sqrt{\frac{v_{hnf}}{a}}$ is the curvature parameter, $\lambda_p = \frac{l v_{hnf}}{u_o k^* \rho_{hnf}}$ is the porosity parameter, $\delta = \frac{T_w - T_\infty}{T_\infty}$ is the temperature difference, and $M = \frac{\sigma_f B_0^2}{a \rho_{hnf}}$ is the magnetic parameter. Figure 2 shows the nanoparticles' shape with shape factors.

Also,

$$\varphi_1 = (1 - \phi_2)^{2.5}(1 - \phi_1)^{2.5} \left[\left((-\phi_1 + 1) + \phi_1 \frac{\rho_{s1}}{\rho_f} \right) + \frac{\rho_{s2}}{\rho_f} \phi_2 \right], \quad (11)$$

$$\varphi_2 = (1 - \phi_2)^{2.5}(1 - \phi_1)^{2.5}, \quad (12)$$

$$\varphi_3 = \left((-\phi_1 + 1) + \phi_1 \frac{(\rho C_p)_{s1}}{(\rho C_p)_f} \right) (-\phi_2 + 1) + \phi_2 \frac{(\rho C_p)_{s2}}{(\rho C_p)_f}, \quad (13)$$

$$\varphi_4 = \frac{k_{nf}}{k_f}. \quad (14)$$

Nanoparticles type	Shape	Shape factor
Sphere		3.0
Bricks		3.7
Cylinders		4.9

Figure 2. Nanoparticle shapes with shape factors.

5. Physical Quantities

Extensive research on heat and mass transfer is conducted across industries such as polymer manufacturing, food processing, healthcare, and information technology, utilizing advanced thermo-fluid dynamic techniques. Numerical simulations are essential in this context for examining physical characteristics such as heat-exchange flows and skin friction. As stated below, the pertinent physical parameters are the Sherwood number Sh at the surface, the Nusselt number Nu , and the coefficient of skin friction C_f :

$$\frac{Nu}{Re^{\frac{1}{2}}} = -\varphi_4 \left(1 + \frac{4}{3} Rd \right) \theta'(0), \quad (15)$$

$$\frac{Sh}{Re^{\frac{1}{2}}} = -\phi'(0), \quad (16)$$

$$Re^{\frac{1}{2}} C_f = \frac{1}{\varphi_2} \left(\alpha_1 f''(0) + \frac{\alpha_1 \alpha_2}{3} (f''(0))^3 \right). \quad (17)$$

6. Numerical Solution

The set of boundary conditions (BCs) and ordinary differential equations (ODEs) formulated in this study is highly nonlinear and involves two-point boundary constraints, making them challenging to solve analytically. Therefore, the non-dimensional ODEs are numerically solved using the BVP4C solver. Equations (7)–(9), which, to meet the boundary criteria, are converted into a system of first-order ODEs and solved using the Runge–Kutta technique, are third-order in f and second-order in θ and ϕ . The shooting methodology in conjunction with the fourth-order Runge–Kutta (RK4) method was used to numerically address the revised boundary value issue. Since the RK4 technique offers dependable accuracy for nonlinear ODE systems at a reasonable computing cost, it was used to integrate the resulting initial value problem. The boundary requirements at infinity were satisfied within a specified tolerance of 10^{-6} after the unknown initial conditions were systematically changed. A grid independence test was performed to check the accuracy of the numerical findings obtained using MATLAB's bvp4c. Simulations were performed with increasing mesh points (100, 300, 500, 700, and 1000). Beyond 500 points, fluctuations in skin friction and Nusselt number were less than 0.01%, indicating numerical stability. This approach provides great solution precision and does not depend on complex separation, as do other numerical techniques. The flowchart of the numerical scheme is depicted in Figure 3. Let us make the following assumption to simulate the flow problem using this iterative method:

$$\begin{aligned} F(1) &= f(\eta); F(2) = f'(\eta); F(3) = f''(\eta); FF(1) = f'''(\eta); F(4) = \theta(\eta); \\ F(5) &= \theta'(\eta); FF(2) = \theta''(\eta); F(6) = \phi(\eta); F(7) = \phi'(\eta); FF(3) = \phi''(\eta), \end{aligned}$$

Then Equations (7)–(10) can be written as

$$FF(1) = \frac{-\left(\frac{4}{3}\alpha_2\gamma(1+2\eta\gamma)F(3)^3 + \varphi_1(-F(2)^2 + F(1)F(3) - FrF(2)^2) + 2\alpha_1\gamma S(3) - \varphi_2 MF(2)\sin^2(\beta) - \lambda_p F(2)\right)}{\alpha(1+2\eta\gamma) + \alpha_2(1+2\eta\gamma)^2 F(3)^2}, \quad (18)$$

$$FF(2) = \frac{-1}{(1+2\eta\gamma)\left(\varphi_4 + \frac{4}{3}Rd\right)} \left[Pr\varphi_3 F(1)F(5) + 2\gamma\left(\varphi_4 + \frac{4}{3}Rd\right)F(5) \right], \quad (19)$$

$$FF(3) = -\frac{1}{(1+2\eta\gamma)} \left(S_c F(7)F(1) + 2F(7)\gamma - S_c \sigma(1 + \delta F(4))^n \exp\left(\frac{-E}{F(4)\delta + 1}\right) F(6) \right), \quad (20)$$

$$\begin{cases} F(1) = 0, & F(2) = 1, & F(7) = 1, & F(5) = -\frac{Bi}{\varphi_4}(1 - F(4)) & \text{at } \eta = 0, \\ F(6) \rightarrow \infty, & F(4) \rightarrow \infty, & F(2) \rightarrow \infty, & & \text{as } \eta \rightarrow \infty. \end{cases} \quad (21)$$

Dimensionless Parameters

The dimensionless parameter ranges were intended to simulate realistic physical conditions for technical applications such as electronics cooling, heat exchangers, and nuclear systems. The magnetic parameter $M = 0$ –5 ranges from modest to significant Lorentz-force effects found in laboratory and industrial MHD settings. The radiation value $Rd = 0.3$ –1.2 represents moderate thermal radiation commonly found in high-temperature activities. The Biot number $Bi = 0.1$ –1.0 indicates mild to high convective heat exchange at the surface. The activation energy value $Ea = 0.5$ –3.0 indicates low- to high-energy reaction barriers for chemical and catalytic processes. These intervals were chosen to capture both the minimal and major impacts of each mechanism, guaranteeing that the computed findings are physically plausible and practically useful.

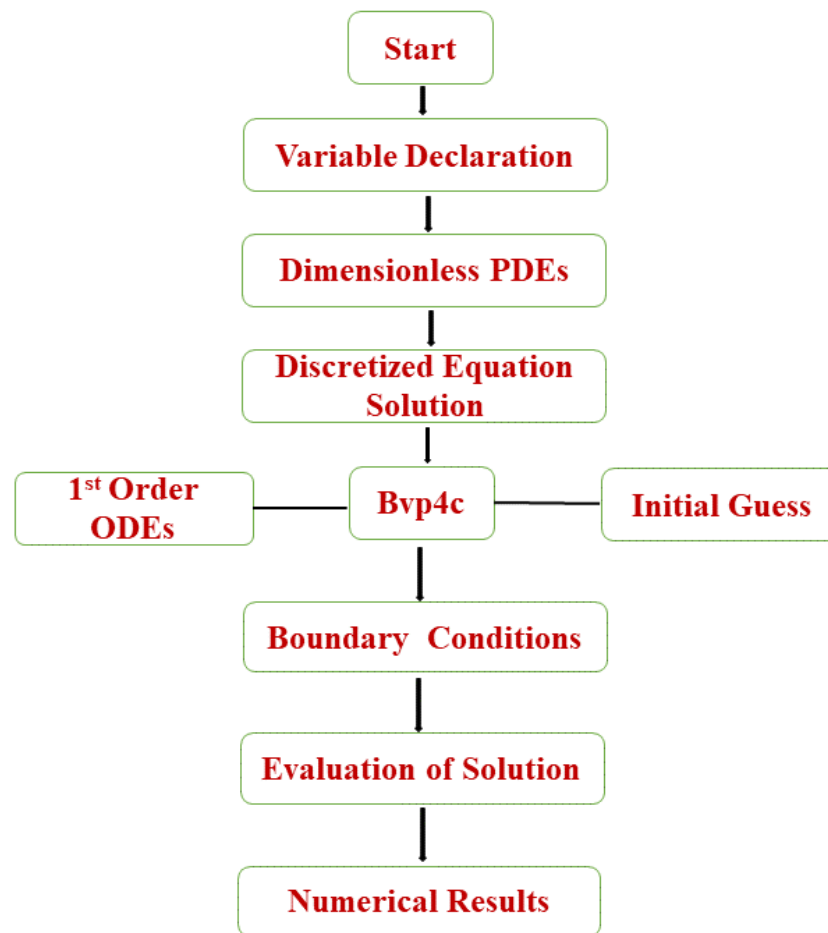


Figure 3. Flow chart of the numerical procedure.

7. Results and Discussion

The influence of governing factors such as mixed convection parameter β_T , curvature γ , magnetic field M , and Prandtl number Pr on essential physical features such as skin friction, rate of heat transfer, and velocity is explored in this study. To simulate the behavior of these parameters, the system of nonlinear ODEs (7)–(9) and boundary conditions (10) is numerically solved in MATLAB (R2020b) using a shooting approach with the Runge–Kutta scheme. The fixed values of the physical parameters are taken into account as follows.

7.1. Velocity Profile

Figures 4–10 show the effects of Prandtl fluid parameters α_1 and α_2 , β , γ , M , Fr , and λ_p , respectively, on velocity distribution. Throughout the investigation, solid lines indicate the propagation of hybrid nanofluid $SWCNT - TiO_2$ /kerosene, whereas dotted lines indicate the flow of hybrid nanofluid $MWCNT - Cu$ /kerosene. The influence of the Prandtl fluid parameters α_1 and α_2 on the velocity profiles is demonstrated in Figures 4 and 5. The velocity fluctuation of α_1 is seen in Figure 4. As α_1 rises, the velocity profile for both fluids rises as well. The physical explanation for this happening is that it reduces the viscosity of the fluid, lowering friction while increasing fluid velocity. The variation of α_2 with velocity is depicted in Figure 5. The velocity description increases as α_2 increases, with $MWCNT - Cu$ /kerosene achieving a peak speed higher than $SWCNT - TiO_2$ /kerosene. The effects of β on both hybrid nanofluids with a velocity distribution are shown in Figure 6. As the value of the parameter β rises, the velocity field decreases. In fact, reducing the impact of gravity reduces the velocity profile. Figure 7 illustrates how the velocity profile is enhanced by the curvature variable γ . As the cylinder radius rises, the curvature parameter

decreases (inverse relationship). A shorter cylinder radius, on the other hand, has a greater curvature influence, which enhances momentum transmission and velocity enhancement in the boundary layer, as illustrated in Figure 7. As the magnetic parameter M increases, Figure 8 illustrates that the velocity profile drops significantly. Physically, increasing the magnetic parameter M enhances the Lorentz force, which opposes fluid motion and converts kinetic energy into heat. This resistance raises temperature and lowers velocity, a phenomenon useful for magnetic control in MHD reactors but unfavorable for cooling systems. The main features of the velocity field with rising Fr values are shown in Figure 9. Figure 10 shows that increasing the inputs of the porosity parameter λ_p improves the velocity profile.

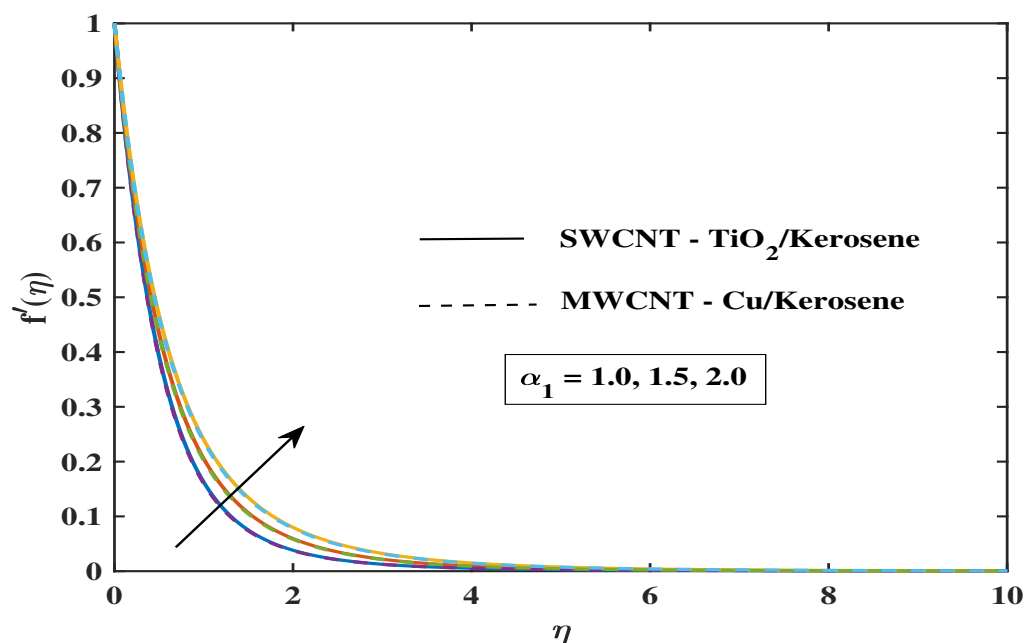


Figure 4. Effect of the Prandtl fluid parameter α_1 on the dimensionless temperature profile $f'(\eta)$.

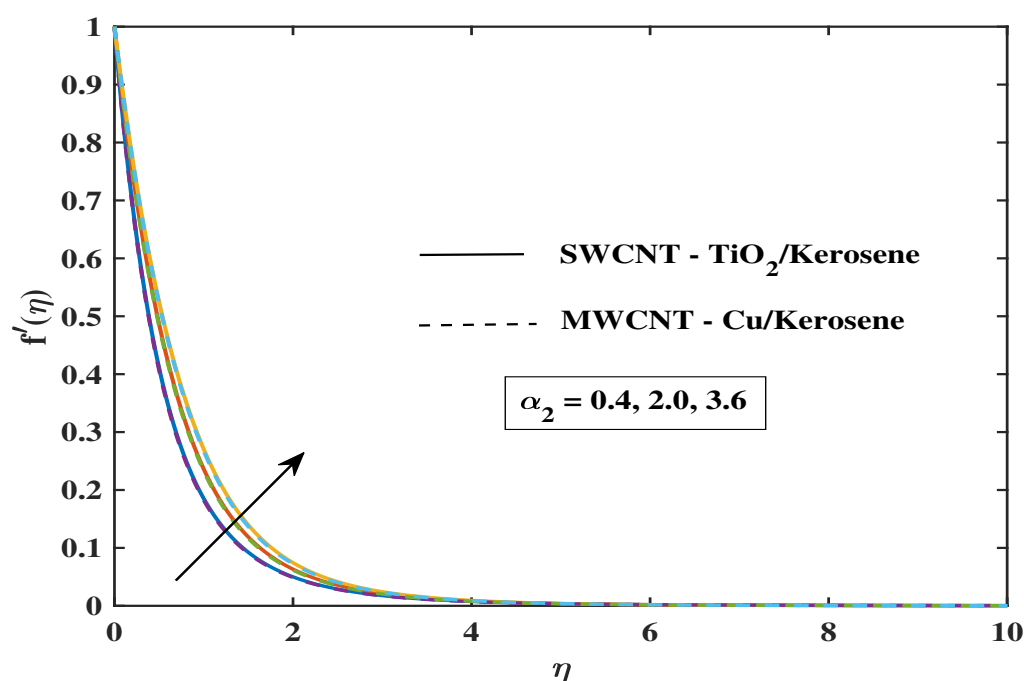


Figure 5. Effect of the Prandtl fluid parameter α_2 on the dimensionless temperature profile $f'(\eta)$.

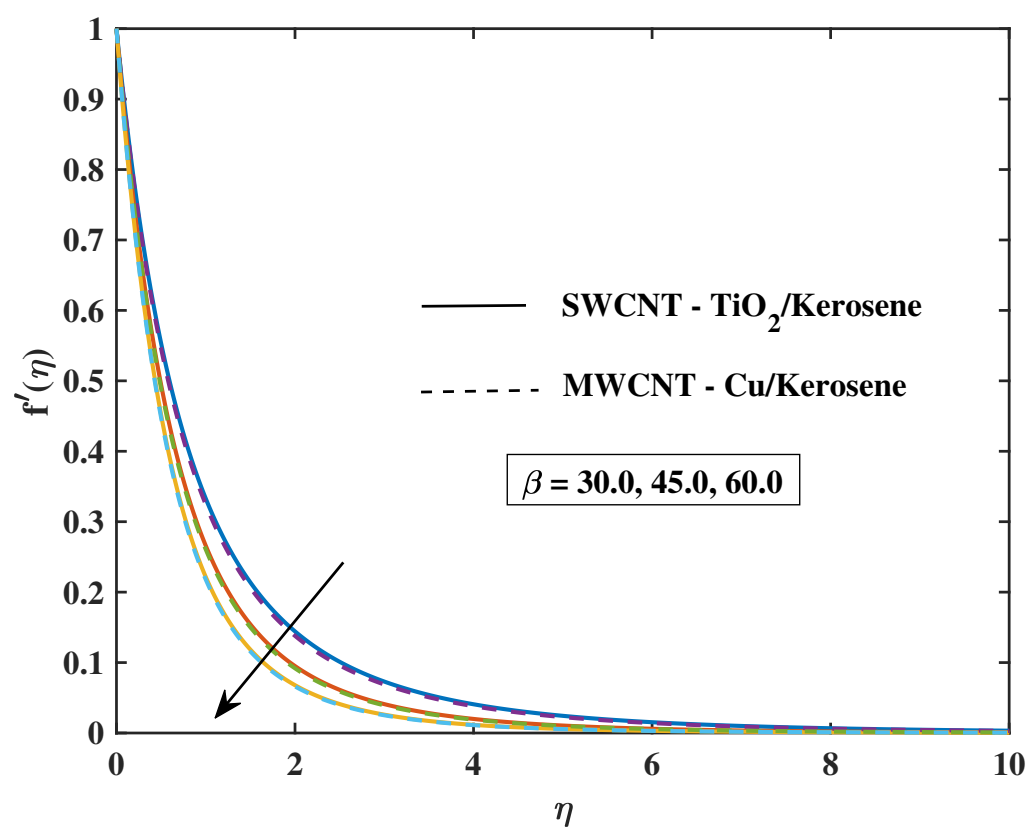


Figure 6. Effect of the Inclination parameter β on the dimensionless temperature profile $f'(\eta)$.

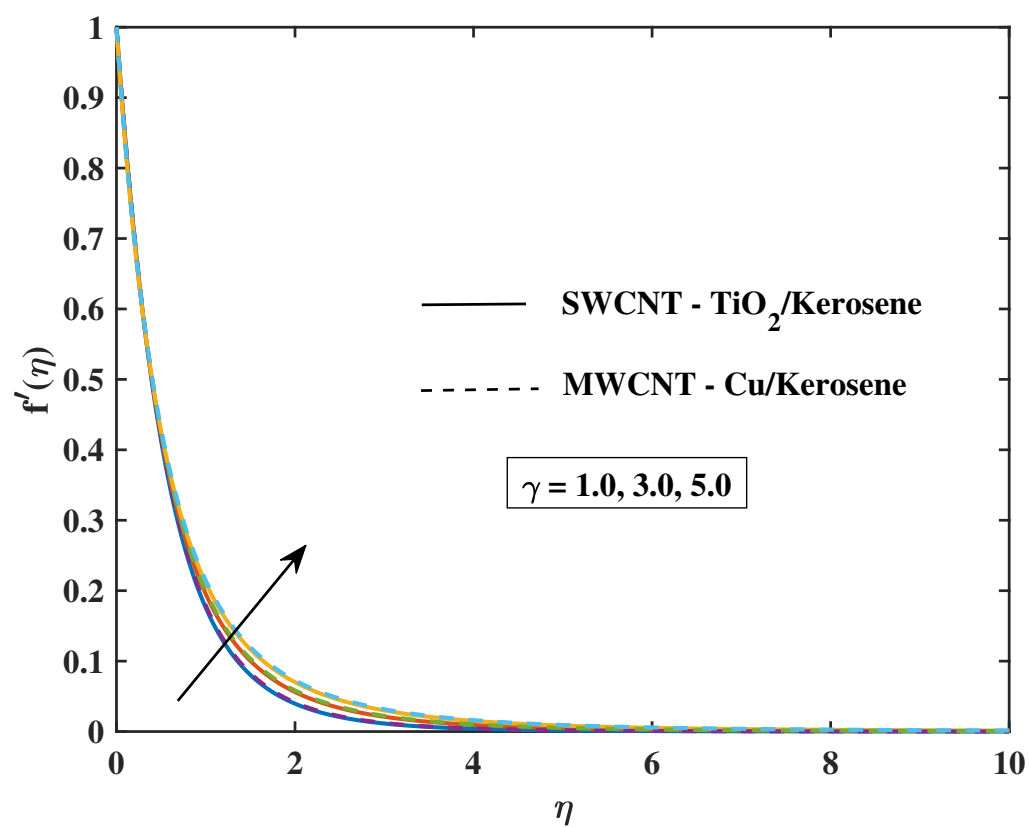


Figure 7. Effect of the Curvature parameter γ on the dimensionless temperature profile $f'(\eta)$.

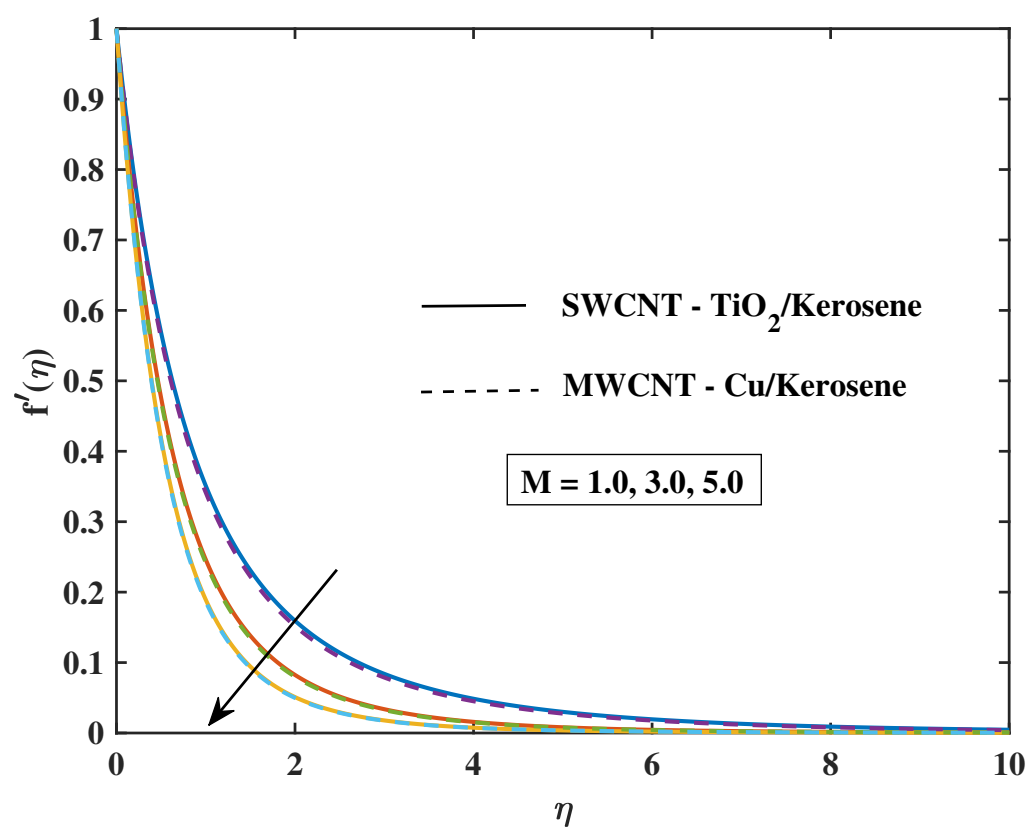


Figure 8. Effect of the Magnetic parameter M on the dimensionless temperature profile $f'(\eta)$.

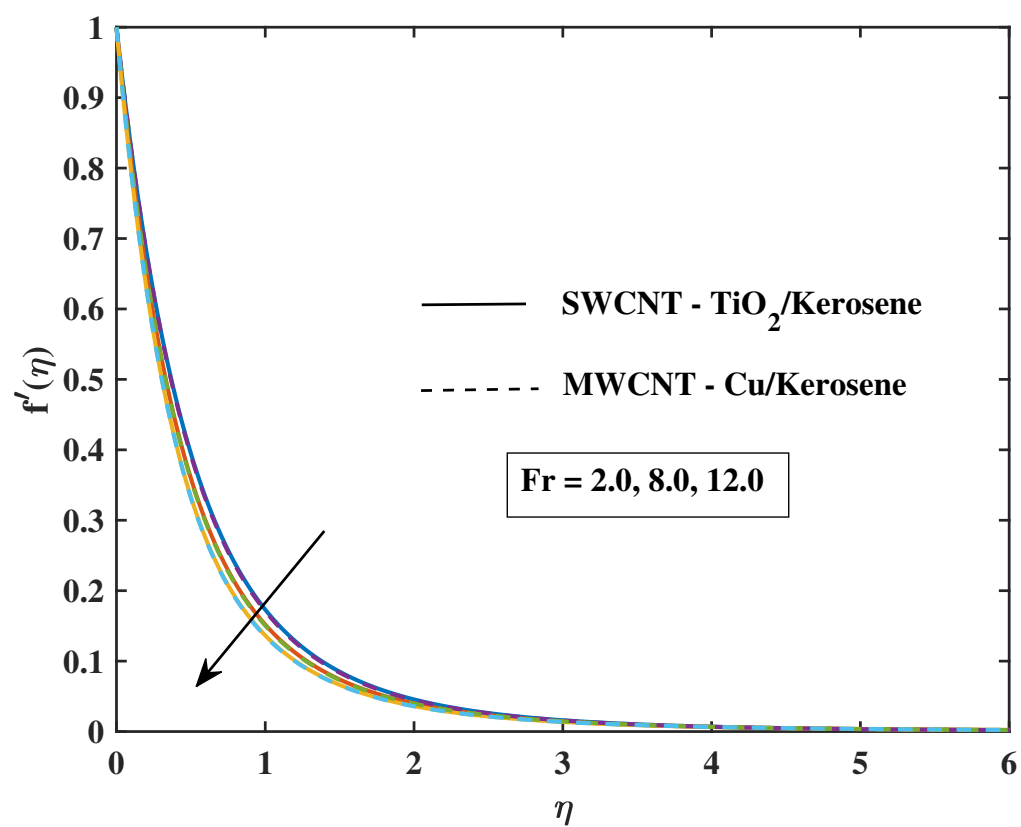


Figure 9. Effect of the Forchheimer parameter Fr on the dimensionless temperature profile $f'(\eta)$.

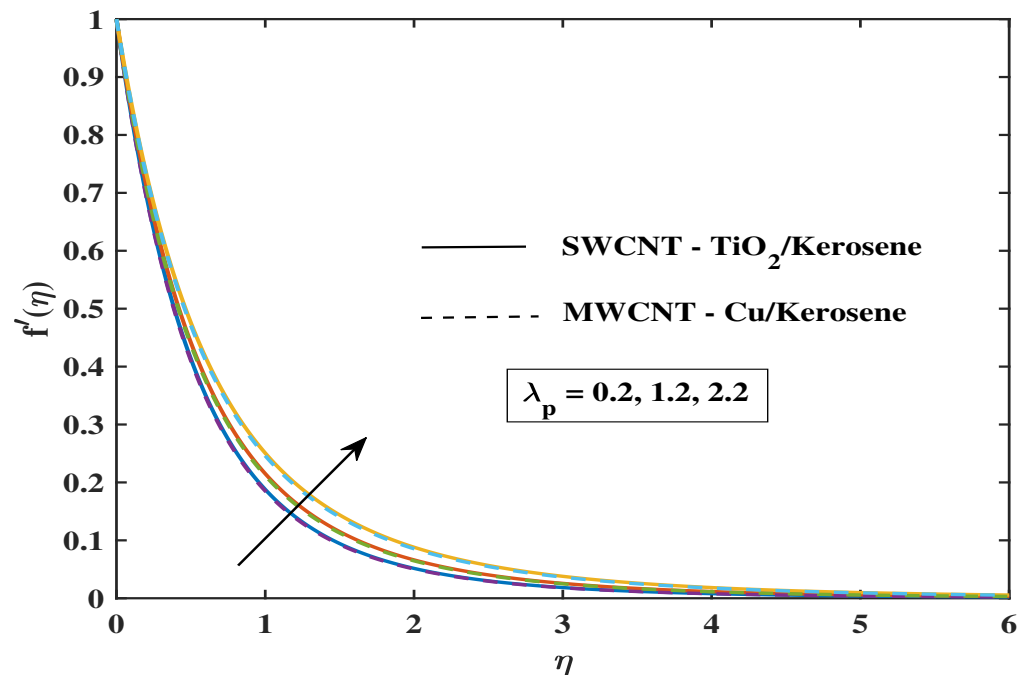


Figure 10. Effect of the Porosity parameter λ_p on the dimensionless temperature profile $f'(\eta)$.

7.2. Temperature Profile

The comparative analysis of thermal profiles for both hybrid nanofluids (SWCNT – TiO₂ and MWCNT – Cu) against multiple variables is shown in Figures 11–18. The Prandtl hybrid nanofluid temperature fluctuations are displayed in Figure 11 depending on its variable α_1 . As the quantity of α_1 increases, the amount of heat decreases. Despite the fact that their values were the same at one point in the graph, the SWCNT – TiO₂ fluid had a lower entropy value than the MWCNT – Cu fluid. This effect happens as a result of low temperatures dropping hybrid nanofluid movement and increasing the system's volatility. The temperature change following the implementation of α_2 is shown in Figure 12. The temperature decreases as the value of α_2 increases. The event occurs because the flow velocity increases, reducing the transfer of heat from the substrate. Figure 13 illustrates how β affects thermal profiles. As the inclination angle increases, the buoyant force acting on the fluid weakens, leading to reduced flow velocity. The slower motion enhances thermal dispersion and overall heat transfer, which is particularly beneficial for thermal management in radiators, cooling systems, and electronic devices. Adjusting the inclination angle, thus, provides an effective means to optimize temperature control and system efficiency. The variation in the temperature profile with respect to the Biot number Bi is illustrated in Figure 14. Because the Biot number is made up of heat and mass transfer parameters, increasing the values of Bi enhances the thermal boundary layer. As a result, the temperature of the fluid increases. As the curvature constant increases, Figure 15 illustrates how the temperature profile improves. Physically, friction creates Lorentz energy, which also provides heat energy; as a consequence, the thermal profile is improved. With respect to the magnetic parameter M , Figure 16 shows the thermal fluctuation ($\theta(\eta)$). The kinetic energy created by the shortening of the fluid's movement is turned into heat energy, and the temperature profile increases. The impact of thermal radiation parameter on temperature distribution for both hybrid nanofluids is depicted in Figure 17. For both circumstances, a direct relationship exists between and anywhere in the boundary layer area. According to physics, when the parameter of heat radiation increases, the mean absorption coefficient drops. As a result, the temperature profile has been enhanced. This effect is vital for high-temperature systems

such as turbines and aerospace shields, where radiative heat transfer prevails. Figure 18 depicts the effect of heat profiles with different shapes of nanoparticles. The heat transfer is higher for cylinder-type nanoparticles compared to sphere and brick-type nanoparticles.

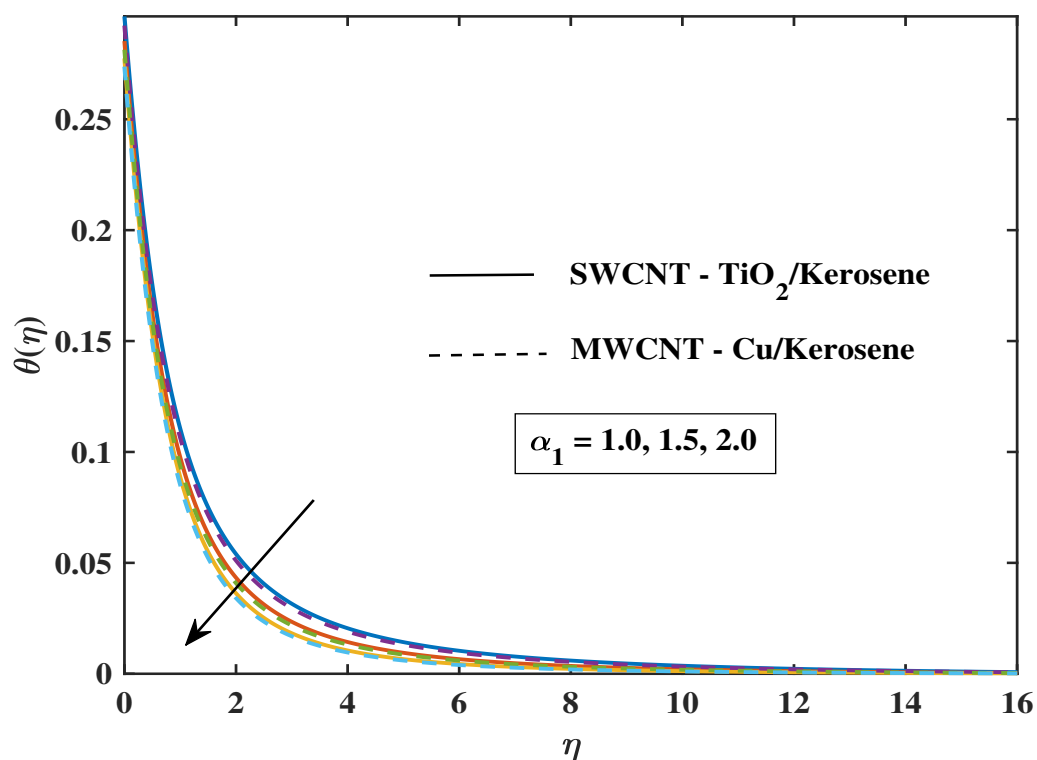


Figure 11. Effect of the Prandtl fluid parameter α_1 on the dimensionless temperature profile $\theta(\eta)$.

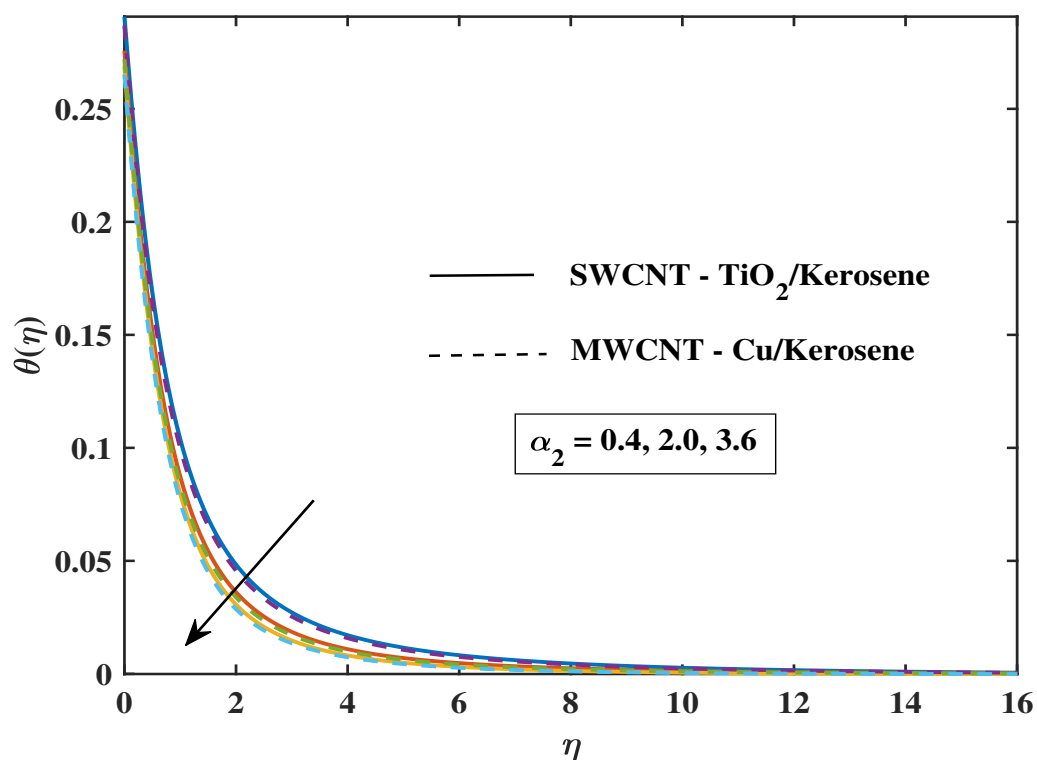


Figure 12. Impact of the Prandtl fluid parameter α_2 on the dimensionless temperature profile $\theta(\eta)$.

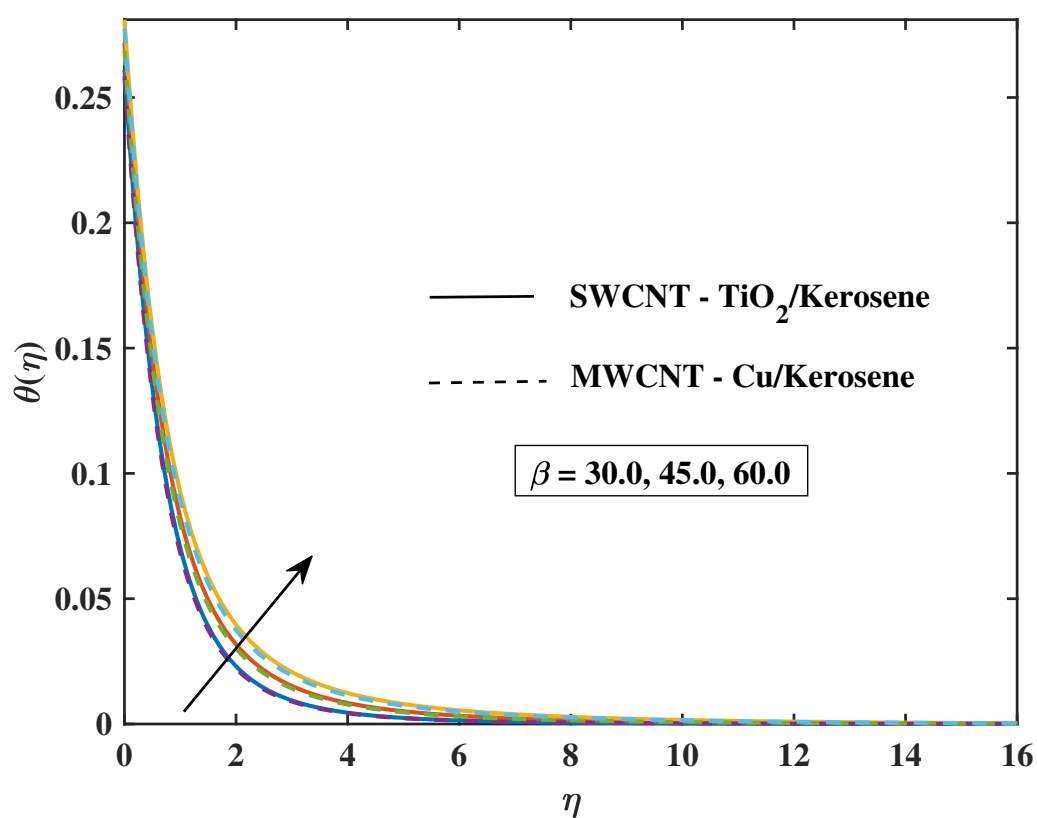


Figure 13. Effect of the Inclination angle parameter β on the dimensionless temperature profile $\theta(\eta)$.

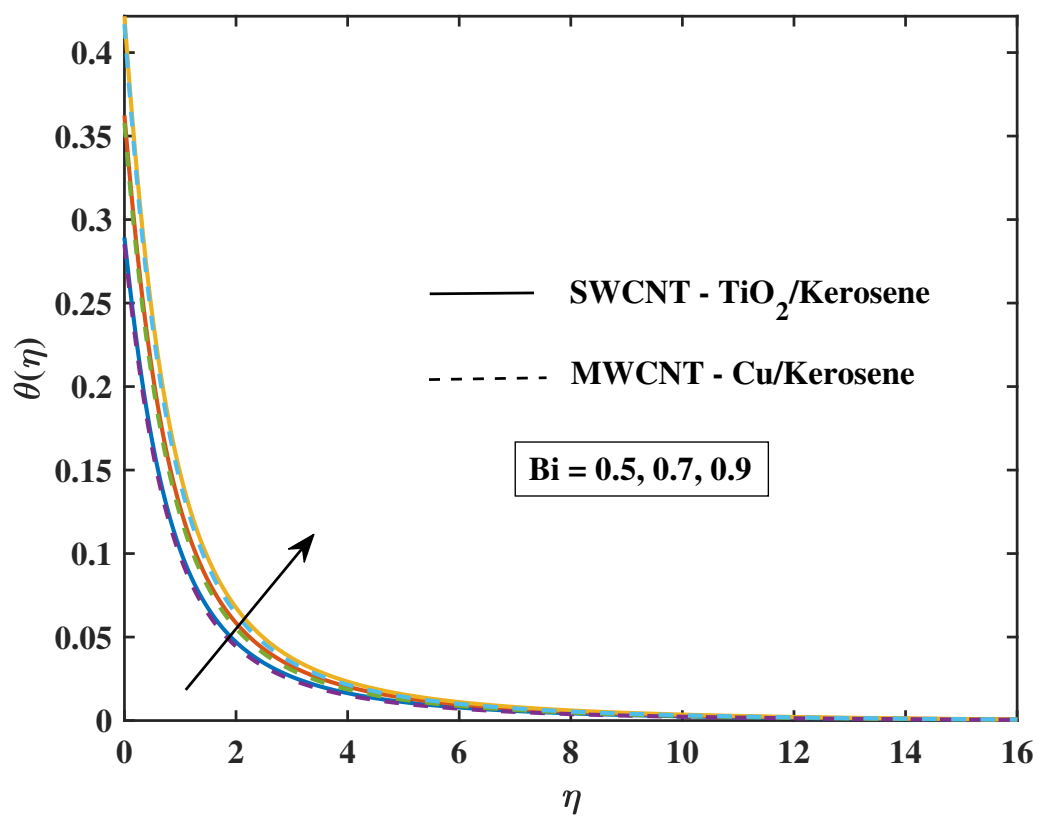


Figure 14. Impact of the Biot parameter Bi on the dimensionless temperature profile $\theta(\eta)$.

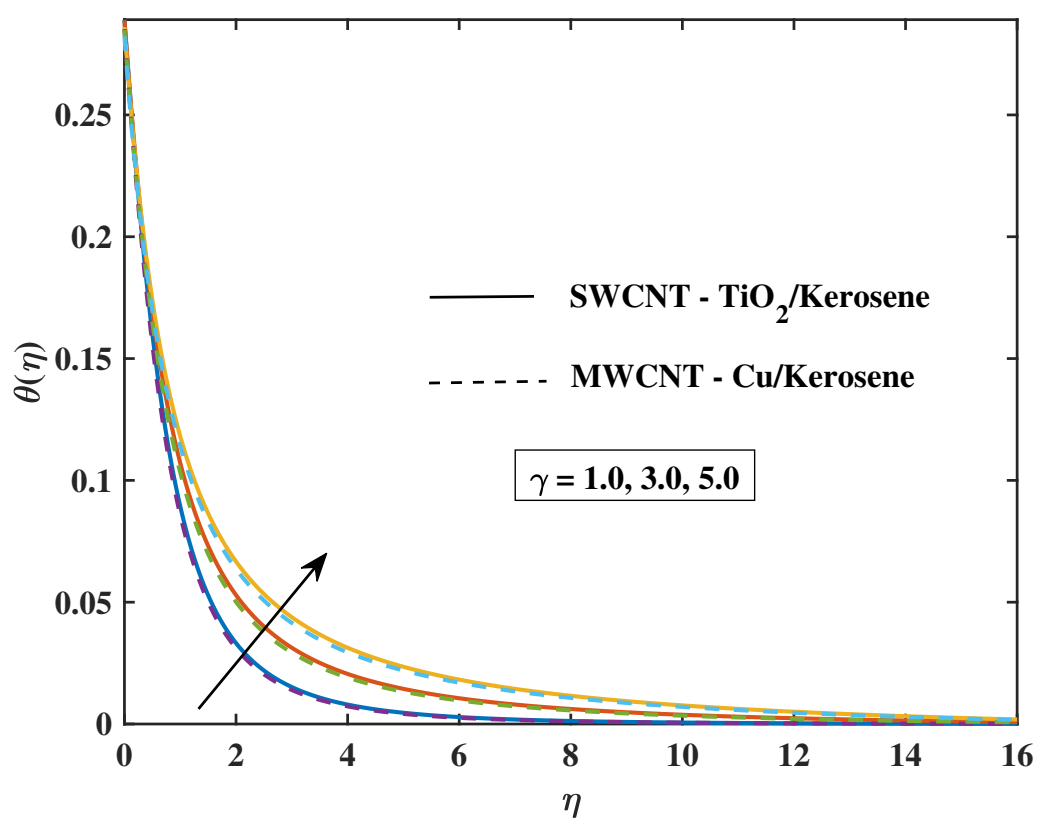


Figure 15. Effect of the Curvature parameter γ on the dimensionless temperature profile $\theta(\eta)$.

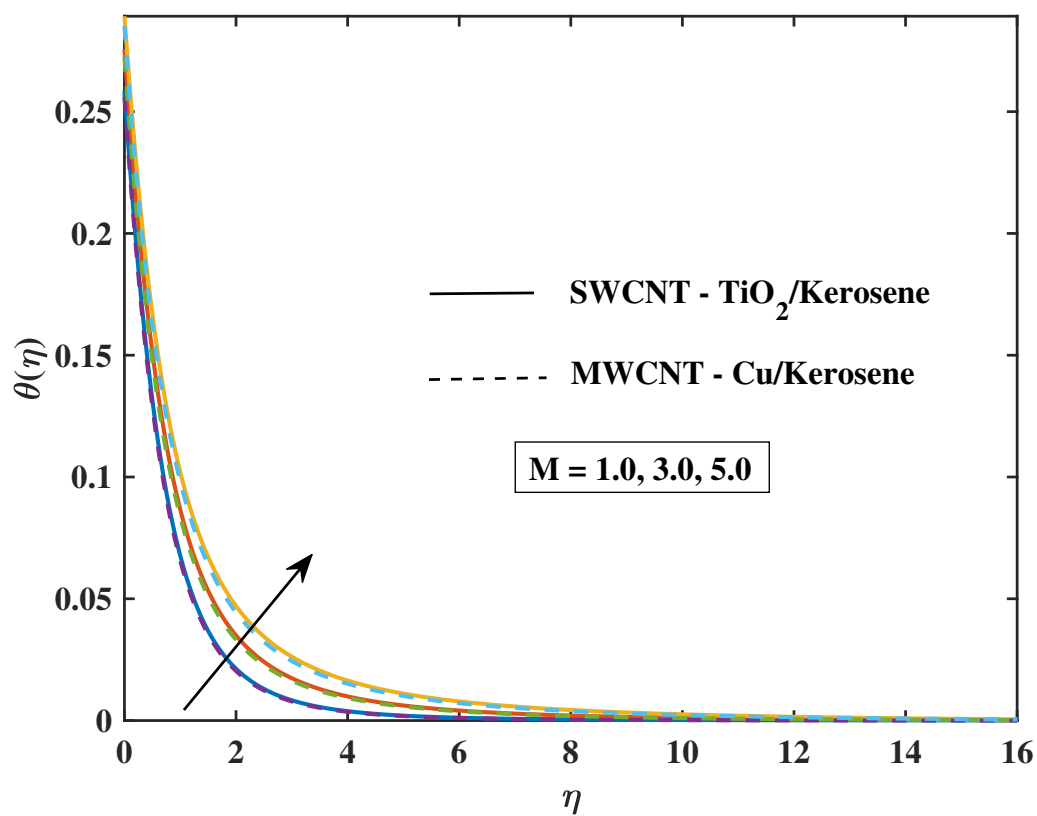


Figure 16. Impact of the Magnetic parameter M on the dimensionless temperature profile $\theta(\eta)$.

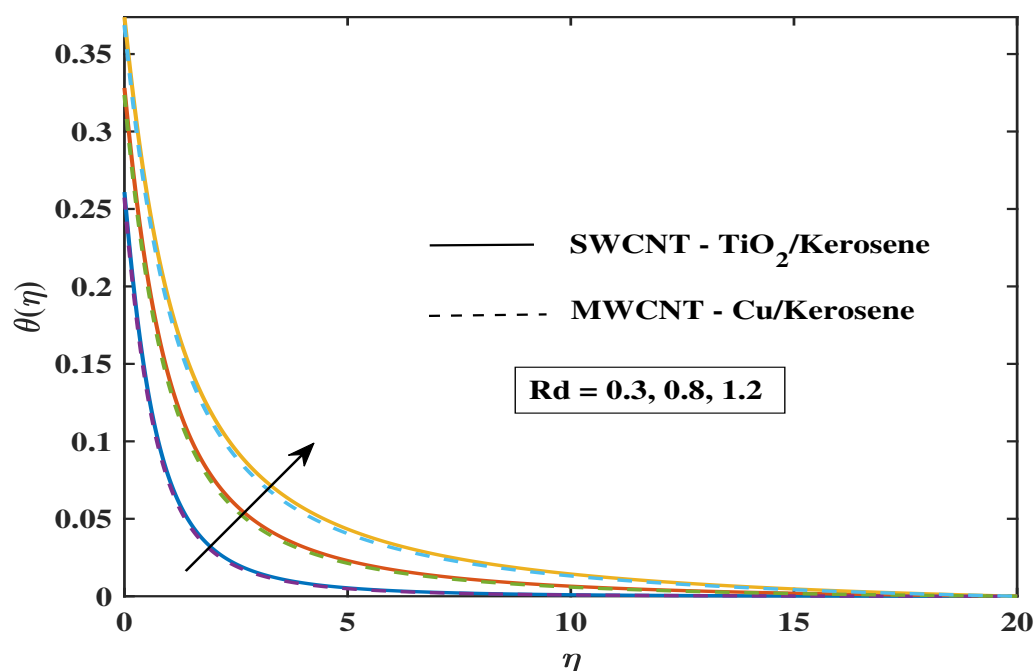


Figure 17. Effect of the Radiation parameter Rd on the dimensionless temperature profile $\theta(\eta)$.

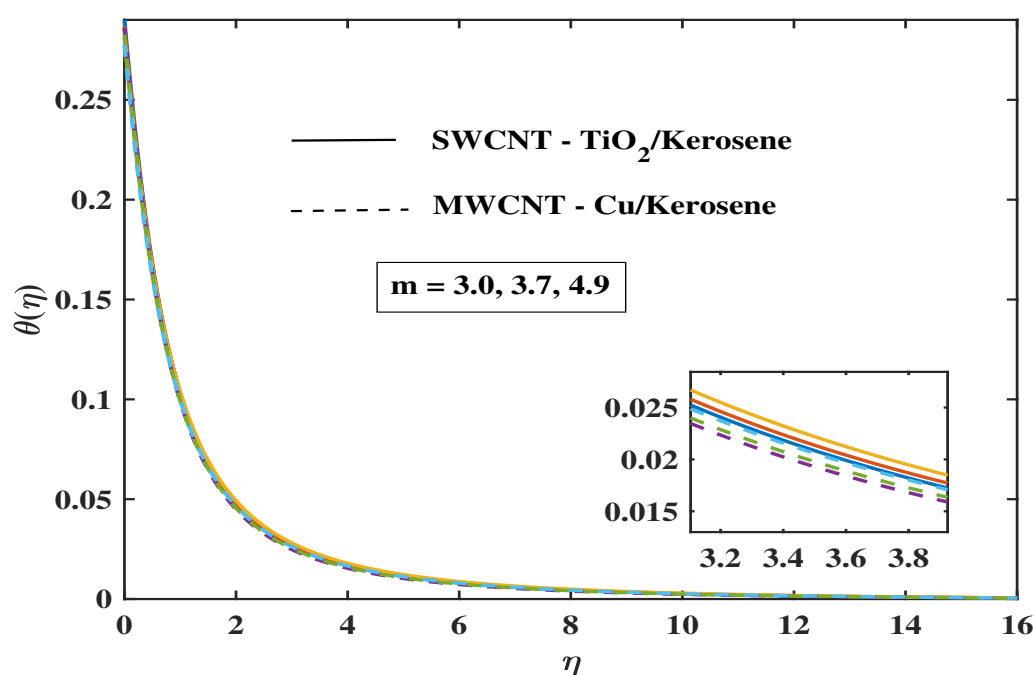


Figure 18. Impact of the Shape factor parameter m on the dimensionless temperature profile $\theta(\eta)$.

7.3. Concentration Profile

Figures 19–22 detail the features of γ , M , Sc , and Ea on the concentration profile, accordingly. Figure 19 shows the effects of the curvature parameter γ on the concentration profile. The concentration profile increases as the γ value increases. Greater porosity enhances heat removal in porous cooling systems; however, increased radiation Rd or curvature γ boosts temperature, which is pertinent to high-temperature heat exchangers. The concentration profile patterns for the magnetic factor M are displayed in Figure 20. The magnetic parameter M represents the ratio of magnetic to viscous forces, where higher M values intensify the Lorentz (opposing) force. This added resistance slows the fluid

motion, thickens the thermal boundary layer, and results in a higher concentration profile. Figure 21 shows the concentration profile for various values of Sc for both hybrid nanofluids. We discover that with increasing Sc values, the concentration profile decreases in both situations. The concentration profile for different Ea values of both hybrid nanofluids is shown in Figure 22. As Ea grows, the exponential decay reduces the reaction rate, enabling more solute to collect and, therefore, raising the concentration profile. The activation energy affects the response rate in the concentration equation by an exponential component of Arrhenius type. Greater activation energy limits particle diffusion, mimicking temperature-dependent transport in catalytic and cracking processes.

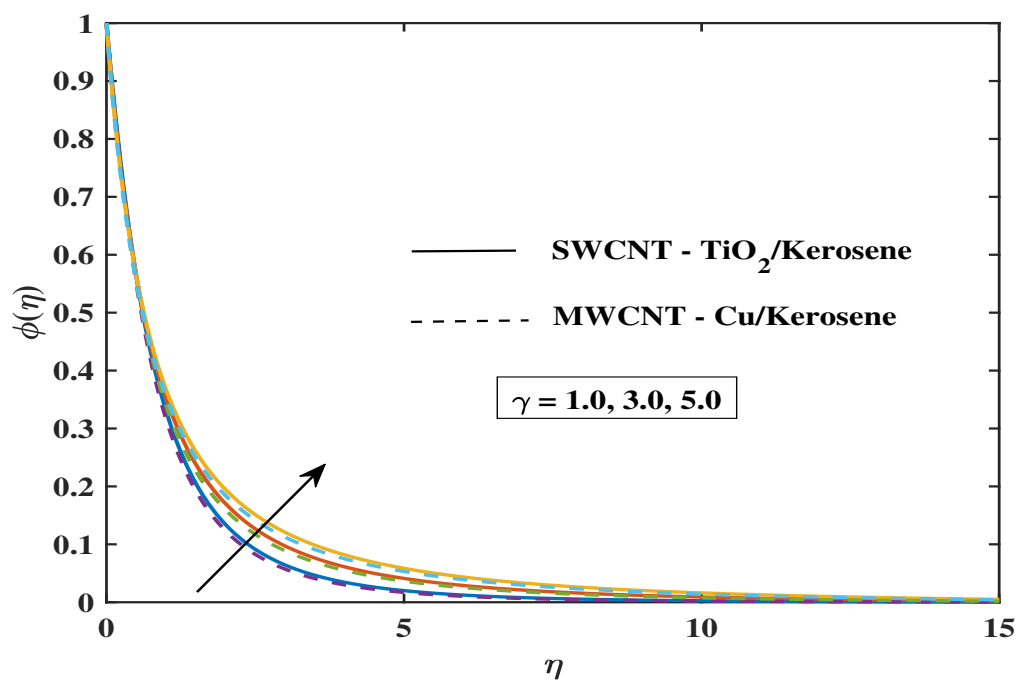


Figure 19. Effect of the Curvature parameter γ on the concentration profile $\phi(\eta)$.

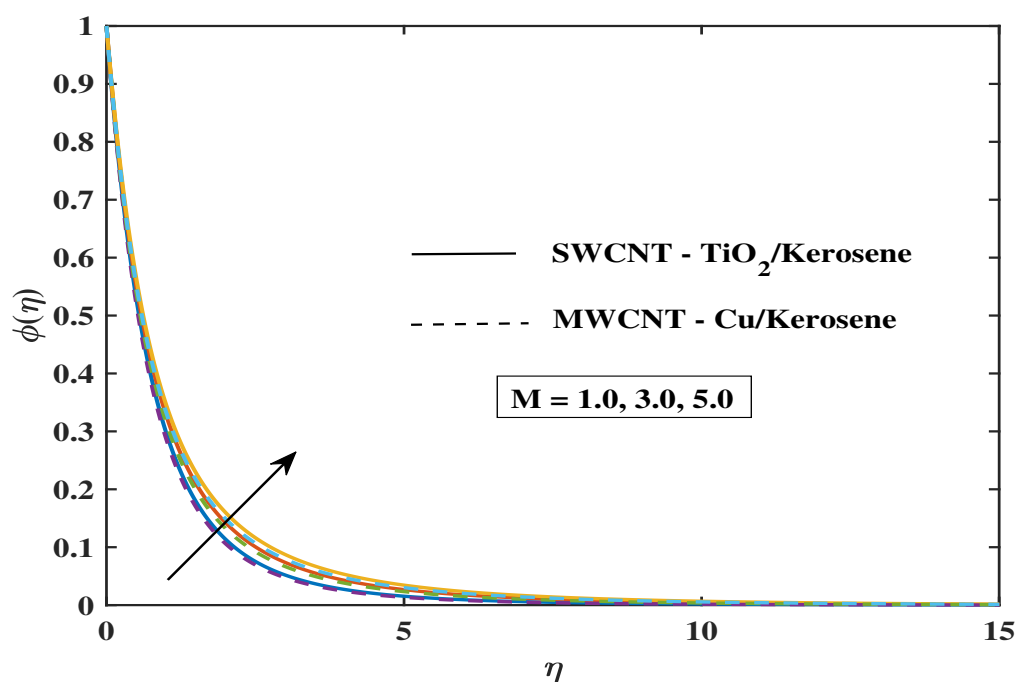


Figure 20. Effect of the Magnetic parameter M on the concentration profile $\phi(\eta)$.

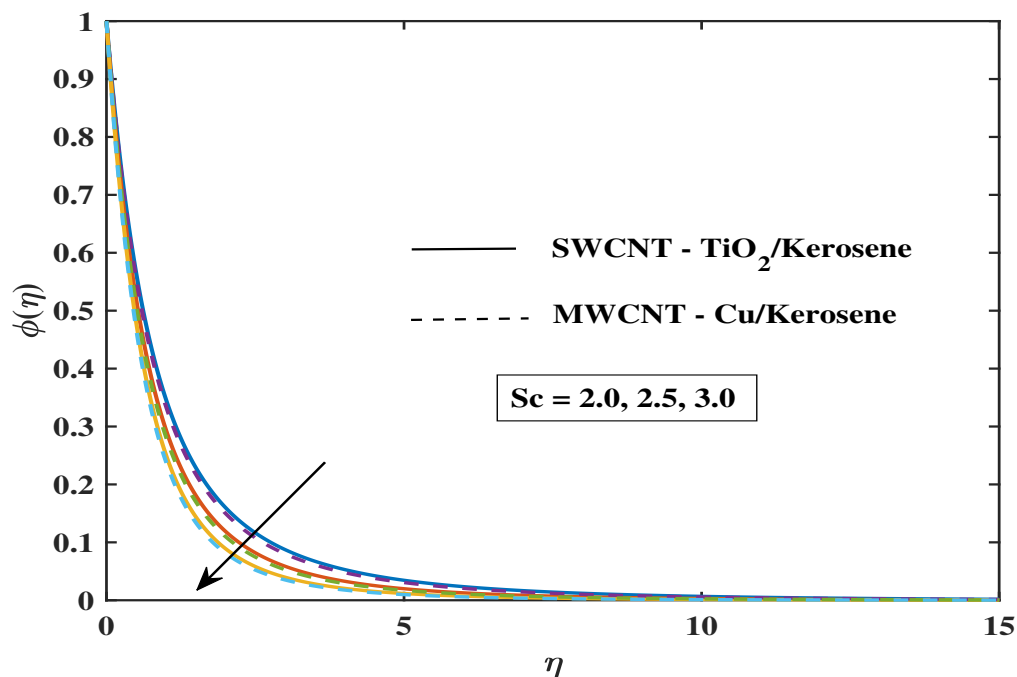


Figure 21. Effect of the Schmidt parameter Sc on the concentration profile $\phi(\eta)$.

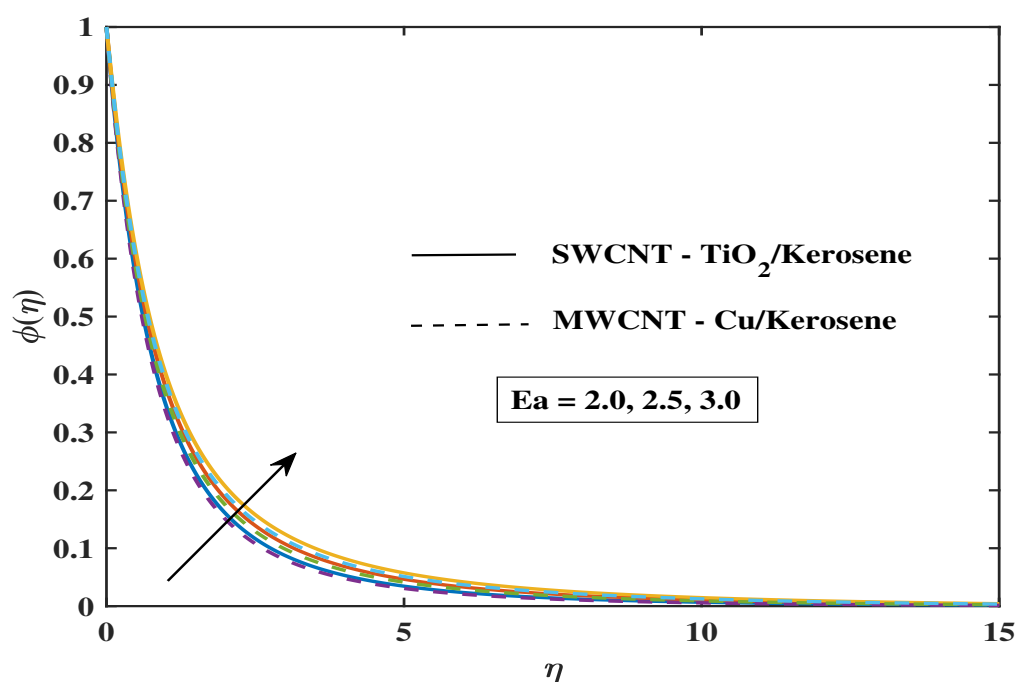


Figure 22. Effect of the Activation energy parameter Ea on the concentration profile $\phi(\eta)$.

7.4. Model Validation

To confirm the reliability of the present numerical framework, a validation exercise was carried out by simplifying the hybrid nanofluid model to the limiting case of pure kerosene ($\phi_1 = \phi_2 = 0$). This reduction converts the governing equations to those describing a conventional Newtonian fluid under identical boundary and physical conditions. The computed values of the skin friction coefficient ($C_f Re_x^{\frac{1}{2}}$) and local Nusselt number ($Nu_x Re_x^{-\frac{1}{2}}$) were compared with the previously published data of Nadeem et al. [9] and Patil & Kulkarni [28]. The comparison results are summarized in Table 4. The outcomes

show excellent consistency between the present solution and the literature, with an average deviation of less than 2%. This high level of agreement validates the accuracy of the employed MATLAB (R2020b) BVP4C solver, confirming that the mesh refinement, step size, and tolerance values are adequately chosen to achieve numerical convergence. Moreover, the validation of our findings with the existing literature is shown in Table 5 for different values of the curvature parameter. Table 5 shows that our results are consistent with the existing literature.

Table 4. Comparison of the present numerical results with previous studies for the case of pure kerosene flow ($\phi_1 = \phi_2 = 0$).

Parameter	Present Study	Nadeem et al. [9]	Patil & Kulkarni. [28]	Deviation (%)
$Nu_x Re_x^{-1/2}$	0.5331	0.5425	0.5280	<1.9
$C_f Re_x^{1/2}$	3.4102	3.4628	3.3870	<1.7

Table 5. Skin friction validation for different (γ) curvature estimations.

γ	Farooq et al. [47]	Song et al. [48]	This Study	CPU Time (s)
0.0	−1.0000	−1.0000	−1.0000	2.760355
0.25	−1.0943743	−1.0943742	−1.094377	2.746383
0.5	−1.1887304	−1.1887303	−1.188727	3.298879
0.75	−1.2818245	−1.2818242	−1.281833	2.205832
1.0	−1.4593752	−1.4593751	−1.459372	2.797052

7.5. Applications and Relevance

The present model is directly relevant to several energy and engineering applications in which hybrid nanofluids are used to improve thermal control. Such systems include nuclear heat exchangers, aerospace fuel cooling loops, and electronics thermal management units, where magnetic fields, radiation, and porous structures often coexist. Realistic parameter ranges were considered:

- Magnetic field strength ($M = 1\text{--}5$) corresponds to fields of 0.1–0.5 T, typical of MHD cooling channels.
- Porosity values ($K_p = 0.1\text{--}0.5$) match metallic or ceramic porous inserts used in compact heat exchangers.
- Nanoparticle volume fractions ($\phi \leq 0.04$) are within experimentally stable limits for SWCNT – TiO_2 and MWCNT – Cu dispersions.

The numerical outcomes of skin friction, Nusselt number, and Sherwood number are shown in Tables 6, 7, and 8, respectively. Table 6 showed that as the value of α_1 , α_2 , Fr , and β increases, the number of skin friction also increases. However, it falls when the porosity parameter rises. Higher porosity reduces fluid flow resistance, which lowers shear stress and skin friction. This explains the drop in behavior as the porosity parameter increases. Table 7 shows that as the value of α_1 , α_2 , Rd , Bi , and Γ increases, Nusselt numbers also increase, but they fall when the magnetic parameter M increases. Because fluid velocity is dampened by the resistive force (Lorentz force) induced by the magnetic field, the Nusselt number falls as the magnetic parameter increases. This lowers the Nusselt number by decreasing the efficiency of convective heat transport. Table 8 depicts that as the value of α_1 , α_2 , Sc , and Γ increases, Sherwood numbers also increase. However, when the activation energy and magnetic parameter rise, it shows a decreasing tendency. Since the magnetic field reduces mass transfer performance by suppressing fluid motion through the Lorentz force, the Sherwood number falls as the magnetic parameter and activation energy increase.

Higher activation energy also reduces the concentration gradient and, as a result, the Sherwood number by slowing the diffusion-driven reaction rate.

Table 6. Numerical values of $-f''(0)$ for different values of parameters.

α_1	α_2	M	λ_p	Fr	β	$-f''(0)$ (SWCNT – TiO ₂)/Kerosene	$-f''(0)$ (MWCNT – Cu)/Kerosene
1.0	0.5	2.0	0.5	1.0	90.0	3.4059	3.4449
1.2						3.8797	3.9250
1.4						4.3078	4.3588
1.3	0.3					3.8710	3.9143
	0.6					4.1963	4.2466
	0.9					4.4463	4.5018
	0.5	1.5				2.6710	2.7355
		2.5				3.1231	3.1810
		3.5				3.5339	3.5871
		2.0	0.5			3.9782	4.0274
			1.0			3.7719	3.8228
			1.5			3.5583	3.6112
			0.2	2.5		4.4370	4.4828
				3.0		4.5467	4.5917
				3.5		4.6550	4.6993
				1.0	30.0	2.5487	2.6154
					45.0	3.1231	3.1810
					60.0	3.6319	3.6840

Table 7. Numerical values of $\theta'(0)$ for different values of parameters.

α_1	α_2	M	Rd	Bi	γ	$\theta'(0)$ (SWCNT – TiO ₂)/Kerosene	$\theta'(0)$ (MWCNT – Cu)/Kerosene
1.0	0.5	2.0	0.5	0.5	0.5	0.5600	0.5634
1.2						0.5640	0.5672
1.4						0.5674	0.5705
1.3	0.3					0.5629	0.5661
	0.6					0.5669	0.5701
	0.9					0.5698	0.5730
	0.5	1.5				0.5858	0.5880
		2.5				0.5792	0.5818
		3.5				0.5734	0.5763
		2.0	0.4			0.5339	0.5367
			0.7			0.6258	0.6297
			1.0			0.7077	0.7131
			0.5	0.4		0.4803	0.4826
				0.6		0.6419	0.6461
				0.8		0.7720	0.7780
				0.5	0.3	0.5657	0.5689
					0.6	0.5661	0.5693
					0.9	0.5683	0.5714

Table 8. Numerical values of $\phi'(0)$ for different values of parameters.

α_1	α_2	M	Sc	Ea	γ	$\phi'(0)$ (SWCNT – TiO ₂)/Kerosene	$\phi'(0)$ (MWCNT – Cu)/Kerosene
1.0	0.5	2.0	2.0	0.9	0.5	1.1996	1.1966
1.2						1.2090	1.2061
1.4						1.2178	1.2148
1.3	0.3					1.2057	1.2027
	0.6					1.2169	1.2139
	0.9					1.2255	1.2226
	0.5	1.5				1.2806	1.2759
		2.5				1.2550	1.2511
		3.5				1.2355	1.2322
		0.5	2.2			1.2761	1.2731
			2.4			1.3365	1.3333
			2.6			1.3947	1.3914
			2.0	2.1		1.1921	1.1891
				2.4		1.1353	1.1323
				2.7		1.0886	1.0856
				2.0	0.3	1.1410	1.1379
					0.6	1.2492	1.2463
					0.9	1.3541	1.3514

8. The Artificial Neural Network Modeling (ANN)

Artificial neural networks (ANNs), which are modeled after the network of linked neurons in the human brain, are widely used in modern computer systems. A supervised feed-forward artificial neural network (ANN) was employed to predict the heat and mass transfer characteristics obtained from the numerical solution of the governing equations. The network was implemented in MATLAB (R2020b) (Neural Network Toolbox) using the Levenberg–Marquardt backpropagation algorithm due to its rapid convergence for nonlinear regression problems. The input layer consisted of five neurons, corresponding to the principal dimensionless parameters of the model: magnetic parameter (M), porosity parameter (λ_p), radiation parameter (Rd), Prandtl fluid coefficients (α_1, α_2), while the output layer contained two neurons representing the local Nusselt number ($Nu_x Re_x^{-\frac{1}{2}}$) and skin-friction coefficient ($C_f Re_x^{\frac{1}{2}}$). A single hidden layer was found sufficient after network sensitivity analysis. The optimal configuration comprised 10 neurons in the hidden layer using the tansig (hyperbolic tangent) activation function, while the output layer employed a purelin (linear) transfer function. The hybrid nanofluid responses derived from the bvp4c numerical solver were predicted using an artificial neural network (ANN) as a stand-in. With Latin Hypercube Sampling, a total of 1000 simulation samples were produced, which were split into 70% training, 15% validation, and 15% testing sets. Before training began, all variables were standardized. Although the current study emphasizes an artificial neural network (ANN) model, future research might look at other or hybrid data-driven methodologies, including Support Vector Machines (SVMs), Gaussian Process Regression (GPR), and Physics-Informed Neural Networks (PINNs). These mathematical frameworks can increase physical interpretability, predictive generalization, and estimation of uncertainty in complicated nanofluidic systems. The anticipated data is captured and stored in the output layer. Through interconnections, data moves from the input layer through further layers, where corrections are performed in response to the error ratio between the goal and anticipated values. Backpropagation is the technique of sending data back to the input layer to reduce errors. Successful ANN training depends on ef-

efficient data optimization. While too much data might result in overfitting and memory problems, too little data may make it more difficult for the network to understand the correlations between variables. The hidden neurons in the current ANN model were given the tansigmoid (tansig) activation function, which is as follows: $f(x) = \frac{1}{1+e^{-x}}$. Complex interactions between the controlling parameters and thermal responses can be captured by the network, thanks to this nonlinear mapping. A linear (purelin) activation function was used for the output layer since it is ideal for forecasting continuous variables like temperature, velocity, and Nusselt number. Tansig and purelin were selected as the combination to guarantee effective training and consistent prediction accuracy. The ANN model included 15 input parameters and three outputs $f'(\eta)$, $\theta(\eta)$, and $\phi(\eta)$. The Levenberg–Marquardt method was used to train a feed-forward network with one hidden layer of 20 neurons. In total, 10% of the data in this study was utilized for testing, 10% for validation, and 80% for model training. The ANN training progress for the parameters λ , Rd , and Ea is shown in Figures 22–24. Several training measures are shown visually, such as regression analysis, autocorrelation, function fit, training states, mean squared errors (MSEs), and error histograms. The training method effectively reduces the MSE for each example until reaching a stable threshold, where more training results in decreasing returns. The high correlation value reflects the smooth and strongly monotonic dependence of heat and momentum transport parameters on the governing dimensionless numbers. This behavior was verified by retraining the model with independently generated data ($\pm 5\%$ parameter perturbation), which produced consistent performance metrics ($R = 0.9987$, $MSE = 2.0 \times 10^{-5}$). The convergence of the test and validation MSEs shows that the model has excellent generalization capabilities and does not overfit. To improve the resilience and dependability of the ANN model, data pre-processing methods, including normalization and outlier identification, were used. By performing predictions with over 90% computational savings and less than 1.5% relative error, the ANN model is shown to be more applicable in real-time control settings when compared to the BVP4C model. The porosity parameter's performance analysis is shown in Figure 23. The network's performance throughout epochs is displayed in Figure 23a, demonstrating the training data's strong validation. Figure 23b demonstrates the variations in the μ parameter, validation results, and gradient behavior. As epochs rise, the error lowers, but the validation check remains constant, exhibited by the gradient (9.9841×10^{-8}) and $\mu 10^{-8}$ continuously dropping throughout iterations. The network reaches its peak performance at epoch 332 with a mean square error (MSE) of about 2.3568×10^{-9} . The error histograms in Figure 23c, where the error almost exactly matches the zero-error reference line, further illustrate the small discrepancy between the output and target of the network, with a value of about 4.6×10^{-6} . The difference that is negative is also a little greater than the difference that is positive. Figure 23d shows a regression plot for λ with a correlation coefficient R of about one, omitting randomness ($R = 0$). The high R -value ($R = 1$) and the small number of empty spaces in the regression plot, which guarantee that missing data has the least influence, demonstrate the network's correctness and efficacy. Figure 23f displays the function fit, whereas Figure 23e displays the autocorrelation. The fit lines closely resemble the desired output with little bias, demonstrating outstanding generalization and optimal accuracy.

In Figure 24, the thermal radiation parameter's performance analysis is displayed. Figure 24a displays the network's performance over the epochs, indicating the training data's high validation. Figure 24b illustrates the modifications to the μ variable, validation tests, and gradient. The gradient $\frac{9.9741}{10^8}$ and $\mu = 10^{-8}$ consistently decline throughout iterations, indicating that the error lowers as epochs grow, but the accuracy of the check stays constant. At epoch 282, the network's mean square error (MSE) is around 1.2298×10^{-8} , signifying peak performance. Further illustrating the small discrepancy between the net-

work's target and output is the error histograms in Figure 24c, where the error almost exactly matches the zero-error reference line, with a value of around 4.51×10^{-5} . Additionally, there is a slight variation between the positive and negative. When the correlation coefficient R is close to 1, excluding randomness ($R = 0$), the regression plot for Rd is displayed in Figure 24d. Figure 24f displays the function fit, whereas Figure 24e displays the autocorrelation. The fit lines closely resemble the desired output with little bias, demonstrating outstanding generalization and optimal accuracy.

Figure 25 displays the activation energy parameter performance analysis. The robust validation of the training data is demonstrated by Figure 25a, which displays the network performance throughout the epochs. Figure 25b displays the gradient, μ variable, and validation test adjustments. As epochs rise, the error lowers, but the authenticity check remains constant, as evidenced by the gradient (9.98×10^{-8}) and $\mu 10^{-9}$ consistently dropping throughout iterations. The network attains its highest functionality at epoch 688, with a mean square error (MSE) of around 3.3141×10^{-10} . The little discrepancy between the network's output and target is further illustrated by the error histograms in Figure 25c, where the error almost matches the reference line with zero errors, with a value of about 1.08×10^{-7} . Additionally, there is a slight variation between the positive and negative. The correlation coefficient R is about 1, excluding randomness ($R = 0$) in the regression plot for Ea , which is displayed in Figure 25d. Figure 25f displays the function fit, whereas Figure 25e displays the autocorrelation. The fit lines closely resemble the desired output with little bias, demonstrating outstanding generalization and optimal accuracy. The outcomes of the ANN and bvp4c approaches are contrasted for a range of controlling parameter values in Figure 26. It is readily apparent that the ANN model can reliably reproduce the bvp4c approach findings throughout a range of λ , Rd , and Ea values.

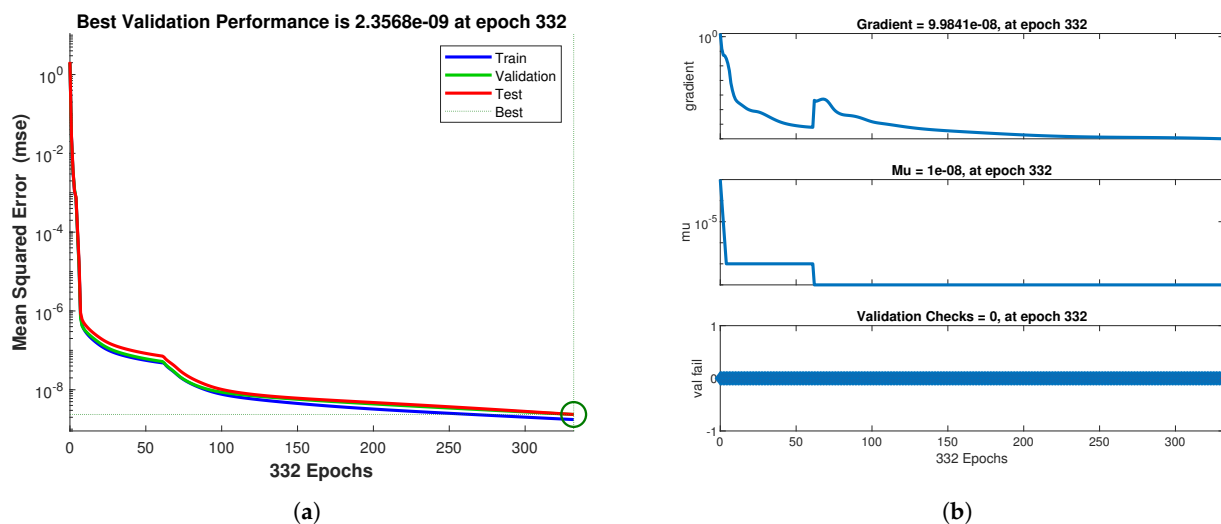


Figure 23. Cont.

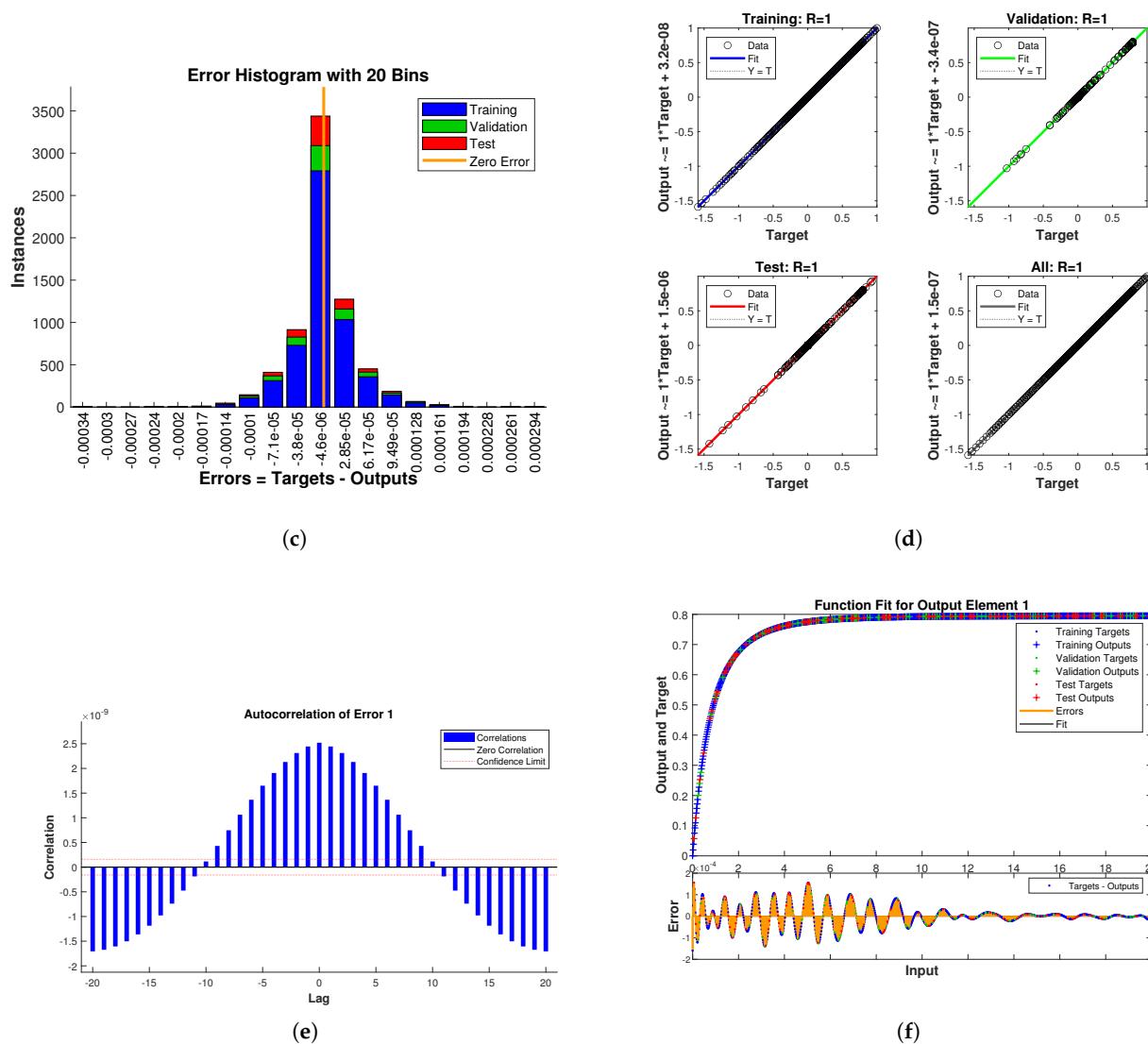


Figure 23. ANN training for porosity parameter (a): Mean Squared Error; (b): Training States; (c) Error Histogram; (d): Regression Analysis; (e): Auto-correlation; and (f): Function Fit.

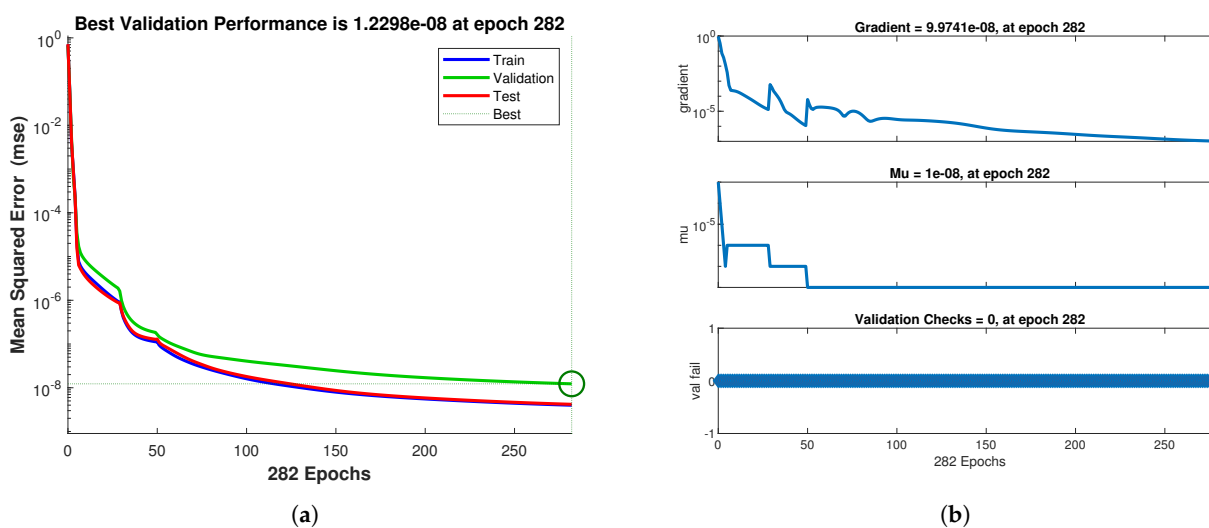


Figure 24. Cont.

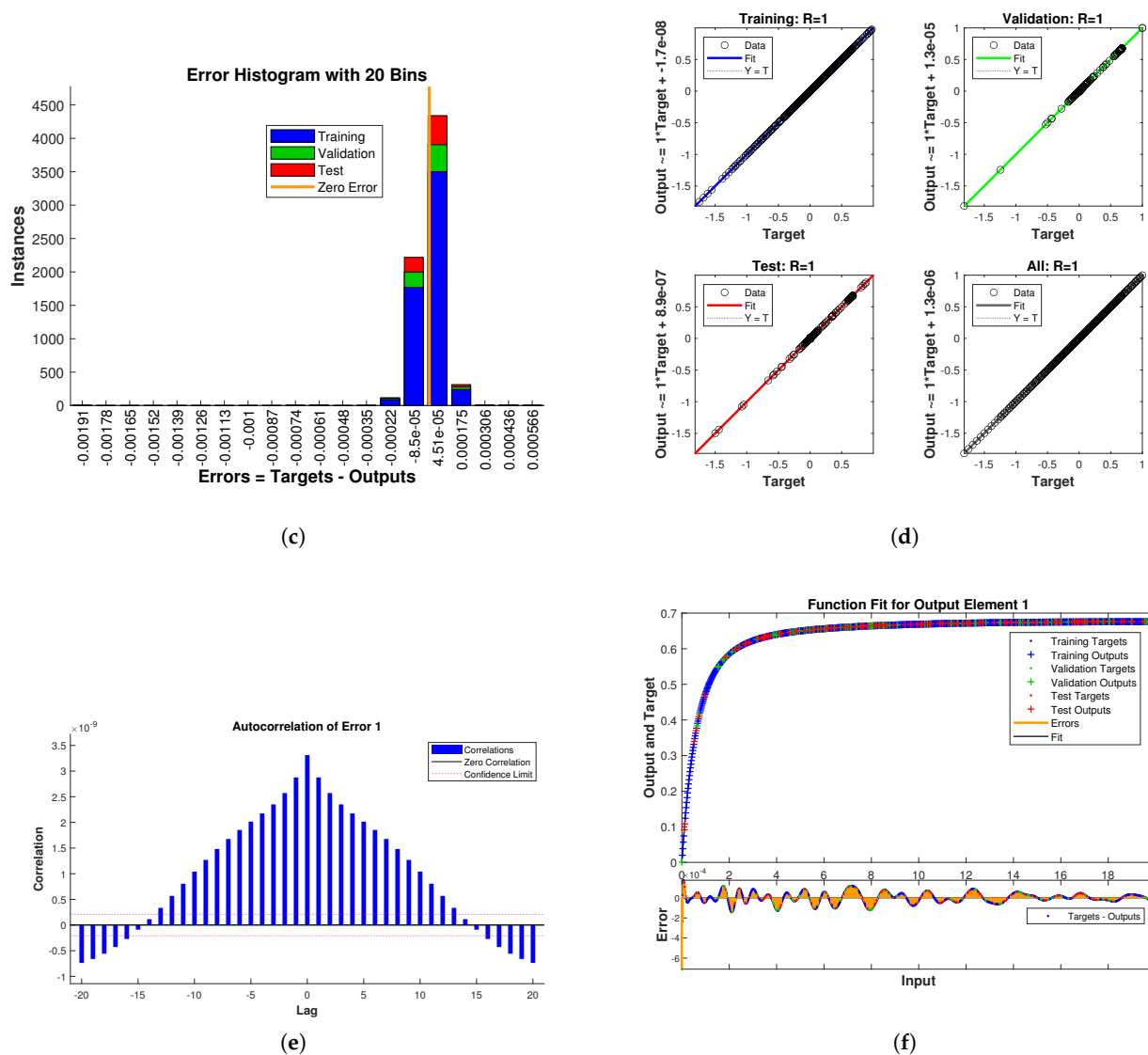


Figure 24. ANN training for thermal radiation parameter (a): Mean Squared Error; (b): Training States; (c) Error Histogram; (d): Regression Analysis; (e): Auto-correlation; and (f): Function Fit.

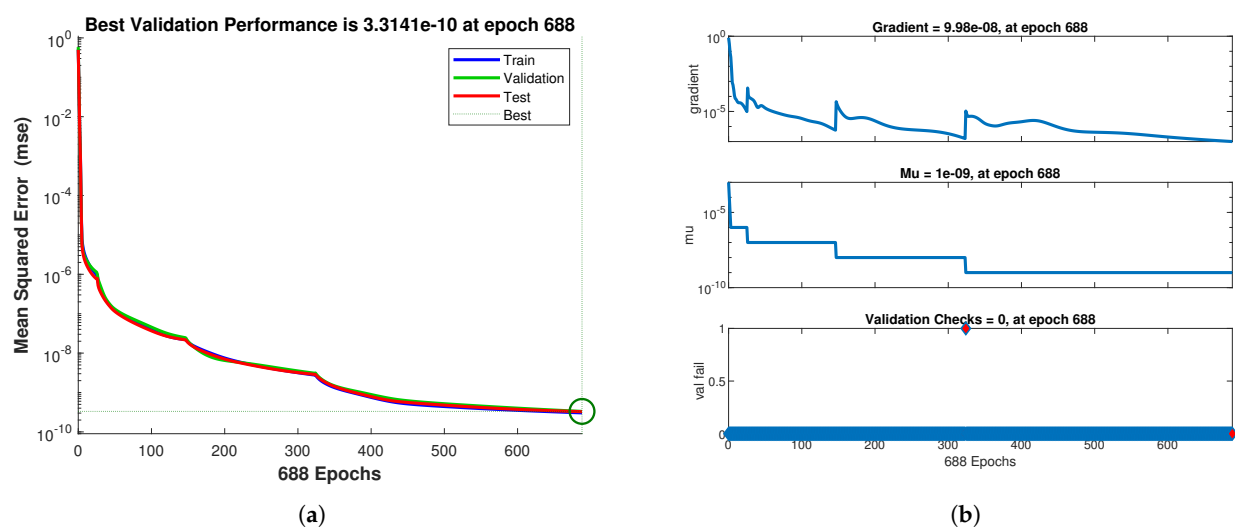


Figure 25. Cont.

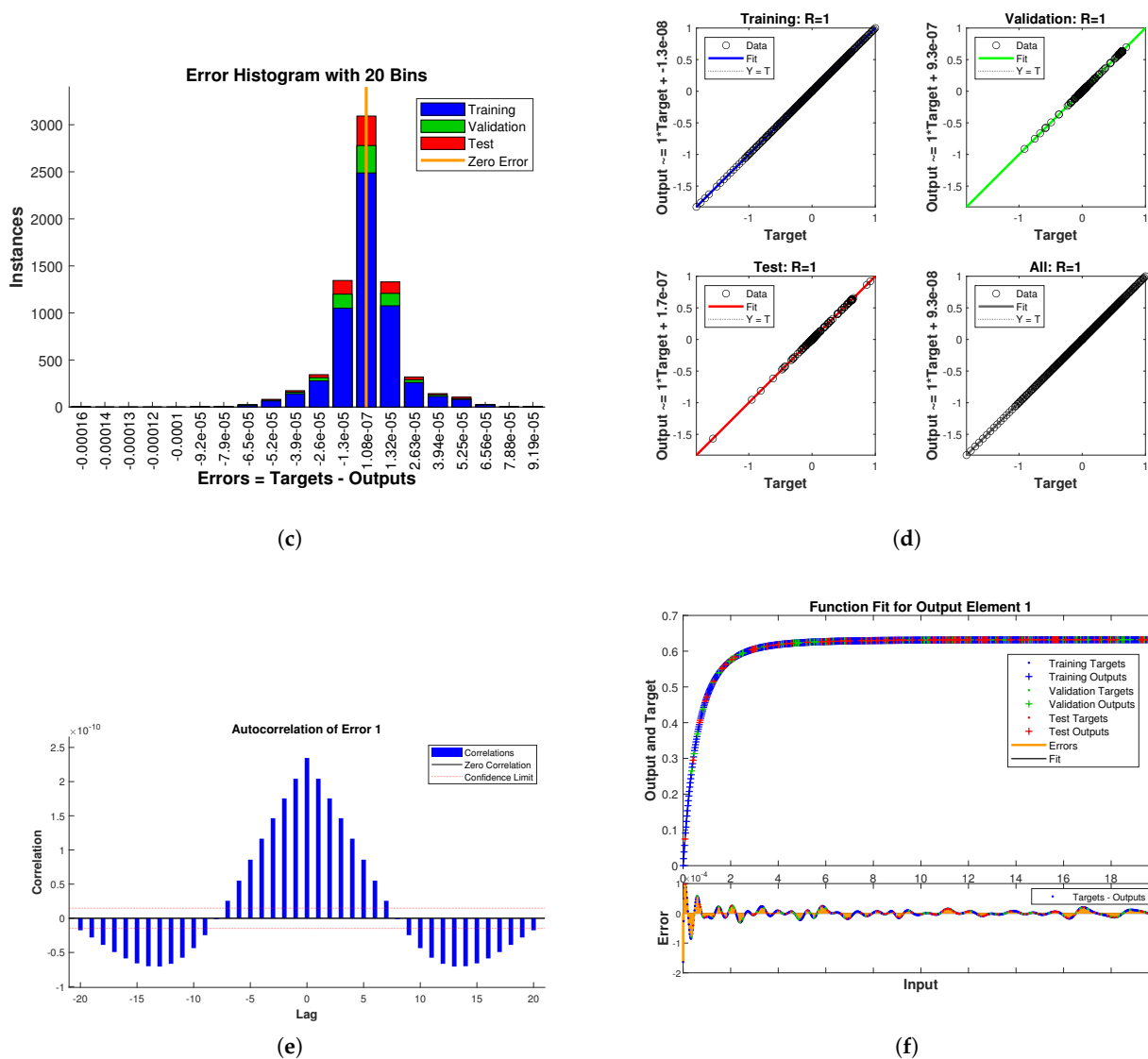


Figure 25. ANN training for activation energy parameter (a): Mean Squared Error; (b): Training States; (c) Error Histogram; (d): Regression Analysis; (e): Auto-correlation; and (f): Function Fit.

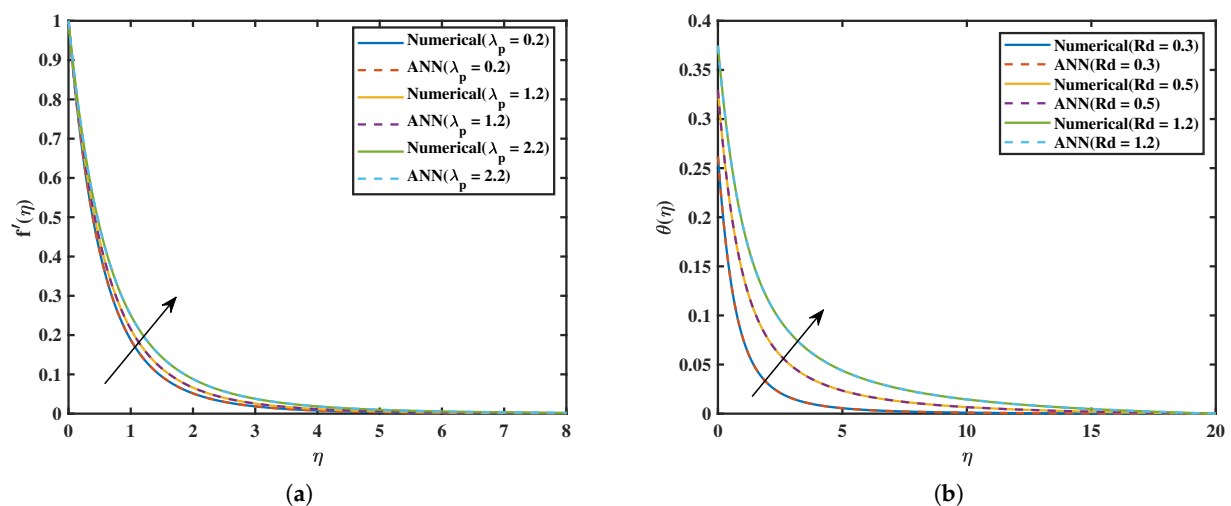


Figure 26. Cont.

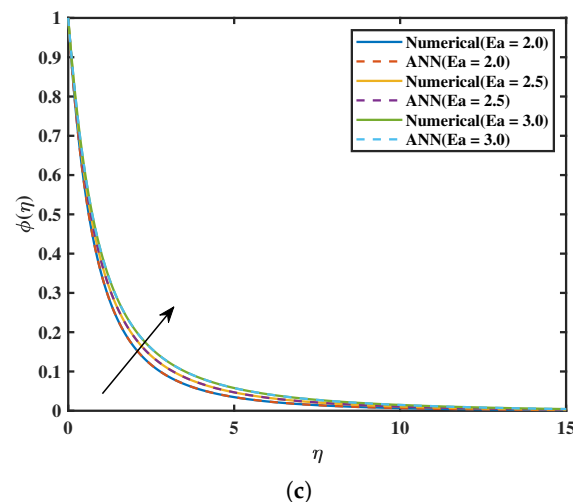


Figure 26. Comparison of Bvp4c and ANN for various parameters (a): Velocity for λ_p ; (b): Temperature for Rd ; and (c): Concentration for Ea .

9. Concluding Remarks

In comparison to the *MWCNT – Cu/kerosene* nanofluid under the same operating circumstances, the *SWCNT – TiO₂/kerosene* hybrid nanofluid showed a 23.6% increase in heat transfer rate (Nusselt number). While the addition of heat radiation increased the temperature field by over 12%, the addition of a magnetic field decreased the velocity by almost 15%. When compared to straight BVP4C simulations, the suggested ANN-numerical hybrid strategy reduced computing time by 90% while keeping prediction error below 1.5%. These numerical results confirm that the ANN framework is a dependable and effective real-time optimization tool for thermal system applications.

- The fluid's velocity drops as the values of magnetic parameter M , β , and Fr enhance, but it climbs when the values of α_1 , α_2 , λ_p , and γ improve.
- The *SWCNT – TiO₂/kerosene* hybrid exhibits superior heat transfer performance (higher Nusselt number) compared with the *MWCNT – Cu/kerosene* case across the considered parameter ranges.
- A modification in the angle of inclination, Biot number, curvature, magnetic parameter, and thermal radiation enhances the thermal profile.
- The temperature of both hybrid nanofluids decreases as the Prandtl fluid parameters increase.
- The shape factor strongly affects heat transfer: cylindrical nanoparticles (CNT-type) enhance thermal conductivity through elongated conduction paths, whereas brick-shaped particles yield weaker enhancement.
- Within the studied context, the created artificial neural network (ANN) model predicts outcomes with great efficiency and accuracy. Despite being tested using a brand-new hybrid nanofluid, it was able to accurately forecast heat transfer.
- The ANN surrogate (single hidden layer, 20 neurons, Levenberg–Marquardt training) reproduced the bvp4c results with <1.5% error and achieved a $\approx 93.8\%$ reduction in computation time, enabling real-time prediction and design optimization.
- Sherwood numbers rise in tandem with the values of α_1 , α_2 , Sc , and γ . On the other hand, it has a declining trend when M and Ec increase.
- As α_1 , α_2 , Fr , and β grow, so does the number of skin friction. Conversely, when the porosity parameter increases, it decreases.
- Nusselt numbers rise with increasing values of α_1 , α_2 , Rd , Bi , and γ , but they decrease with increasing magnetic parameter M .

- Future research might focus on transient and three-dimensional expansions of the model, multi-objective optimization utilizing the trained ANN surrogate, and experimental measurements of hybrid nanofluid stability and heat transport in cylindrical geometries.

Future Directions

To improve heat transfer efficiency, future studies might investigate tri-hybrid nanofluids, turbulent flow effects, and other base fluids. While examining changing magnetic fields and optimization strategies may yield useful industry insights, experimental validation and 3D simulations might increase the model's accuracy.

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Nomenclature

ρ_{nf}	Nanofluid density ($\text{kg}\cdot\text{m}^{-3}$)	σ	Electrical conductivity ($\text{S}\cdot\text{m}^{-1}$)
f	Velocity Profile	Pr	Prandtl Number
θ	Dimensionless temperature	(x, y)	Cartesian coordinates (m)
ν_{nf}	Kinematic viscosity of nanofluid (m^2s^{-1})	α_1	Curvature parameter
T_∞	Temperature away from the surface (K)	Rd	Radiation parameter
T	Non-dimensional temperature (K)	Sh	Sherwood number
C_f	Skin-friction factor	T_w	Temperature at surface (K)
K_{nf}	Thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$)	ρc_p	Heat capacity ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
Fr	Darcy forchheimer coefficient	Ea	Activation energy ($\text{J}\cdot\text{mol}^{-1}$)
ϕ	Dimensionless concentration	β	Inclination angle parameter
$Re(x)$	Reynolds number	Nu_x	Nusselt Number
h_1	Connective variable	D_T	Diffusion variable
(u, v)	Velocity components (m/s)	Sc	Schmidt Number
D_B	Brownian Diffusion factor	T_m	Mean fluid temperature
δ	porosity coefficient (s^{-1})	ψ	Stream function
σ	Electrical conductivity	c_p	Specific heat ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$)
δ	Temperature difference parameter	Bi	Biot number

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