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Coefficient Bounds for Two Subclasses of Analytic Functions Involving a Limacon-Shaped Domain

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Abstract: In the current exploration, we defined new subclasses of analytic functions, namely $R_{lim}(l, \nu)$ and $C_{lim}(l, \nu)$, defined by subordination linked with a Limacon-shaped domain. We found a few initial coefficient bounds and Fekete–Szegő inequalities for the functions in the above-stated new classes. The corresponding results have been derived for the function h^{-1} . Additionally, we discuss the Poisson distribution as an application of our consequences.

Keywords: analytic; starlike; subordination; Fekete–Szegő functional; Hankel determinant; convolution; limacon domain

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1. Introduction and Motivation

Consider the class $\mathcal{A}$ that signifies the family of all holomorphic functions $h(\zeta)$ normalized by $h(0) = 0$, $h'(0) = 1$ defined in the domain of open unit disc $\Delta := \{\zeta \in \mathbb{C} : |\zeta| < 1\}$. In view of the above normalization, the function $h(\zeta)$ admits a “Taylor–Maclaurin” series expansion given by

$$h(\zeta) = \zeta + \sum_{n=2}^{\infty} h_n \zeta^n \quad (\zeta \in \Delta) \quad (1)$$

where h_n are the coefficients of the function $h(\zeta)$.

The subclass of $\mathcal{A}$ that is univalent in Δ is denoted by $\mathcal{S}$. For two functions $g, h \in \mathcal{A}$, we call the function h subordinate to another function g , or g as superordinate to h , represented as $h \prec g$ if there exists a function $\omega(\zeta)$ with $\omega(0) = 0$ and $|\omega(\zeta)| < 1$, such that

$$h(\zeta) = g(\omega(\zeta)) \quad (\zeta \in \Delta). \quad (2)$$

In other words, for a function $h \in \mathcal{A}$ specified by (1), Pommerenke [1,2] introduced the k th Hankel determinant as:

$$H_{k,n}(h) = \begin{vmatrix} h_n & h_{n+1} & \cdots & h_{n+k-1} \\ h_{n+1} & h_{n+2} & \cdots & h_{n+k} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n+k-1} & h_{n+k} & \cdots & h_{n+2(k-1)} \end{vmatrix} \quad (n, k \in \mathbb{N} = 1, 2, 3 \dots).$$

It is constructive to identify whether certain coefficient functionals connected to the function f are bounded in the disc Δ or not. In particular, Noor [3] investigated the asymptotic behavior as $n \rightarrow \infty$ of $H_{k,n}(f)$. In [1], Pommerenke discusses a few of the applications of Hankel determinants for learning about the presence of singularities. Various researchers have also considered the determinant for different subclasses of $\mathcal{A}$ in various ways, and their consequences are presented in the literature survey. For details, interested readers can see [4–10]. For the choice of “ $k = 2$, $n = 1$ ” and for the choice of “ $k = n = 2$ ”, we have

$$H_{2,1}(h) = \begin{vmatrix} 1 & h_2 \\ h_2 & h_3 \end{vmatrix} = h_3 - h_2^2,$$

and

$$H_{2,2}(h) = \begin{vmatrix} h_2 & h_3 \\ h_3 & h_4 \end{vmatrix} = h_2h_4 - h_3^2.$$

Note that $H_{2,1}(h)$ and $H_{2,2}(h)$ are popularly known special cases for Fekete–Szegő inequality for the choice of $\mu = 1$ and the Hankel determinant, respectively.

Various subfamilies involving the $\mathcal{S}$ class of univalent functions that had developed during the period of Bieberbach conjecture are, respectively, the classes of functions that are starlike, convex, close-to-convex, and so on. In 1992, Ma and Minda [11] introduced and defined two new classes of analytic function classes, namely:

$$\mathcal{S}^*(\phi) = \left\{ h \in \mathcal{A} : \frac{\xi h'(\xi)}{h(\xi)} \prec \phi(\xi) \quad (\xi \in \Delta) \right\}$$

and

$$\mathcal{C}(\phi) = \left\{ h \in \mathcal{A} : 1 + \frac{\xi h''(\xi)}{h'(\xi)} \prec \phi(\xi) \quad (\xi \in \Delta) \right\}$$

where the function $\phi(\xi)$ is an analytic function with a positive real part on Δ with $\phi(0) = 1$, $\phi'(0) > 0$, which maps the unit disc Δ onto a starlike region with respect to 1 and is symmetric with respect to the real axis. Various subclasses of starlike and convex functions satisfying the condition of the Ma and Minda type for different choices of the function ϕ have been studied by different researchers.

Sokół and Stankiewicz [12] studied and discussed the starlike class $\mathcal{S}_L^*$ connected with the right-half of the lemniscate of Bernoulli $(u^2 + v^2)^2 - 2(u^2 - v^2) = 0$. In the sense of Ma and Minda, one can express $\mathcal{S}_L^* := \mathcal{S}^*(\sqrt{1 + \xi})$. Further, a function $h \in \mathcal{S}_L^*$ is known as a Sokół and Stankiewicz starlike function. Sokół [13] introduced one more important class, $\mathcal{S}_{q_c}^* := \mathcal{S}^*(q_c)$, where $q_c(\xi) = \sqrt{1 + c\xi}$ with $c \in (0, 1]$. For $c \in (0, 1)$, the function $q_c(\xi)$ maps the unit disk Δ onto the interior of right loop of the Cassinian ovals $(u^2 + v^2)^2 - 2(u^2 - v^2) = c^2 - 1$. There are many classes discussed in detail by specializing the function $\phi(\xi)$ fulfilling the conditions of Ma and Minda. The classes are listed herewith in the paragraph below. Mendiratta et al. [14] introduced the class $\mathcal{S}_{RL}^* := \mathcal{S}^*(\phi_{RL})$ with

$$\phi_{RL}(\xi) = \sqrt{2} - (\sqrt{2} - 1) \sqrt{\frac{1 - \xi}{1 + 2(\sqrt{2} - 1)\xi}}.$$

It is observed that the function $\phi_{RL}(\xi)$ maps Δ onto the region enclosed by the left-half of the shifted lemniscate of Bernoulli $((u - \sqrt{2})^2 + v^2)^2 - 2((u - \sqrt{2})^2 - v^2) = 0$. The class of function $\mathcal{S}_{\zeta}^* := \mathcal{S}^*(\zeta + \sqrt{1 + \zeta^2})$ was introduced and discussed by Raina and Sokół [15] with further investigations in [16–18]. We remark that, in this case, the function $\phi_{\zeta}(\xi) = \zeta + \sqrt{1 + \zeta^2}$ maps Δ onto the crescent-shaped region $\{w \in \mathbb{C} : |w^2 - 1| < 2|w|, \Re w > 0\}$. An examination of the class $\mathcal{S}_{e^{\zeta}}^* := \mathcal{S}^*(e^{\zeta})$ was introduced and investigated by Mendiratta et al. [19], while $\mathcal{S}_C^* := \mathcal{S}^*(1 + \frac{4\xi}{3} + \frac{2\xi^2}{3})$, coupled with the cardioid $(9u^2 + 9v^2 - 18u + 5)^2 - 16(9u^2 + 9v^2 - 6u + 1) = 0$, a heart-shaped curve, was

investigated in detail by Sharma et al. [20]. Kumar and Ravichandran [21] considered and discussed the function class $\mathcal{S}_R^* := \mathcal{S}^*(\phi_0)$, associated with a rational function $\phi_0(\xi)$

$$\phi_0(\xi) := 1 + \frac{\xi}{k} \left(\frac{k + \xi}{k - \xi} \right) = 1 + \frac{1}{k}\xi + \frac{2}{k^2}\xi^2 + \cdots, \quad (k = 1 + \sqrt{2}). \quad (3)$$

Yunus et al. [22] studied the class $\mathcal{S}_{lim}^* := \mathcal{S}^*(1 + \sqrt{2}\xi + \xi^2/2)$ associated with a limaçon domain $(4u^2 + 4v^2 - 8u - 5)^2 + 8(4u^2 + 4v^2 - 12u - 3) = 0$, and Kargar et al. [23] considered and discussed a function class

$$\mathcal{B}\mathcal{S}(\alpha) := \mathcal{S}^* \left(1 + \frac{\xi}{1 - \alpha\xi^2} \right), \quad 0 \leq \alpha < 1,$$

a starlike class associated with the Booth lemniscate. For $0 \leq \alpha < 1$, Khatter et al. [24] discussed the classes

$$\mathcal{S}_{\alpha,e}^* := \mathcal{S}^*(\alpha + (1 - \alpha)e^\xi) \text{ and } \mathcal{S}_{\alpha,L}^* := \mathcal{S}^*(\alpha + (1 - \alpha)\sqrt{1 + \xi}),$$

that involve the exponential function and the lemniscate of Bernoulli. Trivially, for the choice of $\alpha = 0$, the above classes reduce to the function classes $\mathcal{S}_e^*$ and $\mathcal{S}_L^*$, respectively. A recent investigation about the starlike class $\mathcal{S}_{SG}^* := \mathcal{S}^*(\phi_{SG})$, associated with the sigmoid function $\phi_{SG}(\xi) = \frac{2}{1 + e^{-\xi}}$, was systematically performed by Goel and Kumar [25]. In some recent papers of [26,27], as well as [28], certain subclasses of $\mathcal{A}$ were defined by means of the subordination that $\xi h'(\xi)/h(\xi) \prec \phi(\xi)$ for $\xi \in \mathbb{D}$, where ϕ was not necessarily univalent.

For instance, the coefficient inequalities of the function associated with petal type domains were considered by Malik et al. [29]. Yunus et al. [22] (see also [30,31]) introduced a region bounded by a bean-shaped limaçon domain as follows:

$$\omega(\Delta) = \{w = x + iy : (4x^2 + 4y^2 - 8x - 5)^2 + 8(4x^2 + 4y^2 - 12x - 3) = 0\}.$$

Assume that $l : \Delta \rightarrow \mathbb{C}$ is defined by

$$l(\xi) = 1 + \sqrt{2}\xi + \frac{1}{2}\xi^2. \quad (4)$$

Relation (4) is selected so that Limaçon $l(\xi)$ is in the bean shape.

Motivated by the works of the above-mentioned researchers, and using the concept of subordination between the two analytic functions, we here introduce generalized classes of analytic functions satisfying the following subordination conditions.

Definition 1. Let $0 \leq \nu \leq 1$. Let $h \in \mathcal{A}$. Then, h is said to belong to the class $\mathcal{R}_{lim}(l, \nu)$ if the following condition is satisfied:

$$[h'(\xi)]^\nu \left[\frac{\xi h'(\xi)}{h(\xi)} \right]^{1-\nu} \prec l(\xi) \quad (\xi \in \Delta) \quad (5)$$

Note that

$$\mathcal{R}_{lim}^*(l) = \mathcal{R}_{lim}(l, 0) = \{h \in \mathcal{A} : \frac{\xi h'(\xi)}{h(\xi)} \prec l(\xi) \quad (\xi \in \Delta)\},$$

and

$$\mathcal{R}_{lim}^{**}(l) = \mathcal{R}_{lim}(l, 1) = \{h \in \mathcal{A} : h'(\xi) \prec l(\xi) \quad (\xi \in \Delta)\}.$$

We also define another new class as follows.

Definition 2. Let $0 \leq \nu \leq 1$. If $h \in \mathcal{A}$, then h is said to be in the class $\mathcal{C}_{lim}(l, \nu)$, if it satisfies the subordination condition:

$$[h'(\xi)]^\nu \left[1 + \frac{\xi h''(\xi)}{h'(\xi)} \right]^{1-\nu} \prec l(\xi) \quad (\xi \in \Delta). \quad (6)$$

It may be noted that for $\nu = 0$ and $\nu = 1$, we obtain the following classes respectively:

$$\mathcal{C}_{lim}^*(l) = \mathcal{C}_{lim}(l, 0) = \{h \in \mathcal{A} : 1 + \frac{\xi h''(\xi)}{h'(\xi)} \prec l(\xi) \quad (\xi \in \Delta)\},$$

and

$$\mathcal{C}_{lim}^{**}(l) = \mathcal{C}_{lim}(l, 1) = \{h \in \mathcal{A} : h'(\xi) \prec l(\xi) \quad (\xi \in \Delta)\}.$$

The objective of the present article is to investigate the coefficient inequalities, Fekete–Szegő functional $|h_3 - \mu h_2^2|$ for both the real parameter and complex parameter μ and coefficient inequality for the inverse function h^{-1} for the above classes. We also discuss Poisson distribution as an application to our results.

2. A Set of Preliminaries

Let us denote by $\mathcal{P}$ the class of functions $q(\xi)$ that are holomorphic with a positive real part in Δ and having a representation of the form:

$$q(\xi) = 1 + \sum_{n=1}^{\infty} q_n \xi^n \quad (\xi \in \Delta). \quad (7)$$

For our current investigations, the following results in the form of lemmas are required, and are stated as follows.

Lemma 1 (see [32]). Let $q \in \mathcal{P}$ and be of the form (7). Then, for $\forall n \in \mathbb{N} := \{1, 2, 3, \dots\}$,

$$|q_n| \leq 2. \quad (8)$$

Lemma 2 (see [11]). Let $q \in \mathcal{P}$ and be of the form (7). If μ , is any complex number, then we have

$$|q_2 - \mu q_1^2| \leq 2 \max\{1, |2\mu - 1|\}. \quad (9)$$

Lemma 3 ([11]). Let $q \in \mathcal{P}$ and be of the form (7). Then,

$$|q_2 - \nu q_1^2| \leq \begin{cases} -4\nu + 2 & \nu \leq 0, \\ 2 & 0 \leq \nu \leq 1, \\ 4\nu - 2 & \nu \geq 1. \end{cases} \quad (10)$$

3. Coefficient Bounds and the Fekete–Szegő Estimate

We start the section with the following theorem, where in we obtained the first two Taylor–Maclaurin coefficients for the function class $\mathcal{R}_{lim}(l, \nu)$ and $\mathcal{C}_{lim}(l, \nu)$.

Theorem 1. Let $h \in \mathcal{A}$ in the form (1) belong to the class $\mathcal{R}_{lim}(l, \nu)$. Then, we have

$$|h_2| \leq \frac{\sqrt{2}}{1 + \nu}, \quad (11)$$

$$|h_3| \leq \frac{\sqrt{2}}{(2 + \nu)} \max \left\{ 1, \frac{5 - \nu^2}{2\sqrt{2}(1 + \nu)^2} \right\}. \quad (12)$$

Proof. Let the function h defined in (1) be in the class $\mathcal{R}_{lim}(l, \nu)$. By virtue of Definition 1, there exists a function $\omega(\xi)$, which is analytic and satisfies the condition of the Schwarz lemma such that

$$[h'(\xi)]^\nu \left[\frac{\xi h'(\xi)}{h(\xi)} \right]^{1-\nu} = l(\omega(\xi)) \quad (\xi \in \Delta). \quad (13)$$

Let $q \in \mathcal{P}$. By using the Schwarz function $\omega(\xi)$, and applying the definition of subordination, we can write

$$q(\xi) = \frac{1 + \omega(\xi)}{1 - \omega(\xi)} = 1 + q_1\xi + q_2\xi^2 + q_3\xi^3 + \dots,$$

which, once for all, gives

$$\begin{aligned} \omega(\xi) = Q(\xi) &= \frac{q_1\xi + q_2\xi^2 + q_3\xi^3 + \dots}{2 + q_1\xi + q_2\xi^2 + \dots} \\ &= \frac{1}{2}(q_1\xi + q_2\xi^2 + q_3\xi^3 + \dots) \left(1 + \frac{q_1\xi + q_2\xi^2 + \dots}{2} \right)^{-1} \\ &= \frac{q_1}{2}\xi + \left(\frac{q_2}{2} - \frac{q_1^2}{4} \right)\xi^2 + \left(\frac{q_1^2}{8} - \frac{1}{2}q_1q_2 + \frac{q_3}{2} \right)\xi^3 + \dots \end{aligned} \quad (14)$$

where $Q(\xi) = (q(\xi) - 1)/(q(\xi) + 1)$. Using the relation given in (14) in the representation of $l(\omega(\xi))$, we obtain

$$\begin{aligned} l(\omega(\xi)) &= l\left(\frac{q(\xi) - 1}{q(\xi) + 1}\right) = 1 + \frac{q_1}{\sqrt{2}}\xi + \left[\frac{1}{\sqrt{2}}\left(q_2 - \frac{q_1^2}{2}\right) + \frac{q_1^2}{8} \right]\xi^2 \\ &\quad + \left[\frac{1}{\sqrt{2}}\left(q_3 - q_1q_2 + \frac{q_1^3}{4}\right) + \frac{q_1}{4}\left(q_2 - \frac{q_1^2}{2}\right) \right]\xi^3 + \dots \end{aligned} \quad (15)$$

On the other hand, from (1), it is trivial and can be derived that

$$\begin{aligned} [h'(\xi)]^\nu \left[\frac{\xi h'(\xi)}{h(\xi)} \right]^{1-\nu} &= 1 + (1 + \nu)h_2\xi + (2 + \nu)\left(h_3 - \frac{1 - \nu}{2}h_2^2\right)\xi^2 \\ &\quad + \frac{(3 + \nu)}{6}[6h_4 - 6(1 - \nu)h_2h_3 + (1 - \nu)(2 - \nu)h_2^3]\xi^3 + \dots \end{aligned} \quad (16)$$

Now, by using (15) and (16) in (13), and then by equating the coefficients of ξ and ξ^2 on both sides, we easily obtain

$$h_2 = \frac{q_1}{\sqrt{2}(1 + \nu)}, \quad (17)$$

$$\begin{aligned} (2 + \nu)\left(h_3 - \frac{1 - \nu}{2}h_2^2\right) &= \frac{1}{\sqrt{2}}\left(q_2 - \frac{q_1^2}{2}\right) + \frac{q_1^2}{8} \\ \Rightarrow h_3 &= \frac{1}{\sqrt{2}(2 + \nu)}\left(q_2 - \frac{2\sqrt{2}(1 + \nu)^2 + (\nu^2 - 5)}{4\sqrt{2}(1 + \nu)^2}q_1^2\right) \\ &= \frac{1}{\sqrt{2}(2 + \nu)}[q_2 - \gamma_1q_1^2], \end{aligned} \quad (18)$$

where

$$\gamma_1 = \frac{2\sqrt{2}(1 + \nu)^2 + (\nu^2 - 5)}{4\sqrt{2}(1 + \nu)^2}.$$

For h_2 , utilizing (8) of Lemma 1 in (17), we obtain

$$|h_2| \leq \frac{\sqrt{2}}{1+\nu}. \quad (19)$$

For h_3 , applying (9) of Lemma 2 in (18), one will obtain

$$|h_3| \leq \frac{\sqrt{2}}{(2+\nu)} \max \left\{ 1, \left| \frac{5-\nu^2}{2\sqrt{2}(1+\nu)^2} \right| \right\}. \quad (20)$$

The proof of Theorem 1 is thus completed. $\square$

Theorem 2. Let the function $h \in \mathcal{A}$ of the form (1) be in the class $\mathcal{C}_{lim}(l, \nu)$. Then,

$$|h_2| \leq \frac{1}{\sqrt{2}}, \quad (21)$$

$$|h_3| \leq \frac{\sqrt{2}}{3(2-\nu)} \max \left\{ 1, \frac{5-4\nu}{2\sqrt{2}} \right\}. \quad (22)$$

Proof. Let the function h given by (1) be in the class $\mathcal{C}_{lim}(l, \nu)$. Then, from Definition 2, there exists an analytic function $\omega(\xi)$ satisfying the condition of the Schwarz lemma such that

$$[h'(\xi)]^\nu \left[1 + \frac{\xi h''(\xi)}{h'(\xi)} \right]^{1-\nu} = l(\omega(\xi)) \quad (\xi \in \Delta). \quad (23)$$

From (1), it can be easily derived that

$$\begin{aligned} [h'(\xi)]^\nu \left[1 + \frac{\xi h''(\xi)}{h'(\xi)} \right]^{1-\nu} &= 1 + 2h_2\xi + [3(2-\nu)h_3 - 4(1-\nu)h_2^2]\xi^2 \\ &\quad + [8(1-\nu)h_2^3 - 18(1-\nu)h_2h_3 - 4(3-2\nu)h_4]\xi^3 + \dots \end{aligned} \quad (24)$$

Using (15) and (24) in (23), and then by equating the coefficients of ξ and ξ^2 on both sides, we easily obtain

$$h_2 = \frac{q_1}{2\sqrt{2}}, \quad (25)$$

$$\begin{aligned} 3(2-\nu)h_3 - 4(1-\nu)h_2^2 &= \frac{1}{\sqrt{2}} \left(q_2 - \frac{q_1^2}{2} \right) + \frac{q_1^2}{8} \\ \implies h_3 &= \frac{1}{3\sqrt{2}(2-\nu)} \left[q_2 - \frac{4\nu + 2\sqrt{2} - 5}{4\sqrt{2}} \right] q_1^2. \end{aligned} \quad (26)$$

Making use of (8) of Lemma 1 and (9) of Lemma 2, respectively, gives the bounds of h_2 and h_3 . This proves the result of Theorem 2. $\square$

The next theorems give Fekete–Szegő inequality for the class $\mathcal{R}_{lim}(l, \nu)$ and $\mathcal{C}_{lim}(l, \nu)$ when μ is both complex and real.

Theorem 3. Let the function $h \in \mathcal{A}$ belong to the class $\mathcal{R}_{lim}(l, \nu)$. Then, for any complex number μ , we have

$$|h_3 - \mu h_2^2| \leq \frac{\sqrt{2}}{2+\nu} \max \left\{ 1, \left| \frac{(\nu^2 - 5) + 4(2+\nu)\mu}{2\sqrt{2}(1+\nu)^2} \right| \right\} \quad (27)$$

Proof. From relations (17) and (18), we obtain

$$|h_3 - \mu h_2^2| = \frac{1}{\sqrt{2}(2+\nu)} [q_2 - \gamma_2 q_1^2], \quad (28)$$

where

$$\gamma_2 = \frac{2\sqrt{2}(1+\nu)^2 + (\nu^2 - 5) + 4(2+\nu)\mu}{4\sqrt{2}(1+\nu)^2}. \quad (29)$$

An application of Lemma 2 to relation (29) gives

$$|h_3 - \mu h_2^2| \leq \frac{\sqrt{2}}{2+\nu} \max \left\{ 1, \left| \frac{(\nu^2 - 5) + 4(2+\nu)\mu}{2\sqrt{2}(1+\nu)^2} \right| \right\}.$$

This essentially completes the proof of Theorem 3. $\square$

Theorem 4. If the function $h \in \mathcal{A}$ belongs to the function class $\mathcal{R}_{lim}(l, \nu)$, then for any real number μ , we have

$$|h_3 - \mu h_2^2| \leq \begin{cases} -\frac{(\nu^2-5)+4(2+\nu)\mu}{2(1+\nu)^2(2+\nu)} & \mu \leq -\frac{(\nu^2-5)+2\sqrt{2}(1+\nu)^2}{4(2+\nu)} \\ \frac{\sqrt{2}}{2+\nu} & -\frac{2\sqrt{2}(1+\nu)^2+\nu^2-5}{4(2+\nu)} \leq \mu \leq \frac{2\sqrt{2}(1+\nu)^2-\nu^2+5}{4(2+\nu)} \\ \frac{(\nu^2-5)+4(2-\nu)\mu}{2(1+\nu)^2(2+\nu)} & \mu \geq \frac{2\sqrt{2}(1+\nu)^2-\nu^2+5}{4(2+\nu)}. \end{cases} \quad (30)$$

Proof. An application of Lemma 3 to relation (28) gives the desired estimates, as stated in the theorem. This completes the proof. $\square$

Theorem 5. Let the function $h \in \mathcal{A}$ belong to the class $\mathcal{C}_{lim}(l, \nu)$. Then, for any complex number μ , we have

$$|h_3 - \mu h_2^2| \leq \frac{\sqrt{2}}{3(2-\nu)} \max \left\{ 1, \left| \frac{(4\nu - 5) + 3(2-\nu)\mu}{2\sqrt{2}} \right| \right\}. \quad (31)$$

Proof. From (25) and (26), we obtain

$$|h_3 - \mu h_2^2| = \frac{1}{3\sqrt{2}(2-\nu)} |q_2 - \gamma_3 q_1^2| \quad (32)$$

where

$$\gamma_3 = \frac{(4\nu + 2\sqrt{2} - 5) + 3(2-\nu)\mu}{4\sqrt{2}} \quad (33)$$

Application of Lemma 2 to (32) yields the required bound. The proof of Theorem 5 is thus completed. $\square$

Theorem 6. Assume that $h \in \mathcal{C}_{lim}(l, \nu)$. Then, for a real μ , we have

$$|h_3 - \mu h_2^2| \leq \begin{cases} -\frac{\nu(4-3\mu)+6\mu-5}{6(2-\nu)} & \mu \leq -\frac{4\nu+2\sqrt{2}-5}{3(2-\nu)} \\ \frac{\sqrt{2}}{3(2-\nu)} & -\frac{4\nu+2\sqrt{2}-5}{3(2-\nu)} \leq \mu \leq \frac{2\sqrt{2}-4\nu+5}{3(2-\nu)} \\ \frac{\nu(4-3\mu)+6\mu-5}{6(2-\nu)} & \mu \geq \frac{2\sqrt{2}-4\nu+5}{3(2-\nu)}. \end{cases} \quad (34)$$

Proof. Making use of Lemma 3 in relation (32) yields the estimate as mentioned in the theorem. This completes the proof. $\square$

4. Coefficient Inequalities for the Function h^{-1}

Theorem 7. If the function $h \in \mathcal{R}_{lim}(l, \nu)$, given by (1) and $h^{-1}(w) = w + \sum_{n=2}^{\infty} l_n w^n$, is the analytic continuation of the Δ of the inverse function of h with $|w| < r_0$, where $r_0 > \frac{1}{4}$ (the radius of the Koebe domain), then, for any complex number ν , we have

$$|l_2| \leq \frac{\sqrt{2}}{1 + \nu}, \quad (35)$$

$$|l_3| \leq \frac{\sqrt{2}}{2 + \nu} \left[q_2 - \frac{8(2 + \nu) + 2\sqrt{2}(1 + \nu)^2 + (\nu^2 - 5)}{4\sqrt{2}(1 + \nu)^2} \right] \quad (36)$$

and

$$|l_3 - \nu l_2^2| \leq \frac{\sqrt{2}}{(2 + \nu)} \max \left\{ 1, \left| \frac{(\nu^2 - 5) + 4(2 + \nu)(2 - \nu)}{4\sqrt{2}(1 + \nu)^2} \right| \right\}. \quad (37)$$

Proof. Since

$$h^{-1}(w) = w + \sum_{n=2}^{\infty} l_n w^n \quad (38)$$

is the inverse of h , we have

$$h^{-1}(h(\xi)) = h(h^{-1}(\xi)) = \xi. \quad (39)$$

From (39), we have

$$h^{-1}(\xi + \sum_{n=2}^{\infty} h_n \xi^n) = \xi \quad (40)$$

From (38) and (40), we have

$$\xi + (h_2 + l_2)\xi^2 + (h_3 + 2h_2l_2 + l_3)\xi^3 + \dots = \xi. \quad (41)$$

Comparing the coefficients of ξ^2 and ξ^3 on both sides of (41), we obtain

$$l_2 = -h_2 \quad (42)$$

and

$$l_3 = -h_3 - 2h_2l_2 = 2h_2^2 - h_3. \quad (43)$$

Taking the values of h_2 and h_3 from (17) and (18) in (42) and (43), we obtain

$$l_2 = -\frac{q_1}{\sqrt{2}(1 + \nu)} \quad (44)$$

and

$$l_3 = -\frac{1}{\sqrt{2}(2 + \nu)} \left[q_2 - \frac{8(2 + \nu) + 2\sqrt{2}(1 + \nu)^2 + (\nu^2 - 5)}{4\sqrt{2}(1 + \nu)^2} q_1^2 \right]. \quad (45)$$

The bound for l_2 can be obtained by using (8) of Lemma 1. Further, an application of Lemma 2 to (45) gives

$$|l_3| \leq \frac{\sqrt{2}}{2 + \nu} \max \left\{ 1, \left| \frac{8(2 + \nu) + (\nu^2 - 5)}{2\sqrt{2}(1 + \nu)^2} \right| \right\}$$

Moreover, for any complex number ν , we have

$$|l_3 - \nu l_2^2| = \frac{1}{\sqrt{2}(2 + \nu)} \left| q_2 - \frac{2\sqrt{2}(1 + \nu)^2 + \nu^2 - 5 + 4(2 + \nu)(2 - \nu)}{4\sqrt{2}(1 + \nu)^2} q_1^2 \right|. \quad (46)$$

Making use of Lemma 2 in (46) yields

$$|l_3 - \nu l_2^2| \leq \frac{\sqrt{2}}{2+\nu} \max \left\{ 1, \frac{|(\nu^2 - 5) + 4(2 + \nu)(2 - \nu)|}{4\sqrt{2}(1 + \nu)^2} \right\}.$$

This completes the proof. $\square$

Theorem 8. If the function $h \in \mathcal{C}_{lim}(l, \nu)$ given by (1) and $h^{-1}(w) = w + \sum_{n=2}^{\infty} l_n w^n$ is the analytic continuation of the Δ of the inverse function of h , with $|w| < r_0$, where $r_0 > \frac{1}{4}$ (the radius of the Koebe domain), then, for any complex number ν , we have

$$|l_2| \leq \frac{1}{\sqrt{2}}, \quad (47)$$

$$|l_3| \leq \frac{\sqrt{2}}{3(2 - \nu)} \max \left\{ 1, \left| \frac{7 - 2\nu}{2\sqrt{2}} \right| \right\} \quad (48)$$

and

$$|l_3 - \nu l_2^2| \leq \frac{\sqrt{2}}{3(2 - \nu)} \max \left\{ 1, \frac{|7 - 2\nu - 3(2 - \nu)\nu|}{2\sqrt{2}} \right\}. \quad (49)$$

Proof. Proceeding Theorem 7 and substituting the values of h_2 and h_3 from (25) and (26) into (42) and (43), we obtain

$$l_2 = -\frac{q_1}{2\sqrt{2}} \quad (50)$$

and

$$l_3 = -\frac{1}{3\sqrt{2}(2 - \nu)} \left[q_2 - \frac{7 + 2\sqrt{2} - 2\nu}{4\sqrt{2}} q_1^2 \right]. \quad (51)$$

The bound of l_2 can be obtained by virtue of Lemma 1. Further, the application of Lemma 2 to (51) gives the required estimate.

Moreover, for complex parameter ν , we have

$$|l_3 - \nu l_2^2| = \frac{1}{3\sqrt{2}} \left| q_2 - \frac{7 - 6\nu + 2\sqrt{2} + \nu(3\nu - 2)}{4\sqrt{2}} q_1^2 \right| \quad (52)$$

Using Lemma 2 in (52) gives the result. The proof of Theorem 8 is thus completed. $\square$

5. Application of the Poisson Distribution

Now, we discuss the application of the Poisson distribution to the results obtained for the function classes $\mathcal{R}_{lim}(l, \nu)$ and $\mathcal{C}_{lim}(l, \nu)$.

Definition 3. Let X be a discrete random variable. Then, we say that X is said to follow a Poisson distribution with parameter λ if the probability mass function of the random variable X is $p(x)$, where $p(x)$ is given by

$$p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, x = 1, x = 2, x = 3, \dots \quad (53)$$

Porwal [33] introduced a power series whose coefficients are probabilities of Poisson distribution. That is,

$$P(\lambda, \xi) = \xi + \sum_{n=2}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} e^{-\lambda} \xi^n \quad (\lambda > 0; \xi \in \Delta). \quad (54)$$

Applying the ratio test, one can establish that the radius of the convergence of the series defined in (54) is infinity.

Let us introduce an $S^\lambda : \mathcal{A} \rightarrow \mathcal{A}$, defined by

$$\begin{aligned} S^\lambda h(\xi) &= P(\lambda, \xi) * h(\xi) \\ &= \xi + \sum_{n=2}^{\infty} \frac{\lambda^{n-1}}{(n-1)!} e^{-\lambda} h_n \xi^n \\ &= \xi + \sum_{n=2}^{\infty} \eta_n h_n \xi^n \end{aligned} \quad (55)$$

where $\eta_n = \eta_n(\lambda) = \frac{\lambda^{n-1} e^{-\lambda}}{(n-1)!}$. It can be shown that the operator introduced in (55) is linear. Here, $*$ denotes the Hadamard product or convolution between two analytic functions.

Now, we define the class $\mathcal{R}_{lim}(l, \nu, \eta)$ and $\mathcal{C}_{lim}(l, \nu, \eta)$ as follows:

$$\mathcal{R}_{lim}(l, \nu, \eta) = \{h \in \mathcal{A} : S^\lambda h \in \mathcal{R}_{lim}(l, \nu)\} \quad (56)$$

and

$$\mathcal{C}_{lim}(l, \nu, \eta) = \{h \in \mathcal{A} : S^\lambda h \in \mathcal{C}_{lim}(l, \nu)\} \quad (57)$$

where $\mathcal{R}_{lim}(l, \nu)$ and $\mathcal{C}_{lim}(l, \nu)$ are defined in Definitions 1 and 2, respectively.

Continuing as in Theorems 1 and 3, we can establish the coefficient bounds, as well as the Fekete–Szegő functional, for the class $\mathcal{R}_{lim}(l, \nu, \eta)$ by using the analogous bounds for the function of the class $\mathcal{R}_{lim}(l, \nu)$.

Theorem 9. Let $0 \leq \nu \leq 1$ and $S^\lambda h$ be given by (55). If $h \in \mathcal{R}_{lim}(l, \nu, \eta)$ then, for any $\mu \in \mathbb{C}$, we have the inequality

$$|h_3 - \mu h_2^2| \leq \frac{\sqrt{2}}{(2+\nu)\eta_3} \max \left\{ 1, \left| \frac{4\mu(2+\nu)\eta_3 - (5-\nu^2)\eta_2^2}{2\sqrt{2}(1+\nu)^2\eta_2^2} \right| \right\}. \quad (58)$$

Proof. Since $h \in \mathcal{R}_{lim}(l, \nu, \eta)$, it follows from (56) that

$$\left[(S^\lambda h(\xi))' \right]^\nu \left[\frac{\xi (S^\lambda h(\xi))'}{(S^\lambda h(\xi))} \right]^{1-\nu} = l(\omega(\xi)) \quad (\xi \in \Delta). \quad (59)$$

From (55), we obtain

$$\begin{aligned} \left[(S^\lambda h(\xi))' \right]^\nu \left[\frac{\xi (S^\lambda h(\xi))'}{(S^\lambda h(\xi))} \right]^{1-\nu} &= 1 + (1+\nu)\eta_2 h_2 \xi + (2+\nu) \left(\eta_3 h_3 - \frac{1-\nu}{2} \eta_2^2 h_2^2 \right) \xi^2 \\ &\quad + \frac{(3+\nu)}{6} [6\eta_4 h_4 - 6(1-\nu)\eta_2 h_2 \eta_3 h_3 + (1-\nu)(2-\nu)\eta_2^3 h_2^3] \xi^3 + \dots \end{aligned} \quad (60)$$

Applying (15) and (60) to (59), and then by equating the corresponding coefficients of ξ and ξ^2 , one can obtain

$$h_2 = \frac{q_1}{\sqrt{2}\eta_2(1+\nu)}, \quad (61)$$

and

$$h_3 = \frac{1}{\sqrt{2}(2+\nu)\eta_3} \left[q_2 - \frac{(1+2\sqrt{2})\nu^2 + 4\sqrt{2}\nu + (2\sqrt{2}-5)}{4\sqrt{2}(1+\nu)^2} q_1^2 \right]. \quad (62)$$

From (61) and (62), we obtain

$$h_3 - \mu h_2^2 = \frac{1}{\sqrt{2}(2+\nu)\eta_3} [q_2 - \nu q_1^2] \quad (63)$$

where

$$\nu = \frac{[2\sqrt{2}(1+\nu)^2 + (\nu^2 - 5)]\eta_2^2 + 4(2+\nu)\eta_3\mu}{4\sqrt{2}(1+\nu)^2\eta_2^2}. \quad (64)$$

The application of Lemma 2 to (63) gives the desired result and, thereby, proof of the theorem is also completed. $\square$

The Fekete–Szegő inequality for the class $\mathcal{R}_{lim}(l, \beta, \eta)$ is stated in the next theorem for the case of a real μ .

Theorem 10. Let $0 \leq \nu \leq 1$ and $h \in \mathcal{R}_{lim}(l, \nu, \eta)$. For any real number μ , we have

$$|h_3 - \mu h_2^2| \leq \begin{cases} -\frac{(\nu^2-5)\eta_2^2+4(2+\nu)\eta_3\mu}{2(1+\nu)^2(2+\nu)\eta_2^2\eta_3} & \mu \leq -\frac{[2\sqrt{2}(1+\nu)^2+(\nu^2-5)]\eta_2^2}{4(2+\nu)\eta_3} \\ \frac{\sqrt{2}}{(2+\nu)\eta_3} & -\frac{[2\sqrt{2}(1+\nu)^2+(\nu^2-5)]\eta_2^2}{4(2+\nu)\eta_3} \leq \mu \leq \frac{[2\sqrt{2}(1+\nu)^2-(\nu^2-5)]\eta_2^2}{4(2+\nu)\eta_3} \\ \frac{(\nu^2-5)\eta_2^2+4(2+\nu)\eta_3\mu}{2(1+\nu)^2(2+\nu)\eta_2^2\eta_3} & \mu \geq \frac{[2\sqrt{2}(1+\nu)^2-(\nu^2-5)]\eta_2^2}{4(2+\nu)\eta_3} \end{cases}.$$

Proof. An application of Lemma 3 to Relation (63) gives the desired estimate. This completes the proof. $\square$

In the next theorem, from the analogous coefficient estimates for the class $\mathcal{C}_{lim}(l, \beta)$, we ascertain the coefficient bounds and Fekete–Szegő functional for the class $\mathcal{C}_{lim}(l, \beta, \eta)$.

Theorem 11. Let $0 \leq \nu \leq 1$ and μ be any complex number. Let $S^\lambda h$ be given as in (55). If $h \in \mathcal{C}_{lim}(l, \nu, \eta)$,

$$|h_3 - \mu h_2^2| \leq \frac{\sqrt{2}}{3(2-\nu)\eta_3} \max \left\{ 1, \left| \frac{3\mu(2-\nu)\eta_3 - (5-4\nu)\eta_2^2}{2\sqrt{2}\eta_2^2} \right| \right\}. \quad (65)$$

Proof. Since $h \in \mathcal{C}_{lim}(l, \nu, \eta)$, it follows from (57) that

$$\left[(S^\lambda h(\xi))' \right]^\nu \left[1 + \frac{\xi(S^\lambda h(\xi))''}{(S^\lambda h(\xi))'} \right]^{1-\nu} = l(\omega\xi) \quad (\xi \in \Delta). \quad (66)$$

From (55), we obtain

$$\begin{aligned} \left[(S^\lambda h(\xi))' \right]^\nu \left[1 + \frac{\xi(S^\lambda h(\xi))''}{(S^\lambda h(\xi))'} \right]^{1-\nu} &= 1 + 2\eta_2 h_2 \xi + [3(2-\nu)\eta_3 h_3 - 4(1-\nu)\eta_2^2 h_2^2] \xi^2 \\ &\quad + [8(1-\nu)\eta_2^3 h_2^3 - 18(1-\nu)\eta_2 \eta_3 h_2 h_3 - 4(3-2\nu)\eta_4 h_4] \xi^3 + \dots \end{aligned} \quad (67)$$

Applying (15) and (67) to (66), and then comparing the corresponding coefficients of ξ and ξ^2 , we easily see that

$$h_2 = \frac{q_1}{2\sqrt{2}\eta_2}, \quad (68)$$

and

$$h_3 = \frac{1}{3\sqrt{2}(2-\nu)\eta_3} \left[q_2 - \frac{(4\nu + 2\sqrt{2} - 5)}{4\sqrt{2}} q_1^2 \right]. \quad (69)$$

The bounds of h_2 and h_3 can be obtained by using Lemma 1 and Lemma 2 in Relations (68) and (69), respectively. Moreover,

$$|h_3 - \mu h_2^2| = \frac{1}{3\sqrt{2}(2-\nu)\eta_3} \left| q_2 - \frac{(4\nu + 2\sqrt{2} - 5)\eta_2^2 + 3\mu(2-\nu)\eta_3}{4\sqrt{2}\eta_2^2} q_1^2 \right|. \quad (70)$$

The result follows by virtue of Lemma 2, which completes the proof. $\square$

Application of Lemma 3 to Equation (70), when μ is real, gives the Fekete–Szegő inequality for the class $\mathcal{C}_{lim}(l, \nu, \eta)$ as follows:

Theorem 12. Let μ be real and $0 \leq \nu \leq 1$. Further, let $S^\lambda h$ be given by (55). If $h \in \mathcal{C}_{lim}(l, \nu, \eta)$, then

$$|h_3 - \mu h_2^2| \leq \begin{cases} -\frac{(4\nu-5)\eta_2^2+3\mu(2-\nu)\eta_3}{6(2-\nu)\eta_2^2\eta_3} & \mu \leq -\frac{(4\nu+2\sqrt{2}-5)\eta_2^2}{3(2-\nu)\eta_3}, \\ \frac{\sqrt{2}}{3(2-\nu)\eta_3} & -\frac{(4\nu+2\sqrt{2}-5)\eta_2^2}{3(2-\nu)\eta_3} \leq \mu \leq \frac{(2\sqrt{2}-4\nu+5)\eta_2^2}{3(2-\nu)\eta_3}, \\ \frac{(4\nu-5)\eta_2^2+3\mu(2-\nu)\eta_3}{6(2-\nu)\eta_2^2\eta_3} & \mu \geq \frac{(2\sqrt{2}-4\nu+5)\eta_2^2}{3(2-\nu)\eta_3}. \end{cases}$$

Concluding Remarks: In the present article, by making use of a limaçon-shaped domain, we introduced two subclasses of analytic functions, namely $\mathcal{R}_{lim}(l, \nu)$ and $\mathcal{C}_{lim}(l, \nu)$. We investigated the coefficient bounds and the Fekete–Szegő functional associated with the limaçon domain for the said class. Apart from this, we established the coefficient estimates and the familiar Fekete–Szegő estimates for the inverse function class h^{-1} . We highlighted the Poisson distribution as an application of the main results.

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References

1. Pommerenke, C. On the coefficients and Hankel determinants of univalent functions. *J. Lond. Math. Soc.* **1966**, *41*, 111–122. [CrossRef]
2. Pommerenke, C. On the Hankel determinant of univalent function. *Mathematika* **1967**, *14*, 108–112. [CrossRef]
3. Noor, K.I. Hankel determinant problem for the class of functions with bounded boundary rotation. *Rev. Roum. Math. Pures. Appl.* **1983**, *28*, 731–739.
4. Arif, M.; Raza, M.; Tang, H.; Hussain, S.; Khan, H. Hankel determinant of order three for familiar subsets of analytic functions related with sine function. *Open Math.* **2019**, *17*, 1615–1630. [CrossRef]
5. Barukab, O.M.; Arif, M.; Abbas, M.; Khan, S.A. Sharp bounds of the coefficient results for the family of bounded turning functions associated with a petal-shaped domain. *J. Funct. Spaces* **2021**, *2021*, 5535629. [CrossRef]
6. Darus, M.; Shanmugam, T.N.; Sivasubramanian, S. Fekete–Szegő inequality for a certain class of analytic functions. *Mathematica* **2007**, *49*, 29–34.
7. Mohapatra, R.N.; Panigrahi, T. Second Hankel determinant for a class of analytic functions defined by Komatu integral operator. *Rend. Mat. Appl.* **2020**, *41*, 51–58.
8. Murugusundaramoorthy, G.; Bulboacă, T. Hankel determinants for new subclasses of analytic functions related to a shell shaped region. *Mathematics* **2020**, *8*, 1041. [CrossRef]
9. Shi, L.; Ali, I.; Arif, M.; Cho, N.E.; Hussain, S.; Khan, H. A study of third Hankel determinant problem for certain subfamilies of analytic functions involving cardioid domain. *Mathematics* **2019**, *7*, 418. [CrossRef]
10. Shi, L.; Srivastava, H.M.; Arif, M.; Hussain, S.; Khan, H. An investigation of the third Hankel determinant problem for certain subfamilies of univalent functions involving the exponential function. *Symmetry* **2019**, *11*, 598. [CrossRef]
11. Ma, W.C.; Minda, D. A unified treatment of some special classes of univalent functions. In *Proceeding of the International Conference on Complex Analysis, Tianjin, China, 1992*; International Press Inc.: Somerville, MA, USA, 1992.
12. Sokół, J.; Stankiewicz, J. Radius of convexity of some subclasses of strongly starlike functions. *Zesz. Nauk. Politech. Rzesz. Mat.* **1996**, *19*, 101–105.
13. Sokół, J. On some subclass of strongly starlike functions. *Demonstr. Math.* **1998**, *31*, 81–86.

14. Mendiratta, R.; Nagpal, S.; Ravichandran, V. A subclass of starlike functions associated with left-half of the lemniscate of Bernoulli. *Int. J. Math.* **2014**, *25*, 1450090. [[CrossRef](#)]
15. Raina, R.K.; Sokół, J. On coefficient estimates for a certain class of starlike functions. *Hacet. J. Math. Stat.* **2015**, *44*, 1427–1433. [[CrossRef](#)]
16. Raina, R.K.; Sokół, J. Some properties related to a certain class of starlike functions. *Comptes Rendus Math.* **2015**, *353*, 973–978. [[CrossRef](#)]
17. Raina, R.K.; Sharma, P.; Sokół, J. Certain Classes of Analytic Functions Related to the Crescent-Shaped Regions. *J. Contemp. Math. Anal.* **2018**, *53*, 355–362. [[CrossRef](#)]
18. Sokół, J.; Thomas, D.K. Further results on a class of starlike functions related to the Bernoulli lemniscate. *Houst. J. Math.* **2018**, *44*, 83–95.
19. Mendiratta, R.; Nagpal, S.; Ravichandran, V. On a subclass of strongly starlike functions associated with exponential function. *Bull. Malays. Math. Sci. Soc.* **2015**, *38*, 365–386. [[CrossRef](#)]
20. Sharma, K.; Jain, N.K.; Ravichandran, V. Starlike functions associated with a cardioid. *Afr. Mat.* **2016**, *27*, 923–939. [[CrossRef](#)]
21. Kumar, S.; Ravichandran, V. A subclass of starlike functions associated with a rational function. *Southeast Asian Bull. Math.* **2016**, *40*, 199–212.
22. Yunus, Y.; Halim, S.A.; Akbarally, A.B. Subclass of starlike functions associated with a limaçon. *AIP. Conf. Proc.* **2018**, *1974*, 30023.
23. Kargar, R.; Ebadian, A.; Sokół, J. On Booth lemniscate and starlike functions. *Anal. Math. Phys.* **2019**, *9*, 143–154. [[CrossRef](#)]
24. Khatter, K.; Ravichandran, V.; Kumar, S.S. Starlike functions associated with exponential function and lemniscate of Bernoulli. *Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A Mat.* **2019**, *113*, 233–253. [[CrossRef](#)]
25. Goel, P.; Sivaprasad Kumar, S. Certain class of starlike functions associated with modified sigmoid function. *Bull. Malays. Math. Sci. Soc.* **2020**, *43*, 957–991. [[CrossRef](#)]
26. Dziok, J.; Raina, R.K.; Sokół, J. On alpha-convex functions related to shell-like functions connected with Fibonacci numbers. *Appl. Math. Comput.* **2011**, *218*, 966–1002.
27. Dziok, J.; Raina, R.K.; Sokół, J. Certain results for a class of convex functions related to a shell-like curve connected with Fibonacci numbers. *Comput. Math. Appl.* **2011**, *61*, 2605–2613. [[CrossRef](#)]
28. Jurasirńska, R.; Sokół, J. Some problems for certain family of starlike functions. *Math. Comput. Model.* **2012**, *55*, 2134–2140. [[CrossRef](#)]
29. Malik, S.N.; Mahmood, S.; Raza, M.; Farman, S.; Zainab, S. Coefficient inequalities of function associated with petal type domains. *Mathematics* **2018**, *6*, 298. [[CrossRef](#)]
30. Masih, V.S.; Kanas, S. Subclasses of Starlike and Convex Functions Associated with the Limaçon Domain. *Symmetry* **2020**, *12*, 942. [[CrossRef](#)]
31. Murugusundaramoorthy, G.; Shakeel, A.; Kutbi, M.A. Coefficient bounds of Kamali-type starlike functions related with a limaçon-shaped domain. *J. Function Spaces* **2021**, *2021*, 4395574. [[CrossRef](#)]
32. Duren, P.L. *Univalent Functions*; Grundlehren der Mathematischen Wissenschaften; Springer: New York, NY, USA, 1983; Volume 259.
33. Porwal, S. An application of a Poisson distribution series on certain analytic function. *J. Complex Anal.* **2014**, *2014*, 984135. [[CrossRef](#)]

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