symmetry

MDPI

Article

Dynamical Chiral Symmetry Breaking in Quantum Chromo Dynamics: Delicate and Intricate

Reinhard Alkofer

Institute of Physics, University of Graz, NAWI Graz, Universitätsplatz 5, 8010 Graz, Austria; reinhard.alkofer@uni-graz.at

Abstract: Dynamical chiral symmetry breaking (DχSB) in quantum chromo dynamics (QCD) for light quarks is an indispensable concept for understanding hadron physics, i.e., the spectrum and the structure of hadrons. In functional approaches to QCD, the respective role of the quark propagator has been evident since the seminal work of Nambu and Jona-Lasinio has been recast in terms of QCD. It not only highlights one of the most important aspects of DχSB, the dynamical generation of constituent quark masses, but also makes plausible that DχSB is a robustly occurring phenomenon in QCD. The latter impression, however, changes when higher n-point functions are taken into account. In particular, the quark–gluon vertex, i.e., the most elementary n-point function describing the full, non-perturbative quark–gluon interaction, plays a dichotomous role: It is subject to DχSB as signalled by its scalar and tensor components but it is also a driver of DχSB due to the infrared enhancement of most of its components. Herein, the relevant self-consistent mechanism is elucidated. It is pointed out that recently obtained results imply that, at least in the covariant gauge, DχSB in QCD is located close to the critical point and is thus a delicate effect. In addition, requiring a precise determination of QCD's three-point functions, DχSB is established, in particular in view of earlier studies, by an intricate interplay of the self-consistently determined magnitude and momentum dependence of various tensorial components of the gluon–gluon and the quark–gluon interactions.

Keywords: QCD; chiral symmetry; dynamical symmetry breaking; dyson-schwinger equations; quark–gluon vertex

check for updates

Citation: Alkofer, R. Dynamical Chiral Symmetry Breaking in Quantum Chromo Dynamics: Delicate and Intricate. *Symmetry* **2023**, *15*, 1787. https://doi.org/10.3390/sym15091787

Academic Editor: Dubravko Klabučar

Received: 3 July 2023
Revised: 4 September 2023
Accepted: 10 September 2023
Published: 18 September 2023

1. Introduction

Investigations of QCD with the aim of gaining an understanding of hadron physics have been undertaken since QCD was formulated almost 50 years ago [1]. The recent review [2] summarises in more than 700 pages quite a number of highlights arising from these studies. With its almost 5000 references, it makes clear how much this area of research has matured. Nevertheless, it is agreed upon by the community that the understanding of several essential features of QCD and their implications for hadron physics is far from being satisfactory.

In the following short notes, I focus on a very specific property of QCD, namely, the approximate chiral symmetry of the light quarks and how it is dynamically broken. Despite the importance of DχSB for the phenomenological consequences with respect to the spectrum and structure of hadrons, I am concentrating herein on the underlying mechanisms for DχSB, or more precisely, on a detailed analysis within the picture that a super-critically strong attraction between massless fermions triggers DχSB, see, e.g., [3–5]. To this end, one may note that more than 60 years ago Nambu and Jona-Lasinio realised that in four spacetime dimensions a certain coupling strength has to be exceeded for DχSB to occur [6,7].

At this point, a disclaimer is in order. Herein, I will summarise and briefly review some investigations of DχSB, the choice of which is based on my own attempts within this field of research. By no means it is intended to disregard different approaches to the topic which are based on complementary techniques and/or pictures (as, e.g., an explanation of chiral symmetry breaking by considering ensembles of QCD vacua containing lumps of gluon fields with non-vanishing topological winding number densities). Additionally, given the wealth of literature on DχSB, even if one restricts oneself (i) to the picture of a super-critically strong attraction as underlying mechanism and (ii) to functional methods, it is impossible within such a short synopsis as the one presented here to discuss or even mention all relevant research on this topic. Such omissions are also in accord with the intention of the presented discussion: to provide evidence that DχSB in QCD is quite delicate, its manifestations in the properties of quarks and quark–gluon interactions exhibit many facets, and the interplay between those features makes DχSB an intricate idiosyncrasy of QCD.

2. How Robust Is DχSB in QCD?

2.1. The Nambu–Jona-Lasinio Picture

The seminal papers by Nambu and Jona-Lasinio [6,7] introduced the notion of DχSB in analogy to the shortly-before-formulated BCS model of superconductivity [8,9]. The generic idea of Nambu and Jona-Lasinio has been that massless (light) nucleons interact via a four-fermion interaction which in turn leads to massive nucleons and (almost) massless pions as (pseudo-) Goldstone bosons. As their starting point was to describe nucleons as massless Dirac fermions interacting via a $SU_L(2) \times SU_R(2)$ chirally invariant interaction, the dynamics of their model respects chiral symmetry but the ground state symmetry was broken down to a vector $SU_{L+R}(2)$ symmetry. Therefore, given the three-dimensional coset space, three pseudo-scalar massless, resp., light excitations arise as (would-be) Goldstone bosons, the pions.

From this, one can take away three important lessons:

- In contradistinction to spontaneous symmetry breaking, the mechanism of dynamical symmetry breaking introduces a dichotomous nature for the (would-be) Goldstone bosons; they are not only Goldstone bosons but at the same time bound states of a highly collective nature. This is true for the original picture based on nucleons but, of course, also if one starts with light quarks interacting at the tree-level in a chirally symmetric way, see, e.g., the discussion in [10].
- DχSB implies the generation of dynamical masses for originally massless and/or light fermions. This solves the puzzle of why in the quark model for the light quarks the so-called constituent quark masses at the order of $\gtrsim$350 MeV are required instead of the much smaller current quark masses.
- In contradistinction to non-relativistic superconductivity where Cooper pairs are formed at arbitrary small couplings [9] (NB: As a matter of fact, this statement is only true in the mean-field approximation. When taking into account fluctuations also, a certain minimal coupling is required to form Cooper pairs.), DχSB in four spacetime dimensions only takes place if the coupling exceeds a critical value.

Although these three statements are correct, they alone provide an incomplete picture. Before explicating in which sense the second statement has to be augmented, it is instructive to have a closer look at the third one. In the chiral limit, any order parameter will show as a function of the coupling a non-analyticity at the critical value of the coupling. For light quarks, i.e., in the case of approximate chiral symmetry, one has a cross-over characterised by a rapid change of the would-be order parameter. This is illustrated in Figure 1, in which for a calculation within a Nambu–Jona-Lasinio model the constituent quark mass is shown as a function of the four-fermion coupling; for the details, see Ref. [10]. This calculation seems to imply that the physical point is such that the corresponding coupling is much larger than the physical one, and correspondingly all order parameters would depend only mildly on the precise value of the coupling. In the following, I will argue that this behaviour seen in a Nambu–Jona-Lasinio model (and certain truncations to QCD) is not correct for QCD. The most important effect of this is the resulting sensitivity of all chiral order parameters on the precise value of the quark–quark interaction strength.

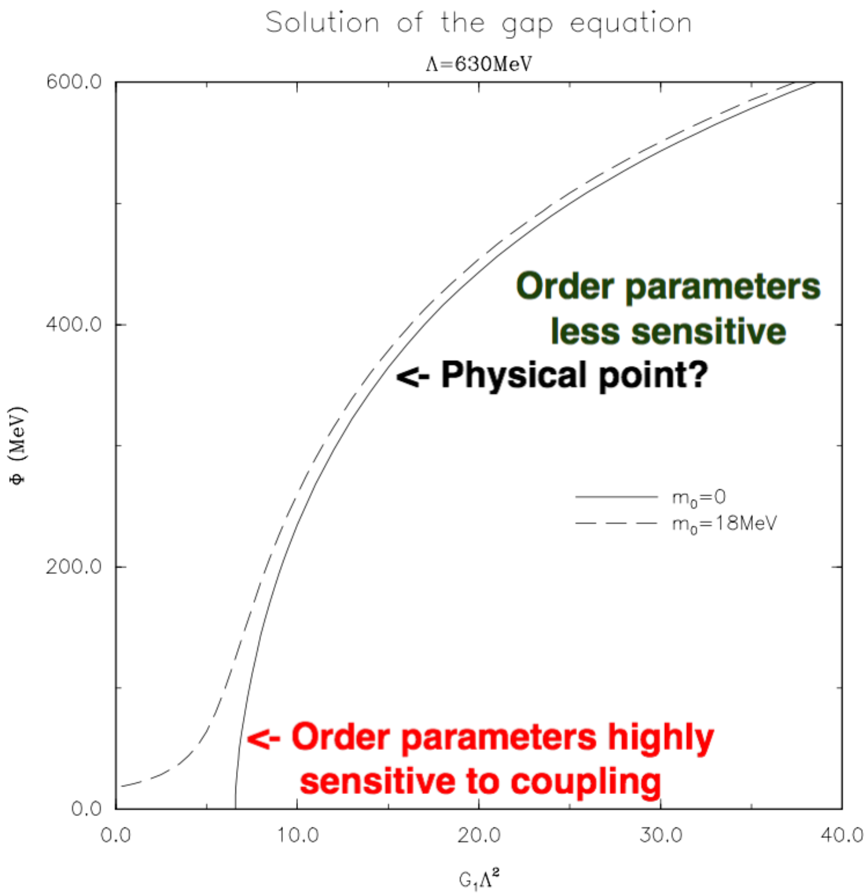

Figure 1. An example for the generated constituent quark mass as a function of the coupling within a Nambu–Jona-Lasinio model calculation. (Adapted from Ref. [10].)

2.2. On the Dyson–Schwinger/Bethe–Salpeter Approach in Rainbow-Ladder Truncation

As DχSB is a non-perturbative phenomenon, methods beyond perturbation theory are needed to investigate it. If it comes to the study of dynamical symmetry breaking, an approach based on Dyson–Schwinger and Bethe–Salpeter equations has been widely employed, see, e.g., the textbook [5] for an introduction. In particular, this approach has been used widely in the context of QCD and hadron physics as documented by a number of reviews [11–17].

An essential element in this approach is the choice of a symmetry-preserving truncation of the infinite set of equations for the n-point correlation functions. Within a Poincaré-covariant setting (implying, at least implicitly, the choice of a covariant gauge, cf. the discussion below in Section 2.4), the simplest non-trivial of such approximations is the rainbow-ladder truncation. It has this name because the infinitely many re-summed diagrams look like rainbows for the quark propagator's Dyson–Schwinger equation and like ladders for the mesons' bound state equations, the Bethe–Salpeter equations.

Since Ref. [18], several hundred solutions of the quark propagator's Dyson–Schwinger equation in rainbow approximation have been published, and since Refs. [19,20], a similar number of solutions for the pion Bethe–Salpeter equation in ladder approximation have been described in the literature. For many but not all hadrons, such an approximation works astonishingly well, see, e.g., Ref. [16] for a detailed discussion and Refs. [21–25] for some examples of beyond-rainbow-ladder calculations (NB: For a study of the functional renormalisation group taking a dynamical quark–gluon vertex into account, see, e.g., [26]).

For the purpose of these notes, the use of the rainbow-ladder truncation will not allow to resolve the issues raised in the preceding section. The reason is quite simple: In this truncation, the quark–gluon vertex is given by a model, and for technical reasons the employed models are incomplete. Important aspects of the effect of DχSB on the quark–gluon interaction are thereby excluded by assumption. Phrased otherwise, one cannot find what one excludes by approximation.

A further "twist" of the rainbow-ladder truncation lies in the reduction to the tree-level tensor component in the quark–gluon vertex followed by a fitting of the overall interaction strength to phenomenological data. This then leads, as argued in the next section, to an overestimation of the coupling strength between quarks and gluons.

2.3. On the Onset of the Conformal Window

It is evident from hadron phenomenology that DχSB takes place in QCD. For a Gedankenexperiment, let us consider a gauge theory with N_f massless (or light) fermions in the fundamental representation of the gauge group. If N_f is small, the anti-screening caused by the gauge bosons dominates, and consequently the running coupling will increase when tuning the scale from larger to smaller scales. Eventually, it will exceed the critical coupling, and DχSB will take place. At very large N_f, the screening caused by the fermions will dominate, and asymptotic freedom will be lost.

However, in between these two extremes there will exist an interval for N_f where the balance between the anti-screening due to the gauge bosons and the screening due to the fermions is such that the anti-screening effect wins only so slightly against the screening. Correspondingly, the coupling will increase when lowering the scale but only so weakly that the critical coupling is never exceeded. Thus, DχSB will not take place. In the chiral limit, such a theory possesses an infrared fixed point; it will be effectively scale-invariant in the deep infrared. For that reason, the corresponding interval for N_f is called the conformal window.

Although the above-described generic picture has been verified by studies based on coupled Dyson–Schwinger equations [27,28], the critical value for the numbers of flavours at which the conformal window sets in N_f^{crit}, is severely underestimated when compared to studies employing other methods, see, e.g., [29–32]. The decisive hint why Dyson–Schwinger studies in rainbow-ladder truncation show such a deficiency comes from the sensitivity of N_f^{crit} on the quark–gluon vertex if one goes (slightly) beyond the rainbow-ladder truncation. This behaviour makes plain that the distribution of the overall quark–gluon interaction strength in the sub-GeV region over several of the quark–gluon vertex tensor structures, as happens without any doubt in QCD, is essential in an understanding of how the increased screening by an increasing number of massless, resp., light, quark flavours drives the system into a chirally symmetric phase with an IR fixed point.

2.4. A Note on Gauge Dependence

Since the seminal work by Curtis and Pennington [33], it has become evident how important the fermion–gauge-boson vertex is in achieving gauge independence in the Dyson–Schwinger approach. Although for QED substantial progress has been achieved, see, e.g., [34–36] and references therein, in the studies of the role of the fermion-photon vertex for gauge independence, the corresponding question in QCD, namely, on the impact of the quark–gluon vertex on the gauge (in-)dependence of hadron observables, has proven to be an extremely hard question. Even the much more humble question of how the different tensors of the quark–gluon vertex may depend on the gauge parameter within the class of linear covariant gauges and how this will effect the underlying mechanism for DχSB in this class of gauges seems beyond reach given the status of Dyson–Schwinger studies of the Yang–Mills sector in the linear covariant gauge, see, e.g., [37–39].

Therefore, although the question of whether DχSB in QCD is delicate and intricate only in the Landau gauge and might be a robust phenomenon in other gauges is highly interesting, it will likely remain unanswered in the next years. Nevertheless, in view of the insights which may be gained in studying the role of the quark–gluon vertex and its impact on DχSB in different gauges, an extension of the approach based on Nielsen identities (as performed in [39]) to the quark sector is certainly desirable. One might also apply the technique of interpolating gauges [40–43] to relate the existing Landau and Coulomb gauge results on DχSB. Until such studies succeed, the analysis described herein will only be applicable to QCD in the Landau gauge.

3. Correlation Functions in the Yang–Mills Sector

In order to investigate the interplay between the quark propagator and the quark–gluon vertex within functional methods, one needs to be able to determine the propagators and the three-point functions in the Yang–Mills sector accurately. In the last two decades, there has been enormous progress in this direction, see, e.g., the reviews [44–46], and it is fair to say that in the Landau gauge, the gluon and the ghost propagators as well as the three-gluon and the ghost-gluon vertex are well understood.

Hereby, two features are important.

First, the gluon propagator's renormalisation function as function of the gluon virtuality p^2 displays on the space-like side a maximum slightly below one GeV, and then decreases towards the infrared. This unusual behaviour not only signals a strongly reduced spectral dimension [47] and relates the gluon long-range properties to non-vanishing p^2 [47,48] but it also leads to the fact that the gluon propagator alone, i.e., without quark–gluon vertex dressings, is much too small in the sub-GeV region to trigger DχSB, see, e.g., the discussion in [49].

Second, the three-gluon vertex gets suppressed towards the infrared, and the corresponding form factors display in the most accurate available calculations even a zero at small values of p^2. As in the Dyson–Schwinger equation for the quark–gluon vertex, the three-gluon vertex turns out to be decisive in determining the infrared enhancement of the quark–gluon vertex form factors, which in turn determines the size and the proximity to criticality of DχSBṪhese two observations together explain why in QCD in Landau gauge DχSB is so delicate in distinction from models ignoring these two facts.

4. Quark Propagator and Quark–Gluon Vertex

4.1. Structure of the Quark–Gluon Vertex

The arguments provided above elucidate the special role of the quark–gluon vertex in the description of DχSB in the Landau gauge. Unfortunately, this vertex possesses a rich structure, and it is exactly the interplay between parts of this structure which turn out to be relevant for the physics of DχSB .

There is one straightforward property of the fully dressed quark–gluon vertex: To the best of our knowledge, it possesses the same colour structure as its tree-level counterpart.

When it comes to flavour, and in particular to the dependence on the current quark mass, a careful assessment of the properties of the substructures is in order. To this end, one notes first that in the Landau gauge, only the parts of the vertex are relevant which are strictly transverse to the gluon momentum. As the quark–gluon vertex transforms as four-vector under Lorentz and as a Dirac matrix under spin rotations, this leaves one with eight possible tensor structures, each tensor structure being multiplied with a form factor depending on three Lorentz-invariant variables which in turn are built from the three involved momenta.

Instead of choosing immediately a definite basis for these eight tensors, it is worthwhile to discuss some generic aspects first. The Feynman integrals for the form factor multiplying the tree-level tensor are ultraviolet divergent, and thus this one form factor needs renormalisation. Choosing the other seven tensors orthogonal to the tree-level, the corresponding form factors are determined from ultraviolet-finite expressions, and correspondingly they decrease power-like for large momenta. This leads to the expectation, later confirmed by calculations, that these form factors are only sizeable if at least one of the involved momenta is small, i.e., in the sub-GeV region.

The eight tensors of the transverse part of the quark–gluon vertex can grouped according to their behaviour under chiral transformations: four of them are chirally symmetric, and thus they will be generically non-vanishing even in the chiral limit and the symmetric

Wigner–Weyl phase of chiral symmetry. In that latter case, the form factors of the other four chirally non-symmetric tensor structures vanish. In the Nambu–Goldstone phase, they will be dynamically generated, and phrased otherwise this exactly means that DχSB also includes the generation of chirality-violating scalar and tensor quark–gluon interaction. As can be seen below, they are quite sizeable.

If it comes to the dependence of the quark–gluon vertex on the current quark mass, i.e., on the explicit breaking of chiral symmetry, this distinction between the chirally symmetric and non-symmetric parts lead to a quite astonishing behaviour of the latter components. The Feynman diagrams for the quark–gluon vertex contain at least one quark propagator within a loop. Of course, this quark propagator goes to zero as the quark mass goes to infinity. Therefore, naively one might conclude that the fully dressed quark–gluon vertex will approach the tree-level one for larger and larger current quark masses. However, one has to take into account that the chirally non-symmetric form factors by the mere virtue of their transformation properties will also have a factor of at least one current quark mass and/or dynamically generated constituent quark mass in the numerator. Therefore, the suppression by the current quark mass in the denominator of the integrand introduced via the quark propagator can and generically will be canceled.

As a matter of fact, this mechanism is already at work in QED w.r.t. the Pauli term and the resulting anomalous magnetic moments (g-2): There is a cancelation of factors of the fermion mass in the QED contributions to, e.g., the (g-2) of the electron and the muon.

4.2. Dynamical Generation of Scalar and Tensorial Quark–Gluon Interactions

In the following, the above statements will be quantified on the basis of the results obtained in [50], see also [51–53] (The interested reader will find figures of the quark–gluon vertex's form factors in these references.) I want to emphasise here that the corresponding results of other groups would have been equally valid; the choice is only based on the availability of the data. Additionally, to be concise within this short note only results in the chiral limit will be discussed.

The following kinematics is chosen:
Gluon momentum: $k^\mu = p^\mu - q^\mu$ with p^μ outgoing and q^μ incoming quark momentum.
Define furthermore:

(i) Normalised gluon momentum:

$$\hat{k}^\mu := k^\mu / \sqrt{k^2}.$$

(ii) Averaged quark momentum; $\frac{1}{2}(p^\mu + q^\mu)$, project it transverse to gluon momentum and normalise it

$$s^\mu := (\delta^{\mu\nu} - \hat{k}^\mu \hat{k}^\nu)\frac{1}{2}(p^\nu + q^\nu)\,, \quad \hat{s}^\mu = s^\mu / \sqrt{s^2}\,.$$

As a three-point function, the quark–gluon vertex, or more precisely the factors multiplying the tensors in a decomposition, depend on three Lorentz invariants, and we choose them to be p^2, q^2, and $p \cdot q$. The transverse part of the quark–gluon vertex is expanded then in the form

$$\Gamma^\mu_{trans}(p, q; k) = \sum_{j=1}^{8} g_i(p^2, q^2; p \cdot q)\rho^\mu_i\,, \tag{1}$$

and in the following we will approximate the transverse part of the quark–gluon vertex. First, as the angular dependence turns out to be weak, we will neglect it. The functions $g_i(p^2, q^2; p \cdot q)$ are symmetric in p and q; therefore, we will substitute them by functions $g_i(\bar{p}^2)$ of only the averaged momentum-squared, i.e., $\bar{p}^2 = \frac{1}{2}(p^2 + q^2)$. The model functions $g_i(\bar{p}^2)$ are fitted to the numerical results at symmetric momenta, $g(p^2, p^2; p \cdot q = 0)$ obtained from a coupled set of quark propagator and quark–gluon vertex Dyson–Schwinger equations in the chiral limit with a model for the three-gluon vertex; see [50–53] for more details.

Hereby, it turns out that g_1, g_2, $g_3 \propto g_2$, and $g_4 = g_7$ are important whereas based on the underlying results for g_5 and g_8, it is safe to neglect these two functions.

Employing that to numerical accuracy, $g_4 = g_7$, and observing that $g_3 \propto g_2$ in the sense that $1.45\, g_2(p^2, p^2, 0) + g_3(p^2, p^2; 0)$ is for all momenta smaller than 0.08, one is left **with effectively three tensor structures.**

1. Tree-level tensor structure (with $x = \bar{p}^2 /\ 1\ \mathrm{GeV}^2$):
   $\rho_1^\mu = \gamma_T^\mu = (\delta^{\mu\nu} - \hat{k}^\mu \hat{k}^\nu)\gamma^\mu$, with
   $g_1(\bar{p}^2) = 1 + (1.6673 + 0.2042x)/(1 + 0.6831x + 0.0008509x^2)$.
   Of course, the tree-level tensor structure is allowed in the chirally symmetric phase.
2. The further sizeable chirally symmetric tensor structure is given by:
   $\rho_4^\mu + \rho_7^\mu = \hat{\not{k}}\ \hat{s}^\mu + \not{\hat{s}}\ \hat{\not{k}}\ \gamma_T^\mu$, with
   $g_4(\bar{p}^2) = g_7(\bar{p}^2) = 2.589x/(0.8587 + 3.267x + x^2)$.
3. The one important tensor structure due to (dynamical or explicit) chiral symmetry breaking is a combination of $\rho_2^\mu = i\hat{s}^\mu$ and $\rho_3^\mu = i\hat{\not{k}}\gamma_T^\mu$.

The corresponding form factors are $g_3(\bar{p}^2) = 0.3645x/(0.01867 + 0.3530x + x^2)$, $g_2(\bar{p}^2) = -g_3(\bar{p}^2)/1.45$, and the latter relation also fixes the relative weight of the second and the third component in the expansion (1).

Hereby, ρ_2^μ is a Dirac scalar (i.e., proportional to the unit matrix), and ρ_3^μ a rank-2 tensor.

Therefore, the one main conclusion of this section is that in QCD in Landau gauge, a **scalarand tensorial quark–gluon interaction** is dynamically generated. Phrased otherwise, non-perturbatively fully dressed gluons interact with quarks as if they had a spin-0 and spin-2 component.

4.3. The Coupled System and Its Lessons for DχSB

Putting all the above pieces together, one realises that a description of DχSB in QCD in Landau gauge and based on the fully dressed quark, gluon, and ghost propagators as well the fully dressed three-point functions, displays quite an elaborate web of self-consistent interdependencies. Contrary to what was assumed in the early days of QCD, namely, that the gluon propagator is the main driver of a robust version of DχSB, it turns out that the intricate interplay between all the involved functions puts the whole system close to criticality. Although amongst these functions, the quark–gluon vertex is the richest in structure, it is the one quantity which allows to improve our understanding of the complicated way the fully dressed gluon interacts with fully dressed quarks in the strongly interacting domain.

From a bird's eye perspective, this should not come as a surprise. It is obvious from the experimental results in hadron physics that thresholds which are apparent in scattering cross-sections stem from intermediate hadron resonances. Despite its rich structure, the quark–gluon vertex is still the simplest among all the QCD correlation functions which could seed such dependencies. Together with an understanding of how the kinetic terms for hadrons might emerge from the QCD degrees of freedom (for a corresponding discussion, see, e.g., [54]) this opens up the possibility to map out the wealth of hadron physics with less than a dozen functions derived from QCD. Therefore, the richness of these functions and of the equations determining them should not come as a surprise.

5. Conclusions and Outlook

In this short note, I argued that the view on DχSB in QCD needs to take into account the results obtained over the last two decades for the correlation functions of gluons and quarks. Having been seduced by some older results to believe that DχSB in strong interactions is a robust phenomenon (due to the reason that interactions are strong), the more recent results urge us to re-think this point of view: It looks much more as if DχSB is delicate and intricate.

At this point, one might argue that this distinction between robust and straightforward vs. delicate and intricate might only be an interpretational one. In my opinion, there are at least three reasons to pay attention to the view advocated here. The first one is within hadron physics itself. Being aware of the sensitivity in the description of DχSB provides some guidance in understanding which hadron observables will inherit this sensitivity on the details of the underlying quark and glue dynamics. In this respect, the question of the formation of a hadron provides quite likely one of the main examples of an intricate process. Second, quite a number of models beyond the Standard Model as, e.g., technicolor, exploit a potential proximity to the lower end of the conformal window to generate a "walking" coupling and correspondingly a vast separation of scales. Needless to say, an understanding of the transition to the conformal window and the physics therein (as well as close to it) will build on the details of the fate of chiral symmetry in this parameter domain. Third (but not least), I would like to remind the reader that the Standard Model possesses another chiral transition triggered by the Higgs–Yukawa couplings and happening at the electroweak scale. (Some insight into how intricately these two chiral transitions intertwine can be inferred from the recent investigation reported in ref. [55]). Therefore, a deepened insight into the chiral properties of the Standard Model fermions will always need to include the very nature of DχSB within QCD.

Funding: This research received no external funding.

Data Availability Statement: The data used for fitting the quark-gluon vertex form factors are available from the author on request via e-mail.

Acknowledgments: It is a pleasure to cordially thank all colleagues who collaborated with me on the topics presented here. I am in particular grateful for the insights gained in my many respective discussions with Per Amund Amundsen, William Detmold, Gernot Eichmann, Christian Fischer, Markus Hopfer, Markus Huber, Felipe Llanes-Estrada, Axel Maas, Pieter Maris, Angel Miramontes, Mario Mitter, Jan Pawlowski, Hugo Reinhardt, Alexandre Salas-Bernardez, Helios Sanchis-Alepuz, Lorenz von Smekal, Milan Vujinovic, Herbert Weigel, Richard Williams, Andreas Windisch, Fabian Zierler, and Daniel Zwanziger. I thank the Open Access Funding by the University of Graz.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Fritzsch, H.; Gell-Mann, M.; Leutwyler, H. Advantages of the Color Octet Gluon Picture. *Phys. Lett. B* **1973**, *47*, 365–368. [CrossRef]
2. Gross, F.; Klempt, E.; Brodsky, S.J.; Buras, A.J.; Burkert, V.D.; Heinrich, G.; Jakobs, K.; Meyer, C.A.; Orginos, K.; Strickland, M.; et al. 50 Years of Quantum Chromodynamics. *arXiv* **2022**, arXiv:2212.11107.
3. Miransky, V.A. Dynamic Mass Generation and Renormalizations in Quantum Field Theories. *Phys. Lett. B* **1980**, *91*, 421–424. [CrossRef]
4. Fomin, P.I.; Gusynin, V.P.; Miransky, V.A.; Sitenko, Y.A. Dynamical Symmetry Breaking and Particle Mass Generation in Gauge Field Theories. *Riv. Nuovo Cimento* **1983**, *6N5*, 1–90.
5. Miransky, V.A. *Dynamical Symmetry Breaking in Quantum Field Theories*; World Scientific: Sinagapore, 1994.
6. Nambu, Y.; Jona-Lasinio, G. Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I. *Phys. Rev.* **1961**, *122*, 345–358. [CrossRef]
7. Nambu, Y.; Jona-Lasinio, G. Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II. *Phys. Rev.* **1961**, *124*, 246–254. [CrossRef]
8. Bardeen, J.; Cooper, L.N.; Schrieffer, J.R. Microscopic theory of superconductivity. *Phys. Rev.* **1957**, *106*, 162. [CrossRef]
9. Bardeen, J.; Cooper, L.N.; Schrieffer, J.R. Theory of superconductivity. *Phys. Rev.* **1957**, *108*, 1175–1204. [CrossRef]
10. Alkofer, R.; Reinhardt, H. *Chiral Quark Dynamics*; Lecture Notes in Physics Monographs; Springer: Berlin/Heidelberg, Germany, 1995; Volume 33. [CrossRef]
11. Roberts, C.D.; Williams, A.G. Dyson-Schwinger equations and their application to hadronic physics. *Prog. Part. Nucl. Phys.* **1994**, *33*, 477–575. [CrossRef]
12. Roberts, C.D.; Schmidt, S.M. Dyson-Schwinger equations: Density, temperature and continuum strong QCD. *Prog. Part. Nucl. Phys.* **2000**, *45*, S1–S103. [CrossRef]
13. Alkofer, R.; von Smekal, L. The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states. *Phys. Rept.* **2001**, *353*, 281. [CrossRef]

14. Maris, P.; Roberts, C.D. Dyson-Schwinger equations: A Tool for hadron physics. *Int. J. Mod. Phys. E* **2003**, *12*, 297–365. [CrossRef]
15. Bashir, A.; Chang, L.; Cloet, I.C.; El-Bennich, B.; Liu, Y.X.; Roberts, C.D.; Tandy, P.C. Collective perspective on advances in Dyson-Schwinger Equation QCD. *Commun. Theor. Phys.* **2012**, *58*, 79–134. [CrossRef]
16. Eichmann, G.; Sanchis-Alepuz, H.; Williams, R.; Alkofer, R.; Fischer, C.S. Baryons as relativistic three-quark bound states. *Prog. Part. Nucl. Phys.* **2016**, *91*, 1–100. [CrossRef]
17. Sanchis-Alepuz, H.; Williams, R. Recent developments in bound-state calculations using the Dyson–Schwinger and Bethe–Salpeter equations. *Comput. Phys. Commun.* **2018**, *232*, 1–21. [CrossRef]
18. Fukuda, R.; Kugo, T. Schwinger-Dyson Equation for Massless Vector Theory and Absence of Fermion Pole. *Nucl. Phys. B* **1976**, *117*, 250–264. [CrossRef]
19. Jain, P.; Munczek, H.J. Calculation of the pion decay constant in the framework of the Bethe-Salpeter equation. *Phys. Rev. D* **1991**, *44*, 1873–1879. [CrossRef]
20. Munczek, H.J.; Jain, P. Relativistic pseudoscalar q anti-q bound states: Results on Bethe-Salpeter wave functions and decay constants. *Phys. Rev. D* **1992**, *46*, 438–445. [CrossRef]
21. Williams, R.; Fischer, C.S.; Heupel, W. Light mesons in QCD and unquenching effects from the 3PI effective action. *Phys. Rev. D* **2016**, *93*, 034026. [CrossRef]
22. Vujinovic, M.; Alkofer, R. Low-energy spectrum of an SU(2) gauge theory with dynamical fermions. *Phys. Rev. D* **2018**, *98*, 095030. [CrossRef]
23. Miramontes, A.S.; Sanchis Alepuz, H.; Alkofer, R. Elucidating the effect of intermediate resonances in the quark interaction kernel on the timelike electromagnetic pion form factor. *Phys. Rev. D* **2021**, *103*, 116006. [CrossRef]
24. Gao, F.; Papavassiliou, J.; Pawlowski, J.M. Fully coupled functional equations for the quark sector of QCD. *Phys. Rev. D* **2021**, *103*, 094013. [CrossRef]
25. Miramontes, A.S.; Alkofer, R.; Fischer, C.S.; Sanchis-Alepuz, H. Electromagnetic and strong isospin breaking in light meson masses. *Phys. Lett. B* **2022**, *833*, 137291. [CrossRef]
26. Cyrol, A.K.; Mitter, M.; Pawlowski, J.M.; Strodthoff, N. Nonperturbative quark, gluon, and meson correlators of unquenched QCD. *Phys. Rev. D* **2018**, *97*, 054006. [CrossRef]
27. Hopfer, M.; Fischer, C.S.; Alkofer, R. Running coupling in the conformal window of large-Nf QCD. *JHEP* **2014**, *11*, 035. [CrossRef]
28. Zierler, F.; Alkofer, R. On correlation functions of gauge theories close to and in the conformal window. **2023**, *to be published*.
29. DeGrand, T. Lattice tests of beyond Standard Model dynamics. *Rev. Mod. Phys.* **2016**, *88*, 015001. [CrossRef]
30. Witzel, O. Review on Composite Higgs Models. *Proc. Sci.* **2019**, *LATTICE2018*, 006. [CrossRef]
31. Gies, H.; Jaeckel, J. Chiral phase structure of QCD with many flavors. *Eur. Phys. J. C* **2006**, *46*, 433–438. [CrossRef]
32. Lee, J.W. Conformal window from conformal expansion. *Phys. Rev. D* **2021**, *103*, 076006. [CrossRef]
33. Curtis, D.C.; Pennington, M.R.; Walsh, D.A. On the gauge dependence of dynamical fermion masses. *Phys. Lett. B* **1990**, *249*, 528–530. [CrossRef]
34. Jia, S.; Pennington, M.R. How gauge covariance of the fermion and boson propagators in QED constrain the effective fermion-boson vertex. *Phys. Rev. D* **2016**, *94*, 116004. [CrossRef]
35. Albino, L.; Bashir, A.; Mizher, A.J.; Raya, A. Electron-photon vertex and dynamical chiral symmetry breaking in reduced QED: An advanced study of gauge invariance. *Phys. Rev. D* **2022**, *106*, 096007. [CrossRef]
36. Guzmán, V.M.B.; Bashir, A. One-loop fermion-photon vertex in arbitrary gauge and dimensions: A novel approach. *Phys. Rev. D* **2023**, *107*, 073008. [CrossRef]
37. Aguilar, A.C.; Binosi, D.; Papavassiliou, J. Yang-Mills two-point functions in linear covariant gauges. *Phys. Rev. D* **2015**, *91*, 085014. [CrossRef]
38. Aguilar, A.C.; Binosi, D.; Papavassiliou, J. Schwinger mechanism in linear covariant gauges. *Phys. Rev. D* **2017**, *95*, 034017. [CrossRef]
39. Napetschnig, M.; Alkofer, R.; Huber, M.Q.; Pawlowski, J.M. Yang-Mills propagators in linear covariant gauges from Nielsen identities. *Phys. Rev. D* **2021**, *104*, 054003. [CrossRef]
40. Fischer, C.S.; Zwanziger, D. Infrared behaviour and running couplings in interpolating gauges in QCD. *Phys. Rev. D* **2005**, *72*, 054005. [CrossRef]
41. Alkofer, R.; Fischer, C.S.; Reinhardt, H.; von Smekal, L. On the infrared behavior of gluons and ghosts in ghost antighost symmetric gauges. *Phys. Rev. D* **2003**, *68*, 045003. [CrossRef]
42. Capri, M.A.L.; Sobreiro, R.F.; Sorella, S.P.; Thibes, R. Renormalizability of a generalized gauge fixing interpolating among the Coulomb, Landau and maximal Abelian gauges. *Ann. Phys.* **2007**, *322*, 1776–1789. [CrossRef]
43. Andraši, A.; Taylor, J.C. Renormalization in a Landau-to-Coulomb interpolating gauge in Yang–Mills theory. *Ann. Phys.* **2021**, *431*, 168551. [CrossRef]
44. Huber, M.Q. Nonperturbative properties of Yang–Mills theories. *Phys. Rept.* **2020**, *879*, 1–92. [CrossRef]
45. Huber, M.Q. Correlation functions of Landau gauge Yang-Mills theory. *Phys. Rev. D* **2020**, *101*, 114009. [CrossRef]
46. Ferreira, M.N.; Papavassiliou, J. Gauge Sector Dynamics in QCD. *Particles* **2023**, *6*, 312–363. [CrossRef]
47. Kern, W.; Huber, M.Q.; Alkofer, R. Spectral dimension as a tool for analyzing nonperturbative propagators. *Phys. Rev. D* **2019**, *100*, 094037. [CrossRef]

48. Fischer, C.S.; Gruter, B.; Alkofer, R. Solving coupled Dyson-Schwinger equations on a compact manifold. *Ann. Phys.* **2006**, *321*, 1918–1938. [CrossRef]
49. Fischer, C.S.; Alkofer, R. Nonperturbative propagators, running coupling and dynamical quark mass of Landau gauge QCD. *Phys. Rev. D* **2003**, *67*, 094020. [CrossRef]
50. Windisch, A. Features of Strong Quark Correlations at Vanishing and Non-Vanishing Density. Ph.D. Thesis, University of Graz, Graz, Austria, 2014.
51. Hopfer, M.; Windisch, A.; Alkofer, R. The Quark-Gluon Vertex in Landau gauge QCD. *Proc. Sci.* **2012**, *ConfinementX*, 073. [CrossRef]
52. Alkofer, R.; Eichmann, G.; Fischer, C.S.; Hopfer, M.; Vujinovic, M.; Williams, R.; Windisch, A. On propagators and three-point functions in Landau gauge QCD and QCD-like theories. *Proc. Sci.* **2013**, *QCD-TNT-III*, 003. [CrossRef]
53. Blum, A.L.; Alkofer, R.; Huber, M.Q.; Windisch, A. Three-point vertex functions in Yang-Mills Theory and QCD in Landau gauge. *EPJ Web Conf.* **2017**, *137*, 03001. [CrossRef]
54. París-López, J.; Sanchis-Alepuz, H.; Alkofer, R. On the Dynamic Generation of Kinetic Terms for Mesonic Bound States. *Acta Phys. Polon. Supp.* **2018**, *11*, 495–500. [CrossRef]
55. Gies, H.; Schmieden, R.; Zambelli, L. Interplay of Chiral Transitions in the Standard Model. *arXiv* **2023**, arXiv:2306.05943.