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Three Convergence Results for Iterates of Nonlinear Mappings in Metric Spaces with Graphs

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Abstract: In 2007, in our joint work with D. Butnariu and S. Reich, we proved that if for a self-mapping of a complete metric that is uniformly continuous on bounded sets all its iterates converge uniformly on bounded sets, then this convergence is stable under the presence of small errors. In the present paper, we obtain an extension of this result for self-mappings of a metric space with a graph.

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1. Introduction

During the last sixty years, many results have been obtained in the fixed-point theory of nonlinear operators in complete metric spaces [1–15]. The first result in this area of research is Banach's celebrated theorem [16], which shows the existence of a unique fixed point of a strict contraction. This area of research includes the analysis of the asymptotic behavior of (inexact) iterates of a nonexpansive operator and their convergence to its fixed points. This research is also devoted to feasibility, common fixed points, iterative methods and variational inequalities with numerous applications in engineering and the medical and natural sciences [17–24].

In our joint paper with D. Butnariu and S. Reich [5], we proved that if for a self-mapping of a complete metric that is uniformly continuous on bounded sets all its iterates converge uniformly on bounded sets, then this convergence is stable under the presence of a small errors. In our present work, we obtain an extension of this result for self-mappings of a metric space with a graph. We also obtain a convergence result for a contractive-type mapping in a metric space with a graph.

It should be mentioned that nonexpansive mappings in metric spaces with graphs have recently been studied in [10,25–34].

2. The First and the Second Main Results

Assume that (X, ρ) is a metric space. For every point $u \in X$ and each nonempty set $D \subset X$, set

$$\rho(u, D) := \inf\{\rho(u, v) : \ v \in D\}.$$

For every point $u \in X$ and each number $r > 0$, put

$$B(u, r) := \{v \in X : \ \rho(u, v) \leq r\}.$$

For every operator $S : X \to X$, set $S^0(u) = u$ for all $u \in X$, $S^1 = S$ and $S^{i+1} = S \circ S^i$ for every nonnegative integer i. We denote the set of all fixed points of S by $F(S)$.

Assume that G is a graph such that $V(G) \subset X$ is the set of all its vertices and the set $E(G) \subset X \times X$ is the set of all its edges. We also assume that

$$(x, x) \in E(G), \ x \in X.$$

The graph G is identified with the pair $(V(G), E(G))$.
Fix $\theta \in X$.
Assume that $A : X \to X$ is a mapping and that the following assumptions hold:
(A1) There exists a unique point $x_A \in X$ satisfying $A(x_A) = x_A$.
(A2) $A^n(x) \to x_A$ as $n \to \infty$ uniformly over all bounded subsets of X.
(A3) A is bounded on bounded subsets of X.
(A4) For each $\epsilon, M > 0$ there exists $\delta > 0$ such that for each $x, y \in B(\theta, M)$ satisfying

$$(x, y) \in E(G) \text{ and } \rho(x, y) \leq \delta$$

the relations

$$(A(x), A(y)) \in E(G) \text{ and } \rho(A(x), A(y)) \leq \epsilon$$

are valid.

The next result is proved in Section 3.

Theorem 1. *Assume that K is a nonempty bounded subset of X and that $\epsilon > 0$. Then, there exist $\delta > 0$ and a natural number N such that for each integer $n \geq N$ and each sequence $\{x_i\}_{i=0}^n \subset X$, which satisfies*

$$x_0 \in K$$

and

$$\rho(A(x_i), x_{i+1}) \leq \delta \text{ and } (A(x_i), x_{i+1}) \in E(G)$$

for each integer $i \in \{0, \ldots, n-1\}$, the inequalities

$$\rho(x_i, x_A) \leq \epsilon, \; i = N, \ldots, n$$

and

$$\rho(x_i, A^i(x_0)) \leq \epsilon, \; i = 0, \ldots, 2N$$

hold.

Since Theorem 1 holds for any positive ϵ, it easily implies the following result.

Corollary 1. *Assume that $\{x_i\}_{i=0}^\infty$ is a bounded sequence such that*

$$\lim_{i\to\infty} \rho(A(x_i), x_{i+1}) = 0$$

and that $(A(x_i), x_{i+1}) \in E(G)$ for all integers $i \geq 0$. Then, $\lim_{i\to\infty} x_i = x_A$.

The next result is also proved in Section 3.

Theorem 2. *Assume that $\epsilon > 0$. Then, there exist $\delta > 0$ such that for each sequence $\{x_i\}_{i=0}^\infty$, which satisfies*

$$\rho(x_0, x_A) \leq \delta$$

and

$$\rho(A(x_i), x_{i+1}) \leq \delta \text{ and } (A(x_i), x_{i+1}) \in E(G)$$

for each integer $i \geq 0$, the inequality

$$\rho(x_i, x_A) \leq \epsilon$$

holds for each integer $i \geq 0$.

It should be mentioned that our results are obtained for a large class of operators. They cover the case when $E(G) = X \times X$, which was considered in [5], the class of nonexpansive mappings $A : X \to X$ on a metric space X with graphs satisfying

$$\rho(A(x), A(y)) \leq \rho(x, y)$$

for each $(x, y) \in E(G)$. It also contains the class of monotone nonexpansive mappings [35,36] and the class of uniformly locally nonexpansive mappings [37].

3. Proofs of Theorems 1 and 2

3.1. Proof of Theorem 1

We may assume without loss of generality that

$$\epsilon < 1/4,\ B(x_A, 8) \subset K. \tag{1}$$

Assumption (A2) implies that there exists an integer $N \geq 8$ for which

$$\rho(A^n(x), x_A) \leq \epsilon/4 \text{ for each integer } n \geq N \text{ and each } x \in K. \tag{2}$$

Assumption (A3) implies that $A^m(K)$ is bounded for all integers $m \geq 1$. Thus, there is $S > 0$ for which

$$A^i(K) \subset B(x_A, S),\ i = 0, \ldots, 2N. \tag{3}$$

By induction and (A4), there exists $\{\gamma_i\}_{i=0}^{2N} \subset (0, \infty)$ such that

$$\gamma_{2N} = \epsilon(16N)^{-1} \tag{4}$$

and that for each $i = 0, \ldots, 2N-1$,

$$\gamma_i \leq \gamma_{i+1}(4N)^{-1} \tag{5}$$

and

$$\begin{gathered}(A(x), A(y)) \in E(G) \text{ and } \rho(A(x), A(y)) \leq (4N)^{-1}\gamma_{i+1} \\ \text{for all } x, y \in B(x_A, S+4) \text{ satisfying } \rho(x, y) \leq \gamma_i,\ (x, y) \in E(G).\end{gathered} \tag{6}$$

Set

$$\delta = \gamma_0/2. \tag{7}$$

Lemma 1. *Assume that* $\{z_i\}_{i=0}^{2N} \subset X$ *satisfies*

$$z_0 \in K \tag{8}$$

and for each $i = 0, \ldots, 2N-1$,

$$\rho(z_{i+1}, A(z_i)) \leq \delta,\ (A(z_i), z_{i+1}) \in E(G). \tag{9}$$

Then,

$$\rho(z_i, A^i(z_0)) \leq \epsilon,\ i = 0, \ldots, 2N$$

and

$$\rho(z_i, x_A) \leq \epsilon,\ i = N, \ldots, 2N.$$

Proof. In view of (7) and (9),

$$\rho(z_1, A(z_0)) \leq \gamma_0,\ (A(z_0), z_1) \in E(G). \tag{10}$$

Equations (3) and (10) imply that

$$A(z_0) \in B(x_A, S),\ z_1 \in B(x_A, S+1). \tag{11}$$

It follows from (6), (7), (9) and (11) that

$$\rho(A^2(z_0), A(z_1)) \leq \gamma_1 (4N)^{-1},\ (A^2(z_0), A(z_1)) \in E(G).$$

By (9),

$$(A(z_1), z_2) \in E(G),\ \rho(A(z_1), z_2) \leq \gamma_0 \leq \gamma_1 (4N)^{-1}.$$

Assume that $k \in \{1, \dots, 2N-1\}$ and that for each $i \in \{1, \dots, k\}$

$$(A^{k-i+1}(z_{i-1}), A^{k-i}(z_i)) \in E(G) \tag{12}$$

and

$$\rho(A^{k-i+1}(z_{i-1}), A^{k-i}(z_i)) \leq \gamma_{k-1}. \tag{13}$$

(By (9), Equations (12) and (13) hold for $k = 1$.) By (3), (4), (8) and (13),

$$\rho(A^k(z_0), x_A) \leq S \tag{14}$$

and for each $p \in \{1, \dots, k\}$,

$$\begin{aligned} \rho(A^k(z_0), A^{k-p}(z_p)) &\leq \sum_{i=0}^{p-1} \rho(A^{k-i}(z_i), A^{k-i-1}(z_{i+1})) \\ &\leq p\gamma_{k-1} \leq 2N\gamma_{2N} \leq \epsilon/8. \end{aligned} \tag{15}$$

It follows from (14) and (15) that for each $p \in \{1, \dots, k\}$,

$$\rho(x_A, A^{k-p}(z_p)) \leq S+1, \tag{16}$$

$$\rho(A^k(z_0), z_k) \leq \epsilon/8. \tag{17}$$

By (6), (13) and (16), for each $i \in \{1, \dots, k\}$,

$$(A^{k-i+2}(z_{i-1}), A^{k-i+1}(z_i)) \in E(G),$$

and

$$\rho(A^{k-i+2}(z_{i-1}), A^{k-i+1}(z_i)) \leq \gamma_k.$$

In view of (9),

$$(A(z_k), z_{k+1}) \in E(G), \qquad \rho(A(z_k), z_{k+1}) \leq \gamma_k.$$

Thus, the assumption made for k also holds for $k+1$. Therefore, we showed by induction that our assumption holds for $k = 1, \dots, 2N$ and that for all $k = 1, \dots, 2N$,

$$\rho(A^k(z_0), z_k) \leq \epsilon/8. \tag{18}$$

By (2), (8) and (18), for each $i \in \{N, \dots, 2N\}$,

$$\rho(z_i, x_A) \leq \rho(z_i, A^i(z_0)) + \rho(A^i(z_0), x_A) \leq \epsilon/8 + \epsilon/4.$$

Lemma 1 is proved. □

Let us complete the proof of Theorem 1. Assume that $n \geq N$ is an integer and that the sequence $\{x_i\}_{i=0}^{n} \subset X$ satisfies

$$x_0 \in K \tag{19}$$

and for every $i \in \{0, \ldots, n-1\}$,

$$\rho(x_{i+1}, A(x_i)) \leq \delta, \qquad (A(x_i), x_{i+1}) \in E(G). \tag{20}$$

In $n \leq 2N$, then the assertion of Theorem 1 follows from Lemma 1. Therefore, we may assume without loss of generality that

$$n > 2N.$$

Lemma 1 implies that

$$\rho(x_j, x_A) \leq \epsilon, \ j = N, \ldots, 2N. \tag{21}$$

We prove that

$$\rho(x_j, x_A) \leq \epsilon, \ j = N, \ldots, n.$$

Assume the contrary. Then, there exists an integer $q \in (2N, n]$ such that

$$\rho(x_q, x_A) > \epsilon. \tag{22}$$

By (21) and (22), we may assume without loss of generality that

$$\rho(x_j, x_A) \leq \epsilon, \ j \in \{2N, \ldots, q-1\}. \tag{23}$$

Define

$$z_j = x_{j+q-N}, \ j = 0, \ldots, N, \ z_{j+1} = A(z_j), \ j = N, \ldots, 2N-1. \tag{24}$$

We show that $\{z_i\}_{i=0}^{2N}$ satisfies the assumptions of Lemma 1. By (20) and (24), we need only to show that $z_0 \in K$. In view of (21), (23) and (24),

$$z_0 = x_{q-N}, \ \rho(z_0, x_A) \leq \epsilon$$

and

$$z_0 \in K.$$

Lemma 1 and (24) imply that

$$\rho(x_A, x_q) = \rho(x_A, z_N) \leq \epsilon.$$

This contradicts (22). The contradiction we have reached completes the proof of Theorem 1.

3.2. *Proof of Theorem 2*

Proof. We may assume that $\epsilon < 1$. Set

$$K = B(x_A, 4).$$

Theorem 1 and the continuity of A at x_A imply that there exist

$$\delta \in (0, \epsilon/2)$$

and a natural number N such that the following property holds:

(a) For each integer $n \geq N$ and each sequence $\{x_i\}_{i=0}^{n} \subset X$ that satisfies

$$x_0 \in K$$

and

$$\rho(A(x_i), x_{i+1}) \leq \delta \text{ and } (A(x_i), x_{i+1}) \in E(G)$$

for each integer $i \in \{0, \ldots, n-1\}$, the inequalities

$$\rho(x_i, x_A) \leq \epsilon/8, \; i = N, \ldots, n, \tag{25}$$

$$\rho(x_i, A^i(x_0)) \leq \epsilon/8, \; i = 0, \ldots, 2N \tag{26}$$

and

$$\rho(A^i(x_0), x_A) \leq \epsilon/8, \; i = 0, \ldots, 2N \tag{27}$$

hold.

Assume that an integer $n \geq N$ and that a sequence $\{x_i\}_{i=0}^{n} \subset X$ satisfy

$$\rho(x_0, x_A) \leq \delta$$

and for each integer $i \in \{0, \ldots, n-1\}$,

$$\rho(A(x_i), x_{i+1}) \leq \delta \text{ and } (A(x_i), x_{i+1}) \in E(G).$$

Then, by property (a), Equations (25)–(27) hold. By (26) and (27), for each $i \in \{0, \ldots, N\}$,

$$\rho(x_i, x_A) \leq \rho(x_i, A^i(x_0)) + \rho(A^i(x_0), x_A) < \epsilon.$$

Theorem 2 is proved. □

4. The Third Main Result

Assume that (X, ρ) is a complete metric space and G is a graph such that $V(G) \subset X$ is the set of all its vertices and the set $E(G) \subset X \times X$ is the set of all its edges. We also assume that the space X is bounded:

$$D := \sup\{\rho(x, y) : \; x, y \in X\} < \infty.$$

Assume that Q is a natural number Q such that the following assumption holds:

(A5) For each $x, y \in X$ there exist $x_0, \ldots, x_q \in X$ such that $q \leq Q$,

$$x_0 = x, \; x_q = y,$$

$$(x_i, x_{i+1}) \in E(G), \; i = 0, \ldots, q-1.$$

Assume that $A : X \to X$ and that $\phi : [0, \infty) \to [0, 1]$ is a decreasing function such that

$$\phi(t) < 1 \text{ for all } t > 0 \tag{28}$$

and that the following assumption holds:

(A6) For all $x, y \in X$, if $(x, y) \in E(G)$, then $(A(x), A(y)) \in E(G)$ and

$$\rho(A(x), A(y)) \leq \phi(\rho(x, y))\rho(x, y).$$

We prove the following result.

Theorem 3. *There exists $x_A \in X$ such that $A^n(x) \to x_A$ as $n \to \infty$ uniformly for $x \in X$. Moreover, if A is continuous at x_A, then $A(x_A) = x_A$.*

Proof. Let $\epsilon \in (0,1)$. In order to prove our theorem, it is sufficient to show that there exists a natural number p such that for each $x, y \in X$,

$$\rho(A^p(x), A^p(y)) \leq \epsilon.$$

Choose an integer

$$p > 1 + \epsilon^{-1}Q^2D(1 - \phi(\epsilon Q^{-1})). \tag{29}$$

Let $x, y \in X$. By (A5), there exist an integer $q \leq Q$ and $x_0, \ldots, x_q \in X$ such that

$$x_0 = x, \ x_q = y, \tag{30}$$

$$(x_i, x_{i+1}) \in E(G), \ i = 0, \ldots, q-1. \tag{31}$$

It order to complete the proof, it is sufficient to show that there exists $j \in \{0, \ldots, p\}$ such that

$$\rho(A^j(x_i), A^j(x_{i+1})) \leq \epsilon/Q, \ i = 0, \ldots, Q-1.$$

Assume the contrary. Then, for each $j \in \{0, \ldots, p\}$,

$$\max\{\rho(A^j(x_i), A^j(x_{i+1})) : \ i = 0, \ldots, q-1\} > \epsilon/Q.$$

Let $j \in \{0, \ldots, p\}$. In view of the equation above, there exists

$$i_j \in \{0, \ldots, q-1\}$$

such that

$$\rho(A^j(x_{i_j}), A^j(x_{i_j+1})) > \epsilon/Q. \tag{32}$$

Assumption (A6) and (31) imply that

$$\rho(A^{j+1}(x_i), A^{j+1}(x_{i+1})) \leq \rho(A^j(x_i), A^j(x_{i+1})), \ i = 0, \ldots, q-1. \tag{33}$$

Assumption (A6) and (31), (32) imply that

$$\rho(A^{j+1}(x_{i_j}), A^{j+1}(x_{i_j+1})) \leq \phi(\rho(A^j(x_{i_j}), A^j(x_{i_j+1})))\rho(A^j(x_{i_j}), A^j(x_{i_j+1}))$$

$$\leq \phi(\epsilon/Q)\rho(A^j(x_{i_j}), A^j(x_{i_j+1}))$$

and

$$\begin{aligned} &\rho(A^j(x_{i_j}), A^j(x_{i_j+1})) - \rho(A^{j+1}(x_{i_j}), A^{j+1}(x_{i_j+1})) \\ &\geq (1 - \phi(\epsilon/Q))\rho(A^j(x_{i_j}), A^j(x_{i_j+1})) \geq (1 - \phi(\epsilon/Q))\epsilon/Q. \end{aligned} \tag{34}$$

By (33) and (34),

$$\sum_{i=0}^{q-1} \rho(A^j(x_i), A^j(x_{i+1}) - \sum_{i=0}^{q-1} \rho(A^{j+1}(x_i), A^{j+1}(x_{i+1})) \geq (1 - \phi(\epsilon/Q))\epsilon/Q. \tag{35}$$

In view of (35),

$$QD \geq \sum_{i=0}^{q-1} \rho(x_i, x_{i+1})$$

$$\geq \sum_{i=0}^{q-1} \rho(x_i, x_{i+1}) - \sum_{i=0}^{q-1} \rho(A^p(x_i), A^p(x_{i+1}))$$

$$= \sum_{j=0}^{p-1} (\sum_{i=0}^{q-1} \rho(A^j(x_i), A^j(x_{i+1})) - \sum_{i=0}^{q-1} \rho(A^{j+1}(x_i), A^{j+1}(x_{i+1})))$$

$$\geq p\epsilon Q^{-1}(1 - \phi(\epsilon/Q))$$

and

$$p \leq \epsilon^{-1} Q^2 D(1 - \phi(\epsilon/Q)).$$

This contradicts (29). The contradiction we have reached completes the proof of Theorem 3. □

5. Conclusions

In this paper, we study the behaviour of inexact iterates of a self-mapping A of a metric space with a graph. Assuming that A is bounded on bounded sets and that it uniformly converges on bounded sets to a unique fixed point, we show that this convergense is stable under the presence of computational errors. A prototype of our results for self-mappings of a metric space without graphs was obtained in our joint paper with D. Butnariu and S. Reich [5]. It should be mentioned that our results are obtained for a large class of operators. They cover the case when $E(G) = X \times X$, which was considered in [5], the class of nonexpansive mappings $A :\to X$ on a metric space X with graphs satisfying

$$\rho(A(x), A(y)) \leq \rho(x, y)$$

for each $(x, y) \in E(G)$. This also contains the class of monotone nonexpansive mappings [35,36] and the class of uniformly locally nonexpansive mappings [37].

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