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Optimizing Mean Estimators with Calibrated Minimum Covariance Determinant in Median Ranked Set Sampling

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Abstract: Calibration methods enhance estimates by modifying the initial design weights, for which supplementary information is exploited. This paper first proposes a generalized class of minimum-covariance-determinant (MCD)-based calibration estimators and then presents a novel class of MCD-based calibrated estimators under a stratified median-ranked-set-sampling (MRSS) design. Further, we also present a double MRSS version of generalized and novel classes of estimators. To assess and compare the performance of the generalized and novel classes of estimators, both real and artificial datasets are utilized. In the presented practical scenarios and real-world applications, we utilize information from a dataset comprising 800 individuals in Turkey from 2014. These data include body mass index (BMI) as the primary variable of interest and age values as auxiliary variables. The BMI results shows that the proposed estimators ($\bar{y}_{PM_I} = 581.1897$, $\bar{y}_{PaM_I} = 544.8397$) have minimum and ($\bar{y}_{PM_{II}} = 669.1822$, $\bar{y}_{PaM_{II}} = 648.2363$) have maximum PREs in the case of single and double MRSS for odd sample sizes. Similarly, ($\bar{y}_{PM_I} = 860.0099$, $\bar{y}_{PaM_I} = 844.7803$) have minimum and ($\bar{y}_{PM_{II}} = 974.5859$, $\bar{y}_{PaM_{II}} = 953.7233$) have maximum PREs in the case of single and double MRSS for even sample sizes. Additionally, we conduct a simulation study using a symmetric dataset.

Keywords: median ranked set sampling; double median ranked set sampling; auxiliary information; minimum covariance determinant estimators; calibration



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1. Introduction

In numerous investigations conducted in real-world settings, particularly in the fields of ecology and environmental research, the main focus variable (referred to as Y) can be challenging to directly observe due to factors such as high costs, procedures that require a significant amount of labor, intrusiveness, or the possibility of harming the subjects under study. Despite the challenges and complexities involved in data collection, it is often relatively simple and cost-effective to rank the sampled units. To demonstrate this point, let us take the case of calliphoridae flies as an example. These flies possess an inherent survival mechanism that enables them to rapidly detect and inhabit a source of food, such as a decaying body, soon after it has perished. In investigations conducted after death, forensic entomologists often depend on the larvae of these flies to estimate the post-mortem interval. The larvae cease feeding once they reach their maximum size. By observing the volume of their intestinal contents (as their anterior intestine remains empty during further development), forensic entomologists can accurately determine the post-mortem interval. However, using radiographic techniques to assess changes in the maggots' intestinal contents presents challenges (Sharma et al. [1]). On the other hand, since the larvae appear to continuously grow in length, measuring and ranking their length is relatively straightforward. Another example can be found in a health-related research endeavor aiming to obtain an average estimate of the cholesterol level of a population. Rather than performing intrusive blood tests on every participant in the sample, it is

possible to visually rank the subjects based on their weights, and blood samples can be collected from only a small number of individuals.

McIntyre [2] provided the initial proposal of RSS. The RSS procedure can be described as follows: A sample is selected from a population N using simple random sampling (SRS). Each unit in this sample undergoes evaluation based on subjective criteria. Only the smallest unit is measured, while the rest are disregarded. Similarly, a second sample is selected, and only the second-smallest unit is measured, while the rest are disregarded. The process of selecting a new sample and measuring the subsequent smallest unit is repeated until the desired sample size is achieved.

Since its inception, ranked set sampling (RSS) has garnered significant attention from researchers and remains an active area of study. While it originated in horticulture with McIntyre's foundational work in 1952, RSS has expanded its applications and is now being utilized in commercial settings. To delve deeper into the intricacies of RSS, interested readers can consult the works of Chen et al. [3], Hassan et al. [4], Bouza [5], Nagy et al. [6], and Benchiha et al. [7]. Shahzad et al. [8] successively used the ranked and true observations of auxiliary variables for mean estimation in MRSS. The three-fold use of auxiliary variables was suggested by Shahzad et al. [9] for mean estimation in MRSS. Bushan et al. [10] defined difference-type estimators in RSS. Muttalak [11] introduced a variation of RSS known as MRSS in order to estimate population means. Muttalak demonstrated that MRSS yields more precise estimates than RSS. In MRSS, rather than measuring the k th ($k = 1, 2, \dots, \theta$) minimum observation, the median of each sample within a cycle is measured. Essentially, MRSS can be seen as an adapted form of RSS designed to improve the accuracy of estimation.

The classic ratio estimator is widely recognized and commonly used in sampling theory to estimate population means (Oral and Oral [12]). Expanding upon this estimator, Al-Omari [13] introduced novel ratio-type estimators that incorporate the MRSS scheme. Later, Koyuncu [14] extended the concepts introduced by Al-Omari [13] and developed different types of estimators. However, all these efforts focused on ratio and difference-type mean estimation within the framework of MRSS. It is worth noting that these estimators rely on traditional descriptive statistics measures. However, no studies have been conducted on calibrated mean estimation under MRSS using robust covariance matrices, such as the MCD matrix. Thus, this study represents an initial step toward developing robust calibrated mean estimators within the MRSS framework.

The structure of this document is as follows: Section 2 begins by introducing the calibration technique and presenting the modified estimators for stratified MRSS. In Sections 3 and 4, a fresh set of estimators is introduced under single and double MRSS schemes. In Section 5, a comprehensive simulation analysis is carried out to compare the effectiveness of the suggested estimators with alternative methods. The Section 6 offers concluding remarks that summarize the findings of this paper.

2. Generalized Class of Calibrated MCD-Based Estimators

MCD estimation was defined by Rousseeuw [15]. To estimate multivariate locations and dispersion with a high-breakdown point, it is necessary to assess the determinant of the Σ_m (variance–covariance matrix). When Σ_m is a positive semi-definite matrix with dimensions of $n_\varphi \times n_\varphi$ and P_1 positive eigenvalues, the determinant represents the product of these eigenvalues. Hence, a small determinant value indicates the presence of linear patterns in the data. The MCD method involves considering all subsets of size n_φ from a dataset and calculating the determinant of Σ_m for each subset. The MCD estimators are obtained by selecting the subset with the lowest determinant along with the typical $1 \times P_1$ mean vector and its corresponding $P_1 \times P_1$ Σ_m matrix. These estimators are discussed in the study by Muthukrishnan and Mahesh [16]. Note that estimators related to central tendency and dispersion can also be improved in the MCD framework by using auxiliary information.

Incorporating auxiliary information has the potential to greatly enhance the mean estimators. In various real-life scenarios, a linear correlation can be observed between a

study variable Y and an auxiliary variable X (see Shahzad et al. [17] and Abbasi et al. [18]). As an example, consider the association between depression and suicide, where individuals with severe depression are more likely to commit suicide compared to those without depression (Johnson et al. [19]). Additionally, we can consider the established direct and positive correlation between body mass index (BMI) and total cholesterol levels (Schroder et al. [20]). These scenarios demonstrate how auxiliary variables can provide valuable information and contribute to more accurate mean estimation.

Zaman and Bulut [21] introduced the concept of MCD-based mean estimation using auxiliary information. Shahzad et al. [22] extended their work on handling missing observations. Zaman and Bulut [23] also introduced MCD-based variance estimators. To learn more about MCD-based mean and variance estimation within simple and stratified sampling designs, readers can refer to Zaman and Bulut [24,25]. However, to the best of our knowledge, no attention has been paid to MCD-based calibrated mean estimators in a stratified MRSS design.

Calibration is a technique used for the development of modified weights. Calibration estimation is a core methodology that seeks to refine initial weights by minimizing a designated measure of distance while incorporating auxiliary data. In the corresponding literature, scholars have investigated the application of calibration weighting in stratification to improve the precision of population parameter estimates. The generation of new calibration weights relies on two key components: (1) a distance function and (2) constraints. These two components serve as the basis for constructing enhanced calibration weights. Since the study variable and the auxiliary variables exhibit a strong correlation, effective weights for the auxiliary variable are expected to be effective for the research variable as well. Building on the pioneering work of Deville and Sarndal [26], many researchers have explored calibration estimation using different calibration constraints in survey sampling (see [14,27–30]). Drawing inspiration from these significant studies, we propose mean estimators using MRSS under the MCD framework.

In a stratified MRSS sampling design, we draw a random sample of size n_φ without replacement from a population with a size of N_φ in stratum φ (where $\varphi = 1, 2, \dots, L$). Let $(X_{i(1)}, Y_{i[1]}), (X_{i(2)}, Y_{i[2]}), \dots, (X_{i(n_\varphi)}, Y_{i[n_\varphi]})$ represent the order statistics of $X_{i(1)}, X_{i(2)}, \dots, X_{i(n_\varphi)}$ and the imperfectly ranked order of $Y_{i[1]}, Y_{i[2]}, \dots, Y_{i[n_\varphi]}$ for the units in the φ -th stratum, where $(\cdot)$ and $[\cdot]$ indicate perfect ranking for X and imperfect ranking for Y , respectively. We denote the units measured using MRSS as $m(O)$ for odd sample sizes and $m(E)$ for even sample sizes.

Define the observed units $m(O)$ for the case of an odd sample size in the φ -th stratum as $(X_{1(\frac{n_\varphi+1}{2})}, Y_{1[\frac{n_\varphi+1}{2}]}), (X_{2(\frac{n_\varphi+1}{2})}, Y_{2[\frac{n_\varphi+1}{2}]}), \dots, (X_{n_\varphi(\frac{n_\varphi+1}{2})}, Y_{n_\varphi[\frac{n_\varphi+1}{2}]})$. Let

$$\bar{x}_{st(m(O))} = \sum_{\varphi=1}^L W_\varphi \bar{x}_{\varphi(m(O))} \quad \text{and} \quad \bar{y}_{st(m(O))} = \sum_{\varphi=1}^L W_\varphi \bar{y}_{\varphi(m(O))} \quad (1)$$

$$\bar{y}_{\varphi(m(O))} = \frac{1}{n_\varphi} \sum_{i=1}^{n_\varphi} Y_{i[\frac{n_\varphi+1}{2}]} \quad \text{and} \quad \bar{x}_{\varphi(m(O))} = \frac{1}{n_\varphi} \sum_{i=1}^{n_\varphi} X_{i(\frac{n_\varphi+1}{2})} \quad (2)$$

denote the overall averages in the φ th strata in Equation (1) and the sample averages in the φ th stratum in Equation (2). Below,

$$(X_{1(\frac{n_\varphi}{2})}, Y_{1[\frac{n_\varphi}{2}]}), (X_{2(\frac{n_\varphi}{2})}, Y_{2[\frac{n_\varphi}{2}]}), \dots, (X_{\frac{n_\varphi}{2}(\frac{n_\varphi}{2})}, Y_{\frac{n_\varphi}{2}[\frac{n_\varphi}{2}]}), (X_{\frac{n_\varphi}{2}+1(\frac{n_\varphi}{2})}, Y_{\frac{n_\varphi}{2}+1[\frac{n_\varphi}{2}]}), (X_{\frac{n_\varphi}{2}+2(\frac{n_\varphi}{2})}, Y_{\frac{n_\varphi}{2}+2[\frac{n_\varphi}{2}]}), \dots, (X_{n_\varphi(\frac{n_\varphi}{2})}, Y_{n_\varphi[\frac{n_\varphi}{2}]})$$

$$\bar{x}_{st(m(E))} = \sum_{\varphi=1}^L W_\varphi \bar{x}_{\varphi(m(E))} \quad \text{and} \quad \bar{y}_{st(m(E))} = \sum_{\varphi=1}^L W_\varphi \bar{y}_{\varphi(m(E))} \quad (3)$$

$$\bar{x}_{\varphi(m(E))} = \frac{1}{n_{\varphi}} \left(\sum_{i=1}^{\frac{n_{\varphi}}{2}} X_{i(\frac{n}{2})} + \sum_{i=\frac{n_{\varphi}+2}{2}}^{n_{\varphi}} X_{i(\frac{n+2}{2})} \right) \quad \text{and} \quad \bar{y}_{\varphi(m(E))} = \frac{1}{n_{\varphi}} \left(\sum_{i=1}^{\frac{n_{\varphi}}{2}} Y_{i[\frac{n_{\varphi}}{2}]} + \sum_{i=\frac{n_{\varphi}+2}{2}}^{n_{\varphi}} Y_{i[\frac{n_{\varphi}+2}{2}]} \right), \quad (4)$$

are the observed units, i.e., $m(E)$ overall averages in the φ th strata Equation (3) and the sample averages in the φ th stratum Equation (4), for an even sample size. Note that $W_{\varphi} = \frac{N_{\varphi}}{N}$ is the stratum weight.

Now, we provide a generalized class of calibration estimators under an MRSS design, which are expressed as

$$\bar{y}_{TM(i)} = \sum_{\varphi=1}^L \Psi_{\varphi} \bar{y}_{\varphi(m(j))}, \quad \text{for } i = 1, 2 \quad (5)$$

and are subject to the following constraints:

$$\sum_{\varphi=1}^L \Psi_{\varphi} = \sum_{\varphi=1}^L W_{\varphi}, \quad (6)$$

$$\sum_{\varphi=1}^L \Psi_{\varphi} \hat{\Delta}x_{\varphi(m(j))} = \sum_{\varphi=1}^L W_{\varphi} \Delta x_{\varphi(m)} \quad (7)$$

where Ψ_{φ} is the calibrated weight, and j denotes odd and even sample sizes for MRSS, i.e., ($j = O, E$). Defining the Lagrange function with its multipliers $\eta_{1(m(j))}$ and $\eta_{2(m(j))}$ yields

$$\Delta_{(m(j))} = \sum_{\varphi=1}^L \frac{(\Psi_{\varphi} - W_{\varphi})^2}{\tau_{\varphi} W_{\varphi}} - 2\eta_{1(m(j))} \left[\sum_{\varphi=1}^L \Psi_{\varphi} - \sum_{\varphi=1}^L W_{\varphi} \right] - 2\eta_{2(m(j))} \left[\sum_{\varphi=1}^L \Psi_{\varphi} \hat{\Delta}x_{\varphi(m(j))} - \sum_{\varphi=1}^L W_{\varphi} \Delta x_{\varphi(m)} \right]. \quad (8)$$

$\sum_{\varphi=1}^L \frac{(\Psi_{\varphi} - W_{\varphi})^2}{\tau_{\varphi} W_{\varphi}}$ is the chi-square distance function, where τ_{φ} is 1 or a reciprocal of any known characteristics of auxiliary information. Through the following calculation, $\frac{\delta \Delta_{(m(j))}}{\delta \Psi_{\varphi}} = 0$, we obtain

$$\Psi_{\varphi} = W_{\varphi} + \tau_{\varphi} W_{\varphi} \left[\eta_{2(m(j))} \hat{\Delta}x_{\varphi(m(j))} + \eta_{1(m(j))} \right], \quad (9)$$

By inserting (9) into (6) and (7), we obtain

$$\eta_{1(m(j))} = - \frac{\sum_{\varphi=1}^L W_{\varphi} \left[\Delta x_{\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right]^2}, \quad (10)$$

$$\eta_{2(m(j))} = \frac{\sum_{\varphi=1}^L W_{\varphi} \left[\Delta x_{\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right]^2}. \quad (11)$$

By substituting $\eta_{1(m(j))}$ and $\eta_{2(m(j))}$ into (9), we obtain

$$\Psi_{\varphi} = W_{\varphi} + \tau_{\varphi} W_{\varphi} \frac{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right] \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right]^2} \sum_{\varphi=1}^L W_{\varphi} [\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}], \quad (12)$$

Inserting Ψ_{φ} into $\bar{y}_{TM}$ yields the calibrated mean estimator of the study variable

$$\bar{y}_{TM(i)} = \sum_{\varphi=1}^L W_{\varphi} \bar{y}_{\varphi(m(j))} + \frac{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right]^2} \sum_{\varphi=1}^L W_{\varphi} [\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}]. \quad (13)$$

This estimator can be rewritten as

$$\bar{y}_{TM(i)} = \bar{y}_{st(m(j))} + \hat{b}_j \left[\sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \right], \quad (14)$$

where

$$\hat{b}_j = \frac{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right]^2}. \quad (15)$$

$$\bar{y}_{TM(i)} = \begin{cases} \bar{y}_{st(m(O))} + \hat{b}_{(O)} \left[\sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(O))}) \right] & \text{when } n \text{ is odd} \\ \bar{y}_{st(m(E))} + \hat{b}_{(E)} \left[\sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(E))}) \right] & \text{when } n \text{ is even.} \end{cases} \quad (16)$$

Note that in a generalized class, $\Delta x_{\varphi(m)}$ be any known characteristic of an auxiliary variable. By replacing $\Delta x_{\varphi(m)}$ with some known parameters, for instance, the arithmetic mean $\bar{X}_{\varphi}$ and the coefficient of variation Cx_{φ} , we can obtain the estimators reported in [17,29,30]. The cited authors defined these estimators under simple and MRSS designs. We adapt their work under the MCD framework, as shown in Table 1. Further, many other estimators can be developed by replacing $\Delta x_{\varphi(m)}$ with some known population characteristics.

Table 1. Family members of all classes.

MRSS Estimators	τ_{φ}	Double MRSS Estimators	$(\hat{\Delta} x_{\varphi(m(j))}, \Delta x_{\varphi(m)})$
$\bar{y}_{TM(1)_I}$	1	$\bar{y}_{TaM(1)_I}$	$(\bar{x}_{\varphi(m(j))}, \bar{X}_{\varphi(m)})$
$\bar{y}_{TM(1)_{II}}$	$1/\hat{C}_{x\varphi(m(j))}$	$\bar{y}_{TaM(1)_{II}}$	$(\bar{x}_{\varphi(m(j))}, \bar{X}_{\varphi(m)})$
$\bar{y}_{TM(1)_{III}}$	$1/\bar{x}_{\varphi(m(j))}$	$\bar{y}_{TaM(1)_{III}}$	$(\bar{x}_{\varphi(m(j))}, \bar{X}_{\varphi(m)})$
$\bar{y}_{TM(2)_I}$	1	$\bar{y}_{TaM(2)_I}$	$(\hat{C}x_{\varphi(m(j))}, Cx_{\varphi(m)})$
$\bar{y}_{TM(2)_{II}}$	$1/\hat{C}_{x\varphi(m(j))}$	$\bar{y}_{TaM(2)_{II}}$	$(\hat{C}x_{\varphi(m(j))}, Cx_{\varphi(m)})$
$\bar{y}_{TM(2)_{III}}$	$1/\bar{x}_{\varphi(m(j))}$	$\bar{y}_{TaM(2)_{III}}$	$(\hat{C}x_{\varphi(m(j))}, Cx_{\varphi(m)})$
$\bar{y}_{PM_I}$	1	$\bar{y}_{PaM_I}$	$(\hat{C}x_{\varphi(m(j))}, Cx_{\varphi(m)})$
$\bar{y}_{PM_{II}}$	$1/\hat{C}_{x\varphi(m(j))}$	$\bar{y}_{PaM_{II}}$	$(\hat{C}x_{\varphi(m(j))}, Cx_{\varphi(m)})$
$\bar{y}_{PM_{III}}$	$1/\bar{x}_{\varphi(m(j))}$	$\bar{y}_{PaM_{III}}$	$(\hat{C}x_{\varphi(m(j))}, Cx_{\varphi(m)})$

3. Novel Class of Calibrated MCD-Based Estimators

Taking inspiration from Koyuncu [14,28], we define the following MCD-based mean estimator under stratified MRSS:

$$\bar{y}_{PM} = \sum_{\varphi=1}^L \Psi_{\varphi} \bar{y}_{\varphi(m(j))}, \quad (17)$$

which is subject to the following constraints:

$$\sum_{\varphi=1}^L \Psi_{\varphi} \bar{x}_{\varphi(m(j))} = \sum_{\varphi=1}^L W_{\varphi} \bar{X}_{\varphi(m)} \quad (18)$$

$$\sum_{\varphi=1}^L \Psi_{\varphi} \hat{\Delta}x_{\varphi(m(j))} = \sum_{\varphi=1}^L W_{\varphi} \Delta x_{\varphi(m)} \quad (19)$$

$$\sum_{\varphi=1}^L \Psi_{\varphi} = \sum_{\varphi=1}^L W_{\varphi}, \quad (20)$$

The Lagrange function is defined with its multipliers $\eta_{1(m(j))}$, $\eta_{2(m(j))}$, and $\eta_{3(m(j))}$

$$\begin{aligned} \Delta_{(m(j))} = & \sum_{\varphi=1}^L \frac{(\Psi_{\varphi} - W_{\varphi})^2}{\tau_{\varphi} W_{\varphi}} - 2\eta_{1(m(j))} \left[\sum_{\varphi=1}^L \Psi_{\varphi} \bar{x}_{\varphi(m(j))} - \sum_{\varphi=1}^L W_{\varphi} \bar{X}_{\varphi(m)} \right] \\ & - 2\eta_{2(m(j))} \left[\sum_{\varphi=1}^L \Psi_{\varphi} \hat{\Delta}x_{\varphi(m(j))} - \sum_{\varphi=1}^L W_{\varphi} \Delta x_{\varphi(m)} \right] - 2\eta_{3(m(j))} \left[\sum_{\varphi=1}^L \Psi_{\varphi} - \sum_{\varphi=1}^L W_{\varphi} \right]. \end{aligned} \quad (21)$$

Through the following calculation, $\frac{\delta \Delta_{(m(j))}}{\delta \Psi_{\varphi}} = 0$, we obtain

$$\Psi_{\varphi} = W_{\varphi} + \tau_{\varphi} W_{\varphi} \left[\eta_{1(m(j))} \bar{x}_{\varphi(m(j))} + \eta_{2(m(j))} \hat{\Delta}x_{\varphi(m(j))} + \eta_{3(m(j))} \right], \quad (22)$$

By substituting (22) into (18)–(20), we obtain a system of equations containing three equations:

$$G_{(3 \times 3)} \eta_{(3 \times 1)} = F_{(3 \times 1)}, \quad (23)$$

where

$$\eta_{(3 \times 1)} = \begin{bmatrix} \eta_{1(m(j))} \\ \eta_{2(m(j))} \\ \eta_{3(m(j))} \end{bmatrix},$$

$$F_{(3 \times 1)} = \begin{bmatrix} \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \\ \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))}) \\ 0 \end{bmatrix},$$

$$G_{(3 \times 3)} = \begin{bmatrix} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \\ \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\ \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \end{bmatrix}.$$

By solving Equation (23), we obtain

$$\eta_{1(m(j))} = \frac{D_{1(m(j))}}{H}, \quad \eta_{2(m(j))} = \frac{D_{2(m(j))}}{H}, \quad \eta_{3(m(j))} = \frac{D_{3(m(j))}}{H},$$

where

$$\begin{aligned} D_{1(m(j))} &= \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2 \\ &\quad + \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right), \\ D_{2(m(j))} &= \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right)^2 \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \\ &\quad + \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right), \\ D_{3(m(j))} &= \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\ &\quad + \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right). \end{aligned}$$

$$\begin{aligned}
H = & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right)^2 \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2 - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2 \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \\
& + 2 \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right).
\end{aligned}$$

Substituting these values into (22) and (17) yields

$$\bar{y}_{PM} = \bar{y}_{st(m(j))} + \eta_{1(m(j))} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) + \eta_{2(m(j))} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) + \eta_{3(m(j))} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right), \quad (24)$$

$$= \sum_{\varphi=1}^L W_{\varphi} \bar{y}_{\varphi(m(j))} + \hat{b}_1 \left[\sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(j))}) \right] + \hat{b}_2 \left[\sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))}) \right], \quad (25)$$

where

$$\hat{b}_{1(j)} = \frac{D_{4(m(j))}}{H}, \quad \hat{b}_{2(j)} = \frac{D_{5(m(j))}}{H},$$

where

$$\begin{aligned}
D_{4(m(j))} = & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2 \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right),
\end{aligned}$$

$$\begin{aligned}
D_{5(m(j))} = & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right)^2.
\end{aligned}$$

$$\bar{y}_{PM} = \begin{cases} \bar{y}_{st(m(O))} + \hat{b}_{1(O)} \left[\sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(O))}) \right] + \hat{b}_{2(O)} \left[\sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(O))}) \right] & \text{when } n \text{ is odd} \\ \bar{y}_{st(m(E))} + \hat{b}_{1(E)} \left[\sum_{\varphi=1}^L W_{\varphi} (\bar{X}_{\varphi(m)} - \bar{x}_{\varphi(m(E))}) \right] + \hat{b}_{2(E)} \left[\sum_{\varphi=1}^L W_{\varphi} (\Delta x_{\varphi(m)} - \hat{\Delta} x_{\varphi(m(E))}) \right] & \text{when } n \text{ is even.} \end{cases} \quad (26)$$

4. Double-Stratified MRSS

In practical situations where the population mean of an auxiliary variable is not known, the double sampling method can be utilized to estimate the population mean. In this section, we specifically focus on a scenario where the mean of the auxiliary variable is unavailable. Our approach builds upon the framework established by Al-Omari [13] and Koyuncu [14] for the double MRSS design. However, we adapt their framework for a two-stage (double) MRSS design, where simple random sampling is employed in the first stage, followed by MRSS in the second stage. It is important to note that in the φ th stratum, $n_{a\varphi} = n_{\varphi}^2$ represents the sample size for the first phase, and n_{φ} represents the sample size for the second phase. Let $\bar{x}_{a\varphi(m)}$, $\hat{\Delta}_{xa\varphi(m)}$ denote the sample characteristics of the auxiliary variable in the first phase. In contrast, $\bar{x}_{\varphi(m(j))}$, $\bar{y}_{\varphi(m(j))}$ and $\hat{\Delta}_{x\varphi(m(j))}$ represent the sample characteristics of the auxiliary variable and the study variable in the second phase within the φ th stratum.

4.1. Generalized Class of Calibrated MCD-Based Estimators

The generalized class of estimators under two-stage MRSS is given below

$$\bar{y}_{TaM(i)} = \sum_{\varphi=1}^L \Psi a_{\varphi} \bar{y}_{\varphi(m(j))}, \quad (27)$$

which is subject to the following constraints:

$$\sum_{\varphi=1}^L \Psi a_{\varphi} = \sum_{\varphi=1}^L W_{\varphi}, \quad (28)$$

$$\sum_{\varphi=1}^L \Psi a_{\varphi} \hat{\Delta}_{x\varphi(m(j))} = \sum_{\varphi=1}^L W_{\varphi} \hat{\Delta}_{xa\varphi(m)} \quad (29)$$

Defining the Lagrange function with its multipliers $\eta_{1(m(j))}$ and $\eta_{2(m(j))}$ yields

$$\Delta_{(m(j))} = \sum_{\varphi=1}^L \frac{(\Psi a_{\varphi} - W_{\varphi})^2}{\tau_{\varphi} W_{\varphi}} - 2\eta_{1(m(j))} \left[\sum_{\varphi=1}^L \Psi a_{\varphi} - \sum_{\varphi=1}^L W_{\varphi} \right] - 2\eta_{2(m(j))} \left[\sum_{\varphi=1}^L \Psi a_{\varphi} \hat{\Delta}_{x\varphi(m(j))} - \sum_{\varphi=1}^L W_{\varphi} \hat{\Delta}_{xa\varphi(m)} \right]. \quad (30)$$

By virtue of performing the following calculation, $\frac{\delta \Delta_{(m(j))}}{\delta \Psi_{\varphi}} = 0$, we obtain

$$\Psi a_{\varphi} = W_{\varphi} + \tau_{\varphi} W_{\varphi} \left[\eta_{2(m(j))} \hat{\Delta}_{x\varphi(m(j))} + \eta_{1(m(j))} \right], \quad (31)$$

By inserting (31) into (28) and (29), we obtain

$$\eta_{1(m(j))} = - \frac{\sum_{\varphi=1}^L W_{\varphi} [\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(j))}] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]^2}, \quad (32)$$

$$\eta_{2(m(j))} = \frac{\sum_{\varphi=1}^L W_{\varphi} [\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(j))}] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]^2}. \quad (33)$$

By substituting $\eta_{1(m(j))}$ and $\eta_{2(m(j))}$ into (31), we obtain

$$\Psi a_{\varphi} = W_{\varphi} + \tau_{\varphi} W_{\varphi} \frac{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] \hat{\Delta}_{x\varphi(m(j))} - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]^2} \sum_{\varphi=1}^L W_{\varphi} [\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(j))}], \quad (34)$$

Inserting Ψa_{φ} into $\bar{y}_{TaM}$ yields

$$\bar{y}_{TaM(i)} = \sum_{\varphi=1}^L W_{\varphi} \bar{y}_{\varphi(m(j))} + \frac{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \hat{\Delta}_{x\varphi(m(j))} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]^2} \sum_{\varphi=1}^L W_{\varphi} [\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(j))}]. \quad (35)$$

This estimator can be rewritten as

$$\bar{y}_{TaM(i)} = \bar{y}_{st(m(j))} + \hat{b}_j \left[\sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(j))}) \right], \quad (36)$$

where

$$\hat{b}_j = \frac{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \hat{\Delta}_{x\varphi(m(j))} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right]}{\left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right] \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right] - \left[\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}_{x\varphi(m(j))} \right]^2}. \quad (37)$$

$$\bar{y}_{TaM(i)} = \begin{cases} \bar{y}_{st(m(O))} + \hat{b}_O \left[\sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(O)))} \right] & \text{when } n \text{ is odd} \\ \bar{y}_{st(m(E))} + \hat{b}_E \left[\sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}_{x\varphi(m(E)))} \right] & \text{when } n \text{ is even.} \end{cases} \quad (38)$$

4.2. Novel Class of Calibrated MCD-Based Estimators

The proposed estimator under stratified MRSS is given below:

$$\bar{y}_{PaM} = \sum_{\varphi=1}^L \Psi_{a\varphi} \bar{y}_{\varphi(m(j))}, \quad (39)$$

which is subject to the following constraints:

$$\sum_{\varphi=1}^L \Psi_{a\varphi} \bar{x}_{\varphi(m(j))} = \sum_{\varphi=1}^L W_{\varphi} \bar{x}_{a\varphi(m)} \quad (40)$$

$$\sum_{\varphi=1}^L \Psi_{a\varphi} \hat{\Delta}_{x\varphi(m(j))} = \sum_{\varphi=1}^L W_{\varphi} \hat{\Delta}_{xa\varphi(m)} \quad (41)$$

$$\sum_{\varphi=1}^L \Psi_{a\varphi} = \sum_{\varphi=1}^L W_{\varphi}, \quad (42)$$

Defining the Lagrange function with its multipliers $\eta_{1(m(j))}$, $\eta_{2(m(j))}$, and $\eta_{3(m(j))}$ yields

$$\Delta_{(m(j))} = \sum_{\varphi=1}^L \frac{(\Psi_{a\varphi} - W_{\varphi})^2}{\tau_{\varphi} W_{\varphi}} - 2\eta_{1(m(j))} \left[\sum_{\varphi=1}^L \Psi_{a\varphi} \bar{x}_{\varphi(m(j))} - \sum_{\varphi=1}^L W_{\varphi} \bar{x}_{a\varphi(m)} \right] - 2\eta_{2(m(j))} \left[\sum_{\varphi=1}^L \Psi_{a\varphi} \hat{\Delta}_{x\varphi(m(j))} - \sum_{\varphi=1}^L W_{\varphi} \hat{\Delta}_{xa\varphi(m)} \right] - 2\eta_{3(m(j))} \left[\sum_{\varphi=1}^L \Psi_{a\varphi} - \sum_{\varphi=1}^L W_{\varphi} \right]. \quad (43)$$

Through the following calculation, $\frac{\delta \Delta_{(m(j))}}{\delta \Psi_{\varphi}} = 0$, we obtain

$$\Psi_{a\varphi} = W_{\varphi} + \tau_{\varphi} W_{\varphi} \left[\eta_{1(m(j))} \bar{x}_{\varphi(m(j))} + \eta_{2(m(j))} \hat{\Delta}_{x\varphi(m(j))} + \eta_{3(m(j))} \right], \quad (44)$$

Substituting (44) into (40)–(42) yields a system of equations containing three equations. We can present this system of equations in matrix form, as follows

$$G_{(3 \times 3)} \eta_{(3 \times 1)} = F_{(3 \times 1)}, \quad (45)$$

where

$$\eta_{(3 \times 1)} = \begin{bmatrix} \eta_{1(m(j))} \\ \eta_{2(m(j))} \\ \eta_{3(m(j))} \end{bmatrix},$$

$$F_{(3 \times 1)} = \begin{bmatrix} \sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))}) \\ \sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))}) \\ 0 \end{bmatrix},$$

$$G_{(3 \times 3)} = \begin{bmatrix} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta}x_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \\ \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right) \\ \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right) & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \end{bmatrix}$$

By solving Equation (45), we obtain

$$\eta_{1(m(j))} = \frac{D_{1(m(j))}}{H}, \quad \eta_{2(m(j))} = \frac{D_{2(m(j))}}{H}, \quad \eta_{3(m(j))} = \frac{D_{3(m(j))}}{H},$$

where

$$\begin{aligned} D_{1(m(j))} &= \sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right)^2 \\ &\quad + \sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right), \end{aligned}$$

$$\begin{aligned} D_{2(m(j))} &= \sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta}x_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right)^2 \\ &\quad - \sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \\ &\quad + \sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))}) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}x_{\varphi(m(j))} \right), \end{aligned}$$

$$\begin{aligned}
D_{3(m(j))} = & \sum_{\varphi=1}^L W_{\varphi} \left(\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \\
& - \sum_{\varphi=1}^L W_{\varphi} \left(\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\
& + \sum_{\varphi=1}^L W_{\varphi} \left(\hat{\Delta} x_{a\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& - \sum_{\varphi=1}^L W_{\varphi} \left(\hat{\Delta} x_{a\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right).
\end{aligned}$$

$$\begin{aligned}
H = & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right)^2 \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2 - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2 \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \\
& + 2 \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right).
\end{aligned}$$

By substituting these values into (44) and (39), we obtain

$$\begin{aligned}
\bar{y}_{PaM} = & \bar{y}_{st(m(j))} + \eta_{1(m(j))} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) + \eta_{2(m(j))} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) + \eta_{3(m(j))} \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right), \\
= & \sum_{\varphi=1}^L W_{\varphi} \bar{y}_{\varphi(m(j))} + \hat{b}_1 \left[\sum_{\varphi=1}^L W_{\varphi} \left(\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(j))} \right) \right] + \hat{b}_2 \left[\sum_{\varphi=1}^L W_{\varphi} \left(\hat{\Delta} x_{a\varphi(m)} - \hat{\Delta} x_{\varphi(m(j))} \right) \right], \quad (46)
\end{aligned}$$

where

$$\hat{b}_{1(j)} = \frac{D_{4(m(j))}}{H}, \quad \hat{b}_{2(j)} = \frac{D_{5(m(j))}}{H},$$

where

$$\begin{aligned}
D_{4(m(j))} = & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta}^2 x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right)^2,
\end{aligned}$$

$$\begin{aligned}
D_{5(m(j))} = & \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \right) \\
& + \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))}^2 \right) \\
& - \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \hat{\Delta} x_{\varphi(m(j))} \bar{y}_{\varphi(m(j))} \right) \left(\sum_{\varphi=1}^L \tau_{\varphi} W_{\varphi} \bar{x}_{\varphi(m(j))} \right)^2.
\end{aligned}$$

$$\bar{y}_{PaM} = \begin{cases} \bar{y}_{st(m(O))} + \hat{b}_{1(O)} \left[\sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(O))}) \right] + \hat{b}_{2(O)} \left[\sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta} x_{\varphi(m(O))}) \right] & \text{when } n \text{ is odd} \\ \bar{y}_{st(m(E))} + \hat{b}_{1(E)} \left[\sum_{\varphi=1}^L W_{\varphi} (\bar{x}_{a\varphi(m)} - \bar{x}_{\varphi(m(E))}) \right] + \hat{b}_{2(E)} \left[\sum_{\varphi=1}^L W_{\varphi} (\hat{\Delta}_{xa\varphi(m)} - \hat{\Delta} x_{\varphi(m(E))}) \right] & \text{when } n \text{ is even.} \end{cases} \quad (47)$$

Note that just like the generalized class, we generate different members of the proposed family by replacing $\Delta x_{\varphi(m)}$ with some known characteristic of an auxiliary variable and different values of τ_{φ} , as shown in Table 1.

5. Numerical Illustration

5.1. Simulation Design

The objective of the simulation experiments in this section is to gain valuable insights into the effectiveness and efficiency of the estimators $\bar{y}_{PM_I}$, $\bar{y}_{PM_{II}}$, $\bar{y}_{PM_{III}}$, $\bar{y}_{PaM_I}$, $\bar{y}_{PaM_{II}}$, and $\bar{y}_{PaM_{III}}$ compared to the estimators $\bar{y}_{TM(1)_I}$, $\bar{y}_{TM(1)_{II}}$, $\bar{y}_{TM(1)_{III}}$, $\bar{y}_{TaM(1)_I}$, $\bar{y}_{TaM(1)_{II}}$, $\bar{y}_{TaM(1)_{III}}$, $\bar{y}_{TM(2)_I}$, $\bar{y}_{TM(2)_{II}}$, $\bar{y}_{TM(2)_{III}}$, $\bar{y}_{TaM(2)_I}$, $\bar{y}_{TaM(2)_{II}}$, and $\bar{y}_{TaM(2)_{III}}$. For each stratum with size $N_{\varphi} = 1000$, four separate bivariate symmetric (Gaussian) distributions were employed. These distributions were symmetric and characterized by $(\mu_x = 1.99, \mu_y = 3.99)$, with variance–covariance matrices specified as follows:

- Stratum 1

$$\Sigma_{xy} = \begin{bmatrix} 1.00 & 0.88 \\ 0.88 & 1.00 \end{bmatrix}$$

- Stratum 2

$$\Sigma_{xy} = \begin{bmatrix} 1.00 & 0.77 \\ 0.77 & 1.00 \end{bmatrix}$$

- Stratum 3

$$\Sigma_{xy} = \begin{bmatrix} 1.00 & 0.66 \\ 0.66 & 1.00 \end{bmatrix}$$

- Stratum 4

$$\Sigma_{xy} = \begin{bmatrix} 1.00 & 0.33 \\ 0.33 & 1.00 \end{bmatrix}.$$

Taking inspiration from the research conducted by Koyuncu [14,28], we extract samples from the aforementioned stratified population. While Koyuncu utilized stratified simple random sampling (SRS), we modify their framework to suit the stratified MRSS design. To ensure a fair comparison between the adapted estimators and the proposed ones, we obtained diverse samples under the MRSS scheme. We also provide the sizes of the diverse samples, which are listed in Table 2, where D_{a1} , D_{a2} , D_{a3} , D_{a4} , F_{a1} , F_{a2} , F_{a3} , and F_{a4} correspond to the total selected sample sizes for each respective stratum at the first and second stages.

We selected $R_1 = 5000$ samples of sizes $n_\varphi = D_{a1}, D_{a2}, D_{a3}$, and D_{a4} under MRSS within a single-stage framework. For each k th sample, we calculated the estimate $(\hat{q}^{(g1)}, \hat{q}^{(g2)})$ of the population mean μ_y , where

$$\hat{q}^{(g1)} = \bar{y}_{TM(1)_I}, \bar{y}_{TM(1)_{II}}, \bar{y}_{TM(1)_{III}}, \bar{y}_{TM(2)_I}, \bar{y}_{TM(2)_{II}}, \bar{y}_{TM(2)_{III}}.$$

$$\hat{q}^{(g2)} = \bar{y}_{PM_I}, \bar{y}_{PM_{II}}, \bar{y}_{PM_{III}}.$$

Adapting the work of Al-Omari [13] and Koyuncu [14], in the first stage, we selected $R_1 = 5000$ samples of sizes $n_{a\varphi} = n_\varphi \times n_\varphi = D_{a1}, D_{a2}, D_{a3}, D_{a4}$. Then, MRSS samples, i.e., $n_\varphi = F_{a1}, F_{a2}, F_{a3}, F_{a4}$, were chosen from the $n_\varphi \times n_\varphi$ samples in the second (double) stage. Using drawn samples, we calculated the estimate $(\hat{q}^{(g1)}, \hat{q}^{(g2)})$ of the population mean μ_y , where

$$\hat{q}^{(g1)} = \bar{y}_{TaM(1)_I}, \bar{y}_{TaM(1)_{II}}, \bar{y}_{TaM(1)_{III}}, \bar{y}_{TaM(2)_I}, \bar{y}_{TaM(2)_{II}}, \bar{y}_{TaM(2)_{III}}.$$

$$\hat{q}^{(g2)} = \bar{y}_{PaM_I}, \bar{y}_{PaM_{II}}, \bar{y}_{PaM_{III}}.$$

The expressions for the MSEs and PREs are as follows:

$$\text{MSE}(\hat{q}^{(g1)}) = \sum_{k_1=1}^{R_1} (\hat{q}^{(g1)} - \mu_y)^2 / R_1.$$

$$\text{MSE}(\hat{q}^{(g2)}) = \sum_{k_1=1}^{R_1} (\hat{q}^{(g2)} - \mu_y)^2 / R_1.$$

$$\text{PRE}(\hat{q}^{(g1)}, \hat{q}^{(g2)}) = \frac{\text{MSE}(\hat{q}^{(g1)})}{\text{MSE}(\hat{q}^{(g2)})} \times 100,$$

The different sample sizes of the symmetric population are provided in Table [2], and the PREs are provided in Tables 3–6.

Table 2. Details of different sample sizes.

	MRSS ($n_{\varphi_1}, n_{\varphi_2}, n_{\varphi_3}, n_{\varphi_4}$)	Table	Double MRSS ($n_{a\varphi_1}, n_{a\varphi_2}, n_{a\varphi_3}, n_{a\varphi_4}$) ($n_{\varphi_1}, n_{\varphi_2}, n_{\varphi_3}, n_{\varphi_4}$)
D_{a1}	(3, 5, 5, 3)	Table 3	(9, 25, 25, 9)
F_{a1}		Table 3	(3, 5, 5, 3)
D_{a2}	(4, 6, 6, 4)	Table 4	(16, 36, 36, 16)
F_{a2}		Table 4	(4, 6, 6, 4)
D_{a3}	(5, 7, 7, 5)	Table 5	(25, 49, 49, 25)
F_{a3}		Table 5	(5, 7, 7, 5)
D_{a4}	(6, 8, 8, 6)	Table 6	(36, 64, 64, 36)
F_{a4}		Table 6	(6, 8, 8, 6)

Table 3. PRE using (D_{a1}, F_{a1}) .

$\hat{\phi}$	PRE MRSS			$\hat{\phi}$	PRE Double MRSS		
	$\bar{y}_{PM_I}$	$\bar{y}_{PM_{II}}$	$\bar{y}_{PM_{III}}$		$\bar{y}_{PaM_I}$	$\bar{y}_{PaM_{II}}$	$\bar{y}_{PaM_{III}}$
$\bar{y}_{TM(1)_I}$	234.1993	233.9116	234.1762	$\bar{y}_{TaM(1)_I}$	219.1993	218.9116	217.6762
$\bar{y}_{TM(1)_{II}}$	231.9974	233.8001	233.9743	$\bar{y}_{TaM(1)_{II}}$	228.9974	227.8002	227.4743
$\bar{y}_{TM(1)_{III}}$	234.2525	233.9648	234.2294	$\bar{y}_{TaM(1)_{III}}$	228.2525	228.9648	239.7294
$\bar{y}_{TM(2)_I}$	637.2576	636.4615	637.1646	$\bar{y}_{TaM(2)_I}$	635.3576	633.4625	633.8646
$\bar{y}_{TM(2)_{II}}$	631.7327	629.9447	631.6406	$\bar{y}_{TaM(2)_{II}}$	623.7327	623.9447	621.4416
$\bar{y}_{TM(2)_{III}}$	637.7392	636.9424	637.6461	$\bar{y}_{TaM(2)_{III}}$	626.7392	626.9424	623.2461

Table 4. PRE using (D_{a2}, F_{a2}) .

$\hat{\phi}$	PRE MRSS			$\hat{\phi}$	PRE Double MRSS		
	$\bar{y}_{PM_I}$	$\bar{y}_{PM_{II}}$	$\bar{y}_{PM_{III}}$		$\bar{y}_{PaM_I}$	$\bar{y}_{PaM_{II}}$	$\bar{y}_{PaM_{III}}$
$\bar{y}_{TM(1)_I}$	1118.201	1118.629	1118.467	$\bar{y}_{TaM(1)_I}$	997.1999	995.6283	1118.467
$\bar{y}_{TM(1)_{II}}$	1119.995	1119.514	1116.352	$\bar{y}_{TaM(1)_{II}}$	1103.9943	1103.5135	1109.852
$\bar{y}_{TM(1)_{III}}$	1117.985	1115.413	1116.259	$\bar{y}_{TaM(1)_{III}}$	1105.9837	1103.4129	1014.769
$\bar{y}_{TM(2)_I}$	1661.371	1654.876	1664.519	$\bar{y}_{TaM(2)_I}$	1658.3702	1569.9992	1662.219
$\bar{y}_{TM(2)_{II}}$	1748.992	1751.889	1749.367	$\bar{y}_{TaM(2)_{II}}$	1746.9908	1759.9853	1749.867
$\bar{y}_{TM(2)_{III}}$	1662.919	1665.554	1664.981	$\bar{y}_{TaM(2)_{III}}$	1659.9181	1668.5528	1663.471

Table 5. PRE using (D_{a3}, F_{a3}) .

$\hat{\phi}$	PRE MRSS			$\hat{\phi}$	PRE Double MRSS		
	$\bar{y}_{PM_I}$	$\bar{y}_{PM_{II}}$	$\bar{y}_{PM_{III}}$		$\bar{y}_{PaM_I}$	$\bar{y}_{PaM_{II}}$	$\bar{y}_{PaM_{III}}$
$\bar{y}_{TM(1)_I}$	223.9373	225.4532	223.9359	$\bar{y}_{TaM(1)_I}$	201.93719	202.4532	215.9359
$\bar{y}_{TM(1)_{II}}$	227.9226	229.4848	227.9183	$\bar{y}_{TaM(1)_{II}}$	212.92255	213.4848	218.4183
$\bar{y}_{TM(1)_{III}}$	224.2589	225.6875	224.1674	$\bar{y}_{TaM(1)_{III}}$	219.26885	209.6875	217.6674
$\bar{y}_{TM(2)_I}$	1728.5356	1805.4129	1723.6128	$\bar{y}_{TaM(2)_I}$	1716.53561	1791.4129	1718.3119
$\bar{y}_{TM(2)_{II}}$	1821.4715	1899.4269	1816.4787	$\bar{y}_{TaM(2)_{II}}$	1803.47145	1892.4269	1806.9787
$\bar{y}_{TM(2)_{III}}$	1759.3319	1836.5667	1754.4752	$\bar{y}_{TaM(2)_{III}}$	1738.42179	1816.6568	1738.8752

Table 6. PRE using (D_{a4}, F_{a4}) .

$\hat{\phi}$	PRE MRSS			$\hat{\phi}$	PRE Double MRSS		
	$\bar{y}_{PM_I}$	$\bar{y}_{PM_{II}}$	$\bar{y}_{PM_{III}}$		$\bar{y}_{PaM_I}$	$\bar{y}_{PaM_{II}}$	$\bar{y}_{PaM_{III}}$
$\bar{y}_{TM(1)_I}$	883.2887	901.9381	873.6702	$\bar{y}_{TaM(1)_I}$	856.2887	878.9381	865.6702
$\bar{y}_{TM(1)_{II}}$	889.8739	908.7711	879.1745	$\bar{y}_{TaM(1)_{II}}$	874.8739	892.7711	869.6745
$\bar{y}_{TM(1)_{III}}$	882.8997	901.6299	873.2869	$\bar{y}_{TaM(1)_{III}}$	868.8997	886.6299	866.7869
$\bar{y}_{TM(2)_I}$	1586.7028	1622.6578	1568.9113	$\bar{y}_{TaM(2)_I}$	1585.7028	1621.6578	1567.7013
$\bar{y}_{TM(2)_{II}}$	1585.6265	1621.2682	1568.9958	$\bar{y}_{TaM(2)_{II}}$	1583.6265	1629.2682	1566.5858
$\bar{y}_{TM(2)_{III}}$	1593.4205	1629.9797	1575.8494	$\bar{y}_{TaM(2)_{III}}$	1591.3305	1627.9797	1573.2494

5.2. BMI Application

The performance of the $\hat{q}^{(g_2)}$ estimators was further evaluated using real data. In this case, we utilized data from Turkey in 2014 concerning 800 individuals, where body mass index (BMI) served as the study variable, and age acted as the auxiliary variable. The dataset consisted of $N = 800$ observations, with a correlation coefficient of $\rho_{xy} = 0.60$, and mean values of $\mu_y = 23.77$ for the study variable (BMI) and $\mu_x = 30.12$ for the auxiliary variable (age). The coefficient of variation for the auxiliary variable was $C_x = 0.36$, and $C_y = 0.17$ represented the coefficient of variation for the study variable. To stratify the dataset, we divided it into two strata based on gender. For details regarding BMI data, refer to the work conducted by Shahzad et al. [17]. Here are some key properties :

- **Stratum-I** $N_{h1} = 477$, $\rho_{yx_{h1}} = 0.62$, $\mu_{y_{h1}} = 22.36$, $\mu_{x_{h1}} = 27.68$, $C_{x_{h1}} = 0.36$, $C_{y_{h1}} = 0.17$.
- **Stratum-II** $N_{h2} = 323$, $\rho_{yx_{h2}} = 0.49$, $\mu_{y_{h2}} = 25.85$, $\mu_{x_{h2}} = 33.73$, $C_{x_{h2}} = 0.33$, $C_{y_{h1}} = 0.13$.

The BMI data results are given in Tables 3–8. It is evident from Tables 3–8 that the PRE (Percentage of Relative Efficiency) values are greater than 100 for all the proposed estimators, indicating their superior performance compared to the adapted estimators. It is important to note that this conclusion has been drawn based on our simulation study. However, we have confidence that these results hold true in various other settings as well.

Table 7. BMI PRE for odd samples.

$\hat{\phi}$	PRE MRSS			$\hat{\phi}$	PRE Double MRSS		
	$\bar{y}_{PM_I}$	$\bar{y}_{PM_{II}}$	$\bar{y}_{PM_{III}}$		$\bar{y}_{PaM_I}$	$\bar{y}_{PaM_{II}}$	$\bar{y}_{PaM_{III}}$
$\bar{y}_{TM(1)_I}$	581.1897	625.0372	592.5631	$\bar{y}_{TaM(1)_I}$	563.1721	602.1433	573.5220
$\bar{y}_{TM(1)_{II}}$	560.0198	605.8929	580.8859	$\bar{y}_{TaM(1)_{II}}$	544.8397	580.5889	563.7429
$\bar{y}_{TM(1)_{III}}$	602.3596	644.1815	604.2403	$\bar{y}_{TaM(1)_{III}}$	581.5045	623.6977	583.3011
$\bar{y}_{TM(2)_I}$	605.2976	650.0379	616.9057	$\bar{y}_{TaM(2)_I}$	586.9033	626.6819	597.4703
$\bar{y}_{TM(2)_{II}}$	584.1277	630.8936	605.2285	$\bar{y}_{TaM(2)_{II}}$	568.5709	605.1275	587.6912
$\bar{y}_{TM(2)_{III}}$	626.4675	669.1822	628.5829	$\bar{y}_{TaM(2)_{III}}$	605.2357	648.2363	607.2494

Table 8. BMI PRE for even samples.

$\hat{\phi}$	PRE MRSS			$\hat{\phi}$	PRE Double MRSS		
	$\bar{y}_{PM_I}$	$\bar{y}_{PM_{II}}$	$\bar{y}_{PM_{III}}$		$\bar{y}_{PaM_I}$	$\bar{y}_{PaM_{II}}$	$\bar{y}_{PaM_{III}}$
$\bar{y}_{TM(1)_I}$	881.1798	925.0273	892.5136	$\bar{y}_{TaM(1)_I}$	863.1127	902.1334	873.5022
$\bar{y}_{TM(1)_{II}}$	860.0099	905.8830	880.8364	$\bar{y}_{TaM(1)_{II}}$	844.7803	880.5790	863.7231
$\bar{y}_{TM(1)_{III}}$	902.3497	944.1716	904.1908	$\bar{y}_{TaM(1)_{III}}$	881.4451	923.6878	883.2813
$\bar{y}_{TM(2)_I}$	910.8645	955.4416	922.3886	$\bar{y}_{TaM(2)_I}$	892.4915	932.1689	903.0573
$\bar{y}_{TM(2)_{II}}$	889.6946	936.2973	910.7114	$\bar{y}_{TaM(2)_{II}}$	874.1591	910.6145	893.2782
$\bar{y}_{TM(2)_{III}}$	932.0344	974.5859	934.0658	$\bar{y}_{TaM(2)_{III}}$	910.8239	953.7233	912.8364

6. Conclusions

In this paper, our focus was on adapting the estimators proposed by Sinha et al. [29] and Garg and Pachori [30], specifically for the stratified MRSS design. MRSS is a well-established sampling technique that is widely recognized in this field. Additionally, this study introduces new calibrated estimators, based on the MCD, for estimating the population mean in both single- and double-stratified MRSS setups. It is worth noting that novel estimators with three constraints exhibit greater effectiveness compared to generalized estimators with two constraints. To support our findings, a simulation study was conducted. We also have plans to further expand on our present work using diverse RSS designs, for which the insights provided by Koyuncu [14,28] will be taken into consideration.

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