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Langmuir Forcing and Collapsing Subsonic Density Cavitons via Random Modulations

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Abstract: Electrostatic nonlinear random Langmuir structures have been propagated in stochastic magnetospheres, clouds and solar wind. A theoretical description of Langmuir waves can be modeled by Schrödinger and Zakharov models with stochastic terms. It was explained that the stochastic parameter affects the forcing, collapsing in strongly density turbulence and density crystalline structures. The unified method has been implemented to provide new stochastic solutions for a Zakharov system in subsonic limit with noises via the Itô sense. This unified approach provides a variety of advantages, such as avoiding difficult calculations and explicitly providing pivotal solutions. It is easy to use, efficient, and precise. The induced generated energy during the collapsing of solar Langmuir wave bursts and clouds is determined by the solitonic formations. In addition, the collapsing strong turbulence or forcing density crystalline structures depend mainly on stochastic processes. Furthermore, electrostatic waves in clouds that may collapse are represented sometimes as dissipative shapes. So, the results of this investigation could be applicable to observations of energy seeding and collapsing in clouds. This energy is based on the electrostatic field and its related densities' perturbation in subsonic limits. Finally, it has been explored how noise parameters in the Itô sense affect the solar wind Langmuir waves' properties. So, the findings of this discussion may be applicable to real observations of energy collapsing and seeding in clouds.

Keywords: stochastic Zakharov model; Brownian motion; subsonic limit; soliton; stochastic structures

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1. Introduction

Space environment discussions have improved the plasma wave characteristics induced by ponderomotive pressures in dispersive areas [1–4]. The Langmuir structures' turbulences were examined under weak conditions in [1]. Langmuir collapse and wave turbulence in laser plasmas are introduced and investigated using numerical simulations in two and three dimensions [2,3]. The physical model of ions waves and their behaviors through the influence of ponderomotive forces was investigated [5]. The ponderomotive forces that affected charged particle behaviors in high-frequency oscillations in sound electromagnetic nonlinear environments were studied by Shakeel et al. [6]. The properties of Langmuir's freak waves in the electron-positron nonlinear complex fluid are explored in nonlinear coupling modulations and small-amplitude limit. Symmetry can be used to reduce the problem and provide solutions that meet specific boundary constraints [7–9].

Over and above, several nonlinear applied systems can produce the observed stochastic electromagnetic waves [10–14]. In applied science, NPDEs and symmetry are closely related concepts. Symmetry is crucial for comprehending NPDEs in particular. A mathematical object's symmetry is its invariance under specific transformations [15,16]. The development of solitonic solutions for the NPDEs is crucial for understanding nonlinear phenomena and describing physical processes in several scientific fields [17–22]. Mean-field theories can theoretically investigate the presence of stochastic waves in physics applications [23]. The examination of Markovian processes using measured-valued and geometrical methods can provide insight into the strong stochastic processes in a nonlinear state [24,25]. Additionally, parabolic semilinearly (nonlinearly) NPDEs were used to study the nonlinear Fokker–Planck problem model. There are many methods for extracting wave solutions from nonlinear partial differential nonlinear equations (NPDEs) by transforming them into ordinary differential nonlinear equations (ODEs). The majority of these methods are nonclassical symmetry reduction techniques. We seek to resolve the Zakharov equation in the subsonic domain in the Itô sense defined by [6,26]:

$$\begin{aligned} &i\mathfrak{E}_t(x,t) + \frac{1}{2}\mathfrak{E}_{xx}(x,t) - \mathfrak{D}(x,t)\,\mathfrak{E}(x,t) - i\delta\,\mathfrak{E}(x,t)\,B_t = 0\,, \\ &\mathfrak{D}_{tt}(x,t) - \mathfrak{D}_{xx}(x,t) - 2\,(|\,\mathfrak{E}(x,t)\,|^2)_{xx} = 0, \end{aligned} \tag{1}$$

$\mathfrak{E}\,e^{-iw_p t}$ denotes the normalized perturbed electrostatic field oscillations and $\mathfrak{D}(x,t)$ denotes normalized density perturbations. The noise B_t denotes the Brownian times derivative of $B(t)$ and δ represents the noise amplitude [27]. The Zakharov model for electrostatic fields is described using Equation (1). Musher et al. investigated the weak Langmuir [1]. Eliasson et al. investigated the trapping of Langmuir waves in ion holes [28]. It is often noted that Langmuir waves are excited and produced by a variety of nonlinear structures in various envelope forms [29,30]. According to Hess et al. in [31,32], the Langmuir structures in solar wind are the result of many types of solar emissions such as bursts and localized nonlinear wave packets. A Brownian motion is a time-continuous stochastic process [33,34]. The following are the major properties of Brownian motion $\{B(t)\}_{t\geq 0}$:

(i) $B(t), t \geq 0$ is a continuous function of t, t denotes the time and $B(t) \sim N(0,t)$.
(ii) $B(s) - B(t)$ and $B(k) - B(u)$ are independent for $s < t < u < k$.
(iii) $B(t) - B(s)$ has a normal distribution with a mean and variance of zero and $t - s$, respectively, i.e., $B(t) - B(s) \sim \sqrt{t-s}\,N(0,1)$, $N(0,1)$ is a standard normal distribution.

The clouds with complex gravitational instabilities with implications for the creation of dissipative and huge structures in space have been extensively studied [35–38]. The cooling of cloud elements and gravitational collapse of particles of matter can both occur concurrently in clouds and space plasmas due to the radiative effects [35–38]. The temperature and extremely precipitable water in the atmospheric column are impacted by clouds' radiative cooling [39,40]. Wen et al. used the fluid model to study the cooling of laser plasmas and witness shockwaves [39]. On the other hand, it has been demonstrated that the instability of anvil clouds in tropical strong convective processes depends on interactions between clouds and radiation. Furthermore, by stabilizing the atmosphere through wave collapsing and radiative cooling in clouds, it will have an effect on extensive convection [41–43]. In dusty astrophysical self-gravity, the modulating effects of radiative condensations and cooling with their perturbations on the structural propagation parameters have been explored [43]. Furthermore, a number of physical factors, including temperature ratio and grain charge variations, have a significant impact on how clouds are seeded [44–46]. Those variables also determine how electrostatic collapse occurs in dynamical cloud characteristics. Recently, the new theoretical NLSE applications implies an important role in the interpretation of many phenomena and observations that have become of large scientific significance. This equation can be used and applied to compare with the observations of collapsing wave energies and clouds seeding [46–48].

In [26], the authors studied the system of equations for the ion sound and Langmuir waves via the Bernoulli sub-equation approach. Namely, they consider this model in a deterministic case. They also investigated the modulation instability analysis for this model based on the standard linear stability analysis. Baskonus et al. introduced some dark solutions for Equation (1), using the sine-Gordon expansion approach [5], Manafian-presented solitons, kink, periodic and rational solutions for the ion sound and Langmuir waves [49]. Kumar et al. employed the tanh–coth technique and introduced some soliton solutions [50]. In [51], the authors investigated this system in a deterministic case to obtain several solitary wave solutions, using two new analytical methods. The goal of this study is to solve the Zakharov model in a subsonic limit, utilizing a unified method to provide several stochastic solutions. Most standard papers considered the ion sound and Langmuir waves system in a deterministic case. In contrast to these papers, we consider this model in a stochastic case specifically forced by multiplicative noise in the Itô sense. To the best of our knowledge, no previous study has been conducted utilizing this unified method for solving this model in a stochastic sense. The presented solutions in this work demonstrate certain important physics phenomena such as bursty electrostatic waves in cusps areas and solar wind. In comparison to other methods, this unified method has a number of benefits, including the avoidance of challenging calculations and the explicit provision of crucial solutions. It is simple, effective and precise. We also present the potential equation for the Zakharov model of the system dynamical equation.

This study is structured as follows: Section 2 briefly presents the unified method which is used to extract stochastic solutions for Zakharov equations in the subsonic limit via the Itô sense. The Zakharov model in the subsonic limit is described in Section 3. Section 3 provides some vital solutions to the Zakharov model's subsonic limit. Section 4 introduces a closed form of stochastic solutions for Equation (1). The physical explanation for the stochastic solutions is described in Section 5. Finally, a summary of some conclusions is provided in Section 6.

2. Description of the Method

We present the abbreviation of the unified method [52]. Consider the following NLPDEs

$$\Psi(Q, Q_x, Q_t, Q_{xx}, Q_{tt}, Q_{xt}, \ldots) = 0. \tag{2}$$

Utilizing the wave transformation:

$$Q(x,t) = Q(\zeta), \qquad \zeta = x - wt, \tag{3}$$

w is the wave speed in Equation (2) transferred to the following ODE:

$$\Phi(Q, Q', Q'', Q''', \ldots) = 0. \tag{4}$$

The following ODE was created from several applied science models of the form (2):

$$LQ'' + MQ^3 + NQ = 0. \tag{5}$$

The constants L, M, and N are specific constants determined by the constants of the main model and the wave speed.

The families of solutions for Equation (5) are [52]:

1. **Family 1:**

$$Q(\zeta) = \pm\sqrt{\frac{-2N}{M}}\,\text{sech}\left(\pm\sqrt{-\frac{N}{L}}\zeta\right). \tag{6}$$

2. **Family 2:**

$$Q(\zeta) = \pm\sqrt{\frac{-35\,N}{18\,M}}\mathrm{sech}^2\left(\pm\sqrt{-\frac{5\,N}{12\,L}}\,\zeta\right). \tag{7}$$

3. **Family 3:**

$$Q(\zeta) = \pm\sqrt{\frac{-N}{M}}\,\tanh\left(\pm\sqrt{\frac{N}{L}}\zeta\right). \tag{8}$$

3. Subsonic Limit Description

In the subsonic limit, Equation (1) is reduced to

$$i\mathfrak{E}_t(x,t) + \frac{1}{2}\mathfrak{E}_{xx}(x,t) + 2\mid \mathfrak{E}(x,t)\mid^2 \;\mathfrak{E}(x,t) - i\delta\,\mathfrak{E}(x,t)\,B_t = 0. \tag{9}$$

The term $\delta\,\mathfrak{E}(x,t)\,B_t$ denotes a stochastic noise term. Utilizing transformations

$$\mathfrak{E}(x,t) = Q(\zeta)e^{i\varphi+\delta B(t)-\delta^2 t} \tag{10}$$

and

$$\zeta = v\,x + \rho\,t,\;\varphi = cx + \lambda t.$$

Equation (9) is reduced to

$$-\frac{1}{2}c^2Q(\zeta) + 2Q^3(\xi)e^{2\delta B(t)-2\delta^2 t} + \frac{1}{2}v^2Q''(\zeta) - \lambda Q(\zeta) = 0, \tag{11}$$

where ρ, c, λ, and v are constants. Taking the expectation of both sides to Equation (11) yields

$$-\frac{1}{2}c^2Q(\zeta) + 2Q^3(\xi)e^{-2\delta^2 t}\;E(e^{2\delta B(t)}) + \frac{1}{2}v^2Q''(\zeta) - \lambda Q(\zeta) = 0. \tag{12}$$

Since $E(e^{2\delta B(t)}) = e^{2\delta^2 t}$, Equation (12) becomes

$$v^2Q''(\zeta) + 4Q^3(\zeta) - (c^2+2\lambda)Q(\zeta) = 0. \tag{13}$$

On the other hand, we have

$$(c\,v+\rho)Q'(\zeta) - \delta^2Q(\zeta) = 0, \tag{14}$$

which gives

$$Q(\zeta) = e^{\frac{\zeta\delta^2}{c\,v+\rho}}$$

with condition

$$\rho^2\left(-c^2-2\lambda\right) - 2c\rho v\left(c^2+2\lambda\right) + v^2\left(-c^4-2\lambda c^2+\delta^4\right) + 4(c\,v+\rho)^2e^{-\frac{2\eta\delta^2}{c\,v+\rho}} = 0.$$

Equation (13) is a dynamical system of a particle in the potential, which is defined as

$$V = -\frac{c^2Q^2(\zeta)}{2v^2} - \frac{\lambda Q^2(\zeta)}{v^2} + \frac{Q^4(\zeta)}{v^2}. \tag{15}$$

Hence, the exact solution for Equation (13) is:

$$Q(\zeta) = \frac{\sqrt{2}\sqrt{c^2+2\lambda}e^{\frac{\sqrt{c^2+2\lambda}(\rho t+xv)}{v}}}{e^{\frac{2\sqrt{c^2+2\lambda}(\rho t+xv)}{v}}+1}.$$

Consequently, the solutions of Equation (9) and the corresponding electrostatic potential $\mathfrak{E}(x,t)$ and the density distributions $\mathfrak{D}(x,t)$ are

$$\mathfrak{E}(x,t) = \frac{\sqrt{2}\sqrt{c^2+2\lambda}e^{\frac{\sqrt{c^2+2\lambda}(\rho t+xv)}{v}}}{e^{\frac{2\sqrt{c^2+2\lambda}(\rho t+xv)}{v}}+1}e^{i(cx+\lambda t)+\delta B(t)-\delta^2 t}. \tag{16}$$

$$\mathfrak{D}(x,t) = -e^{2\left(\delta B(t)-\delta^2 t\right)}\left|\frac{\sqrt{2}\sqrt{c^2+2\lambda}e^{\frac{\sqrt{c^2+2\lambda}(\rho t+xv)}{v}}}{e^{\frac{2\sqrt{c^2+2\lambda}(\rho t+xv)}{v}}+1}\right|^2. \tag{17}$$

4. Closed-Form Solutions

We produce further stochastic solutions for Equation (9). According to the unified method, the stochastic solutions of Equation (9) are:

1. **Family I:**

$$Q_{1,2}(x,t) = \pm\sqrt{\frac{c^2+2\lambda}{2}}\,\mathrm{sech}\left(\pm\frac{\sqrt{c^2+2\lambda}}{v}(v\,x+\rho\,t)\right). \tag{18}$$

Thus, the solutions for Equation (1) are

$$\mathfrak{E}_{1,2}(x,t) = \pm\sqrt{\frac{c^2+2\lambda}{2}}\,e^{i(cx+\lambda t)+\delta B(t)-\delta^2 t}\,\mathrm{sech}\left(\pm\frac{\sqrt{c^2+2\lambda}}{v}(v\,x+\rho\,t)\right). \tag{19}$$

$$\mathfrak{D}_{1,2}(x,t) = -e^{2\left(\delta B(t)-\delta^2 t\right)}\left|\sqrt{\frac{c^2}{2}+\lambda}\,\mathrm{sech}\left(\frac{\sqrt{c^2+2\lambda}(\rho t+xv)}{v}\right)\right|^2. \tag{20}$$

2. **Family II:**

$$Q_{3,4}(x,t) = \pm\sqrt{\frac{35\,(c^2+2\lambda)}{72}}\mathrm{sech}^2\left(\pm\frac{\sqrt{5\,(c^2+2\lambda)}}{2\sqrt{3}\,v}\,(v\,x+\rho\,t)\right). \tag{21}$$

Thus, the solutions for Equation (1) are

$$\mathfrak{E}_{3,4}(x,t) = \pm\sqrt{\frac{35\,(c^2+2\lambda)}{72}}\,e^{i(cx+\lambda t)+\delta B(t)-\delta^2 t}\,\mathrm{sech}^2\left(\pm\frac{\sqrt{5\,(c^2+2\lambda)}}{2\sqrt{3}\,v}\,(v\,x+\rho\,t)\right). \tag{22}$$

$$\mathfrak{D}_{3,4}(x,t) = -\frac{1}{72}\,e^{i(cx+\lambda t)+\delta B(t)-\delta^2 t}\left|\sqrt{35\,(c^2+2\lambda)}\mathrm{sech}^2\left(\pm\frac{\sqrt{5\,(c^2+2\lambda)}}{2\sqrt{3}\,v}\,(v\,x+\rho\,t)\right)\right|^2. \tag{23}$$

3. **Family III:**

$$Q_{5,6}(x,t) = \pm\frac{\sqrt{c^2+2\lambda}}{2}\tanh\left(\pm\frac{\sqrt{-(c^2+2\lambda)}}{\sqrt{2}\,v}(v\,x+\rho\,t)\right). \tag{24}$$

Thus, the solutions for Equation (1) are

$$\mathfrak{E}_{5,6}(x,t) = \pm\frac{\sqrt{c^2+2\lambda}}{2}\,e^{i(cx+\lambda t)+\delta B(t)-\delta^2 t}\,\tanh\left(\pm\frac{\sqrt{-(c^2+2\lambda)}}{\sqrt{2}\,v}(v\,x+\rho\,t)\right). \tag{25}$$

$$\mathfrak{D}_{5,6}(x,t) = -\,e^{i(cx+\lambda t)+\delta B(t)-\delta^2 t}\left|\frac{\sqrt{c^2+2\lambda}}{2}\tanh\left(\pm\frac{\sqrt{-(c^2+2\lambda)}}{\sqrt{2}\,v}\xi\right)\right|^2. \tag{26}$$

5. Discussion of Results

The subsonic limit of the Zakharov system has been theoretically studied to obtain mathematical forms of electrostatic fields and densities $\mathfrak{E}(x,t)$ and $\mathfrak{D}(x,t)$ as super, envelopes, and kink dissipative structures. These waves self-compressed and become trapped in the voids of self-induced densities. Therefore, the principal mechanism for energy dissipation for extreme turbulence is collapsing soliton–cavitons produced by extremely inhomogeneous Langmuir turbulence.

A Zakharov model (1) with Brownian time-dependent function $B(t)$ and noise term in the Itô sense transformed to Equation (9). The expectation of (9) with $E(e^{2\delta B(t)}) = e^{2\delta^2 t}$ reduced the model to a differential form that was solved to obtain several electrostatic solutions. Equations (16) and (17) represent the exact field $\mathfrak{E}(x,t)$ and density $\mathfrak{D}(x,t)$ for subsonic conditions. The properties of Equation (16) are represented in Figures 1–3. Figure 1 is a geometrical configuration of the random properties of both wave amplitude and energy associated with the occurrence of collapse. Figure 2 denotes the huge soliton-like wave, which begins to collapse with an increase in the random coefficient. In addition, it was noted that the breathers wave has been formed in Figure 3.

On the other hand, to introduce different stochastic solutions, the unified solver method was used for solving Equation (9) [52]. Many stochastic solutions for fields $\mathfrak{E}(x,t)$ and corresponding densities $\mathfrak{D}(x,t)$ are given by Equations (18)–(26). The randomly stochastic dark envelopes, dissipative shocks and shocks such as soliton fields and super structures are obtained.

The stochastic characteristics of Equation (25) are illustrated in Figures 4 and 5. Figure 4 is a geometrical configuration of the tanh solution. The random features of both the wave amplitude and energy for the forced random wave are shown in Figure 4. Figure 5 depicts the super random forced soliton-like wave, which begins to grow with the increasing random coefficient.

In the removal of noise impacts, Equations (16) and (17) read:

$$\mathfrak{E}(x,t) = e^{i(cx+\lambda t)}\,\frac{\sqrt{2}\sqrt{c^2+2\lambda}\,e^{\frac{\sqrt{c^2+2\lambda}(t+x\omega)}{\omega}}}{e^{\frac{2\sqrt{c^2+2\lambda}(t+x\omega)}{\omega}}+1}. \tag{27}$$

$$\mathfrak{D}(x,t) = -e^2\left|\sqrt{\frac{c^2}{2}+\lambda}\,\mathrm{sech}\left(\frac{\sqrt{c^2+2\lambda}(t+x\omega)}{\omega}\right)\right|^2. \tag{28}$$

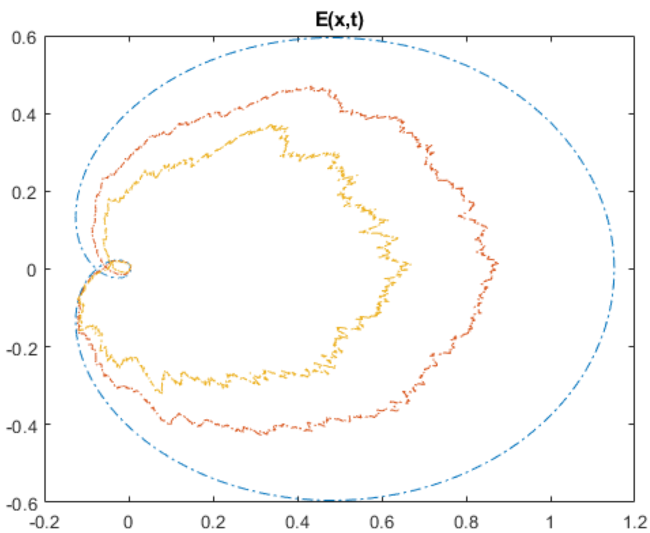

Figure 1. Trajectory of $\mathfrak{E}(x,t)$ for $v = 0.05, \rho = 1, c = 1.5$.

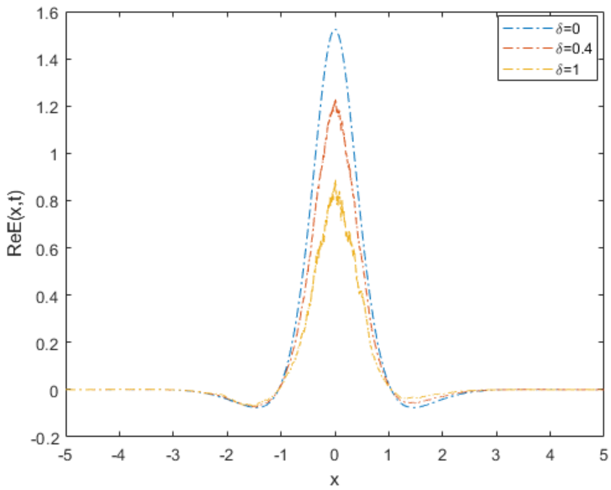

Figure 2. Change of $Re\mathfrak{E}(x,t)$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

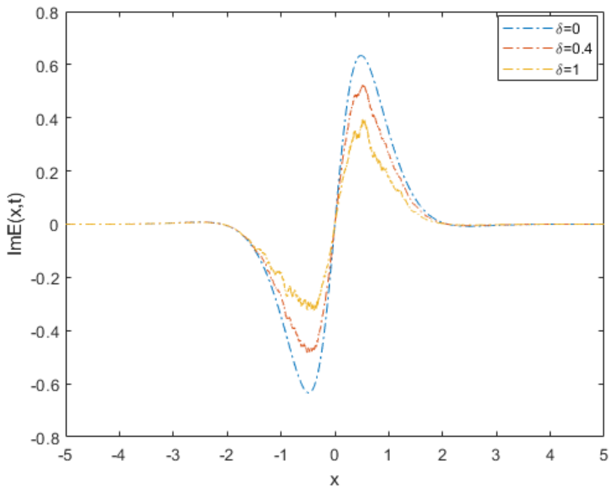

Figure 3. Change of $Re\mathfrak{E}(x,t)$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

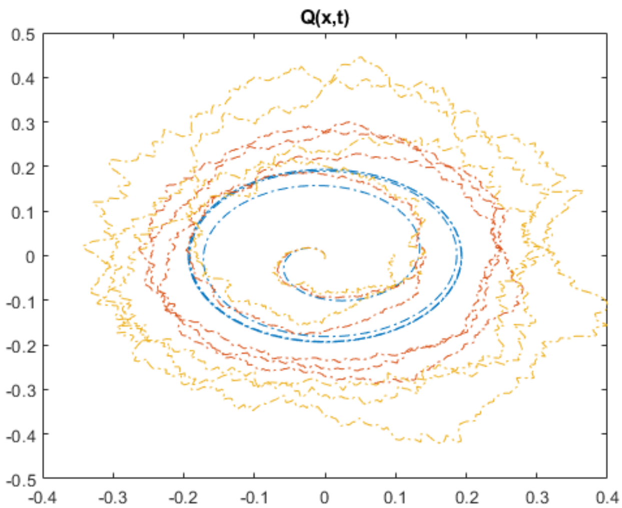

Figure 4. Trajectory of $\mathfrak{E}_5(x,t)$ for $v = 0.05, \rho = 1, c = 1.5$.

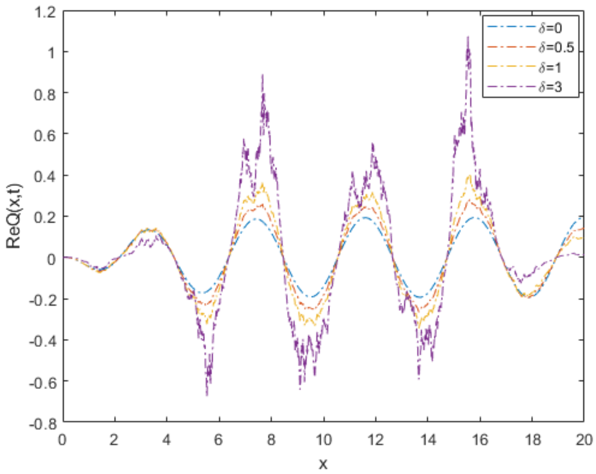

Figure 5. Change of $Re\mathfrak{E}_5(x,t)$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

To discuss the solitary behavior for $\delta = 0$, some important structures are obtained; i.e., the periodical envelope breathers field wave $\mathfrak{E}_1(x,t) = E(x,t)$ and bell shape solitons $\mathfrak{E}_3(x,t) = E(x,t)$ are shown in Figures 6 and 7. In addition, the corresponding super localized behavior of $\mathfrak{D}_3(x,t) = n(x,t)$ is plotted in Figure 8. Finally, the solitary structures of solution $\mathfrak{E}_5(x,t) = E(x,t)$ are depicted in Figures 9–11. The $\mathfrak{E}_5(x,t)$ variations with x and t are given in Figure 9. The dark envelope and localized super soliton are displayed in Figures 10 and 11. The stable stationary soliton $\mathfrak{D}_5(x,t) = n(x,t)$ for a dissipative electrostatic field is depicted in Figure 12. To sum up, the attributes of stochastic subsonic Zakharov model solutions that are modulated with noise affected the energy properties of the obtained electrostatic fields and densities.

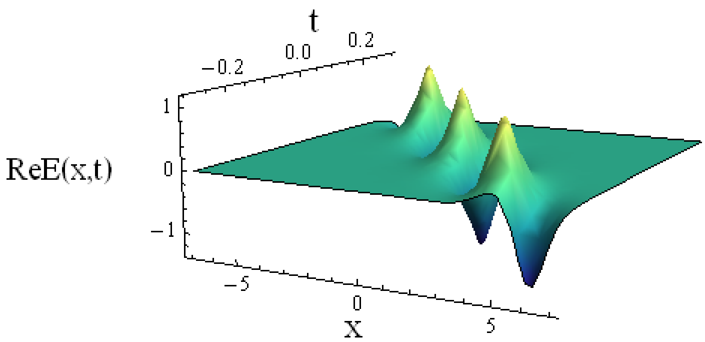

Figure 6. Change of $Re\mathfrak{E}_1(x,t)$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

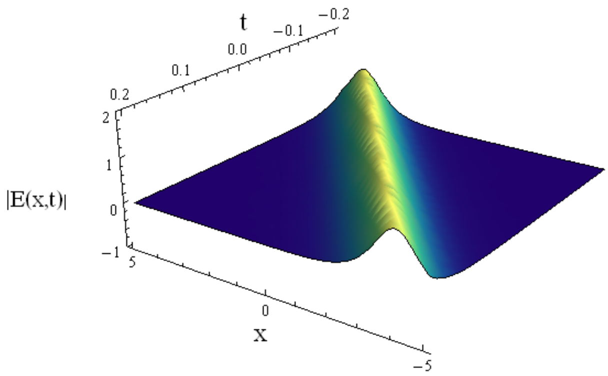

Figure 7. Change of $|\mathfrak{E}_3(x,t)|$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

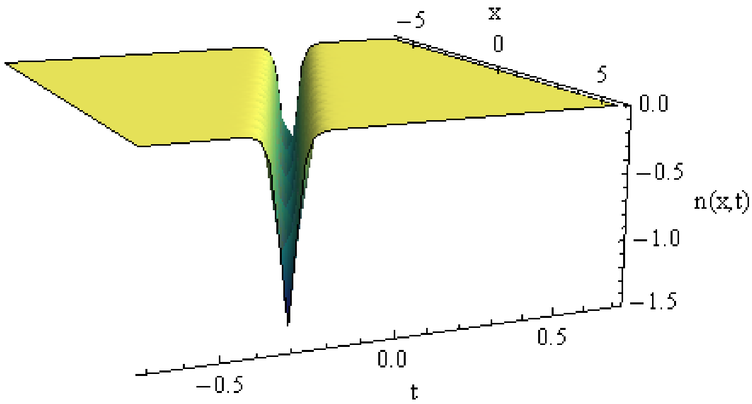

Figure 8. Change of $\mathfrak{D}_1(x,t)$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

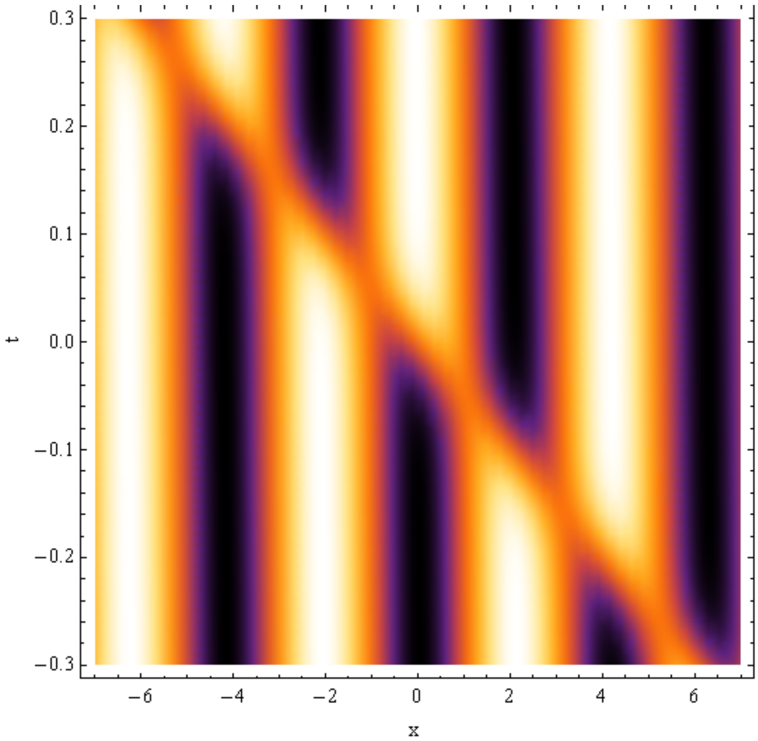

Figure 9. Change of $\mathfrak{E}_5(x,t)$ with x and t for $v = 0.05, \rho = 1, c = 1.5$.

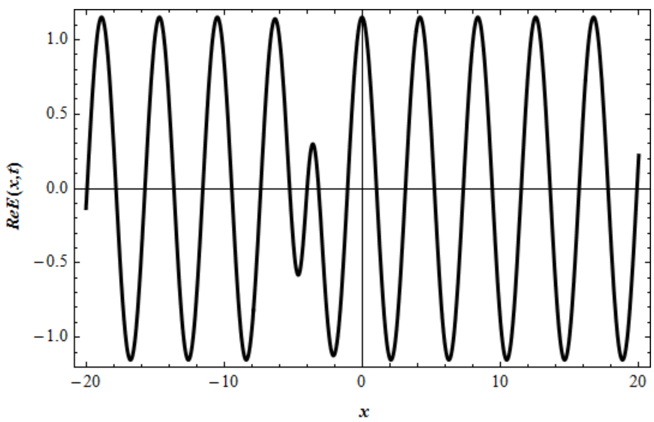

Figure 10. Change of $Re\mathfrak{E}_5(x,t)$ with x for $v = 0.05, \rho = 1, c = 1.5$.

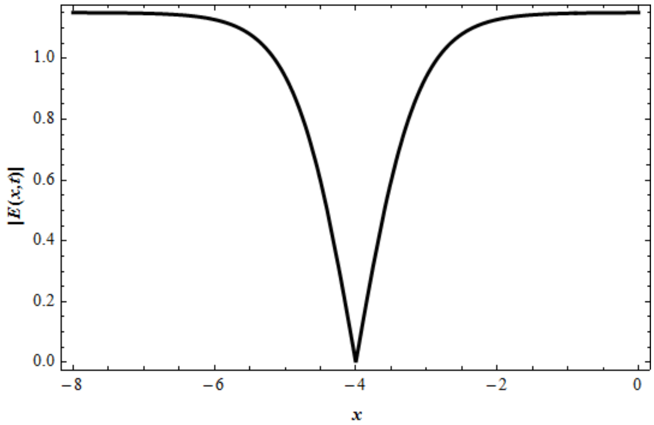

Figure 11. Change of $|\mathfrak{E}_5(x,t)|$ with x for $v = 0.05, \rho = 1, c = 1.5$.

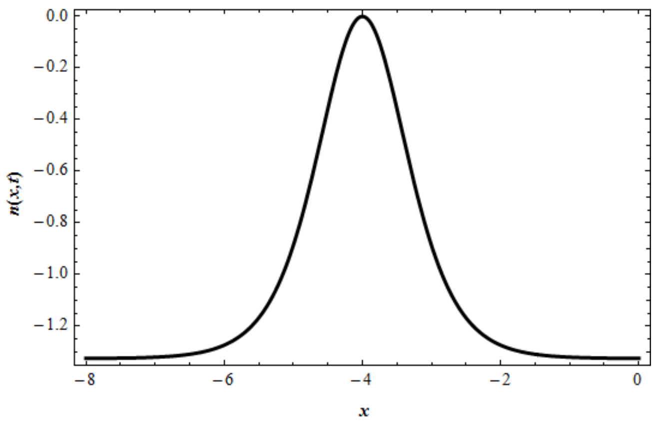

Figure 12. Plot of $\mathfrak{D}_5(x,t)$ with x for $v = 0.05, \rho = 1, c = 1.5$.

6. Conclusions

Important features including exactly solitary, dark envelopes and super behaviors were applied to characterize the Zakharov stochastic model in a subsonic case. The influences of random parameters in the amplitude, energy collapsing and seeding in clouds with types of structural solutions have been examined. It was noted that the stochastic effects may explain some modulations in the obtained collapsing or forced energy of solar Langmuir burst waves. The applications of this theoretical investigation might be used in solar wind energy and seeding in clouds applications.

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