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Quark Matter at High Baryon Density, Conformality and Quarkyonic Matter

Larry McLerran

Institute for Nuclear Theory, University of Washington, Box 351550, Seattle, WA 98195, USA; mclerran@mac.com

Abstract: This paper discusses high-baryon-density quarkyonic matter in the context of recent observations concerning neutron stars and the qualitative reasons why quarkyonic matter explains certain features of the equation of state that arises from these observations. The paper then provides a qualitative discussion of the quarkyonic hypotheses, and the essential features of quarkyonic matter that explain the outstanding features of the equation of state.

Keywords: quarkyonic matter; high baryon density QCD; conformality

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1. Introduction

The physics of high-energy-density matter and heavy-ion collisions has been thoroughly developed over the past 40 years. The recent developments associated with the quarkyonic hypothesis will be discussed, concerning the properties of matter at high baryon density and low temperature. Such matter may occur in neutron stars and intermediate-energy nuclear collisions. There is a wealth of exciting new information concerning neutron stars coming from gravitational wave experiments and neutron star radii measurements, providing strong constraints on the properties of such matter. Furthermore, there is the possibility of extracting complementary information from existing facilities by colliding intermediate-energy heavy ions.

The purpose of this paper is to discuss the motivations of the quarkyonic hypothesis based on the theory of strong interactions, QCD, and the properties of recent neutron star observations. First, recent neutron star observations are discussed, in particular, the extracted equation of state and its conformal properties. Next, the quarkyonic hypothesis is proposed and argued that it has the properties needed to describe the qualitative features of the equation of state. Then, the recently conjectured possible experimental probes in low-energy heavy-ion experiments are discussed. The paper ends with a brief outline of some important theoretical issues which are currently poorly understood concerning quarkyonic matter.

The purpose of this paper is to not only provide a comprehensive review of the properties of high-density, low-temperature baryonic matter, but to also provide a simple and qualitative description of quarkyonic matter and its implications in light of recent observations.

2. The Importance of Neutron Star Studies in Nuclear Physics

The measurements of neutron stars can provide their radii and masses, and in the case of gravitational wave detection, constraints on the quadrupole deformability. These properties can be determined by knowing an equation of state,

$$P = P(\epsilon) \tag{1}$$

where the energy density of cold matter is ϵ and the pressure is P. The equation of state plus the Tolman–Oppenheimer–Volkov equation and the general relativistic equation of hydrostatic equilibrium, can determine these quantities. The equation of state needs to be known up to an energy density one order of magnitude greater than that of nuclear matter.

For energy densities twice the nuclear matter energy density, the equation of state is determined by low-energy nuclear theory, and its generalizations involving effective field theory. At two orders of magnitude greater than the nuclear matter energy density, the equation of state is that of quarks, and is determined by QCD computations [1,2]. At intermediate-energy densities, the equation of state is known from first principle computations. Nevertheless, combining the neutron star observations together with the known properties at high- and low-energy density, it is possible to obtain a good determination of the equation of state.

This empirically determined equation of state is important [3–7]. At nuclear matter density, matter is a non-relativistic liquid of nucleons with a sound velocity $v_s^2 \sim 10^{-2} - 10^{-1}$. To describe neutron stars by a density four times that of nuclear matter, the sound velocity is $v_s^2 \geq 1/3$, where $v_s^2 = 1/3$ is the limit for a relativistic ideal gas. There are indications that the sound velocity exceeds 1/3 in this region. This transition from a non-relativistic fluid to an ultra-relativistic system occurs over a very small range of change of typical particle separation, $r/r_0 \sim (\rho/\rho_0)^{-1/3} \sim 1.6$, such that the transition occurs incredibly rapidly.

The computation of the sound velocity as a function of density [8] is shown in Figure 1. This computation involves many fits to neutron star data, and the probability distribution for various sound velocities is shown (Figure 1).

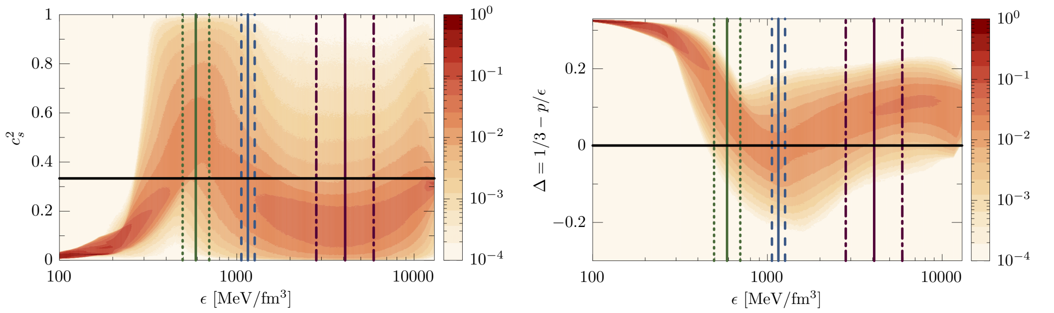

Figure 1. Probability density functions (PDFs) of the speed of sound (**left panel**) and trace anomalies (**right panel**) as functions of the energy density. Vertical lines represent the median and 1σ credibility region for the peak position in c_s^2 (green solid and dotted lines), values at the centre of maximally massive neutron stars (blue solid and dashed lines), and the peak position in χ_B (purple solid and dash-dotted lines). Horizontal, black lines show the conformal values of $c_s^2 = 1/3$ (**left panel**) and $\Delta = 0$ (**right panel**) [8].

Another dimensionless parameter that characterizes the equation of state is the trace anomaly scaled by the energy density [9],

$$\Delta = \frac{1}{3} - \frac{P}{\epsilon}. \tag{2}$$

The trace anomaly is a measure of the deviation from the scale invariance. Determination of the scaled trace anomaly is shown in Figure 1. It is notable that the approximate scale invariance can be achieved at densities found inside neutron stars, particularly because at such densities it is commonly believed that nuclear matter still strongly interacts. Of course, quark matter must asymptotically become scale invariant because of the asymptotic freedom property of QCD; however, this observed precocious scaling of dense matter is quite remarkable. In Figure 2, a contour plot extracted from the analysis of fits to neutron star equations of state in neutron star cores is shown. It is notable that the values centre around the scale invariant $v_s^2 = 1/3$ and $\Delta = 0$.

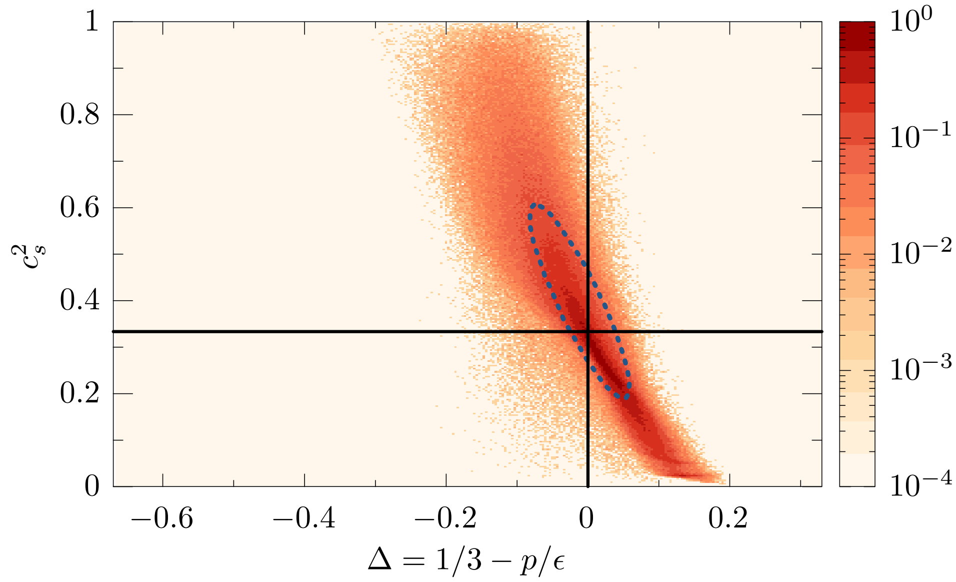

Figure 2. Probability density function (PDF) of the trace anomaly vs. the speed of sound at the centre of maximally stable neutron stars. The dotted blue ellipse marks the 1σ credibility around the mean. The black horizontal and vertical lines mark the conformal values [8].

The trace anomaly and sound velocity are related by

$$v_s^2 = \frac{1}{3} - \Delta - \epsilon \frac{d\Delta}{d\epsilon}. \tag{3}$$

The fact that the sound velocity is large at relatively low densities is therefore another indication that the matter is extremely rapidly approaching the scale invariant limit $\Delta = 0$.

3. Strongly Interacting Matter and the Quarkyonic Hypothesis

The indications described above suggest that matter rapidly approaches an almost conformally invariant distribution of quarks and gluons at densities of about one order of magnitude greater than the density of nuclear matter. Suppose that matter converts between a system of nucleons and quarks at some fixed baryon density. Here the case of isospin zero matter is considered for simplicity, and made from neutrons and protons only (or up and down quarks). To qualitatively understand this transition, let us consider free gasses. The Fermi momenta are denoted as k_F^N and k_F^Q for nucleons and quarks, respectively. We will take the number of quark colours as a parameter to illustrate this point. Since a nucleon is composed of N_c quarks, we might naively expect that $k_F^N = N_c k_F^Q$. On the other hand, the baryon number density is

$$n_B \sim (k_F^N)^3 \sim (k_F^Q)^3. \tag{4}$$

If we take the Fermi momenta of nucleonic matter at a few times the nuclear matter density to be of order Λ_{QCD}, then in the quark matter phase, the Fermi momenta of quarks is also of order Λ_{QCD} and if we try to imagine nucleons made of quarks in this new phase, then they would be relativistic. How can matter make such a drastic change?

A way to resolve such a paradoxical situation is through quarkyonic matter [10,11]. In QCD, nucleons are made of quarks, and discussing which degrees of freedom are quarks and nucleons reflect the desired approximations. At low baryon density, it is a good approximation to treat nucleons as a Fermi gas of nucleons as they occupy low-momentum states. These nucleons are composed of quarks, but the quarks are spread over a momentum scale, which at low momentum is of the order of the QCD scale. However, as one adds more nucleons, these quark states approach an occupation number of order one and this fills the quark states up to a Fermi energy of order of the QCD scale. New nucleons need to be added, but to avoid Pauli blocking of the quarks, they must be added at the Fermi surface corresponding to a nucleon energy of order $N_c \Lambda_{QCD}$. The nucleons and quarks at this point are relativistic. Therefore, there is a transition at a density of order Λ^3_{QCD}, where the degrees of freedom change rapidly from non-relativistic to relativistic.

One can understand quarkyonic matter at high densities, as a Fermi sea of quarks is approximately filled with a Fermi surface of nucleons. However, quark–hadron duality requires that we look at this situation a little closer [12]. A filled Fermi sea of quarks will comprise one baryon for each N_c quarks. It takes N_c quarks to make one baryon. However, at a nucleon momentum of order $N_c k_{quark}$, the occupation number of a filled Fermi gas is N_c^3 times the occupation number of a quark because the De Broglie wavelength of a nucleon is $1/N_c$ compared to a quark. Therefore, the nucleon occupation number with $k_N = N_c k_Q$ composed of quarks in the Fermi sea is $1/N_c^3$. This is true until one approaches the Fermi surface for quarks. Here, one can completely fill nucleon states close to the Fermi energy, $k_F^N = N_c k_F^Q$. The quark constituents are spread out in momentum above the Fermi sea because of the intrinsic momentum of quarks inside the nucleon, making a Fermi surface for quarks whose width is $\Delta k \sim \Lambda_{QCD}$. Therefore, the picture of quarks that arises is a filled Fermi sea of quarks with a Fermi surface of width Λ_{QCD}. This can also be thought of in terms of nucleons as an under-occupied distribution of nucleons with an occupation number of order $1/N_c^3$ followed by this surface of fully occupied nucleons. The nucleon distribution is linked with the quark distribution. This is the under-occupied distribution of nucleons that corresponds to a filled Fermi sea.

Let us summarize this argument in a slightly different way. When we think about quarkyonic matter we must have a dual picture in mind. One picture is the distribution of quarks and the other is the distribution of nucleons. These pictures must be consistent with one another. The quark distribution is a filled Fermi sea of quarks plus a tail above some momentum that falls to zero. In the tail, the occupation number of quarks falls to zero, and its width is of order of the scale of quark momentum in the nucleon wavefunction. In this tail, we can fit nucleon states up to some saturation density. The total density of nucleons is of the same order as when the quark sea first appears, but the local phase space density of nucleons, dn_N/d^3k, is saturated and of order 1. Thus, the nucleons form a shell with a typical momentum $k_N^F \sim N_C k_q^N$. On the other hand, in the quark sea the total quark density is of order $(k_q^F)^3$, and if the nucleons are composed of quarks their baryon density must be that of the quarks, requiring a suppression of the phase space density of nucleons because $n_N \sim (k_N^F)^3/N_C^3 \sim (k_q^F)^3$. Therefore, the bulk distribution of nucleons with momentum less than the nucleon shell is suppressed.

One can model such a picture in various ways to develop a concrete picture with these features [13]. It is also possible to develop explicit quantum mechanical computations of dual description of quarks and nucleons which exhibit these features [14]. The phenomenological model of McLerran and Reddy proposes that nucleons exist on a shell in momentum space whose thickness is determined so that the density of nucleons is fixed at some value above that of nuclear matter [13]. The Fermi sea of quarks and gluons sit below this shell. In the computations of Fujimoto, McLerran and Tojo, the general features of this model are reproduced in a quantum mechanical computation where nucleons are explicitly composed of quarks, and the occupation numbers of the quark and gluon states are constrained to be between zero and one [14]. The rapid rise in the sound velocity is attributed to the transition to quarkyonic matter that occurs when quark occupation numbers approach one. Similar features are extracted from excluded volume models of nuclear matter, where the overlaps of hard cores may be thought of as corresponding to the density at which the occupation numbers of quarks approach one, and the origin of the hard core repulsive interaction is the Fermi exclusion of filled quark states [15].

The reason the sound velocity increases rapidly in the quarkyonic compared to the nucleonic phase is easily understood. The typical Fermi momentum for a density corresponding to the nuclear matter density is of order Λ_{QCD}. Therefore, the sound velocity squared in nuclear matter is of order $v_s^2 \sim \Lambda_{QCD}^2/M_N^2 \sim 1/N_c^2$. For quarkyonic matter it is $v_s^2 \sim \Lambda_{QCD}^2/M_q^2 \sim 1$, meaning the ratio of sound velocities is of order $1/N_c^2$. The transition occurs at approximately a constant baryon density; therefore, the sound velocity and equation of state change quickly from a soft equation of state to a hard equation of state. In models, this change can sometimes correspond to a peak in the sound velocity [13,14].

To summarize, it is possible to reproduce features of the equation of state inferred from neutron star observations using a quarkyonic description. The rapid change in the sound velocity occurs when the matter becomes quarkyonic. At this point, the degrees of freedom become relativistic, although approximate scale invariance is acquired at higher densities.

There is an issue about how one thinks about the transition to quarkyonic matter. Indeed, such a transition might be thought of as a phase transition to quarks. However, it is not a first- or second-order transition, since the sound velocity does not tend to zero at the transition. Indeed, the sound velocity increases rapidly. In fact, the transition is probably not due to de-confinement, since in the large N_c limit the Debye screening length is

$$r_{Debye}^2 \sim N_c/(k_q^F)^2. \tag{5}$$

This is because Debye screening is induced by a one-loop effect, and at a temperature of zero, such a loop can only be made of quarks, supressing the diagram by a factor of N_c. Therefore, the Debye screening length only becomes of the order of the QCD scale when $k_q^F \sim \sqrt{N_c}\Lambda_{QCD}$. On the other hand, the quarkyonic transition occurs at lower density when $k_q^F \sim \Lambda_{QCD}$. Of course, for $N_c = 3$ it is possible that these transitions are poorly separated.

4. Accelerator Experiments and High-Density Baryonic Matter

One should also ask what can be learned from systematic accelerator studies. One question to address is how one might measure a rapidly varying sound velocity as a function of baryon density at low temperatures. This restricts one to low-energy experiments with low-energy accelerators, such as GSI, FAIR, FRIB or RHIC. Two suggestions have been made.

The first involves measuring fluctuations in the baryon number [16]. With a cumulant of the baryon number distribution defined as

$$\kappa_j =<< (n_B - < n_B >)^j >> . \tag{6}$$

This can be rewritten in terms of thermodynamic quantities as

$$\kappa_j = VT^{j-1}\frac{d^jP}{d\mu_B^j}. \tag{7}$$

where T is the temperature, μ_B the baryon number chemical potential, and P is the pressure. These can be combined together at low temperature to give

$$\frac{d\ ln\ v_s^2}{d\ ln\ n_B} + v_s^2 = 1 - \frac{\kappa_1\kappa_3}{\kappa_2^2}. \tag{8}$$

Another suggested method is to determine the sound velocity transport computations. This is easily performed for vector mean field models, where the sound velocity is directly related to the strength of the mean field [17]. The mean field vector potential is the chemical potential for the baryon number. However,

$$v_s^2 = \frac{n_B}{\mu_B dn_B/d\mu_B} \tag{9}$$

and therefore

$$\mu_B = \mu_B(n_B^0)exp\left(\int_{n_B^0}^{n_B} dn'\ \frac{v_s^2(n')}{n'}\right) \tag{10}$$

such that the effects of the vector field can be known from the dependence of the sound velocity on the density.

While the results of such analyses [16,17] are suggestive that there may be large sound velocities achieved at low densities, our understanding is still in its early stages, and with time this will increase. The results presented here are interesting. The sound velocity squared is first seen to decrease and then approach unity at a density close to that of nuclear matter. This is different from the studies on neutron stars. Neutron stars have an isospin to baryon number of near $1/2$ while nuclear matter is closer to zero, so there is no a priori contradiction.

5. Theoretical Issues

Among the theoretical issues, the following are most important:

Can one develop a transport theory method of computations which allows quarkyonic matter to form at high baryon densities in relativistic nuclei collisions? To do this one needs to impose constraints on the nucleon and quark occupation numbers. Perhaps one could simulate nucleonic collisions and then compute the quark content, imposing quark and nucleon occupation numbers between zero and one. This would be sufficient to compute the equation of state for systems that are time-independent. In such a dynamic system, an important issue would be to see how the occupation numbers are distributed and evolve.

Another important question concerns the isospin dependence of the equation of state. For neutron matter, the isospin per nucleon is $1/2$, while for nuclear matter it is zero. In principle, the equation of state may be very different for such systems. It might be that the inferred properties from the equation of state of neutron matter is qualitatively different from the inferred collisions of nuclei with low isospin per nucleon.

How does chiral symmetry manifest in quarkyonic matter? Theoretical studies have been carried out that show the pattern of chiral symmetry restoration may be quite arcane [18]. How does one clearly model such matter? Are there consequences at densities found for neutron stars or accelerator collisions?

How does one generalize quarkyonic matter from a temperature of zero to finite density [19]? This is important to describe low-energy heavy-ion collisions and for the collision dynamics of neutron star collisions.

How does one use field theoretical methods, such as mean field theory, to dynamically demonstrate the formation of quarkyonic matter [20,21]?

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