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Fundamental Asymmetries between Spatial and Temporal Boundaries in Electromagnetics

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Abstract: Time-varying materials bring an extra degree of design freedom compared to their conventional time-invariant counterparts. However, few discussions have focused on the underlying physical difference between spatial and temporal boundaries. In this letter, we thoroughly investigate those differences from the perspective of conservation laws. By doing so, the building blocks of optics and electromagnetics such as the reflection law, Snell's law, and Fresnel's equations can be analogously derived in a temporal context, but with completely different interpretations. Furthermore, we study the unique features of temporal boundaries, such as their nonconformance to energy conservation and causality.

Keywords: asymmetry; temporal boundary; time varying media; reflection law; Snell's law; Fresnel's equations; energy conservation; and causality

1. Introduction

There has been a long history of research interest in methodologies for artificially controlling electromagnetic (EM) wave propagation [1–4]. The rapid development of metamaterials in recent years has provided an unprecedented flexibility for manipulating EM waves. With carefully tailored electric and/or magnetic responses, such metamaterials may be engineered to exhibit extreme values of permittivity ϵ and/or permeability μ [5–7].

The control over EM waves in the vast majority of metamaterials has been accomplished via the spatial distribution of the effective material properties. Reconfigurable and active metamaterials, however, are enabled by the time-variation in their material properties, which are unachievable by conventional materials [8–12]. This class of metamaterials with time-varying properties opens a new realm of possibilities for tailoring EM waves. To be more specific, in addition to considering the spatial distribution of the constituent materials (i.e., $\epsilon(x,y,z)$), one could explore the desired functionalities with greater flexibility by introducing the additional dimension of “time”, i.e., $\epsilon(x,y,z,t)$ [2]. In recent years, researchers have extensively explored these novel “4-D” material systems, with a particular focus on their time-varying aspect. It was found that when an EM wave travels through a medium whose material properties (i.e., ϵ or μ) change suddenly, the wave will split into two waves propagating in opposite directions [13]. This phenomenon is the temporal dual of wave propagation through spatial boundaries, where a wave meets the boundary between two different materials. Since then, many well-known concepts and applications such as homogenization, cloaks, aiming, photonic band gaps, polarization conversion, and quarter-wave transformers have been studied in the temporal domain [14–18].

While temporal boundaries have been researched widely, several fundamental questions still remain unanswered. For example, a temporal version of Fresnel's equation has been developed in [13]. Why are the reflection and transmission coefficients different from its spatial counterpart? As a building block in electromagnetics, does Snell's law also have a temporal equivalent? In [17], the authors found that the wave energy of the proposed temporal system is not constant, and attribute it to external energy sources. Is there another



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perspective for interpreting this energy conservation enigma? Finally, some researchers have pointed out that causality plays an important role in time-varying systems. So, what role does causality play in an actual EM system? Moreover, how does causality shed light on the asymmetry between space and time?

In this paper, we attempt to answer the above questions by comparing the differences between spatial and temporal boundaries. Specifically, we consider two cases: a conventional spatial boundary in a time-invariant system, and a temporal boundary in a spatially invariant system. From Noether's theorem, these two cases correspond to energy and momentum conservation, respectively. First, we compared the momentum conservation between the two cases, and derive the temporal version of the reflection law and Snell's law. Then we revisit Fresnel's equations in the spatial domain and Morgenthaler's equations [19] in the time domain. Furthermore, we discuss the energy conservation issue and causality, which are unique to temporal systems.

2. Momentum Conservation (Reflection Law, Snell's Law)

Here we reexamine the momentum conservation law and establish a framework for how it leads to different conclusions in spatial and temporal systems. Specifically, we consider two scenarios: The first one considers a time-invariant 2D space, whose refractive index is n_1 and n_2 for the region $z > z_0$ and $z < z_0$, respectively. The second one involves an unbounded and homogeneous system where the medium undergoes a sudden permittivity or permeability change at $t = t_0$. The electric field of the incidence wave is defined as $\vec{E} = E_0 e^{-j(\vec{k} \cdot \vec{r} - \omega t)}$, where $\omega = 2\pi/T$ is the angular frequency, and $\vec{k}$ is the wave vector whose magnitude $|\vec{k}| = 2\pi/\lambda$ is the wavenumber (i.e., spatial frequency). Here we do not specify the polarization of the wave because the system is assumed to be isotropic. The angular frequency and wave vector are closely related to the wave energy and momentum through the Planck-Einstein equation and the de Broglie relation, respectively [20].

For the first scenario, the frequency remains constant because of photon energy conservation. Considering $\vec{v} = \omega/\vec{k}$ and $|\vec{v}| = c/n$, we have:

$$\begin{cases} \frac{n_1}{|\vec{k}_r|} = \frac{n_1}{|\vec{k}_{in}|} \\ \frac{n_2}{|\vec{k}_t|} = \frac{n_1}{|\vec{k}_{in}|} \end{cases}, \quad (1)$$

where $\vec{k}_{in}$, $\vec{k}_t$, and $\vec{k}_r$ represent the wave vector of the incident, transmitted, and reflected wave of angle θ_{in} , θ_t , and θ_r , respectively. The magnitude of the wave vector is a function of the refractive index change. To further define the direction of the wave vector, its tangential components are taken into consideration:

$$\begin{cases} |\vec{k}_r| \sin(\theta_r) = |\vec{k}_{in}| \sin(\theta_{in}) \\ |\vec{k}_t| \sin(\theta_t) = |\vec{k}_{in}| \sin(\theta_{in}) \end{cases}. \quad (2)$$

Accordingly, Equations (1) and (2) together comprise the reflection law and Snell's law in the spatial domain.

On the other hand, for the second scenario, the wave vector $\vec{k}$ is constant due to the momentum conservation. In this case it is the frequency that depends on the refractive index change:

$$\begin{cases} n_2 \omega_b = -n_1 \omega_{in} \\ n_2 \omega_f = n_1 \omega_{in} \end{cases}, \quad (3)$$

where ω_{in} , ω_f , and ω_b represent the angular frequency of the incident, forward, and backward wave. It is worth noting that the backward and forward waves have negative and positive frequencies, respectively. This is because the velocity of the forward wave is in the same direction as the incident wave, while that of the backward wave is in the opposite direction.

The different expressions for Snell's law manifest the fundamental asymmetry in electromagnetics between space and time. In the first scenario, the incident and reflected waves appear in the same medium. Therefore, it is the reflection angle that is equal to the incidence angle. On the other hand, in the second scenario, the forward and backward waves exist in the same medium. Consequently, the magnitude of the backward wave velocity is equal to that of the forward wave, rather than the incident wave.

Moreover, Equation (1) in the first scenario is a vector equation. To determine the direction of the wave vector, its tangential components are considered in Equation (2). On the contrary, Equation (3) in the second scenario is a scalar equation. The direction of velocity is decided by the plus or minus sign of the frequency. The medium is spatially homogeneous, allowing the forward and backward waves to propagate in the same (or opposite) direction as the incident wave. In other words, the incident wave is always geometrically normal to the temporal boundary, which is a consequence of the fact that the temporal boundary is perpendicular to all three spatial directions.

3. Transmission and Reflection (Fresnel Equations)

Besides Snell's law, Fresnel's equations represent another important set of rules governing EM waves. In the case of normal incidence, and considering the spatial boundary conditions at $z = z_0$, the transmission and reflection coefficients (*tandr*) can be expressed as:

$$t = \frac{\left| \frac{\vec{E}}{E_{inc}} \right|_t}{\left| \frac{\vec{E}}{E_{inc}} \right|} = \frac{2Z_2}{Z_2 + Z_1}, \quad (4)$$

$$r = \frac{\left| \frac{\vec{E}}{E_{inc}} \right|_r}{\left| \frac{\vec{E}}{E_{inc}} \right|} = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \quad (5)$$

where Z_1 and Z_2 are the wave impedances in medium n_1 and n_1 . This yields the following condition:

$$1 + r = t \quad (6)$$

This expression not only reveals that the boundary condition is implicit in Fresnel's equations, but also indicates that the incidence and reflection appear on the same side of the spatial boundary, while the transmission is on the opposite side.

For a temporal boundary (the second scenario), the field continuity conditions can be expressed as [13,19]:

$$\vec{D} = \epsilon_1 \vec{E}_{in} = \epsilon_2 (\vec{E}_f + \vec{E}_b), \quad (7)$$

$$\vec{B} = \mu_1 \vec{H}_{in} = \mu_2 (\vec{H}_f - \vec{H}_b). \quad (8)$$

Notice that the existence of a temporal boundary does not break the spatial translational symmetry. Therefore, all three components of $\vec{D}$ and $\vec{B}$ are conserved. Hence, it can be shown that [19]:

$$\tau = \frac{\vec{E}_f}{\vec{E}_{in}} = \frac{1}{2} \left(\frac{\epsilon_1}{\epsilon_2} + \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} \right), \quad (9)$$

$$\rho = \frac{\vec{E}_b}{\vec{E}_{in}} = \frac{1}{2} \left(\frac{\varepsilon_1}{\varepsilon_2} - \frac{\sqrt{\mu_1 \varepsilon_1}}{\sqrt{\mu_2 \varepsilon_2}} \right), \quad (10)$$

where τ and ρ are defined as transmission and reflection in the temporal scenario. Equations (7)–(10) were first derived by Morgenthaler [19], and have been widely used to analyze the transmission and reflection of EM waves passing through a temporal boundary. They can be viewed as the analog of Fresnel's equations in the temporal domain.

Interestingly, Equations (9) and (10) can be rewritten in a more suggestive form:

$$\tau = \frac{\varepsilon_1}{\varepsilon_2} \left(\frac{Z_2 + Z_1}{2Z_2} \right), \quad (11)$$

$$\rho = \frac{\varepsilon_1}{\varepsilon_2} \left(\frac{Z_2 - Z_1}{2Z_2} \right). \quad (12)$$

Notice that the first term on the right side of Equations (11) and (12) is equal to $\left| \vec{D}_f \right| / \left| \vec{D}_{in} \right|$ and $\left| \vec{D}_b \right| / \left| \vec{D}_{in} \right|$ respectively. By defining the transmission and reflection coefficients in terms of $\vec{D}$ instead of $\vec{E}$, allows them to be rewritten in an expression very similar to Equations (4) and (5):

$$\tau_d = \frac{\vec{D}_f}{\vec{D}_{in}} = \frac{Z_2 + Z_1}{2Z_2}, \quad (13)$$

$$\rho_d = \frac{\vec{D}_b}{\vec{D}_{in}} = \frac{Z_2 - Z_1}{2Z_2}. \quad (14)$$

These two equations lead to:

$$\tau_d + \rho_d = 1. \quad (15)$$

It can be clearly observed that there is a similarity between Equations (6) and (15).

In short, the temporal form of Fresnel's equations is different from their spatial counterparts, which is partly due to the different boundary conditions employed for the two scenarios. However, there are deeper reasons for this difference. First, the forward and backward waves exist after the temporal boundary, while the incident wave appears before it (notice that this difference also plays a key role in the previous section). Second, it is $\vec{D}$, rather than $\vec{E}$, that is continuous, but the reflection and transmission coefficients are still expressed in terms of electric fields by convention. This difference provides a complete explanation for why in temporal scenarios things become more complicated. That is, we need two independent parameters (ε, μ) to determine the transmission and reflection behavior (see Equations (9) and (10)). While in the spatial scenario, t and r are only a function of the impedance Z .

4. Energy Conservation

Energy conservation is implicit in all time-invariant systems. When a wave propagates through a spatial boundary, the total energy is conserved (*i.e.*, $T + R = t^2 \cdot Z_1 / Z_2 + r^2 = 1$), which can be derived from Equations (4) and (5). On the other hand, the existence of temporal boundaries breaks the time translation symmetry, and wave energy conservation can no longer be guaranteed.

This raises the following questions: does the energy change after a wave meets a temporal boundary? If so, where does the energy change come from? Let us first analyze

this problem from the wave perspective of light. The total power intensity of the forward and backward waves can be defined using the Poynting vector:

$$\frac{I_f}{I_1} = \frac{\left| \vec{E}_f \times \vec{H}_f \right|}{\left| \vec{E}_{in} \times \vec{H}_{in} \right|} \quad (16)$$

$$\frac{I_b}{I_1} = \frac{\left| \vec{E}_b \times \vec{H}_b \right|}{\left| \vec{E}_{in} \times \vec{H}_{in} \right|} \quad (17)$$

Adding Equations (16) and (17) together, while taking into consideration Equations (9) and (10), we find that:

$$I_2 = I_f + I_b = \left(\frac{n_1}{n_2} \right)^2 I_1 \quad (18)$$

where I_1 and I_2 represent the total wave power intensities before and after the temporal boundary. The power intensity decreases if $n_1 < n_2$, and increases if $n_1 > n_2$. Then, from the particle perspective of light, we recognize that the power intensity of EM waves is also defined as the total amount of energy passing through a surface per unit area A , and per unit time t : $I = N\hbar\omega/At$, where N is the number of photons. It further goes as:

$$I = d\hbar\omega \left| \vec{v} \right| \quad (19)$$

where d is the photon density (number of photons per unit volume).

According to the Plank-Einstein relation, the energy of each photon is proportional to the angular frequency ω . Then by considering Equations (3) and (19), we can deduce that when a wave passes through a temporal boundary, the energy and speed of each photon change simultaneously, and each of them contributes n_1/n_2 times to the total energy change. On the other hand, the photon density ρ remains the same.

5. Causality

Causality is a unique property relative to the time domain. An interesting example demonstrating how causality plays a role in the time domain is to consider stacking two boundaries. As in the previous section, we consider two scenarios featuring spatial and temporal boundaries respectively. Figure 1a demonstrates the multi-reflection process when a wave encounters a stacked boundary in the spatial domain. The EM wave bounces back and forth between the two boundaries, and will continue this process ad infinitum if a lossless system is considered [1].

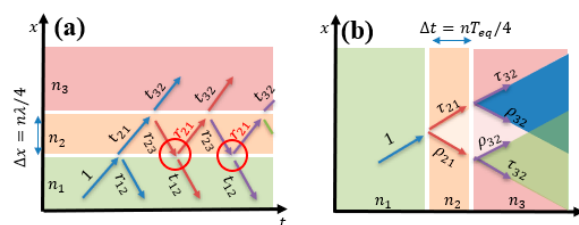


Figure 1. Schematics showing the transmission and reflection process when there are two boundaries. (a) The spatial case. Notice that multi-reflections occur at the interface between the two boundaries. (b) The temporal case. The backward and forward waves are represented by ρ and τ . The incident wave splits into two waves (τ_{21} and ρ_{21}), and together they form the first future light cone (the white region). Those two waves further split into four waves after encountering the second temporal boundary, and create two more future light cones (the blue and green regions). The latter light cones are inside of the former light cone.

On the contrary, as Figure 1b shows, there are no “bouncing” waves between two temporal boundaries, because causality forbids the waves from propagating ‘backwards’ in time. To be more specific, the first temporal boundary causes the incident wave to split into a forward (τ_{21}) and backward (ρ_{21}) term. After a period of time, these two waves propagating in opposite directions encounter the second temporal boundary, and each of them simultaneously splits into two waves (i.e., four waves in total).

Comparing with the temporal case, the total transmission in the spatial case is composed of many extra reflection terms, as depicted in the red circle in Figure 1a. An additional 180° phase change is brought about by this extra reflection, which is not present in the temporal case due to causality. Therefore, the transmission coefficients in the spatial and temporal cases are different: it increases in the former case while decreasing in the latter.

Now, let us validate the above physical interpretation in detail. First, we analyzed a specific system involving impedance matching. In practice, the two spatial boundaries can be separated by a distance: $\Delta x = n\lambda/4$ ($n = 1, 2, 3 \dots$), as depicted in Figure 1a. The total transmission and reflection can be expressed as:

$$t_{total} = t_{21}t_{32}\sum_{m=0}^{\infty}(r_{23}r_{21}e^{j\pi})^m \quad (20)$$

$$r_{total} = r_{12} + t_{21}t_{12}r_{23}e^{j\pi}\sum_{m=0}^{\infty}(r_{21}r_{23}e^{j\pi})^m \quad (21)$$

where $t_{ij} = 2Z_j / (Z_j + Z_i)$ and $r_{ij} = (Z_j - Z_i) / (Z_j + Z_i)$ ($i, j = 1, 2, 3$) is the impedance of the medium with corresponding refractive index n_i . The total transmission and reflection can be decomposed into an infinite sum of terms $(r_{21}r_{23}e^{j\pi})^m$, for which all of them are positive. Therefore, all the partial transmission terms have the same phase, while the partial reflection terms possess a 180° phase difference relative to the first reflection term (r_{12}). Consequently, one can maximize the transmission while minimizing the reflection. This is the mechanism behind the well-known quarter-wavelength impedance matching technique.

The temporal analog of this technique, (i.e., antireflection temporal coatings (ATCs)) was studied in [15,21,22]. The incident wave will split into four components after encountering two temporal boundaries, as shown in Figure 1b. For simplicity, here we assume that $\mu_1 = \mu_2 = \mu_3$. By letting the time duration between the two temporal boundaries satisfy the relation $\Delta t = t_2 - t_1 = \frac{nT_{eq}}{4}$, the forward and backward waves can be expressed as:

$$\tau_{total} = \tau_{21}\tau_{32} + \rho_{21}\rho_{32}e^{j\pi} \quad (22)$$

$$\rho_{total} = \rho_{21}\tau_{32} + \tau_{21}\rho_{32}e^{j\pi} \quad (23)$$

where $\rho_{ij} = 1/2(\varepsilon_i/\varepsilon_j - \sqrt{\varepsilon_i/\varepsilon_j})$ and $\tau_{ij} = \rho_{ij} = 1/2(\varepsilon_i/\varepsilon_j + \sqrt{\varepsilon_i/\varepsilon_j})$. Both terms $\tau_{21}\tau_{32}$ and $\rho_{21}\rho_{32}$ are forward, while $\rho_{21}\tau_{32}$ and $\tau_{21}\rho_{32}$ are backward. Hence, the phase difference of the total forward and backward waves is solely determined by the optical path difference $e^{j\pi}$. Consequently, with the antireflection temporal coating, both the reflection and transmission will be minimized.

The total transmission with and without the ATCs can be derived as:

$$\tau_{w/ATC} = \left(\frac{\varepsilon_1}{\varepsilon_3}\right)^{3/4} \quad (24)$$

$$\tau_{w/oATC} = \frac{1}{2}\left(\frac{\varepsilon_1}{\varepsilon_3} + \sqrt{\frac{\varepsilon_1}{\varepsilon_3}}\right) \quad (25)$$

Obviously, the following condition $\tau_{w/ATC} < \tau_{w/oATC}$ is met if $\varepsilon_1 \neq \varepsilon_3$. This result again indicates that the introduction of ATCs will reduce the total transmission.

Causality forbids the waves from propagating in a time-reversed direction, and consequently will not play a role in time-invariant systems. This subtle distinction partly explains why, although the quarter-wave transformer and its temporal counterpart can both eliminate reflection, the transmission behavior is very different. The example also supports our claim in the energy conservation section. For the spatial case, the total wave energy is conserved (transmission increases and reflection decreases). However, for the temporal case, the total wave energy is not conserved (both the transmission and reflection are reduced).

6. Conclusions

Temporal boundaries in EM seem to be a perfect dual of spatial boundaries in many ways. However, there are some fundamental differences underlying the perceived similarities. Our work has investigated and summarized several important distinctions between spatial and temporal EM boundaries, from several different perspectives. First, it has been shown that spatial and temporal symmetry correspond to momentum and energy conservation, respectively. Second, there are some unique properties in temporal systems, such as causality, that differ from spatial systems. We found that many rules governing the spatial domain (i.e., time-invariant systems) cannot be simply employed directly in the time domain, which sometimes justifies that a more in-depth investigation be carried out. Our findings provide profound insight into the asymmetry between space and time in fundamental EM systems. This work also serves as a guide to designing EM systems that operate solely in the space-invariant scenario, which are analogous to but different from the traditional time-invariant systems.

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References

1. Pozar, D. *Microwave Engineering*, 3rd ed.; John Wiley & Sons: Hoboken, NJ, USA, 2005.
2. Engheta, N. Metamaterials with high degrees of freedom: Space, time, and more. *Nanophotonics* **2020**, *10*, 639–642. [[CrossRef](#)]
3. Mai, W.; Zhu, D.; Gong, Z.; Lin, X.; Chen, Y.; Hu, J.; Werner, D.H. Broadband transparent chiral mirrors: Design methodology and bandwidth analysis. *AIP Adv.* **2019**, *9*, 045305. [[CrossRef](#)]
4. Mai, W.; Kang, L.; Jenkins, R.; Zhu, D.; Mao, C.; Werner, P.L.; Chen, Y.; Werner, D.H. A knotted metamolecule with axisymmetric strong optical activity. *Adv. Opt. Mater.* **2020**, *8*, 2000948. [[CrossRef](#)]
5. Liberal, I.; Engheta, N. Near-zero refractive index photonics. *Nat. Photonics* **2017**, *11*, 149–158. [[CrossRef](#)]
6. Valentine, J.; Zhang, S.; Zentgraf, T.; Ulin-Avila, E.; Genov, D.A.; Bartal, G.; Zhang, X. Three-dimensional optical metamaterial with a negative refractive index. *Nature* **2008**, *455*, 376–379. [[CrossRef](#)]
7. Dani, K.M.; Ku, Z.; Upadhyay, P.C.; Prasankumar, R.P.; Brueck, S.R.J.; Taylor, A.J. Subpicosecond optical switching with a negative index metamaterial. *Nano Lett.* **2009**, *9*, 3565–3569. [[CrossRef](#)]
8. Ullah, Z.; Witjaksono, G.; Nawi, I.; Tansu, N.; Khattak, M.I.; Junaid, M. A review on the development of tunable Ggraphene nanoantennas for terahertz optoelectronic and plasmonic applications. *Sensors* **2020**, *20*, 1401. [[CrossRef](#)]
9. Guo, X.; Ding, Y.; Duan, Y.; Ni, X. Nonreciprocal metasurface with space–time phase modulation. *Light Sci. Appl.* **2019**, *8*, 123. [[CrossRef](#)] [[PubMed](#)]
10. Zhang, L.; Chen, X.Q.; Shao, R.W.; Dai, J.Y.; Cheng, Q.; Castaldi, G.; Galdi, V.; Cui, T.J. Breaking reciprocity with space-time-coding digital metasurfaces. *Adv. Mater.* **2019**, *31*, 1904069. [[CrossRef](#)] [[PubMed](#)]
11. Nagulu, A.; Dinc, T.; Xiao, Z.; Tymchenko, M.; Sounas, D.L.; Alu, A.; Krishnaswamy, H. Nonreciprocal components based on switched transmission lines. *IEEE Trans. Microw. Theory Tech.* **2018**, *66*, 4706–4725. [[CrossRef](#)]
12. Taravati, S.; Chamanara, N.; Caloz, C. Nonreciprocal electromagnetic scattering from a periodically space-time modulated slab and application to a quasisonic isolator. *Phys. Rev. B* **2017**, *96*, 165144. [[CrossRef](#)]

13. Xiao, Y.; Maywar, D.; Agrawal, G. Reflection and transmission of electromagnetic waves at a temporal boundary. *Opt. Lett.* **2014**, *39*, 574–577. [[CrossRef](#)] [[PubMed](#)]
14. Pacheco-Peña, V.; Engheta, N. Effective medium concept in temporal metamaterials. *Nanophotonics* **2019**, *9*, 379–391. [[CrossRef](#)]
15. Pacheco-Peña, V.; Engheta, N. Antireflection temporal coatings. *Optica* **2020**, *7*, 323. [[CrossRef](#)]
16. Pacheco-Peña, V.; Engheta, N. Temporal aiming. *Light Sci. Appl.* **2020**, *9*, 129. [[CrossRef](#)] [[PubMed](#)]
17. Xu, J.; Mai, W.; Werner, D.H. Complete polarization conversion using anisotropic temporal slabs. *Opt. Lett.* **2021**, *46*, 1373–1376. [[CrossRef](#)] [[PubMed](#)]
18. Ramaccia, D.; Toscano, A.; Bilotti, F. Light propagation through metamaterial temporal slabs: Reflection, refraction, and special cases. *Opt. Lett.* **2020**, *45*, 5836. [[CrossRef](#)] [[PubMed](#)]
19. Morgenthaler, F. Velocity modulation of electromagnetic waves. *IEEE Trans. Microw. Theory Tech.* **1958**, *6*, 167–172. [[CrossRef](#)]
20. Shankar, R. *Principles of Quantum Mechanics*; Springer: Berlin/Heidelberg, Germany, 1994.
21. Mai, W.; Xu, J.; Werner, D.H. Antireflection temporal coatings: Comment. *Optica* **2021**, *8*, 824. [[CrossRef](#)]
22. Pacheco-Peña, V.; Engheta, N. Antireflection temporal coatings: Reply. *Optica* **2021**, *8*, 826. [[CrossRef](#)]

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